C. R. Acad. Sci. Paris, t. 328, Série II *b*, p. 869–874, 2000
Mécanique des fluides/*Fluid mechanics*

Erosion and sedimentation of a bump in fluvial flow

Pierre-Yves LAGRÉE

**Laboratoire de modélisation en mécanique, U.M.R. 7607, Université Pierre et Marie Curie, Boîte 162, 4 place Jussieu, 75005 Paris, France
Courriel: pyl@ccr.jussieu.fr**

(Reçu le 2 mars 2000, accepté le 6 septembre 2000)

Abstract. The 2D laminar quasistationary interacting boundary layer flow with mass transport of a suspended sediment is solved over an erodable bump (or dune) in the case of a fluvial régime. It is assumed that if the skin friction goes over a threshold value, the bump is eroded, then, the concentration of sediment in suspension is convected but falls at a constant settling velocity. This changes the shape of the dune, exemples of displacement toward final equilibrium states are presented. © 2000 Académie des sciences/Éditions scientifiques et médicales Elsevier SAS

interacting boundary layer / sedimentation / erosion / asymptotic methods

Érosion et sédimentation d'une dune en régime fluvial

Résumé. *Nous étudions un écoulement 2D stationnaire laminaire en régime fluvial avec le point de vue de la couche limite interactive. Les équations dynamiques et l'équation de la masse sont résolues simultanément. La forme de la dune évolue de par l'arrachement qui se produit lorsque le frottement pariétal dépasse une certaine valeur seuil et de par la sédimentation qui se produit à vitesse constante. Des exemples de déformations de dunes sont présentés. © 2000 Académie des sciences/Éditions scientifiques et médicales Elsevier SAS*

couche limite interactive / sédimentation / érosion / méthodes asymptotiques

Version française abrégée

Nous nous proposons de calculer le déplacement d'une dune (*figure 1*) immergée dans un écoulement fluvial ($Fr < 1$) constituée d'un matériau érodable transporté par l'eau. Ces écoulements, importants pour les problèmes d'environnement, sont très complexes car tous les effets sont liés. Nous prenons le point de vue de type couche limite (mais en nous affranchissant des simplifications de la méthode intégrale [1]) et en introduisant une interaction « fluide parfait/couche limite » pour simplifier les équations de Navier–Stokes ; on suppose la quasi stationnarité (l'érosion et la sédimentation modifient très lentement la forme de la bosse), la laminarité (la turbulence est une modélisation supplémentaire) et la bidimensionnalité (pour la simplicité). La concentration de sédiments en suspension sera supposée assez faible pour que la viscosité reste égale à celle du fluide, de même, la masse volumique du fluide est inchangée par la mise en suspension.

Les équations sont adimensionnées avec les échelles usuelles de couche limite que ce soit pour les équations dynamiques (1) ou les équations de conservation de la masse (4). Les conditions aux limites pour (1) sont l'adhérence et la condition de raccord (2), le profil de Blasius est donné en entrée du domaine. L'interaction forte [3,4] se traduit par la relation liant la vitesse longitudinale, l'épaisseur de déplacement et la forme de la bosse (3).

Note présentée par Évariste SANCHEZ-PALENCIA.

S1620-7742(00)01269-1/FLA

Les conditions aux limites pour l'équation de conservation de la masse (4) gouvernent la dynamique lente de la forme de la bosse (6) : il y a par hypothèse [5–7] arrachement (5) de particules si le frottement pariétal dépasse un seuil (noté τ_s), sinon il y a seulement déposition sur le fond.

La résolution se fait par un schéma aux différences finies. Le couplage est assuré par une méthode « semi inverse ». Si on se donne une bosse initiale et un jeu de paramètres, on observe alors sur la *figure 2* que le frottement (à $\check{t} = 0$) augmente fortement au passage de la bosse, il dépasse le seuil critique bien avant le sommet. La bosse est érodée avant le sommet, il y a déposition ensuite.

