Granular Micro-Structure and Avalanche Precursors

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Using numerical simulation, we show the existence of a sharp transition in the state of a granular slope few degrees before the angle of avalanche, involving the packing micro-structure and avalanche precursors. Interestingly, this transition coincides with the transition to metastability observed experimentally. This questions the emergence of new correlation lengths in relation with this sharp modification of the pile internal state.

1 INTRODUCTION

The stability of granular slopes is related to the existence of two angles. The static angle of avalanche \( \theta_c \) characterizes the maximum slope angle compatible with static equilibrium. Equivalently, \( \theta_c \) is the slope angle for which slope failure and surface avalanche are incipient. The dynamical angle of repose \( \theta_d \) is the angle at which the surface of the flow is stabilized after a surface avalanche was triggered, with \( \theta_d < \theta_c \). The angle of hysteresis, defined as the slope interval \( [\theta_d, \theta_c] \) is typically of few degrees.

The value of \( \theta_c \) reflects the effective properties of static friction of the packing. It is related to the microscopic friction acting at contacts and the geometrical effects due to the shape of the grains. The angle of avalanche is thus dependent on the organization of the packing at the grains scale, and is likely to be sensitive to its intrinsic disorder. This sensitivity is apparent when measuring the value of \( \theta_c \) for one given granular medium over successive avalanches: \( \theta_c \) is not mono-valued, but distributed in a slope interval of few degrees. In this slope interval the trigger of an avalanche is possible everywhere, suggesting a metastable state.

The metastability of granular slopes approaching the angle of avalanche has been investigated experimentally. Daerr & Douady (Daerr & Douady 1999) have measured the size of the perturbation necessary to trigger a surface flow depending on the slope \( \theta \) of the surface. They found that the size of the perturbation tends to zero when the slope tends towards \( \theta_c \), and that no surface flow can be triggered for \( \theta < \theta_d \), thus characterising the slope metastability as a sub-critical transition. Other experiments show that a perturbation of given size applied to a granular pile affects an increasing area of the pile when the latter evolves towards \( \theta_c \) (Deboeuf et al 2003).

These experiments show a profound modification of the state of the pile evolving towards stability limit, which effect is to increase the susceptibility of the pile to avalanching. However, beyond the observation of a metastable state in \( [\theta_d, \theta_c] \), the precise analysis of the state of the granular pile in this interval escapes the experimental tools. In this contribution, we investigate the modification of the internal state of a granular slope using numerical simulation. We are thus able to show a sharp transition in the stress state of the pile for a slope \( \theta \) few degrees lower than \( \theta_c \). This transition coincides with a critical mobilization of contact friction and the emergence of friction induced instabilities, and also coincides with the onset of dilatancy, which are all strong precursors of the destabilization.
Figure 2. Grain $p$ involved in contacts $\alpha$ with its neighbours; $r^\alpha$ is the vector position of the contact $\alpha$, and $f^\alpha$ is the force transmitted at the contact $\alpha$.

Figure 3. Evolution of the macroscopic stress ratio $\Gamma = Q/P$ as a function of the slope angle $\theta$. The stability limit occurs for $\theta = \theta_c \simeq 19.5^\circ$, for which the stress ratio reaches the limit value $\Gamma_c \simeq 0.36$.

2 NUMERICAL SIMULATION

We have applied the contact dynamics algorithm (Jean & Moreau 1992) in 2D to simulate a periodic granular pile composed of 10000 grains with diameter $D$ distributed in a small interval so that $D_{\text{max}}/D_{\text{min}} = 1.5$, preventing long range crystal-like ordering of the packing. The pile is built by a random rain of grains in the gravity field; its width is $200D$ and its height is $\simeq 40D$. The grains are perfectly rigid and interact through a simple Coulomb friction involving the microscopic coefficient of friction $\mu$ at the contacts between the grains. The pile is tilted at a constant rotation rate in the gravity field so that the slope $\theta$ of its free surface increases slowly from initially $0^\circ$ to the static angle of avalanche $\theta_c$ (Figure 1). The quasi-static evolution towards avalanching is the subject of what follows.

