

# THE COHESIVE GRANULAR COLLAPSE AS A CONTINUUM : PARAMETRIZATION STUDY

ANAIS ABRAMIAN<sup>1</sup>, LYDIE STARON<sup>1</sup> AND PIERRE-YVES LAGREE<sup>3</sup>

<sup>1</sup> Institut d'Alembert, Sorbonne Université, CNRS  
4 Place Jussieu, 75252 Paris Cedex 05  
anaïs.abramian@upmc.fr

**Key words:** Granular Materials, Cohesion, Navier-Stokes solver

**Abstract.** The flow of dry granular materials is now well modeled by the  $\mu(I)$ -rheology. However, how cohesion forces between grains alters the flow dynamics, and its rheology, remains elusive. To start modelling such cohesive flows, we first focus on the configuration of a granular collapse, which is a simple benchmark test. Specifically, we compare granular-collapse experiments of cohesive grains with numerical simulations, where we test a simple rheology for the material : the so-called  $\mu(I)$ -rheology, supplemented by a yield stress for cohesion. This document reports the sensitivity of our numerical simulations on the parameters of the rheology, often challenging to measure in experiments.

## 1 INTRODUCTION

Cohesive granular materials are ubiquitous in natural flows, often induced by capillary bridges of water between grains. A recent example of such a flow is the landslide of Llusco, in Peru [1] (Fig. 1). After intense rains, the soil lost cohesion and collapsed, without any seismic activity. This dramatic event reveals our misunderstanding on the static and the dynamics of a cohesive collapse.

We also frequently encounter granular materials in an industrial context, such as in construction, pharmaceuticals or food-processing industry. In this context, cohesive flows can clog conveyors or silos, and



**Figure 1:** Picture of a landslide which occurred in Llusco (Peru), in 2018. This collapse is typical of a cohesive granular flow, featuring a first fracture plane, with secondary fractures transverse to the flow.

then stop a production chain. The need to facilitate transport and handling of powders leads the use of different industrial testers. Among different testers, the Hosokawa tester yields a “flowability” index between 0 and 100, based on a series of experiments such as the angle of repose, the tapped bulk density or the particle-size distribution. However, these measurements remain qualitative, and, even if they enable to compare two powders, they are mostly done in quasistatic conditions. Our objective is to model the dynamics of cohesive granular material and understand the physics which is behind this concept of flowability.

Although the flow of dry granular materials is now relatively well modeled by the  $\mu(I)$ -rheology [2], how cohesion forces affects the rheology and the flow dynamics remains elusive. Most of recent work focused on the wet granular materials, where a small amount of liquid induces cohesion forces by capillary bridges [6]. However, this cohesion force between two ideal spheres depends mainly on the shape of the bridge, and on its position, which are impossible to control experimentally.

To bypass this issue, Gans et al. (2020) [4] use a new cohesion-controlled granular material. It consists of glass beads coated by a polymer, where cohesion of the bulk increases with the polymer thickness. Consequently, the cohesion of the bulk can easily be tuned by varying the quantity of polymer during the preparation. After having characterized the onset of the flow of this new material on an inclined plane, its dynamics is now investigated in another simple configuration: the granular collapse. The material is initially prepared in a column of a given aspect ratio  $a = H/L$  [Fig. 2(a)]. At the beginning of the experiment ( $t = 0$ ), the gate is removed and the column collapses under its own weight.

In parallel, we perform numerical simulations to model the effect of cohesion on the granular collapse. To do so, we consider the material as a fluid of a peculiar rheology: the  $\mu(I)$ -rheology, supplemented by a yield stress  $\tau_c$  which accounts for cohesion,