On trace alors sur la *figure 3* la hauteur de bosse en fonction du temps $\check{t}$. La montée est quasi rectiligne (on retrouve un écoulement dans un convergent). La chute de la dune est incurvée, elle est plus raide que la montée (dans notre modèle il n'y a pas d'intervention d'angle d'équilibre de tas, il pourrait être introduit ultérieurement). La *figure 3* présente la forme finale obtenue pour différentes hauteurs de bosse de même forme. Pour la valeur la plus importante de la hauteur, l'écoulement présente aux premiers instants un petit bulbe de recirculation qui disparaît ensuite.

Ce modèle a l'avantage de mettre beaucoup de mécanismes en compétition et ne fait pas les simplifications intégrales usuelles. Cependant, pour prétendre faire des comparaisons expérimentales, il doit être modifié par au moins l'introduction d'un modèle de turbulence pour les diffusions visqueuse et massique.

---

1. Introduction

Let us consider the deformation of a dune immerged in a fluvial flow. This dune is made of an erodable material wich may be convected and diffused in water. This kind of flow, very important for environmental problems, is very complex because all effects are linked (the flow depends on the shape of the bump which depends on the flow which erodes or deposits sediments on the river bed which modifies again the flow). Those problems are often solved by integral boundary layer theory [1], it is pertinent because all the phenomena take place near the wall. Here we use the framework of the interacting boundary layer theory which allows a strong coupling between the boundary layer and the perfect fluid. The flow is 2D (for sake of simplicty) quasisteady (erosion and sedimentation is a slow process) and it is assumed laminar (turbulence modelisation has to be introduced). The concentration of sediments in the flow is supposed small enough to unaffect viscosity and density of the flow.

All those hypothesis may be removed one after the other, complicating more and more the final numerical resolution.

2. Dynamical aspect: Interacting boundary layer

In *figure 1* we present a rough sketch of the flow and the notations. As usual, we introduce by phenomenological analysis, small parameters to simplify the adimensionalized equations. Navier–Stokes equations are written with u scaled by U_0 (free stream velocity), x scaled by L (bump lenght), y scaled by h_0 (initial water height) and v scaled by $U_0 h L^{-1}$. Time may be L/U_0, but if we call T the scale of the erosion/sedimentation ($t = T\check{t}$, *cf.* (Section 3)), we have: $L/U_0/T \ll 1$, $\check{t}$ is only a parameter associated to the bump shape. Next, we assume that the order of magnitude of the transversal size of the bump and of the boundary layer is the same: $LR_e^{-1/2}$ (where $R_e = U_0 L/\nu$ and with $1 \gg h_0/L \gg R_e^{-1/2}$, let us call $\varepsilon = L h_0^{-1} R_e^{-1/2}$ the ratio of this scale by the initial water height). We assume as well that the scale of the bump length is the same than the length of developement of the boundary layer L. Asymptoticaly (infinite Reynolds number R_e, and so infinitely small bump) we recover a perfect fluid problem (with Froude number $F_r = U_0^2/gh_0$) of uniform constant horizontal velocity ($\bar{u}(\bar{x}, \bar{y}) = 1$, $\bar{v}(\bar{x}, \bar{y}) = 0$, $\bar{p}(\bar{x}, \bar{y}) = (1 - \bar{y})F_r^{-1}$) bounded by a free flat interface ($\bar{h}(\bar{x}) = 1$). We write $\bar{u}(\bar{x}, \bar{y} = 0) = \bar{u}_e(\bar{x})$, the slip velocity at the river bed

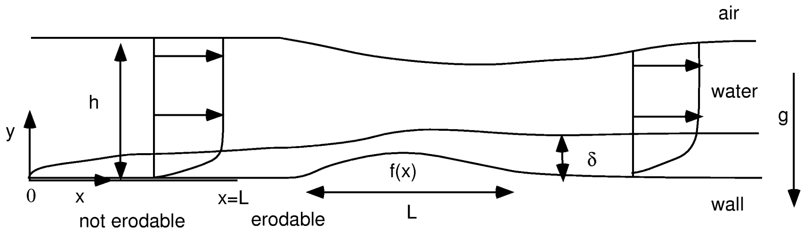

Figure 1. Sketch of the flow, before $\bar{x} = 1$ the bottom is not erodable.