3 MULTI-SCALE ANALYSIS OF THE STRESS

3.1 Definition

To evaluate the level of stress applied to any volume of the packing, we use the concept of grains moment tensor defined in (Moreau 1997). We consider a grain $p$ involved in $N_\alpha$ contacts with neighboring grains, and submitted to the forces $f^\alpha$ applied at the contact points $r^\alpha$ (Figure 2). The moment tensor $M_p$ of the grain $p$ is given by:

$$M_{ij}(p) = \sum_{\alpha=1,N_\alpha} r^\alpha_i f^\alpha_j,$$

where $i$ and $j$ are the space dimensions. Using the additivity of the moment tensor, we compute the tensor $M$ of the pile taking into account all the grains. The moment tensor is related to the stress tensor through the relation $\sigma = M/V$ where $V$ is the volume over which the tensor is evaluated. In the following we consider ratios of stress component so that the definition of the volume is not required. Using the eigen values of the moment tensor $m_1$ and $m_2$, we compute the deviatoric part $Q = (m_1 - m_2)/2$ and the isotropic part $P = (m_1 + m_2)/2$. We form the ratio of these two quantities

$$\Gamma = \frac{Q}{P},$$

i.e. the level of deviatoric stress normalised by the pressure.

3.2 Macroscopic stress and grains stresses

The evolution of the ratio $\Gamma$ evaluated over the whole volume of the pile as a function of the slope angle $\theta$ is reported in Figure 3. While the slope evolves towards $\theta_c$, the linear increase of $\Gamma$ shows the gradual loading of the slope and the increase of shear stress. For $\theta \simeq 19.5^\circ$, this regular increase stops and softening of the material occurs; this indicates that the stability limit is reached, namely that $\theta = \theta_c$. We define the corresponding limit stress state

$$\Gamma_c = \frac{Q}{P}(\theta_c).$$

The mechanism by which the whole pile reaches the limit state $\Gamma_c$ is not obvious. In order to compare this
macroscopic evolution to the evolution of the stress state at the grains scale, we define for each grain the stress ratio
\[ \gamma = \frac{q}{p}, \]
where \( q \) and \( p \) are the deviatoric and isotropic part of the grain moment tensor given by equation (1). We focus on the overloaded grains, namely the grains satisfying \( \gamma \geq \Gamma_c \). Three snapshots of the pile for \( \theta = 0, 10^\circ \) and \( \theta_c \) are shown in Figure 4; on these snapshots, the overloaded grains only are represented in black. In the course of time, we observe a slow structuring of the packing very much reminiscent of a percolation process.

### 3.3 Multi-scale analysis

The structuring of the local stress state corresponds to the slow emergence of areas where the stress state has locally reached the macroscopic limit \( \Gamma_m \). We compute the maximum size \( \ell_c^{\text{max}} \) of these areas in the course of time. To do so, we focus on the overloaded grains; for each one, we consider a circular neighborhood of radius \( \ell_c \) over which the stress state is evaluated. We can thus determine the size of the neighborhood \( \ell_c \) for which the stress state satisfies \( \Gamma = \Gamma_c \). Carrying this operation over all the overloaded grains we are able to determine \( \ell_c^{\text{max}} = \max \{ \ell_c \} \) for any value of the slope \( \theta \). The evolution of \( \ell_c^{\text{max}} \) as a function of \( \theta \) is plotted in Figure 5a. We first observe a slow increase for \( \theta \in [0, 15^\circ] \). At \( \theta \approx 15^\circ \), a rapid jump suddenly brings \( \ell_c^{\text{max}} \) to 30D, namely a size comparable to the packing height (this maximum value 30D is imposed as a cut-off for practical reasons in the computation of \( \ell_c \), which otherwise would reach the pile dimension 40D). Again, this behaviour is reminiscent of a percolation mechanism. Moreover, it clearly establish the existence of a transition in the state of the packing few degrees (here 5\(^\circ\)) before the angle of avalanche \( \theta_c \) is reached. Beyond the fact that this transition in itself is a strong precursor of the avalanche, it changes the internal state of the packing and presumably its susceptibility to avalanching.