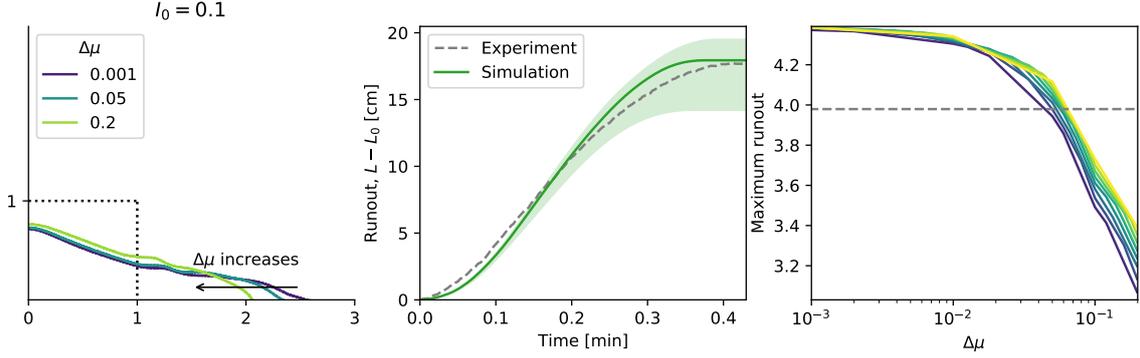
$$\tau = \tau_c + \mu(I)P, \quad \text{with} \quad I = \frac{\dot{\gamma}d}{\sqrt{P/\rho}}, \quad (1)$$

where  $I$  is named the inertial number [5] and  $\mu(I)$  is an empirical relation, which has been validated experimentally and numerically for dry granular materials [3, 7]:

$$\mu(I) = \mu_s + \frac{\mu_s - \mu_2}{I_0/I + 1}, \quad (2)$$

with  $\mu_s$  the friction coefficient, and  $I_0$ ,  $\Delta\mu = \mu_2 - \mu_s$  are empirical parameters. The friction coefficient  $\mu_s$  has been measured experimentally through the inclined-plane configuration, and is constant with the cohesion force, equal to  $\mu_s = 0.4 \pm 0.05$  with these beads. However, the parameters  $I_0$  and  $\Delta\mu$  are difficult to measure experimentally. First,  $\mu_2$  is the friction coefficient between two grains only, which requires small-scale experiments. Then, the measurement of  $I_0$  can only be fitted on the whole rheology, which needs a home-made shear cell, with no-slip conditions on the walls, while the confinement pressure  $P$  can vary. This makes a challenging apparatus.

Consequently, we focus here on the influence of these two parameters ( $I_0$  and  $\Delta\mu = \mu_s - \mu_2$ ) in the numerical resolution of the cohesive granular collapse. Specifically, we investigate their influence on the velocity of the collapse, and on the length of the final deposit (or final runoff). By comparing these results with the experiments, we demonstrate that the dependency of the rheology on  $I$  is necessary to model the collapse correctly. We show, however, that our uncertainty on the rheological parameters can lead about 20% of uncertainty on the final runoff.



**Figure 2:** Variation of  $\Delta\mu$ . (a) Final shape of the deposit as a function of  $\Delta\mu$  for a fixed  $I_0$ . (b) Comparison between an experiment and the corresponding numerical simulation. The upper bound of the green area is  $\Delta\mu = 0$ . The lower bound is  $\Delta\mu = 0.4$ . (c) Variation of the final runout as a function of  $\Delta\mu$  for different  $I_0$ .

## 2 SENSITIVITY TO $\Delta\mu$

A first parameter involved in the granular rheology (1) but not measured experimentally is  $\Delta\mu = \mu_s - \mu_2$ . It is the friction difference between a quasi-static granular flow and a collisional flow. In other words, it is the friction difference between a grains surrounded grains, or and the grain-grain friction. In the experiments of Jop et al. (2006) [2], they found empirically a value of  $\Delta\mu = 0.26$  for glass beads of diameter  $d = 0.53$  mm.