Figure 1. *Une vue de l'écoulement, avant $\bar{x} = 1$ le sol est fixe.*

(in the sequel we keep $\bar{u}_e(\bar{x})$ instead of 1, the reason beeing the 'strong coupling' which will appear at the end of the section). In order to reobtain the no slip condition, a boundary layer is introduced at the wall ($x = L\bar{x}$ and $y = \hat{y}LR_e^{-1/2}$, or $\bar{y} = \varepsilon\hat{y}$). There, the velocities are scaled by $U_0\hat{u}(\bar{x}, \hat{y})$ and $U_0R_e^{-1/2}\hat{v}(\bar{x}, \hat{y})$. In order to remove the transverse variations, the pressure field may be written as the sum of a dynamical and a hydrostatic pressure $\rho U_0^2(\hat{p}(\bar{x}) + F_r^{-1}(L/h_0)\hat{y}R_e^{-1/2})$, then the asymptotic longitudinal velocity matching allows to write $-d_{\bar{x}}\hat{p}(\bar{x}) = \bar{u}_e(\bar{x})d_{\bar{x}}\bar{u}_e(\bar{x})$. Finaly, the boundary layer equations obtained are mapped by the Prandtl transformation. $\tilde{y} = \hat{y} - \hat{f}(\bar{x}, \check{t})$, $\tilde{u} = \hat{u}$, $\tilde{v}(\bar{x}, \tilde{y}) = \hat{v}(\bar{x}, \hat{y}) - \hat{u}(\bar{x}, \hat{y})\frac{\partial \hat{f}(\bar{x})}{\partial \bar{x}}$ which makes the wall 'flat'. The final system is then simply:

$$\frac{\partial}{\partial \bar{x}}\tilde{u} + \frac{\partial}{\partial \tilde{y}}\tilde{v} = 0 \quad \text{and} \quad \tilde{u}\frac{\partial}{\partial \bar{x}}\tilde{u} + \tilde{v}\frac{\partial}{\partial \tilde{y}}\tilde{u} = \bar{u}_e(\bar{x})\frac{\mathrm{d}}{\mathrm{d}\bar{x}}\bar{u}_e(\bar{x}) + \frac{\partial^2}{\partial \tilde{y}^2}\tilde{u} \tag{1}$$

Boundary conditions are no slip condition and asymptotic matching:

$$\tilde{u}(\bar{x}, \tilde{y} = 0) = 0, v(\bar{x}, \tilde{y} = 0) = 0 \quad \text{and} \quad \lim_{\tilde{y}\to\infty} \tilde{u}(\bar{x}, \tilde{y}) = \bar{u}_e(\bar{x}) \tag{2}$$

The entrance velocity $\tilde{u}(\bar{x} = 1, \tilde{y})$ and $\tilde{v}(\bar{x} = 1, \tilde{y})$ is the Blasius one. From (1) we deduce

$$\lim_{\tilde{y}\to\infty}\left(\tilde{v}(\bar{x}, \tilde{y}) + \tilde{y}\frac{\mathrm{d}}{\mathrm{d}\bar{x}}\bar{u}_e(\bar{x})\right) = \frac{\mathrm{d}}{\mathrm{d}\bar{x}}\big(\delta_1(\bar{x})\bar{u}_e\big), \quad \text{with } \delta_1(\bar{x}) = \int_{\tilde{y}=0}^{\tilde{y}=\infty}\left(1 - \frac{\tilde{u}(\bar{x}, \tilde{y})}{\bar{u}_e(\bar{x})}\right)\mathrm{d}\tilde{y},$$

