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Boundary layer separation and asymptotics from 1904 to 1969

À propos de la séparation de la couche limite et de l'analyse asymptotique de 1904 à 1969

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ABSTRACT

The aim of this Note is to follow the spreading of the idea of asymptotic analysis in fluid mechanics (more precisely the laminar boundary layer). We will focus on the actors who worked on the problem of boundary layer separation from 1904 to 1969. This correspond to the invention of the "boundary layer" concept by Prandtl in 1904 and the invention of the "triple deck" in three different papers in 1969. The rationalization of the methods by the GALCIT group allowed one to solve, with exactly the same tools, problems at small and large Reynolds number (or problems with a small parameter). This story starts in Göttingen and goes to California before coming to London and Moscow.

RÉSUMÉ

Le but de cette note est de suivre le transport et la diffusion de l'idée d'analyse asymptotique dans le domaine de l'aéronautique des « couches limites » laminaires. Nous nous concentrerons sur les acteurs qui ont travaillé sur le problème de la séparation de la couche limite. Nous restreignons la période d'étude à l'intervalle allant de 1904 à 1970, principalement parce que cette période s'étend de l'invention du concept de « couche limite » par Prandtl, en 1904, à celle de la « triple couche » (ou « triple pont » en français) dans trois articles différents, en 1969. Par ailleurs, la rationalisation de la méthode intitulée « développements asymptotiques raccordés » a permis de résoudre avec exactement les mêmes outils mathématiques des problèmes à petit et à grand nombre de Reynolds, et plus généralement des problèmes avec un petit paramètre. Cette histoire commence à Göttingen et se déplace en Californie, avant de se poursuivre à Londres et à Moscou.

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Cette « couche limite » dans la période 1904–1969. La notion a été introduite par Ludwig Prandtl à l'université de Göttingen. La montée du nazisme a fait ensuite fuir de nombreux scientifiques aux États-Unis. Parmi eux, Kurt Friedrichs, formé à l'école de Prandtl, synthétise la notion de couche limite en un problème simple. Les idées qu'il a développées pour résoudre ce problème ont été étendues et systématiquement utilisées par un groupe de chercheurs à Los Angeles au Caltech, le GALCIT. Théodore von Kármán, débauché par le prix Nobel Millikan, était le directeur du GALCIT. L'université allemande en général, et celle de Göttingen dont était issu von Kármán en particulier, était caractérisée par une forte interaction entre théorie et pratique ; ce souffle traversa ainsi l'Atlantique entre les deux guerres. Le groupe était constitué de Paco Lagerstrom, Julian Cole et d'autres. Milton Van Dyke, issu de ce groupe, a eu une action particulière par les liens qu'il a tissés à la fois en France, en Angleterre et en URSS. Il a certainement diffusé les idées de la « méthode des développements asymptotiques raccordés ». Cette méthode permet de mieux comprendre la théorie de la couche limite, mais aussi le paradoxe de Stokes à faible vitesse en écoulement très visqueux. Elle permet de comprendre la séparation de la couche limite laminaire. Elle explique ainsi différents paradoxes liés à la viscosité et/ou la séparation (paradoxe « de d'Alembert », paradoxe « de la condition de Brillouin–Villat », paradoxe « de l'influence de l'aval sur l'amont »). Cette dernière théorie, appelée « triple pont » a été établie par K. Stewartson, V. Neiland et A. Messiter simultanément en 1969–1970, à Londres, Moscou et Los Angeles, montrant ainsi que les idées ont évolué indépendamment. Bien entendu, ces idées ont mis du temps à éclore et de nombreux passeurs d'idées ont établi des liens entre les différentes écoles (T. von Kármán, S. Goldstein, J. Lighthill, M. Van Dyke...).

1. Introduction

It is commonly admitted that d'Alembert's paradox was solved by Ludwig Prandtl. D'Alembert's (1717–1783) paradox tells us that, for an incompressible and inviscid potential flow, there is no drag force on a body (of say characteristic size L) moving into a fluid (of viscosity μ and density ρ) at constant velocity U_∞. This is obviously wrong in practice. In 1904, Prandtl presented the breakthrough idea: one has to simplify the Navier–Stokes equations to estimate the corrective effect of viscosity near the body. But other paradoxes arose during the century, linked with viscosity. To solve them, one needed some mathematical formalization, which was done in the 1940s in North America. Systematic applications came in the fifties and sixties.