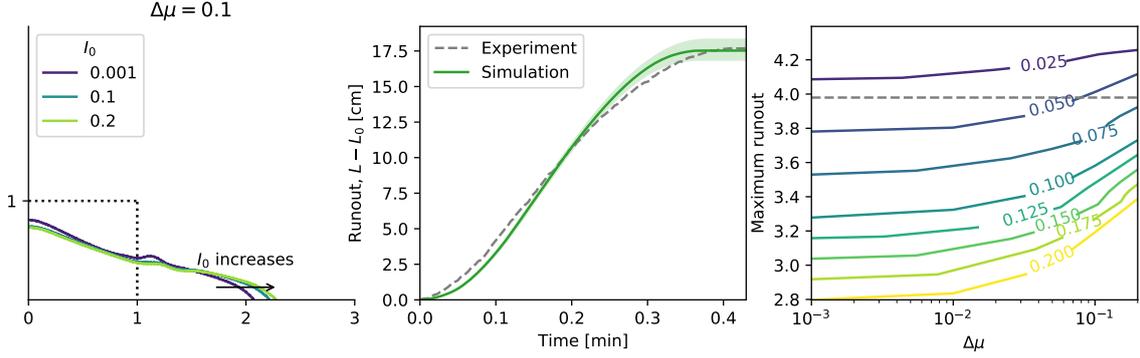
Here, we vary this parameter and study its influence on the granular collapse. We fix the aspect ratio of the column to  $a = 1$ , and set the cohesion through a characteristic length :  $\ell_c = \tau_c / \rho g = 3.6$  cm. Fig. 2(a) shows the initial and final shape of the material. Fixing  $I_0$  to 0.2, we then vary  $\Delta\mu$  from 0 to 0.5 and investigate its influence on the dynamics of the collapse, and on the final shape of the deposit.

After the column flows, the material ends up with a static final shape, featuring a characteristic bump, which was not sheared during the collapse [Fig. 2(a)]. The value of  $\Delta\mu$  qualitatively modifies this final shape as follow. The angle of the front increases when  $\Delta\mu$  increases, and the bump is more pronounced as  $\Delta\mu$  is higher. This makes the position of the front particularly sensitive to  $\Delta\mu$ .

To investigate this sensitivity, plot the time-evolution of the front's position, subtracted by the initial length of the column,  $L - L_i$ , for  $\Delta\mu = 0.2$  [Green line, Fig. 2(b)]. We then compare this numerical result to an experiment in the same configuration [Dashed grey line, Fig. 2(b)].

Now, we vary  $\Delta\mu$  from zero to 0.4 [Green area, Fig. 2(b)]. We observe a qualitatively good agreement, although it is better at the beginning of the collapse than at the end. The final runout is more sensitive to  $\Delta\mu$ , than its derivative, which corresponds to the velocity of the collapse. When  $\Delta\mu$  is small enough, the final runout is the higher. When  $\Delta\mu$  increases, the final runout decreases. Finally, this gives an uncertainty of 25% on the final length of the deposit.

Overall, the variation of the final runout with  $\Delta\mu$  is shown on Fig.2(c). Each line, which corresponds to a different value of  $I_0$ , are close. Thus, the parameter  $\Delta\mu$  seems to control the most the final runout of the collapse, at least more than  $I_0$ . To confirm it, we perform a detailed investigation of the influence of the parameter  $I_0$  in the next section.



**Figure 3:** Variation of  $I_0$ . (a) Final shape of the deposit as a function of  $I_0$  for a fixed  $\Delta\mu$ . (b) Comparison between an experiment and the corresponding numerical simulation. The upper bound of the green area is  $I_0 = 0.6$ . The lower bound is  $I_0 = 0.1$ . (c) Variation of the final runout as a function of  $I_0$ .