the displacement thickness. Coming back to variables $(\bar{x}, \hat{y})$, the term $\varepsilon\hat{y}\frac{\mathrm{d}}{\mathrm{d}\bar{x}}\bar{u}_e(\bar{x})$ matches to the Taylor developpement of the perfect fluid velocity at the wall, so we identify $\varepsilon(\frac{\mathrm{d}}{\mathrm{d}\bar{x}}(\delta_1\bar{u}_e) + \bar{u}_e\frac{\partial \hat{f}}{\partial \bar{x}})$ to be at leading order $\varepsilon\bar{u}_e\frac{\mathrm{d}}{\mathrm{d}\bar{x}}(\delta_1 + \hat{f})$. This later equation may be interpreted as a blowing velocity which perturbes the perfect fluid layer at order ε. The Euler solution is then $\bar{u}(\bar{x}, \bar{y}) = \bar{u}_e(\bar{x})$, $\bar{h}(\bar{x}) = 1 - \varepsilon(1 - F_r^2)^{-1}(\delta_1 + \hat{f})$ and $\bar{p}(\bar{x}, \bar{y}) = (\bar{h}(\bar{x}) - \bar{y})F_r^{-1}$ with

$$\bar{u}_e(\bar{x}) = 1 + \varepsilon\frac{\delta_1 + \hat{f}}{1 - F_r^2} \tag{3}$$

In the fluvial régime that we study ($F_r < 1$), a decrease of the water level is produced at the bump [2]. At this point, all we have done is only a second order boundary layer effect on the perfect fluid. But we use here the framework of the 'interacting boundary layer theory', so we allow a 'mix' of the order of magnitudes 1 and ε from (3) in the matching condition (2). The perfect fluid slips now on the real wall $((\varepsilon)\hat{f}(\bar{x}, \check{t}))$ thickened by the displacement thickness $((\varepsilon)\delta_1(\bar{x}))$. The two layers are now strongly coupled [3]. The ultimate justification is the 'triple deck theory' [4].

3. Transport equation

If we assume that the Schmidt number is nearly one in order to have a mass transport boundary layer of same scale than the momentum boundary layer and that the settling velocity is of the same scale than the transverse boundary layer velocity (written – $\tilde{V}_f < 0$) then the mass conservation of the particles is:

$$\tilde{u}\frac{\partial}{\partial \bar{x}}\tilde{c} + (\tilde{v} - \tilde{V}_f)\frac{\partial}{\partial \tilde{y}}\tilde{c} = S_c^{-1}\frac{\partial^2}{\partial \tilde{y}^2}\tilde{c} \tag{4}$$

Upstream, the concentration is supposed to be zero (there is no previous incoming flow of sediments). The boundary condition for the suspended concentration are then:

$$\tilde{c}(\bar{x} < 1, \tilde{y}) = 0, \quad \tilde{c}(\bar{x}, \tilde{y} \to \infty) = 0, \quad -\frac{\partial \tilde{c}}{\partial \tilde{y}}\Big|_0 = \beta\left(H\left(\frac{\partial \tilde{u}}{\partial \tilde{y}}\Big|_0 - \tau_s\right)\right)\left(\frac{\partial \tilde{u}}{\partial \tilde{y}}\Big|_0 - \tau_s\right)^{\gamma} \tag{5}$$

where $H(x)$ is the Heaviside function, β is of order one and $\gamma = 3/2$ (common value). The latter of (5) is common in the literature of erosion of cohesive sediments (Van Rijn formula, *cf.* Nielsen (1992) [5]), but other formulas may be found. It means that there exists a threshold value of the skin friction if $\frac{\partial \tilde{u}}{\partial \tilde{y}}|_0$ is bigger than this threshold value τ_s, then the flow erodes the bump, else there is no erosion ($\frac{\partial \tilde{c}}{\partial \tilde{y}}|_0 = 0$). Finaly, the net flux of particules at the wall obtained from (4) has two contributions: erosion $S_c^{-1}\frac{\partial \tilde{c}}{\partial \tilde{y}}|_0$ and sedimentation $\tilde{V}_f\tilde{c}|_0$, this total flux deforms the river bed according to [5–7]:

$$\frac{\partial \hat{f}}{\partial \check{t}} = S_c^{-1}\frac{\partial \tilde{c}}{\partial \tilde{y}}\Big|_0 + \tilde{V}_f\tilde{c}|_0 \tag{6}$$

It is of course at this point that the time scale T associated with the preceeding equation is choosen: the deformation is done at a very long scale compared to the hydrodynamic scale (so the flow is quasisteady).

