PARISimulator useful equations

Implicit momentum diffusion

Predicted u-velocity implicit diffusion

$$du_{i,j,k} = \frac{2}{\rho_{i+1,j,k} + \rho_{i,j,k}} \left(diffusion_{xy} + diffusion_{xz} \right) \tag{1}$$

With:

$$\begin{aligned} diffusion_{xy} &= \frac{1}{dy_j} \left[\frac{\mu_{i,j,k} + \mu_{i+1,j,k} + \mu_{i+1,j+1,k} + \mu_{i,j+1,k}}{4} \cdot \frac{v_{i+1,j,k} - v_{i,j,k}}{dxh_i} - \frac{\mu_{i,j,k} + \mu_{i+1,j,k} + \mu_{i+1,j-1,k} + \mu_{i,j-1,k}}{4} \cdot \frac{v_{i+1,j-1,k} - v_{i,j-1,k}}{dxh_i} \right] \\ diffusion_{xz} &= \frac{1}{dz_k} \left[\frac{\mu_{i,j,k} + \mu_{i+1,j,k} + \mu_{i+1,j,k+1} + \mu_{i,j,k+1}}{4} \cdot \frac{w_{i+1,j,k} - w_{i,j,k}}{dxh_i} - \frac{\mu_{i,j,k} + \mu_{i+1,j,k} + \mu_{i+1,j,k-1} + \mu_{i,j,k-1}}{4} \cdot \frac{w_{i+1,j,k-1} - w_{i,j,k-1}}{dxh_i} \right] \end{aligned} \tag{2}$$

Predicted v-velocity implicit diffusion

$$dv_{i,j,k} = \frac{2}{\rho_{i,j+1,k} + \rho_{i,j,k}} \left(diffusion_{yx} + diffusion_{yz} \right) \tag{3}$$

With:

$$\begin{aligned} diffusion_{yx} &= \frac{1}{dx_i} \left[\frac{\mu_{i,j,k} + \mu_{i+1,j,k} + \mu_{i+1,j+1,k} + \mu_{i,j+1,k}}{4} \cdot \frac{u_{i,j+1,k} - u_{i,j,k}}{dyh_j} - \frac{\mu_{i,j,k} + \mu_{i,j+1,k} + \mu_{i-1,j+1,k} + \mu_{i-1,j,k}}{4} \cdot \frac{u_{i-1,j+1,k} - u_{i-1,j,k}}{dyh_j} \right] \\ diffusion_{yz} &= \frac{1}{dz_k} \left[\frac{\mu_{i,j,k} + \mu_{i,j+1,k} + \mu_{i,j+1,k+1} + \mu_{i,j,k+1}}{4} \cdot \frac{w_{i,j+1,k} - w_{i,j,k}}{dyh_j} - \frac{\mu_{i,j,k} + \mu_{i,j+1,k} + \mu_{i,j+1,k-1} + \mu_{i,j,k-1}}{4} \cdot \frac{w_{i,j+1,k-1} - w_{i,j,k-1}}{dyh_j} \right] \end{aligned} \tag{4}$$

Predicted w-velocity implicit diffusion

$$dw_{i,j,k} = \frac{2}{\rho_{i,j,k+1} + \rho_{i,j,k}} \left(diffusion_{zx} + diffusion_{zy} \right) \tag{5}$$

With:

$$
\begin{aligned}
diffusion_{zx} &= \frac{1}{dx_i}\left[\frac{\mu_{i,j,k}+\mu_{i+1,j,k}+\mu_{i+1,j,k+1}+\mu_{i,j,k+1}}{4}\cdot\frac{u_{i,j,k+1}-u_{i,j,k}}{dzh_k}-\frac{\mu_{i,j,k}+\mu_{i-1,j,k}+\mu_{i-1,j,k+1}+\mu_{i,j,k+1}}{4}\cdot\frac{u_{i-1,j,k+1}-u_{i-1,j,k}}{dzh_k}\right] \\
diffusion_{zy} &= \frac{1}{dy_j}\left[\frac{\mu_{i,j,k}+\mu_{i,j+1,k}+\mu_{i,j+1,k+1}+\mu_{i,j,k+1}}{4}\cdot\frac{v_{i,j,k+1}-v_{i,j,k}}{dzh_k}-\frac{\mu_{i,j,k}+\mu_{i,j-1,k}+\mu_{i,j-1,k+1}+\mu_{i,j,k+1}}{4}\cdot\frac{v_{i,j-1,k+1}-v_{i,j-1,k}}{dzh_k}\right]
\end{aligned}
\tag{6}
$$

Explicit momentum diffusion

Predicted u-velocity explicit diffusion

$$du_{i,j,k} = \frac{2}{\rho_{i+1,j,k} + \rho_{i,j,k}} \left(diffusion_{xx} + diffusion_{xy} + diffusion_{xz} \right) \tag{7}$$

