Influence of collision on the flow through in-vitro rigid models of the vocal folds

M. Deverge
Fluid Dynamics Laboratory, Technical University of Eindhoven, Postbus 513, 5600-MB Eindhoven, The Netherlands

X. Pelorson and C. Vilain
Institut de la Communication Parléé, I.N.P.G, 46 av. F. Viallet, 38031 Grenoble cedex 1, France

P.-Y. Lagréé
Laboratoire de Modélisation Mécanique, Université Paris VI, 4 place Jussieu, Tour 66, case 162, 75252 Paris Cedex 05, France

F. Chentouf, J. Willems, and A. Hirschberg
Fluid Dynamics Laboratory, Technical University of Eindhoven, Postbus 513, 5600-MB Eindhoven, The Netherlands

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Measurements of pressure in oscillating rigid replicas of vocal folds are presented. The pressure upstream of the replica is used as input to various theoretical approximations to predict the pressure within the glottis. As the vocal folds collide the classical quasisteady boundary layer theory fails. It appears however that for physiologically reasonable shapes of the replicas, viscous effects are more important than the influence of the flow unsteadiness due to the wall movement. A simple model based on a quasisteady Bernoulli equation corrected for viscous effect, combined with a simple boundary layer separation model does globally predict the observed pressure behavior. © 2003 Acoustical Society of America. [DOI: 10.1121/1.1625933]

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I. INTRODUCTION

Voiced sound production, or phonation, is driven by a modulation of the flow passing through the glottis as a result of the oscillation of the vocal folds. Typically, the fundamental oscillation frequency for a male speaker is of order of $10^2$ Hz, which is much lower than the frequency range perceptually relevant for speech (of order of $10^3$ Hz for most voiced sounds). One can therefore expect that, to simulate this behavior, two different models can be used, one predicting the oscillation of the vocal folds and a second predicting the sound production. As the matter of fact, a simple mechanical model such as a two-mass model or a three-mass model combined with a simplified flow model does indeed predict the self-sustained oscillations of the vocal folds. In such lumped parameter models the mechanics of the vocal folds is approximated by rigid masses attached to springs. The oscillations of this mass-spring system are driven by the difference in hydrodynamic force on the vocal folds during opening and closing phases. The contact between the vocal folds during the closure of the glottis is described as a change in stiffness of the springs. The flow is interrupted but the movement of the vocal folds continues as they are allowed to penetrate each other.

In earlier papers we have verified that, for conditions typical to those encountered during phonation, a quasi-steady incompressible flow model based on the concept of viscous boundary layers appears to be a reasonable approximation of the glottal flow. However, the accuracy of this flow model during the closure of the glottis was left as an open problem. Considering the simplicity of the two-mass model it was thought, in practice, more reasonable to ignore the deviations from such a flow model which will certainly occur when the vocal folds collide. However, the closing phase of the vocal folds is known to be a very important feature of voiced sound production. This is the abrupt event which is needed to generate the higher harmonics which are perceptually relevant for speech. Many aspects of the voice quality or of the “naturalness” of the synthetic sounds can be related to this particular event.

During the closure of the vocal folds the quasi-steady, incompressible boundary layer theory will fail because the flow channel, the glottis, becomes too thin to allow a distinction between a frictionless main flow and viscous boundary layers. Eventually the flow becomes dominated by viscous effects. On the other hand, the volume flux induced by the wall displacement becomes locally larger than the flux driven by the trans-glottal pressure difference. In such a case the flow becomes essentially unsteady.

The goal of this study is to investigate whether both viscous and unsteady phenomena are equally important as well as whether they appear simultaneously. The answer to this question will obviously depend upon the shape of the glottis. We will therefore consider three different shapes. In addition to two rounded models we will also present results for a channel with a uniform height. Such a straight channel is interesting because we can obtain analytical solutions for
the flow equations in three different limits: the steady viscous flow, the unsteady frictionless flow and the unsteady viscous flow. In the steady viscous flow case an integral formulation of the boundary layer theory is used. As shown by Ishizaka\textsuperscript{1} and Van Zon\textsuperscript{11} an analytical formulation can be obtained for a uniform channel. The unsteady frictionless flow is based on the Bernoulli equation. Last, the unsteady viscous flow approximation is obtained by assuming an equilibrium between viscous and pressure forces in a quasiparallel flow. This corresponds to the lubrication theory of Reynolds.\textsuperscript{12}

Finally we will also compare our data with a commonly used flow model. It is based on a correction of the steady Bernoulli equation for viscous effects based on the assumption of a Poiseuille flow.\textsuperscript{1} A more elaborate description, accounting for a nonfixed flow separation point\textsuperscript{13,2} will also be considered.

