DISCHARGE FLOW OF GRANULAR MEDIA FROM SILOS WITH LATERAL ORIFICE : EXPERIMENTS, DISCRETE, AND CONTINUOUS SIMULATIONS

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Summary We compare laboratory experiments, contact dynamics simulations and continuum Navier–Stokes with $\mu(I)$-rheology simulations of the granular flow through a silo with a lateral opening. For 3D and 2D simulations the Hagen-Beverloo law is obtained. For 3D shallow silos, the sidewalls have a crucial role, which has been added in an averaged version of $\mu(I)$-Navier-Stokes allowing to reproduce the deviation from Hagen-Beverloo due to this lateral friction.

INTRODUCTION

Marine sand glass has long been the main tool to measure time. Since then, granular flows outside silos remain always of great industrial interest in many fields (such as in food, pharmaceutical, energy, ... industry). Considering granular media stored in a tank, we want to understand the flow leaking outside it. This could be due to a lateral accidental tearing of the wall (this can represent very schematically a fuel rod in a nuclear power station during some hypothetical accidental conditions, in which case the orifice is lateral). The final modeled configuration is depicted on figure 1 left. In contrast, the usual silo configuration is symmetrical and has the orifice at the bottom. In this usual configuration, in case of a moderate opening of size D, the discharge flow rate Q is well known to follow the Hagen-Beverloo law (resp. in 3D and in 2D):

$$Q_{3D} \sim \rho\sqrt{gD^5} \ \text{ resp. } \ Q_{2D} \sim \rho\sqrt{gD^3}.$$

Recently, there have been several papers reporting on the computation of a continuum flow ([6, 7] starting from a fluid description and [1] starting from a solid point of view) and comparing it to direct simulations of contact dynamics, the continuum theory being the $\mu(I)$-rheology ([4], [3]) in 2D, and the geometrical configuration the usual one. We propose here 2D simulations in a novel configuration and compare contact dynamics with 2D continuum simulations. We propose as well new 3D continuum simulations in order to find the scaling law of the discharge rate in this configuration with a lateral orifice, focussing on the friction of the walls.

EXPERIMENTS AND SIMULATIONS

Experiments

Experiments are performed with a rectangular silo fitted with spherical glass beads (see figure 1 left). The mass flow rate is measured by a precision balance; the top surface of particles is tracked by a fast camera. The controlled parameters are the size of particles d in the range [75μm-1300μm], the width of the silo W in the range [3.5mm-40mm] and the height of orifices D [2.7mm-35mm].

2D Contact Dynamics simulations

We use the LMGC 90 software implementation of the contact dynamics method [5]. The particles, interacting through a dense granular flow, are treated as perfectly rigid and inelastic. Contact dissipation is modeled in terms of a friction coefficient that we set to $\mu_p = 0.4$ between particles and to $\mu_w = 0.5$ with the wall, further details may be found in [8].

2D, 3D, and averaged 2D numerical simulations

The Navier-Stokes simulations are performed with the free solver *Gerris* and its new version *Basilisk* under development. They both use finite volume, projection methods. As two phases are present: a passive surrounding gas and the granular fluid itself, a Volume of Fluid method is used to track the interface. *Basilisk* allows for a resolution of the fully-coupled Poisson–Helmholtz problem (the former code solved dimension by dimension). The simulations are in 2D and in 3D:

$$\underline{\nabla} \cdot \underline{u} = 0,\ \rho\frac{d\underline{u}}{dt} = \underline{\nabla} \cdot \underline{\underline{\sigma}} + \rho\underline{g}, \quad \underline{\underline{\sigma}} = -p\underline{\underline{I}} + 2\eta\underline{\underline{D}} \quad \text{with } \underline{\underline{D}} = \frac{1}{2}(\underline{\nabla u} + \underline{\nabla u}^T),$$