This Note shows different levels interacting all together. First, a geographical one: we first present Göttingen University, and how most of the knowledge was deported due to immigration to the United States and how it came back to Europe. Second, a historical one: this story is part of the twentieth century from World War I, nazism rise, World War II, and Cold War. Third, a conceptual one with different paradoxes: "d'Alembert's paradox", "Stokes' paradox", "Whitehead's paradox", "Brillouin–Villat's condition paradox", "upstream influence paradox", and maybe to a certain extent "Kutta–Jukowski's condition" paradox. Fourth, a human one: the influence of strong characters like L. Prandtl, T. von Kármán, M. Van Dyke, K. Stewartson (and many other that we will meet in this Note) who have built this story.

2. Germany: Göttingen

The story begins in the Prussian Empire, in Göttingen. The university was re-founded with a clear aim of "combined research and training" according to the ideas of Wilhelm Humboldt (1767–1835), the reform movement being initiated by mathematician Felix Klein (1849–1925). He wanted to establish Göttingen as the world's leading mathematics research centre. For this purpose, Klein hired David Hilbert (1862–1943). He remarked as well a young scientist, Ludwig Prandtl (1875–1953), and gave him the chair of Applied Mechanics. He came to Klein's attention after working at M.A.N. company in Nuremberg. He had to design exhaust ducts for ships [1,2]. He observed that Bernoulli's law was not always applicable, and that flow separation was responsible for discrepancies. That is why he re-examined the Navier–Stokes equations instead of the Euler ones (leading to Bernoulli law). In modern notations (those developed after 1904), the key feature is the simplification of the equations of momentum with respect to a vanishingly small parameter: the inverse Re^{-1} of the Reynolds number (defined as usual as $Re = \rho U_\infty L/\mu$). The Navier–Stokes equations simplify for very small inverse of the Reynolds number in two different problems. First, the inviscid one and second, the viscous one, they have different scales in space (boundary layer relative thickness is $Re^{-1/2}$). He solved the problem of the flat plate, generic to any wing, with the help of Paul Blasius (1883–1970). Blasius was his first PhD student in Göttingen [3]. At the time, the simplification was not presented in a clear rational mathematical framework, as it was the first step toward its creation. The treaty of Versailles had forbidden Germans from having any air force. Hence, Prandtl and his collaborators benefited from German important research efforts on wings and gliders (sailplanes). Schlichting's book being the compilation of all this knowledge, as it came from lectures to German engineers in 1941–1942 (preface to the first edition [4]).

Interestingly enough, at the same time, in 1905, the Swede Vagn Ekman (1874–1954) published his theory of the Ekman spiral. This theory was a response to the question of Fridtjof Nansen during his polar expedition (on the ship named *Fram* from 1893 to 1896). He had observed that icebergs did not drift in the direction of the wind, but always deviated with an angle to the right. The explanation of course did not involve the concept of boundary layer, but the equations are very similar, and modern courses treat "Ekman layer" as a "boundary layer" (as we will see after with K. Stewartson, who worked on that problem just before introducing the "Triple Deck"). Ekman in his paper [5] is indebted to Boussinesq (1842–1929), who introduced the notion of "depth of wind current" proportional to $\sqrt{\mu}$ (same dependence in viscosity as Prandtl's boundary layer, which is $L\,Re^{-1/2}$). These developments are neither in Van Dyke's paper nor in Tani's one the roots of the boundary-layer idea [6,7].

With this new tool, several aeronautical problems have been solved at Göttingen University by Prandtl and its students and coworkers. But during that time, was the rise of nazism. A lot of scientists left Göttingen and Germany. Considered as Germans, they were not welcome in France, some found positions in Great Britain, but many of them emigrated to the United States. Some of the great scientists formed at this university are John von Neumann (1903–1954, emigrated to the US in 1930), Max Born (emigrated to England in 1933), Theodor von Kármán (1881–1963 emigrated to the US in 1929), Richard Courant (1888–1972, arrived in the US in 1934 after a stay in Cambridge)... There is a famous word [8] of Hilbert to the nazi Minister of Education, Bernhard Rust, who was very active in purging universities of jews and nazis' opponents. Rust asked whether "the Mathematical Institute really suffered so much because of the departure of the Jews." Hilbert replied, "Suffered? No, it just doesn't exist anymore!"