### 4 AVALANCHE PRECURSORS

#### 4.1 The mobilization of contact friction

The friction acting between the grains is a simple Coulomb law relating the normal and tangential force at contact \( N \) and \( T \) through the relation \( T/N \leq \mu \), where \( \mu \) is the microscopic coefficient of friction. When the Coulomb threshold is reached, namely for \( T/N = \mu \), any relative slip motion between the two grains in contact is possible. Such slip motions create a local dynamical instability which allows for the local reorganization of the packing: collective slip motions and redistribution of the forces. These instabilities release energy and help the pile finding a more stable configuration; however, the perturbation they create may not be damped by the rearrangements of the grains and a surface avalanche might be triggered. The kinetic energy released by these friction-induced instabilities is reported in Figure 6 as a function of the slope angle \( \theta \). We observe peaks increasing in size at the approach of the stability limit. It can be
shown that the occurrence of the dynamical instabilities is related to the mobilization of the contact friction forces (Staron et al. 2002). We define the proportion \( \nu \) of contact satisfying \( T/N = \mu \) (referred to as critical in the following):

\[
\nu = \frac{N_c}{N},
\]

where \( N_c \) is the number of critical contacts and \( N \) is the total number of contacts in the packing. We can show that \( \nu \) is bounded by a saturation value \( \nu_c \) for which the probability of larger scale instabilities becomes higher. This behaviour is apparent when plotting the size of the instability as a function of the proportion of critical contacts (i.e. in the state \( T/N = \mu \)) reached just before the instability (Figure 7). The size of the instability is measured as the proportion of contacts gained or lost in the event. Two clouds of points can be distinguished: a group of frequent small-scale rearrangements for \( \nu \leq \nu_c \), and a group of rare large-scale rearrangements for \( \nu \geq \nu_c \).

In Figure 8 are presented three snapshots of the state of the pile for \( \theta = 10^\circ, 16^\circ \) and \( \theta_c \). The gray scale encode the local density of critical contacts; areas represented in black have reached the state \( \nu \geq \nu_c \), and are thus potential source of friction-induced instabilities. We observe that these areas grow in size and finally span the whole pile when stability limit approaches, following a process again bearing strong analogy with percolation. The evolution of the mean size \( r_{\text{mean}} \) of these areas where \( \nu \geq \nu_c \) is plotted in Figure 5b as a function of the slope angle \( \theta \). A divergence is clearly observed in the interval of slope angle [16°, \( \theta_c \)]. As previously, we observe that the internal state of the pile (here the organization of contact friction) undergoes a dramatic evolution few degrees before the angle of avalanche is reached.

4.2 The dilatancy transition

We measure the variations of the volume of the packing \( \epsilon_V = (V - V_0)/V_0 \), where \( V_0 \) is the initial volume, as a function of the slope angle \( \theta \). These variations are plotted in Figure 9. We first observe a contractant behaviour followed by a dilatant behaviour as is classically observed in dense sands. Remarkably, the transition to dilatant behaviour occurs at \( \theta \approx 15^\circ \), in coincidence with the transition in the stress state and the contact friction mobilization.

5 CONCLUSIONS

Using numerical simulation, we have shown the existence of a sharp transition in the state of a granular slope few degrees before the angle of avalanche. This transition involve the packing micro-structure, namely the local stress and the contact friction. It is accompanied by avalanche precursors i.e. local dynamical instabilities and a dilatant behaviour. This transition involved a mechanism very reminescent of percolation process, thus stressing the analogy between granular slopes loss of stability and phase transition. Interestingly, this transition coincides with a transition towards a metastable state observed experimentally. It is not clearly established whether this sharp modification of the local stress and of the microscopic friction results in the emergence of new correlation lengths in the packing. Nevertheless, it is very tempting to identify the origin of the metastability with these modifications of the internal state of the pile.

REFERENCES