### 3 SENSITIVITY TO $I_0$

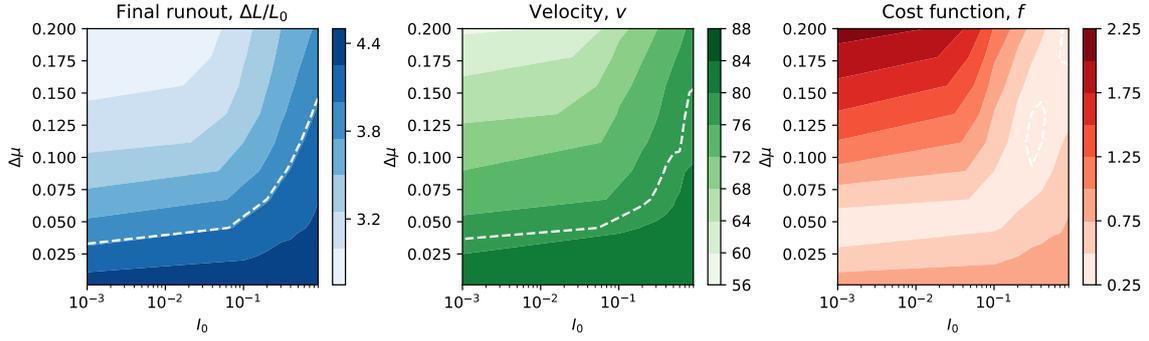
We now test the sensitivity of the flow to a second parameter involved in the rheology (1), which we noted  $I_0$ . This parameter is usually directly fitted on a measured rheology and is not related to any independent measurements. Jop et al. (2006) [2] found empirically the value of  $I_0 = 0.279$  for dry granular materials. To investigate the influence of this parameter on the dynamics of a cohesive collapse, we perform the same procedure than previously. We fix the parameter  $\Delta\mu = 0.2$ , and vary  $I_0$ . First, we observe the influence of this parameter on the final shape of the deposit [Fig.3(a)]. In this case, the local slope at the front is not modified, but this parameter makes the scale of the inertial number vary. This reduces the size of the bump, and therefore the final runout, although slightly.

To measure quantitatively this influence on the final runout, we plot the position of the front as a function of time for the same configuration and compare it to the experiment [Fig.3(b)]. By varying  $I_0$  from  $10^{-3}$  to 0.4, we show the variation of this function as  $I_0$  increases. Once again, the velocity of the collapse is less sensitive than the final runout, which can vary of about 10% in the range of values we explored. Fig.3(c) shows the dependency of the final runout as a function of  $I_0$  for different  $\Delta\mu$ . Here again, this shows that it is  $\Delta\mu$  which is most important in the choice of the parameters. In particular, we superimposed the value of the final runout measured experimentally for the same configuration [dashed grey line, Fig. 3(c)], and deduce that a large range of values for the two parameters  $I_0$  and  $\Delta\mu$  are possible to fit with the final runout.

In the next section, we explore the range for both parameters together for the runout and the velocity and investigate which couple of values fits the experimental data at best.

### 4 SENSITIVITY TO BOTH PARAMETERS

We now vary sequentially  $I_0$  and  $\Delta\mu$ , and compute the corresponding final runout of the collapse. We can then plot the final runout as a function of the two parameters  $\Delta\mu$  and  $I_0$  [given by the colorbar, Fig. 4(a)]. On this map, we superimpose the experimental value, and find a few couples of parameters which could fit at best the final runout [white dashed line, Fig. 4(a)].



**Figure 4:** (a) Map of runout. Dashed white line: experimental value of the final runout. (b) Map of velocity. Dashed white line: experimental value of the final runout. (c) Map of cost function. Dashed white line: minimization of the cost function.

However, it is not because we capture the final runout that our modelisation on the dynamics of the collapse is robust. Indeed, the velocity is another important parameter to model correctly the dynamics. Consequently, we also compute the velocity of the collapse, that we define as the highest slope of the front position, as a function of both parameters, and superimpose the experimental value [Fig. 4(b)]. We find the best fit for approximately the same couples of parameters.

To account for the entire dynamics of the collapse, we measure the difference of the front position at each time, for the experiment and the numerical simulation. In other words, we define a cost function  $f$ , as the integral of the difference:

$$f = \int (r_{num} - r_{exp})^2 dt \quad (3)$$

where  $r = \Delta L/L_0$  is the normalized runout as a function of time. We now want to minimize this function to fit the experiments at best.

We thus plot this function  $f$  for both parameters [Fig. 4(c)]. The minimum value of this function is the dashed white line, and corresponds to the couple of parameters :  $I_0 = 0.35$  and  $\Delta\mu = 0.125$ . Still, these values are in this range of parameters we explored. We must compare these values with the dry granular materials.