3. United States: New York

Among those who crossed the sea was Kurt Friedrichs (1901–1982). He escaped because his fiancee was jewish; he went to New York in 1935 and married there. For numericians, he is famous for the CFL condition he settled in 1928 with two other colleagues from Göttingen: Richard Courant and Hans Lewy (1904–1988 arrived in California in 1935). Friedrichs, after his studies in Göttingen, was assistant of von Kármán in Aachen in 1927. Friedrichs extracted from Prandtl's boundary layer problem the following differential equation (where a is a given constant and where ε is a parameter):

$$\varepsilon f''(y) + f'(y) = a, \text{ with } f(0) = 0 \text{ and } f(1) = 1 \tag{1}$$

The problem is to find the solution to this equation when ε is vanishingly small. Solving with $\varepsilon = 0$ gives a kind of Euler problem: the condition in $y = 0$ is not used to solve the equation, only the condition in $y = 1$ is relevant. Hence, the "external" or "outer" or "distal" solution is $f_{\text{ext}}(y) = ay + (1 - a)$. This strange behaviour, where a small parameter has a huge importance and where a boundary condition is lost, is exactly the same than for the Navier–Stokes equations. If $1/Re = 0$, one has the Euler equations, with an arbitrary slip boundary condition (only the normal component of velocity is zero). The Navier–Stokes equations, valid even if $1/Re$ is smaller and smaller (but not zero), have a no-slip boundary condition (every component of velocity is zero). To solve the problem and remove the singularity at the boundary, one has to rescale the variable y near the origin in order to focus on the behaviour near this boundary. A good choice is to define a small scale δ, so that we stretch $\tilde{y} = y/\delta$. By "dominant balance" (simplify the equation, but not too much), we obtain $\delta = \varepsilon$ (for boundary layer theory: $\delta = LRe^{-1/2}$). At this small scale, the equation reads $\tilde{f}''(\tilde{y}) + \tilde{f}'(\tilde{y}) = 0$ when ε is vanishingly small. The "internal" or "inner" or "proximal" solution is $\tilde{f}(\tilde{y}) = A(1 - e^{-\tilde{y}})$. The solution goes from 0 to a finite value, say A, in this very thin zone. This finite value is supposed to be the value obtained in 0 for f_{ext}, which is $1 - a$. This is the "matching". This double simplification is the signature of a "singular perturbation problem". This simplification gave birth to a technique of approximation of problems with a small parameter called "Matched Asymptotic Expansion". Most of the ideas, including the matching, were introduced in the 1940s by Kurt Friedrichs in a course at New York University and never published as it. Friedrichs and Wolfgang Wasow (1909–1993) introduced the term "singular perturbations" in a 1946 paper. Wasow was born in Switzerland, he studied at the Sorbonne and in Göttingen. After a stay in Cambridge, he went in 1939 to the USA.

4. United States: Los Angeles

The technique was developed in the 1950s, at Caltech Institute of Technology's Guggenheim Aeronautical Laboratory (GALCIT, Los Angeles). Several rich Americans, such has J. Guggenheim and J. Rockefeller, gave money at this time to promote science and let scientists visit each other all over the world (such as von Neumann, Van Dyke, Goldstein...). Theodore von Kármán (1881–1963), who was a former Prandtl PhD student in Göttingen and who was working in Aachen, became director of this structure in 1930. He was recruited by Robert Millikan (the Nobel prize for the elementary charge of electricity). Millikan thought first to bring Prandtl, but he was a bit too old. Millikan's son, Clark Millikan was the next director in 1949. T. von Kármán came to America with a "tradition of deep commitment to the scientific formulation and mathematical solution of real engineering problems" [9]. This is for sure Humboldt's legacy that he developed at GALCIT. A group more devoted on perturbation theory was created in the late 1940s by Paco Lagerstorm (1914–1989) and Julian Cole (1925–1999). The work and the ideas of Saul Kaplun (1924–1964) seem to have been decisive. With many others L. Tilling, G. Latta, J. Kevorkian, A. Messiter, M. Van Dyke…, they became equally involved in asymptotic expansion procedures for more general singular perturbation problems [10–12]. "Nevertheless, Cole generously credits colleagues for various crucial ideas. In fact, in many cases, the seminal contributions were uniquely his" [9].