4. Resolution, results and conclusion

We have to solve at each time step $\check{t}$:
first: a stationnary I.B.L. problem ((1) and (2)) at given bump shape with the coupling relation (3);

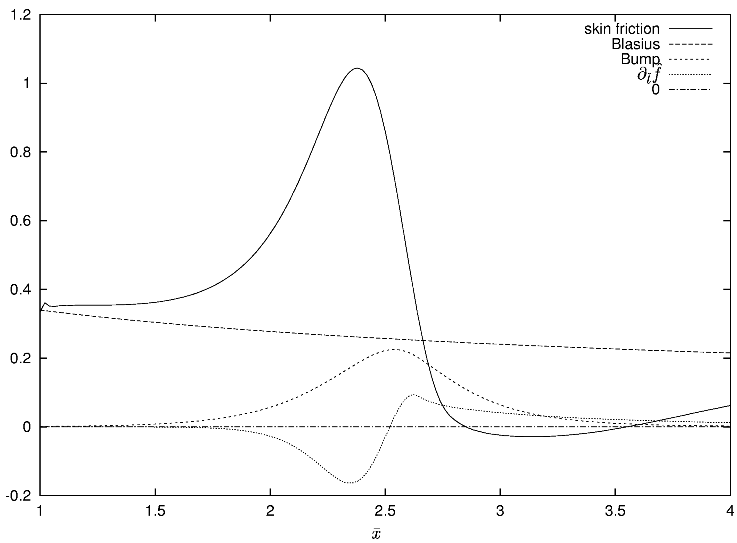

Figure 2. At initial time, the initial bump $\hat{f}(\bar{x}, \check{t} = 0)$ and the associated computed skin friction at the wall $\partial_{\tilde{y}}\tilde{u}$ and total flux of sediments: $\partial_{\check{t}}\hat{f}$.

Figure 2. *Au temps initial, tracé de la distribution de frottement, de la forme initiale de la bosse $\hat{f}(\bar{x}, \check{t} = 0)$ et de la variation de la forme de la bosse $\partial_{\check{t}}\hat{f}$.*

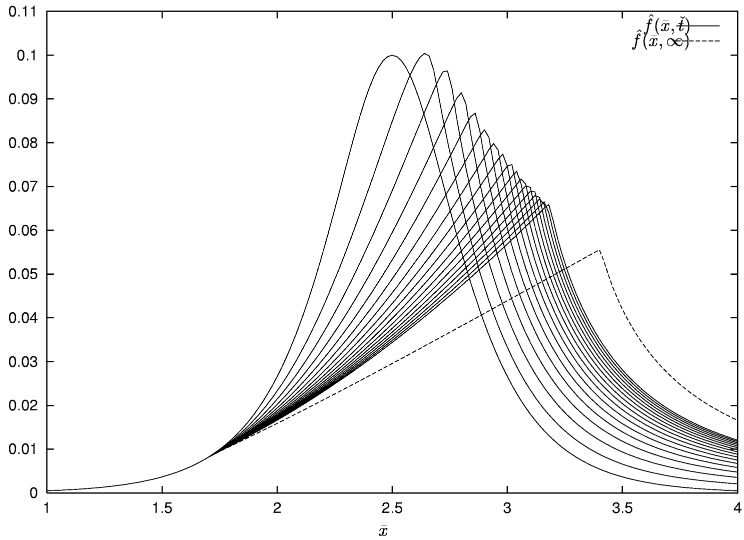

Figure 3. The dune shape ($\hat{f}(\bar{x}, \check{t})$) as a function of time $\check{t} = 0, 1, 2, 3, \ldots, 16, \infty$.

Figure 3. *Évolution de la forme de la bosse en fonction du temps* $\check{t} = 0, 1, 2, 3, \ldots, 16, \infty$.