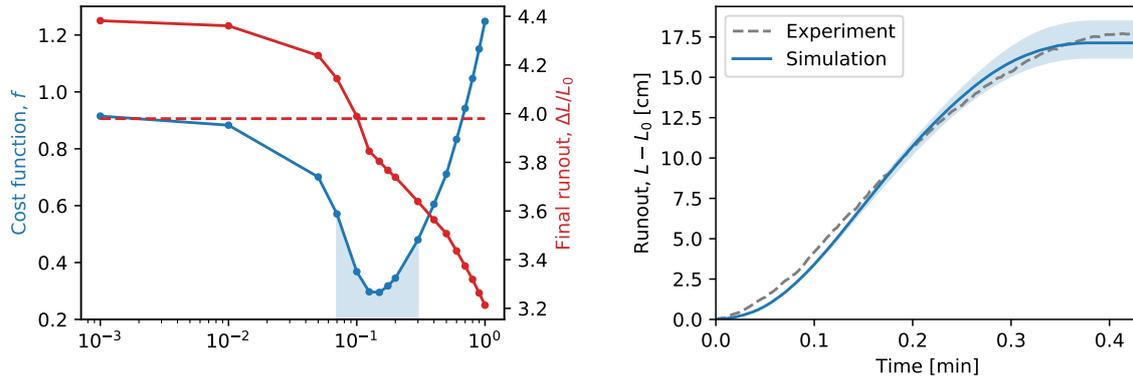
Finally, to avoid dealing with two parameters, we can linearize the rheology and make it vary through one single parameter to facilitate the study, which we do in the next section.

## 5 LINEARIZED RHEOLOGY

For columns of small aspect ratios, we can linearize the rheology on the inertial number, through  $\mu(I)$  as follows:

$$\mu(I) = \mu_s + aI \quad (4)$$

where we define  $a = \Delta\mu/I_0^2$ . The rheology now depends on a unique parameter, that we vary to find the optimal value compared to the one experiment. Here again, we define the cost function and plot it as a function of this parameter  $a$  (Fig. 5). We find that the cost function is minimum for  $a \approx 0.125$ .



**Figure 5:** (a) In blue: cost function  $f$  defined by equation (3) as a function of parameter  $a$  involved in the linearized rheology (4). In red: final runout of the deposit as a function of  $a$ . Dashed blue line : experimental value of the final runout. (b) Position of the front as a function of time. Blue: numerical simulation for  $a = 0.125$ . Dashed grey line : experimental data. The blue area corresponds to the range of  $a$  colored on the panel (a).

## 6 CONCLUSION

In this article, we performed a parametrization study of the cohesive granular rheology. Specifically, we investigated the influence of two parameters involved in the rheology :  $\Delta\mu$  and  $I_0$ . We observed that  $\Delta\mu$  influences the most the final shape of the deposit, and in particular its final runout. We found a few couples of values which fit the experimental runout and velocity at best. Finally, we linearized the rheology, and found in this case a unique value of parameter which fits at best the experiments :

$$\mu(I) = \mu_s + aI \quad (5)$$

where  $a \approx 0.125$ .

This could be generalized to other cases of cohesion and aspect ratios, but this study gives, at least, insights on the parameters involved in the rheology and on the amplitude of our uncertainties.

## REFERENCES

- [1] Roca Guevara, Nicole Katherine. "Impacto Ambiental por deslizamiento de suelo en el Sector Campanayocpata del Centro Poblado de Lutto en Chumbivilcas-Cusco 2018." (2018).
- [2] Jop, P., Forterre, Y., & Pouliquen, O. (2006). A constitutive law for dense granular flows. *Nature*, 441(7094), 727.
- [3] Lacaze, L., & Kerswell, R. R. (2009). Axisymmetric granular collapse: a transient 3D flow test of viscoplasticity. *Physical Review Letters*, 102(10), 108305.
- [4] Gans, A., Pouliquen, O., and Nicolas, M. (2019). A cohesion-controlled granular material. *Physical Review E* 101.3 (2020): 032904.
- [5] GDR MiDi 2004 (2004), On dense granular flows, *Eur. Phys. J. E* **14**, 341-365

- [6] Bocquet, L., Charlaix, E., & Restagno, F. (2002). Physics of humid granular media. *Comptes Rendus Physique*, 3(2), 207-215.
- [7] Lagrée P.-Y., L. Staron & S. Popinet, The granular column collapse as a continuum: validity of a two-dimensional Navier-Stokes model with the  $\mu(I)$  rheology, *J. Fluid Mech.* vol **686**, pp **378-408** (2011)