For small Reynolds numbers, this theory solved the "Stokes paradox" for a flow around a cylinder (and "Whitehead paradox" for flow past a sphere) in 1957 (see [13]). The work was done independently by I. Proudman & J.R.A. Pearson 1957 in Cambridge and P. Kaplun & P. Lagerstrom 1957. In his paper, the English team is clearly grateful to an earlier paper by

P. Kaplun & P. Lagerstrom 1955, and in a foot note acknowledges a conference paper of Kaplun (and that he even "gives an improved approximation for the drag").

For large Reynolds numbers, this theory allowed Milton Van Dyke (1922–2010) to settle the second-order boundary layer theory in 1962 [14]. Note that, in this paper, he refers to the "so-called method of inner and outer expansions" (the same year sees the first occurrence of "matched asymptotic" in Bretherton (born 1935) [15]). Van Dyke's book *Perturbation Methods in Fluid Mechanics*, first published in 1964, is a large view of applications in aeronautics. Most of such works were personal and remained unpublished as said by [16].

5. Europe: Italy, France, Germany

Following d'Alembert's tradition of inviscid fluids with a mathematical point of view, scientists were looking at flow separation with the ideal fluid point of view. Tullio Levi Civita (1873–1941) [17] looked at the wake after a cylinder. Using same ideas, Marcel Brillouin (1854–1948) in 1911 and Henri Villat (1879–1972,) in 1914 [18] gave the condition of smooth departure of the free stream line on a cylinder (see 1953 paper of Isao Imai (1914–2004), from Japan). Unfortunately, this gave a new paradox: the "Brillouin–Villat condition paradox". Considering that the free stream line departing from the cylinder at the separation point with a non-zero angle induces a strong adverse pressure gradient from the ideal fluid, this adverse gradient would have created a separation in the boundary layer before the considered point. If the free stream line is tangent to the cylinder, then the pressure gradient is smooth, but it is always favourable. Hence there is no separation. See [19] and [20] for discussions of this free stream line problem. Note that Villat understood Kutta–Jukowski's condition before them, but he was not published by the reviewer Boussinesq [21,22].

Another paradox linked to ideal fluid boundary layer decomposition was obtained during the war in Germany. But, contrary to this previous one, it was based on experiments. Klaus Oswatitsch (1910–1993) and Karl Wieghardt (1913–1996), both former assistants to Prandtl in Göttingen, observed in 1941 a striking phenomenon that was reproduced in the GALCIT wind tunnel by Hans Liepmann (1914–2009), another German refugee in 1939 in the United States. It was called "upstream influence" paradox. This new paradox appears in supersonic flows: a disturbance which in inviscid theory would affect only downstream conditions is able to exercise an upstream influence through the boundary layer (which itself in boundary layer theory would affect only downstream conditions) [23]. This paradox was first partially explained by J. Lighthill in 1953.

6. Europe: Great Britain

We came back to Europe in the previous section, but Göttingen and GALCIT were still in the loop. Let us stay now in Europe: in Great Britain. As suggested before, this country was maybe a first step to escape from Germany in the 1930s. It is then likely than Göttingen ideas irrigated this country. But, for sure, Milton Van Dyke was awarded of both a Guggenheim Fellowship and a Fulbright grant to spend the academic year 1954–1955 with George Batchelor at Cambridge University. He participated in the creation of the *Journal of Fluid Mechanics* [24], and a paper by him was even the very first article published in the volume 1, issue 1 in May 1956.

An important actor is Keith Stewartson (1925–1983). He was introduced to those problems by Selig Brodetsky (1888–1954) in a lecture given at Cambridge. Brodetsky was born in Ukraine, he escaped from pogrom at the age of six. In 1908, in Cambridge, he received the highest honours. The conservative press was forced to recognise that a son of immigrants surpassed all the local students. He worked in 1923 on the ideal fluid flow around a cylinder (in the same spirit as Levi-Civita, Brillouin, and Villat; see [19] for a extensive discussion of the models for the flow around a cylinder). Stewartson was a student of Leslie Howarth (1911–2001). Howarth got in 1936 his PhD with Sydney Goldstein (1903–1989) in Manchester. In 1937–1938, he met von Kármán and worked with him at GALCIT. He came back in 1949 to Bristol and worked on the stagnation point with simplified boundary layers methods (which allowed one to reach approximate solutions to the von Kármán integral and were very useful at this time). In the 1930s, he obtained a numerical prediction of the position of boundary layer separation and singularity at separation. He introduced the "Howarth transformation" for compressible boundary layers. He wrote in 1959 the chapter "Laminar Boundary Layers" in *Handbuch der Physik*. Sydney Goldstein, his PhD advisor had a huge influence at this time as a teacher and as a purveyor of ideas (Stewartson [25] considering in 1970 that Goldstein [26] in 1930 "introduces the notion of matched asymptotic expansions, logarithms and all, to my knowledge for the first time, and certainly ante-dating by many years the upsurge of interest in the method which occurred during the 1950s.") In 1928, Goldstein was awarded Rockefeller Research Fellowship and spent a year working at the University of Göttingen. In 1929 he obtained a position at the University of Manchester (that of Osborne Reynolds and of Horace Lamb (1849–1934) [27]); he then left for Cambridge in 1931. During the war, he worked on boundary layer theory at the "National Physical Laboratory" near London. The important date is 1948, when he published a work known now as "Goldstein singularity in boundary layer" [28]. As an active zionist, he went to Technion, but four years later, in 1954, he preferred Harvard as there was too much administration in Israel.

