Effect of viscoelasticity of arterial wall on pulse wave: a comparative study on ovine

X.-F. Wang*†, J.-M. Fullana†, P.-Y. Lagrée ‡ † and R. L. Armentano+

† UPMC Univ Paris 06, UMR 7190, Institut Jean Le Rond d'Alembert, F-75005 Paris, France.
‡ CNRS, UMR 7190, Institut Jean Le Rond d'Alembert, F-75005 Paris, France.

+Faculty of Engineering and Natural and Exact Sciences, Favaloro University, Belgrano 1723, C1093AAF Buenos Aires, Argentina

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1.Introduction

Pulse waves of pressure and flow rate in the arterial system can be well captured by 1D models of blood flow. In the 1D models, the mechanics of the arterial wall is taken into account to close the governing equations. Although the viscoelastic behaviour of the wall has been recognized as a fundamental factor for a long time, most 1D simulations in literature adopted elastic models for simplicity. A recent i*n vitro* study [Alastruey et al. 2011] showed that the viscosity has considerable influence on the pulse waves, especially at the peripheral part of the arterial network. However, the vessels in the study were made of polymers which are actually much less viscous than the real arterial wall. There are also some other studies on *in vivo* conditions, such as ref. [Reymond et al. 2009]. But the estimation of the wall viscoelasticity was done by interpolation on limited available data. In this paper, the coefficients of the viscoelastic model were estimated from *in vivo* measurements. The pulse wave in an arterial tree was simulated by a nonlinear 1D blood flow model. The effect of the viscoelasticity of the arterial wall on the pulse wave was investigated.

2. Methods

The experimental data were obtained from a group of eleven ovines. The experimental protocol conformed to the *European Convention for the protection of Vertebrate Animals used for Experimental and Scientific Purposes*. The arterial tree of the ovine was perfused by an artificial heart under general anesthesia. Synchronized recording of pressures and diameters was applied on seven anatomical locations. For more details on the animal experiments, see ref. [Armentano et al. 1995].

The viscoelasticity of a material can be modeled by a Kelvin-Voigt model, where the tensile stress σ is related with the strain stress e by

$$\sigma = Ee + \phi \frac{de}{dt}.$$

In this equation, E is the Young's modulus, and φ is the viscosity coefficient of the material. With the hypothesis of a thin-walled vessel, we derived the constitutive relation between the transmural pressure P (the difference between internal and external pressure) and the radius r of the circular cross-section,

$$P = \frac{Eh}{(1-\eta^2)r_0} - \frac{Eh}{(1-\eta^2)}\frac{1}{r} + \frac{\phi h}{(1-\eta^2)r_0}\frac{dr}{rdt},$$

where r_0 is the undeformed radius, η the Poisson's ratio, and h the thickness of the arterial wall. In this equation, P is in linear relation with two quantities, $1/r$ and $dr/(rdt)$. The coefficients were estimated by linear regression from the time series on the pressures and diameters.

Incorporating this viscoelastic model, the governing equations of the 1D flow can be expressed in terms of the cross-section area A and the flow rate Q ,

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0,$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A} + \frac{\beta}{3\rho}A^{\frac{3}{2}}\right) = -C_f \frac{Q}{A} + C_v \frac{\partial^2 Q}{\partial x^2} + \frac{A}{\rho}\left(\partial_x(\beta\sqrt{A_0}) - \frac{2}{3}\sqrt{A}\,\partial_x \beta\right),$$

where ρ is the density of the fluid and A_0 is the undeformed cross-section area. The two coefficients for elasticity and viscosity are

$$\beta = \frac{\sqrt{\pi}Eh}{(1-\eta^2)A_0} \text{ and } C_v = \frac{\sqrt{\pi}\phi h}{2\rho(1-\eta^2)\sqrt{A_0}}.$$

We thus obtained a hyperbolic-parabolic system and we solved it by an operator splitting method which separates the hyperbolic and the parabolic parts. The hyperbolic part was solved by a Taylor-Galerkin scheme and the parabolic part by a Crank-Nicolson scheme. The implementation of the schemes has been verified by our previous studies [Wang et al.2012, Saito et al. 2011].