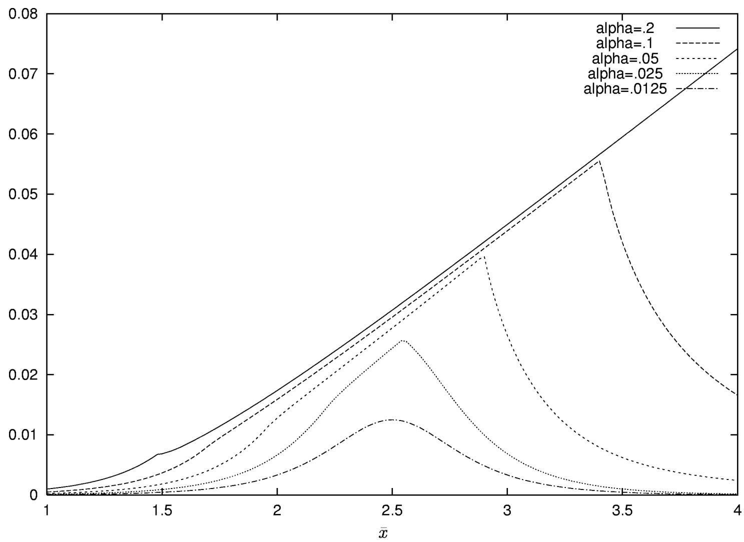

Figure 4. Final dune shapes for different starting values of α.

Figure 4. *Formes finales de dunes pour différentes valeurs de* α.

second: the mass transport equation ((4) and (5)). Thereafter, the shape of the bump is modified according to (6) for the next time step.

This system is solved by a finite differences scheme in 'inverse' way. The perfect fluid is computed in a 'direct' way. The coupling is done by a 'semi inverse' iteration.

At initial time $\check{t} = 0$, we impulsively introduce a bump ($f(\bar{x}, \check{t} = 0) = \alpha / \cosh(4(\bar{x} - 2.5))$). A typical set of order one parameters for the models is: $\alpha = 0.225$, $Fr = 0.6$, $\varepsilon = 500^{-1/2}$, $\beta = 0.8$, $\tau_s = 0.35$, $S_c = 1$ and $\tilde{V}_f = 1$. In *figure 2* we see that the skin friction increases highly before the crest (the Blasius value is remind), and so trespass the threshod value τ_s. The sediments are picked up ($\frac{\partial \tilde{c}}{\partial \tilde{y}}|_0 < 0$), they go in the flow, the dune is eroded. The total flux is then negative before the crest and positive after: the sediments are falling on the lee side (6).

In *figure 3* we draw the shape of the bump at different time step $\check{t}$. The final calculated stationnary bed profile is caracterized by a constant skin friction equal to τ_s ((5) and (6)). The upstream side is nearly linear (convergent channel flow). The lee side has a bigger slope. We note that in this model there is no equilibrium angle (which may be included in the threshold (5)) as a term proportional to $\partial_{\bar{x}} \hat{f}$. In *figure 4* we put the final shape for different α. If $\alpha \geqslant 0.2$, the flow is separated, at first the size of the separation bulb increases because of the stiffening of the bump, but, notice that with this model we are able to compute flow separation. Of course, we can not increase to much α in order to have, as usual, not a too large separation bulb.

The advantage of this model is that a lot of hydrodynamical mecanisms have been put without usual integral simplifications. Of course, the first hypotheses to introduce in the model would be a turbulent stress viscosity and diffusivity and for the river bed it would be interesting to introduce the slope limitation.

Acknowledgements. I would like to thank in particular Pierre Ollier (ENSTA).

References

[1] Zeng J., Lowe D.R., Sedimentology 44 (1997) 67–84.
[2] Baines P.G., Topographic Effects in Stratified Flows, Cambridge University Press, 1995.
[3] Sychev V.V., Ruban A.I., Sychev V.V., Korolev G.L., Asymptotic Theory of Separated Flows, Cambridge University Press, 1998.
[4] Gajjar J., Smith F.T., Mathematika 30 (1983) 77–93.
[5] Nielsen P., Coastal Bottom Boundary Layers and Sediment Transport, World Scientific, 1992.
[6] Fredsoe J., Deigaard R., Mechanical of Coastal Sediment Transport, World Scientific, 1992.
[7] Izumi N., Parker G., J. Fluid Mech. 283 (1995) 341–363.
[8] Akiyama J., Stefan H., J. Hydraulic Engrg. 111 (12) (1985).