Those are the people who influenced Stewartson. He married in 1953 and spent just after one year in Caltech. A complete view of Stewartson work can be found in [29]. The already-mentioned Ekman layer was among Keith Stewartson achievements (in 1957 and 1966). He presented multilayered flows with asymptotic expansion. He was interested in the impulsive motion of a flat plate. This problem of the transition from the Rayleigh solution (far from the leading edge) to the Blasius solution (near the leading edge, but not too much) received a long-lasting interest, from 1951 to 1973. He wrote a

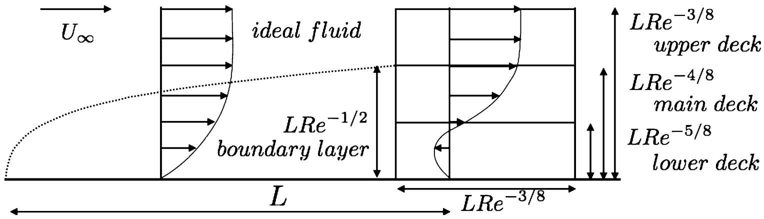

Fig. 1. Sketch of the flow of velocity U_∞ over a plate. If separation occurs at a position of characteristic length L, the boundary layer is of thickness $L\,Re^{-1/2}$ at this length L. The triple-deck structure is of short length $LRe^{-3/8}$. Recirculation of the flow occurs near the wall in the lower deck of thickness $LRe^{-5/8}$.

book on compressible boundary layer in 1964: *The theory of laminar boundary layers in compressible fluids*. The author of this Note bought it by chance for 3£ at Foyles in London in 1989. In the last chapter, "Interaction", there is a summary of the pre-"Triple Deck" ideas, with photos of experiments of the "upstream influence" paradox mentioned before.

Keith Stewartson allowed the ideas of Prandtl to be rejuvenated in Europe. These ideas were spread in Europe at that time through the various actors we met. In its 1928 paper, Prandtl exposed the problem of boundary layer separation. The "upstream influence" paradox was linked with supersonic boundary layer separation. As said before, the main ideas came from James Lighthill (1924–1998), who had a huge influence in fluid mechanics. He found in 1953 the orders or magnitudes and the key mechanisms for supersonic separation. But the full asymptotic problem was solved in 1969 thanks to "Matched Asymptotic Expansion" by K. Stewartson [30], introducing the "Triple Deck Theory" (see Fig. 1). This is a kind of boundary layer, called "lower deck", of shorter length ($L\,Re^{-3/8}$) and thickness ($L\,Re^{-5/8}$) in the boundary layer itself (of thickness $L\,Re^{-1/2}$). That is where separation occurs. Over the lower deck is the "main deck", of thickness $L\,Re^{-4/8} = L\,Re^{-1/2}$. An ideal fluid layer is at the top. It is of length and thickness $L\,Re^{-3/8}$, it is called "upper deck". The path to Triple Deck was not so simple, and ideas were in fact developed from all over the century all over the world: first thanks to Prandtl ideas, second thanks to the diffuse knowledge all around the word, and third thanks to the rationalization efforts of the Caltech GALCIT team. The "Triple Deck" theory solved the "upstream influence" paradox: the mathematical interpretation is that the linearized system admits an eigenvalue. The name "Triple Deck" did not come from English naval tradition directly, but from the decks of the sandwiches (F.T. Smith, private communication).