With:

$$\begin{aligned}
diffusion_{xx} &= \frac{2}{dxh_i} \left(\frac{\mu_{i+1,j,k} \left(u_{i+1,j,k} - u_{i,j,k} \right)}{dx_{i+1}} - \frac{\mu_{i,j,k} \left(u_{i,j,k} - u_{i-1,j,k} \right)}{dx_i} \right) \\
diffusion_{xy} &= \frac{1}{dy_j} \left[\frac{\mu_{i,j,k} + \mu_{i+1,j,k} + \mu_{i+1,j+1,k} + \mu_{i,j+1,k}}{4} \cdot \left(\frac{u_{i,j+1,k} - u_{i,j,k}}{dyh_j} + \frac{v_{i+1,j,k} - v_{i,j,k}}{dxh_i} \right) - \frac{\mu_{i,j,k} + \mu_{i+1,j,k} + \mu_{i+1,j-1,k} + \mu_{i,j-1,k}}{4} \cdot \left(\frac{u_{i,j,k} - u_{i,j-1,k}}{dyh_{j-1}} + \frac{(v_{i+1,j-1,k} - v_{i,j-1,k})}{dxh_i} \right) \right] \\
diffusion_{xz} &= \frac{1}{dz_k} \left[\frac{\mu_{i,j,k} + \mu_{i+1,j,k} + \mu_{i+1,j,k+1} + \mu_{i,j,k+1}}{4} \cdot \left(\frac{u_{i,j,k+1} - u_{i,j,k}}{dzh_k} + \frac{w_{i+1,j,k} - w_{i,j,k}}{dxh_i} \right) - \frac{\mu_{i,j,k} + \mu_{i+1,j,k} + \mu_{i+1,j,k-1} + \mu_{i,j,k-1}}{4} \cdot \left(\frac{u_{i,j,k} - u_{i,j,k-1}}{dzh_{k-1}} + \frac{w_{i+1,j,k-1} - w_{i,j,k-1}}{dxh_i} \right) \right]
\end{aligned} \tag{8}$$

Predicted v-velocity explicit diffusion

$$dv_{i,j,k} = \frac{2}{\rho_{i,j+1,k} + \rho_{i,j,k}} \left(diffusion_{yx} + diffusion_{yy} + diffusion_{yz} \right) \tag{9}$$

With:

$$\begin{aligned}
diffusion_{yx} &= \frac{1}{dx_i} \left[\frac{\mu_{i,j,k} + \mu_{i+1,j,k} + \mu_{i+1,j+1,k} + \mu_{i,j+1,k}}{4} \cdot \left(\frac{u_{i,j+1,k} - u_{i,j,k}}{dyh_j} + \frac{v_{i+1,j,k} - v_{i,j,k}}{dxh_i} \right) - \frac{\mu_{i,j,k} + \mu_{i,j+1,k} + \mu_{i-1,j+1,k} + \mu_{i-1,j,k}}{4} \cdot \left(\frac{u_{i-1,j+1,k} - u_{i-1,j,k}}{dyh_j} + \frac{(v_{i,j,k} - v_{i-1,j,k})}{dxh_{i-1}} \right) \right] \\
diffusion_{yy} &= \frac{2}{dyh_j} \left(\frac{\mu_{i,j+1,k} \left(v_{i,j+1,k} - v_{i,j,k} \right)}{dy_{j+1}} - \frac{\mu_{i,j,k} \left(v_{i,j,k} - v_{i,j-1,k} \right)}{dy_j} \right) \\
diffusion_{yz} &= \frac{1}{dz_k} \left[\frac{\mu_{i,j,k} + \mu_{i,j+1,k} + \mu_{i,j+1,k+1} + \mu_{i,j,k+1}}{4} \cdot \left(\frac{v_{i,j,k+1} - v_{i,j,k}}{dzh_k} + \frac{w_{i,j+1,k} - w_{i,j,k}}{dyh_j} \right) - \frac{\mu_{i,j,k} + \mu_{i,j+1,k} + \mu_{i,j+1,k-1} + \mu_{i,j,k-1}}{4} \cdot \left(\frac{v_{i,j,k} - v_{i,j,k-1}}{dzh_{k-1}} + \frac{w_{i,j+1,k-1} - w_{i,j,k-1}}{dyh_j} \right) \right]
\end{aligned} \tag{10}$$

Predicted w-velocity explicit diffusion

$$dw_{i,j,k} = \frac{2}{\rho_{i,j,k+1} + \rho_{i,j,k}} \left(diffusion_{zx} + diffusion_{zy} + diffusion_{zz} \right) \tag{11}$$

