

## Size Segregation in Bi-disperse Granular Flows

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### Abstract

Using a discrete simulation method, we investigate numerically two-dimensional bi-disperse chute flows formed of a layer of larger grains overlaid by a layer of smaller grains, and analyze their evolution for different slopes and different volume fraction of large beads. As size segregation occurs, the vertical position of the centre of mass of the large beads is shown to increase exponentially with time with a typical time scale decreasing with the volume fraction of large beads. A simple model balancing lift and drag forces acting on large particles recovers this dynamics, successfully predicts the typical time scale of segregation, and allows this typical time scale to be related to the flow dynamics.

### 1. Introduction

Size segregation in granular systems is a frequent phenomenon observed once the distribution of grains size is sufficiently large, and the system undergoes a sufficiently rapid, or long-lasting, shear motion [1, 2, 3, 4]. In this situation, larger grains raise to the surface while smaller one sink to the bottom as a result of "kinematic sieving", leading to heterogeneous patterns and spatial ordering [5, 6]. In the present study, we are interested in characterizing the rise of the larger beads as a results of the flow dynamics, and understanding what controls the efficiency of the mechanism, ie the typical time scale of segregation. To this purpose, discrete numerical simulation is a helpful tool, first because it allows for perfectly well-controlled "experimental" conditions (grain contact properties, flow geometry, flow duration), but mostly because it provides direct knowledge of the state of the system (velocities, forces, volume fraction). Simulating numerical two-dimensional bi-disperse chute flows formed of larger and smaller grains, we analyze their evolution for different slopes and different volume fraction of large

beads.

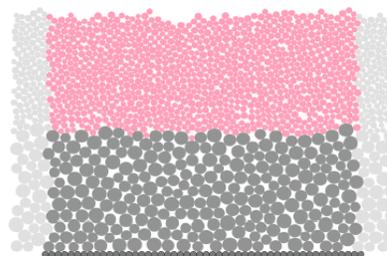


Figure 1: A periodic bi-disperse granular bed in its initial state: a layer of large grains is overlaid by a layer of small grains.

### 2. The numerical flows

Considering a perfectly two-dimensional geometry, we form a granular layer made of two species of grains of circular shape, on an incline of slope  $\theta$  and subjected to gravity. The large beads have a mean diameter  $d_L$  and the small beads have a mean diameter  $d_S$ . The ratio  $d_L/d_S$  was not varied: we chose  $d_L/d_S = 2$ . To ensure long flow durations, periodic boundary conditions were implemented; the width of the simulation cell is  $50d_S$ . We denote  $\xi$  the volume fraction of large beads, ie the ratio of the volume of large beads to the total volume of grains:  $\xi = V_L/(V_S + V_L)$ . Accordingly, the volume fraction of small beads is given by  $(1 - \xi)$ . In the simulations,  $\xi$  was varied between 0.08 and 0.89. The slope of the granular bed  $\theta$  was varied between  $17^\circ 5'$  and  $22^\circ$  to the horizontal, allowing different flow velocities.

### 3. An exponential evolution

The center of mass of the large beads  $y_G$  is plotted as a function of time in Figure 2: it exhibits an

exponential increase towards a saturated segregated state which corresponds to the stationary regime. The exponential rise of the large beads in the flow is a robust feature: the position of the centre of mass of the large grains  $y_G$  obeys the following evolution:

$$y_G(t) = y_0 + (y_\infty - y_0) [1 - \exp(-t/\tau)] \quad (1)$$

where  $y_0$  and  $y_\infty$  are respectively the initial and final value of  $y_G$ , and  $\tau$  is the typical time scale characterizing the segregation process.

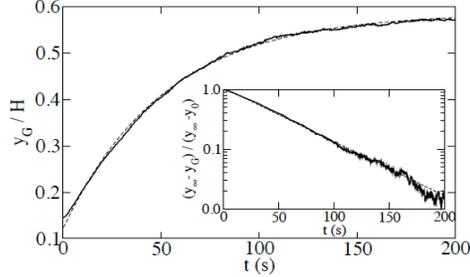


Figure 2: Position of the center of mass of the large beads  $y_G$  as a function of time  $t$  for a granular flow inclined at an angle  $\theta = 18^\circ$  and with a volume fraction of large beads  $\xi = 0.37$ . The dotted line shows the exponential fit. Inset graph: same evolution with respect to initial and final position  $(y_\infty - y_G) / (y_\infty - y_0)$  in a log-lin graph.

#### 4. A Lift-and-drag model

Assuming the existence of a lift force, a possible form for the lift is:

$$F_{lift} = C (y_\infty - y_G) / (y_\infty - y_0) \quad (2)$$

where  $C$  is a function with the following dimension:  $[C] = \rho L^3 T^{-2}$  in a two-dimensional problem.

Due to the lift force  $F_{lift}$ , the large grains rise in the flow. As a result, they experience a drag applied by the smaller grains. By analogy with the drag in viscous fluids, the mean vertical velocity of the flow being zero, the drag force is given by [7]:

$$F_{drag} = D dy_G/dt \quad (3)$$

where  $D$  is a function with the following dimension  $[D] = \rho L^2 T^{-1}$  in a two-dimensional problem. Equating lift and drag force recovers the observation:

$$y_G(t) = y_0 + (y_\infty - y_0) \left( 1 - \exp\left(-\frac{C}{D(y_\infty - y_0)} t\right) \right) \quad (4)$$

Deriving lift and drag expression from dimensional arguments, we can thus make a prediction for the time scale  $\tau$ :

$$\tau = \frac{D(y_\infty - y_0)}{C} \quad (5)$$

This prediction, explicitly discussed in the contribution, proves a good description of the data (Figure 3).

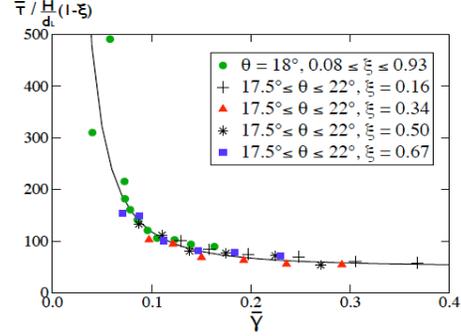


Figure 3: The normalized typical time-scale of segregation  $\bar{\tau} / \frac{H}{d_L} (1 - \xi)$  as a function of the normalized shear rate  $\|\bar{\gamma}\|$ . The plain line shows the prediction  $(1/\|\bar{\gamma}\|^2 + 75)$ .

#### Acknowledgements

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