Edward Fraenkel (born 1927) was at the time another English mathematician involved in Matched Asymptotic Expansions. In his paper [31], he gave a short history of this technique. He cites as well Arthur Erdélyi (1908–1977). He was born in Budapest, like von Kármán. He emigrated in 1937 to Scotland, then became professor at Caltech in 1950. Fraenkel gave the course on Perturbation Methods in Cambridge. Note that the real first name of Fraenkel is Ludwig. He came at the age of seven to Great Britain, emigrating with his father Eduard Fraenkel, who had jewish parents and who was a philologist (Eduard studied in Göttingen).

7. Europe: Soviet Union

We turn now to Soviet Union. Some Russian names have been evocated: Brodetsky and Jukowski. But again, Milton Van Dyke played a big role in this story. He shared with Russian information about the blunt body problem [16] (hypersonic flow). In 1965, he spent three months in Dorodnitsyn's group. Van Dyke had learned Russian, so he was even able to give lectures. Anatoly Dorodnitsyn (1910–1994) defended in 1942 his PhD on compressible flows (so that Stewartson used the Howarth–Dorodnitsyn transformation for the compressible Blasius flow). If, unfortunately, Dorodnitsyn's ideas on computers were a dead end (M. Gorokhovski, private communication), according to Julian Cole ("Perturbation Methods in Applied Mechanics" 1968), he is, with K. Friedrichs, one of the early initiators of "matching". His PhD student was Vladimir Sychev (1924–2016), who had been impressed by Leonid Ivanovitch Sedov (1907–1999). During World War II, Sedov found the Similarity Solution for a blast wave, and until recently, it is believed that Sedov was the principal engineer behind the Soviet Sputnik project. Sychev said that "Sedov's conferences are those which inspire optimism and confidence in victory upon nazis" according to the translation of Sychev Russian-language Wikipedia page. Lev Landau explained some of Sychev ideas in Volume 6 of *A Course of Theoretical Physics*. Triple Deck scaling linking pressure and length are in an exercise (§40 on page 156), but he forgot to quote V. Sychev, who was upset (A. Ruban, private communication). Nevertheless, the "Brillouin–Villat" paradox is solved by Sychev in 1972 [20], thanks to the "Triple Deck" again: the effective separation point lies near (in Triple Deck scales $L\,Re^{-3/8}$) the "Brillouin–Villat" condition point. Vladimir Neiland is another Dorodnitsyn's student in Tsagi. He developed a theory close to "Triple Deck" and published it in the same year 1969 in [32]. Van Dyke Russian translation's book (MIR Editions) is the first reference to the paper. Goldstein, Kaplun, and Stewartson's names are cited without reference in the paper. Neiland (sometimes spelt Neyland) even gave seminars in Europe in the middle of the 1960s at ONERA (V. Neiland, private communication). The translation of Neiland's Russian-language Wikipedia page tells that "According to James Lighthill, with Nikolai Zhukovsky, Andrei Kolmogorov and Lev Landau one of the four scientists of the USSR, who determined the "face" of modern aerodynamics."

We said that there were three different ways to present Triple Deck: Stewartson, Neiland, and now we have to come back again to GALCIT. Arthur Messiter (no dates, retired in 1998) was another PhD student from Julian Cole. He found the scales as well and formulated nearly the same equations published in 1970 (in [33]). Contrary to the supersonic regime examined by Stewartson and Neiland, he looked at the incompressible flow at the trailing edge of a flat plate.

The three points of view, of Neiland (1969), of Messiter (1970), and of Stewarton & Williams (1969) – and even that Sychev (1972) – are very different. This reflects how ideas emerged from Prandtl's ones finally focussed on the "upstream influence paradox" and the boundary layer separation.