With:

$$
\begin{aligned}
diffusion_{zx} &= \frac{1}{dx_i}\left[\frac{\mu_{i,j,k}+\mu_{i+1,j,k}+\mu_{i+1,j,k+1}+\mu_{i,j,k+1}}{4}\cdot\left(\frac{u_{i,j,k+1}-u_{i,j,k}}{dzh_k}+\frac{w_{i+1,j,k}-w_{i,j,k}}{dxh_i}\right)-\frac{\mu_{i,j,k}+\mu_{i-1,j,k}+\mu_{i-1,j,k+1}+\mu_{i,j,k+1}}{4}\cdot\left(\frac{u_{i-1,j,k+1}-u_{i-1,j,k}}{dzh_k}+\frac{(w_{i,j,k}-w_{i-1,j,k}}{dxh_{i-1}}\right)\right]\\
diffusion_{zy} &= \frac{1}{dy_j}\left[\frac{\mu_{i,j,k}+\mu_{i,j+1,k}+\mu_{i,j+1,k+1}+\mu_{i,j,k+1}}{4}\cdot\left(\frac{v_{i,j,k+1}-v_{i,j,k}}{dzh_k}+\frac{w_{i,j+1,k}-w_{i,j,k}}{dyh_j}\right)-\frac{\mu_{i,j,k}+\mu_{i,j-1,k}+\mu_{i,j-1,k+1}+\mu_{i,j,k+1}}{4}\cdot\left(\frac{v_{i,j-1,k+1}-v_{i,j-1,k}}{dzh_k}+\frac{w_{i,j,k}-w_{i,j-1,k}}{dyh_{j-1}}\right)\right]\\
diffusion_{zz} &= \frac{2}{dzh_k}\left(\frac{\mu_{i,j,k+1}\left(w_{i,j,k+1}-w_{i,j,k}\right)}{dz_{k+1}}-\frac{\mu_{i,j,k}\left(w_{i,j,k}-w_{i,j,k-1}\right)}{dz_k}\right)
\end{aligned}
\tag{12}
$$

Momentum convection

QUICK interpolation u-velocity

$work_{i,j,k,1} =$
...to be completed

A matrix: Poisson solver

$$
\begin{aligned}
A_{i,j,k,1} &= \frac{2 \cdot dt \cdot umask_{i-1,j,k}}{dx_i \cdot dxh_{i-1}\left(\rho_{t(i-1,j,k)} + \rho_{t(i,j,k)}\right)} \\
A_{i,j,k,2} &= \frac{2 \cdot dt \cdot umask_{i,j,k}}{dx_i \cdot dxh_{i}\left(\rho_{t(i+1,j,k)} + \rho_{t(i,j,k)}\right)} \\
A_{i,j,k,3} &= \frac{2 \cdot dt \cdot vmask_{i,j-1,k}}{dy_j \cdot dyh_{j-1}\left(\rho_{t(i,j-1,k)} + \rho_{t(i,j,k)}\right)} \\
A_{i,j,k,4} &= \frac{2 \cdot dt \cdot vmask_{i,j,k}}{dy_j \cdot dyh_{j}\left(\rho_{t(i,j+1,k)} + \rho_{t(i,j,k)}\right)} \\
A_{i,j,k,7} &= \sum_{n=1}^{6} A_{i,j,k,n} \\
A_{i,j,k,8} &= -\left[S_{vol} + \frac{u^*_{i,j,k} - u^*_{i-1,j,k}}{dx_i} + \frac{v^*_{i,j,k} - v^*_{i,j-1,k}}{dy_j} + \frac{w^*_{i,j,k} - w^*_{i,j,k-1}}{dz_k}\right]
\end{aligned}
\tag{13}
$$

Where S_{vol} is a volume source such that $\nabla \cdot \overrightarrow{u} = S_{vol}$.

Solving for P

$$
P_{i,j,k} = (1-\beta) \cdot P_{i,j,k} + \frac{\beta}{A_{i,j,k,7}}\left(A_{i,j,k,1} \cdot P_{i-1,j,k} + A_{i,j,k,2} \cdot P_{i+1,j,k} + A_{i,j,k,3} \cdot P_{i,j-1,k} + A_{i,j,k,4} \cdot P_{i,j+1,k} + A_{i,j,k,5} \cdot P_{i,j,k-1} + A_{i,j,k,6} \cdot P_{i,j,k+1} + A_{i,j,k,8}\right) \tag{14}
$$

Correcting velocities

$$
u^{n+1}_{i,j,k} = u^*_{i,j,k} - dt \cdot \frac{2}{\rho_{i+1,j,k} + \rho_{i,j,k}} \frac{p_{i+1,j,k} - p_{i,j,k}}{dxh_i} \tag{15}
$$