3. Results and Discussion

The experimental data were recorded at the seven anatomical locations as shown in Figure 2 (top):

***Corresponding author. Email: xfwang0219@gmail.com**

Ascending Aorta (AA), Proximal Descending aorta (PD), Medial Descending aorta (MD), Distal Descending aorta (DD), Brachiocephalic Trunk (BT), Carotid Artery (CA) and Femoral Artery (FA). In Figure 1, one case of comparison between data and the fitted model is shown (at MD). The model captures the viscoelastic behaviour very well.

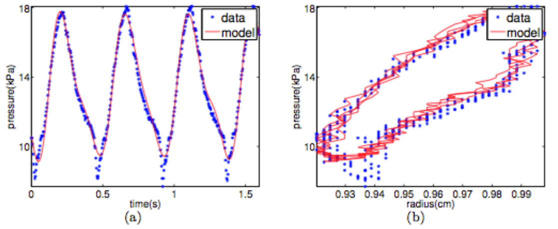

Figure 1. One case of comparison between data (star) and the fitted model (red line) at the location MD: the time series of pressure (a) and the hysteresis loop of radius and pressure (b).

From the coefficients of the constitutive relation, we further evaluated r_0, Eh and φh. The mean values and standard deviations among the group of ovines are shown in Table 1. We note two main patterns about the value of the coefficients. First, descending from the central to the peripheral part of the tree, both stiffness and viscosity values increase. Second, the stiffness and viscosity values at carotid artery are significantly larger than the values at other locations.

Artery	r_0 (cm)	Eh (10^5 Pa cm)	φh (10^4 Pa cm s)
AA	0.9526±0.0659	0.5712±0.0831	0.1002±0.0319
PD	0.8841±0.0180	0.7608±0.0726	0.1283±0.0284
MD	0.8594±0.0190	0.7872±0.0898	0.1652±0.0675
DD	0.8294±0.0138	1.5806±0.1788	0.3015±0.0672
BT	0.8994±0.0996	0.7107±0.1914	0.1382±0.0636
CA	0.4070±0.0263	3.2928±0.4250	0.6367±0.1290
FA	0.2826±0.0160	0.7000±0.1157	0.1259± 0.0392

Table 1. The mean values and standard deviations of the undeformed radii, elasticity and viscosity coefficients of the group of ovines.

Figure 2 shows the history profiles of flow rate and pressure at two locations, one at the central part (MD) and the other at the peripheral part (CA). A half sinusoidal flow rate was imposed at AA as inflow to the network. The amplitude and the frequency of the numerical results are comparable with the measured data. By comparing the results predicted by the two models, with and without the wall viscosity, we clearly observe the smoothing effect of the viscosity on the waveform, especially for the flow rate.

4. Conclusions

We recorded the time series of diameters and pressures at seven locations of ovine arterial tree. The viscoelastic behaviour of the wall was modeled by a Kelvin-Voigt model and the coefficients were derived by linear regression. The 1D simulations of pulse wave with these values showed that the smoothing effect of the viscosity on the waveform is very significant.

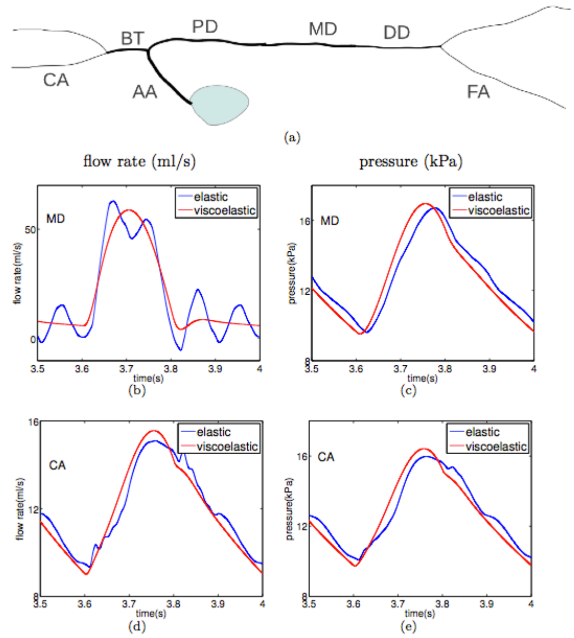

Figure 2. The topology of the ovine arterial tree (top) and the history profiles of flow rate and pressure at two locations.

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