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with the friction $\mu(I)$ depending on the inertial number I, ([4], [3]), see details in [6, 7] ($\mu_s = 0.4$, $\Delta\mu$ = 0.28, I_0=0.4):

$$\eta = \frac{\mu(I)p}{\sqrt{2}D_2}, \text{ with } I = \frac{d\sqrt{2}D_2}{\sqrt{p/\rho}}, \quad \mu(I) = \mu_s + \frac{\Delta\mu}{I_0/I+1}, \text{and} \quad D_2 = \sqrt{\underline{\underline{D}} : \underline{\underline{D}}}\,.$$

Modeling the friction on the wall (proportional to $\mu_w p$) is a problem with the fluid description as we can impose only no slip or slip for the velocity at the wall. To take into account this lateral friction, we average the momentum equation across the width of the silo (in the Hele-Shaw spirit, [2]). This adds $-2(\mu_w p/W)(\overrightarrow{u}/|\overrightarrow{u}|)$ as an averaged additional force from the sidewalls in the momentum equation. We test these three models with both codes.

RESULTS

For the 2D configuration simulations we obtain a good agreement between contact dynamics and Navier–Stokes, and again we recover the Hagen–Beverloo 2D law. Interestingly enough, we found a simple analytical solution for the pressure field (a kind of Flamant solution plus lithostatic pressure), which describes well the initial times. For pure 3D continuum simulations with Navier-Stokes (with slip conditions at the wall), we obtain as well the Hagen–Beverloo 3D law for the flow for aspect ratio of order one and for square or round holes. Changing now the width W of the silo, from the experiments, we identify two regimes, which depend on the ratio of height to width of the orifice. When D/W is smaller than a critical value, $Q \sim D^{3/2}W$, when D/W is larger then $Q \sim W^{3/2}D$ (see figure 1 center). The same trend is observed for 2D cross-averaged simulations (with the additional sidewalls friction term), this is plotted on figure 1 right.

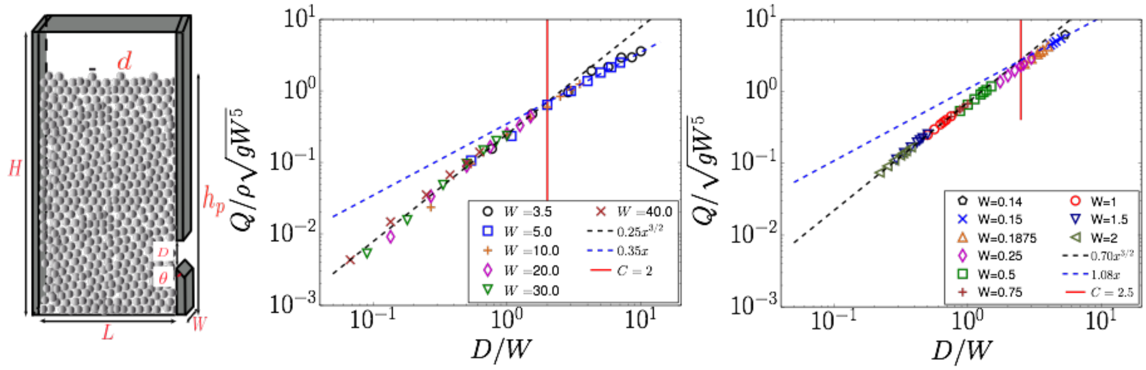

Figure 1: Left, the model configuration: an asymmetrical 3D silo with lateral orifice. When D/W is smaller than a critical value C, we have $Q \sim D^{3/2}W$, like the Hagen–Beverloo 2D scaling. When D/W is larger than C, we have $Q \sim W^{3/2}D$. The trends obtained from the experiments (center) are reproduced with the averaged Navier–Stokes with $\mu(I)$, (right).

CONCLUSIONS

The main result of this work is the identification, both experimentally and numerically, of a new flow regime for thin silos with a lateral opening, where the discharge rate is proportional to D, in contrast to the classical 2D behavior where the discharge rate is in power $3/2$, obtained for thick silos. This work also demonstrates that the continuum $\mu(I)$-rheology describes well granular flows compared to experiments and discrete simulations. It can thus be used as a tool to compute many industrial configurations.

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