8. Europe: France

We will finish with France, because Milton Van Dyke, who definitely played a huge role in this story, came in 1958 to Paris. He had learned French as well, and gave courses on hypersonic flows at the Sorbonne. Walking along the Seine River and the stalls of the "bouquinistes", he found the idea of the book *An album of fluid motion* [16]. Von Kármán made several stays in France, he gave conferences at the Sorbonne and was a regular visitor of ONERA [34]. Paco Lagerstorm (1914–1989), born in Sweden, studied theology and roman languages. He spoke French. In 1946, he was recruited at GALCIT by Hans Liepmann (1914–2009, PhD advisor of Julian Cole, he was in Aachen with Von Kármán, and came thanks to him to the USA). Lagerstorm was a friend of Paul Germain (1920–2009) who went to GALCIT in 1953. Lagerstrom was invited in 1960–1961 to Paris by Germain. B. Mandelbrot, in his biography [35], said that Lagerstrom was "strange and mysterious", and that von Kármán thought he proposed "unphysical" subjects. Lagerstrom is nevertheless one of the main actors from Caltech. He came at the University of Paris (at IHP) in 1960–1961 as a visiting professor in the frame of a Guggenheim Fellowship. He gave a course on asymptotics (rather simple, T. Levy, private communication). Paul Germain [36] was a great enthusiast of the asymptotic methods he discovered thanks to Lagerstorm. In fact, Germain (upset by H. Cartan, who wondered about the utility of his last work in 1945 [34]) was sent by Joseph Pérès (1890–1962) to S. Goldstein's National Physical Laboratory in 1945. That is how he learned fluid mechanics. He met there V. M. Falkner (who solved with Sylvia Skan in 1930 the Prandtl problem with a power law pressure gradient with self-similar solutions). Germain's "best contribution to fluid mechanics" according to himself [37], is Jean-Pierre Guiraud (born 1928). He worked with him on asymptotics mainly at ONERA. Jean-Pierre Guiraud spent six months in 1958 in GALCIT (without going one time at the beach, M. Van Dyke 1990, private communication) and transmitted those ideas to France as well [38]. Germain and Guiraud applied Van Dyke Matched Asymptotic Expansion method at any order on the shock wave problem (1960, 1961). Maybe Paul Germain tried to reproduce at ONERA and in the "Laboratoire de mécanique théorique" the conditions of GALCIT. As J.-P. Guiraud, Radyadour Zeytounian (who wrote many books on asymptotics) was associated with ONERA, where Jean-Claude Le Balleur, Jean Cousteix were hired as well. The two last worked on boundary layer theory with success. Jacques Mauss was a PhD student of W. Ekhaus (who was appointed by the Department of Mechanics of the Sorbonne thanks to Germain), who was himself a student of L. Tilling from GALCIT. It is to be noted that, in his book on perturbation methods, Van Dyke thanks the Frenchmen P.-A. Bois, C. François, P. Germain, and H. Viviand.

9. Conclusion

The "boundary layer" introduced in 1904 is a rational description of the laminar Navier–Stokes equations at large Reynolds numbers. It solved the "d'Alembert" paradox. The "Triple Deck" theory introduced in 1969 is a rational description of the laminar Navier–Stokes equations at large Reynolds number, which allows one to explain flow separation on smooth surfaces in some asymptotic cases. It solved the "upstream influence" paradox and the "Brillouin–Villat condition" paradox. This Triple Deck theory explains the birth of large separation on a cylinder (F.T. Smith 1980 computation of Sychev's problem) as well as leading edge marginal separation (A. Ruban 1982) as a variant. This structure is part of the Tollmien–Schlichting wave (F.T. Smith, 1979). This theory may be an explanation for understanding the Kutta–Jukowski condition: the latter is just the expression of the small $Re^{-3/8}$ Triple Deck structure at the trailing edge of an airfoil (see F.T. Smith's review "On the high Reynolds number theory of laminar flows", IMA, 1982).

The "Matched Asymptotic Expansions" introduced in the years 1940–1950 as mathematical generalizations of boundary layer theory were important to settle the boundary layer theory itself and Triple Deck theory. "Matched Asymptotic Expansions" solved as well the "Stokes–Whitehead" paradox at small Reynolds numbers. Many other problems dealing with a small parameter may be solved with this method; they are presented in the books of P. Lagerstrom, J. Cole and M. Van Dyke (*Fluid Mechanics and singular Perturbation*, 1967, resp. *Perturbation Methods in Applied Mechanics*, 1968, resp. *Perturbation Methods in Fluid Mechanics*, 1964). Most of those problems are "singular", and as J. Hinch [39] said, "interesting problems are often singular."

This story is anchored in the history of the twentieth century through the different Worlds Wars up to Cold War, summarizing all the efforts done to improve military aviation by democracies (for example in France [40]), but unfortunately by totalitarianism in Germany and the Soviet Union. This is a human story in which people emigrated and in which key people like von Kármán, Van Dyke, Lighthill and many anonymous other ones spread their ideas across the borders.

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