In the first section of this paper the dimensionless parameters relevant for our study are discussed. In the second section three theories are described together with the simplified theory based on the equation of Bernoulli corrected for friction losses. In Sec. III a brief description of the setup and of the experimental method used will be presented. The last part of this paper will be devoted to the analysis of the experimental results. First, only steady flows will be considered, this will allow us to evaluate the effects of viscosity. Second, the results obtained for the oscillating glottis will then be presented and discussed.

II. DIMENSIONLESS PARAMETERS AND BASIC ASSUMPTIONS

The experiments presented here have been designed to simulate the conditions typical of voiced sound production. In particular, the pressure differences in the flow are small compared to the atmospheric pressure and the acoustical wave lengths are very large compared to the length $L$ of the glottis. One can therefore assume that the flow is locally incompressible.

We consider the flow through an oscillating rigid model of the vocal folds with length $L$ (in the flow direction) and width $W$. The minimum aperture of the glottis, $h_g$, occurs at $x=x_g$ (henceforth called the throat of the channel) and varies in time between $h_{\text{min}}$ and $h_{\text{max}}$. An estimate for the flow velocity is the velocity $U_B$ calculated from the pressure difference $p_u-p_d$ across the glottis by means of the Bernoulli equation for steady inviscid flows:

$$U_B = \sqrt{\frac{2(p_u-p_d)}{\rho}},$$

where $\rho$ is the air density which we assume to be constant. The pressures $p_u$ and $p_d$ correspond, respectively, to positions just upstream and downstream of the glottis. We thus assume implicitly that $p_d$ is the pressure in the free jet downstream of the glottis. The fact that $h_g/L = O(10^{-1})$ and $h_g/W = O(10^{-1})$ indicates that a quasi-one-dimensional approximation for the flow should be reasonable $\vec{v} = (\mu(x,y,t),0,0)$. This implies that the pressure is approximatively uniform in a cross section normal to the flow direction $p = p(x,t)$. The ratio $h_g/\delta_v$ between the channel height and of the viscous boundary layer thickness $\delta_v = \sqrt{vL/\nu}$, where $\nu$ is the kinematic viscosity, yields an indication for the importance of viscosity. This ratio is related to the Reynolds number $\text{Re}_h = U_B h_g/\nu$,

$$\left(\frac{h_g}{\delta_v}\right)^2 = \text{Re}_h h_g L.$$  \hspace{1cm} (2)

The Reynolds number $\text{Re}_h = U_B h_g/L$, based on the length $L$ of the channel, provides an indication for the onset of turbulence in the glottis. Using values typical of voiced sound production, one gets $\text{Re}_h = O(10^4)$. A laminar flow within the glottis can therefore be expected but the jet formed by flow separation downstream of the glottis will be turbulent.

A measure for the unsteadiness of the flow is the ratio of the volume flux due to the wall movement $f(h_{\text{max}}-h_{\text{min}})WL$, where $f$ is the fundamental frequency of the motion, and the volume flux $U_B h_g W$ driven by the pressure difference $p_u-p_d$ across the glottis,

$$\frac{f(h_{\text{max}}-h_{\text{min}})L}{U_B h_g} = \text{Sr}_L \frac{h_{\text{max}}-h_{\text{min}}}{h_g},$$

where $\text{Sr}_L = fL/U_B$ is the Strouhal number based on the channel length $L$. In the case of a uniform straight channel one obviously has $h_g = h_{\text{min}}$ because a collision $h_{\text{min}} \rightarrow 0$ implies an essentially unsteady flow. In the case of a more complex geometry however the choice of a relevant length scale for $h_g$ is still an open question.

III. THEORETICAL MODELS

A. Introduction

Although the application to channels of arbitrary shapes will be presented, the theoretical models considered here will mainly focus on the case of straight uniform channels. In all cases it is considered that the edges of the inlet are always well rounded so that any singular losses at the inlet are negligible. Because $h_g/W \ll 1$ the flow velocity $u_g$ far upstream of the glottis is neglected and a uniform pressure $p_u$ is assumed. In the case of a uniform channel, at the downstream end of the glottis the edges are considered as sharp so that the flow separation occurs at this fixed point. It is assumed that, in all cases, the pressure in the jet formed by this flow separation is equal to the pressure $p_d$ far downstream of the channel. Last, as explained above and because $h_g/L \ll 1$ and $h_g \ll W$ a quasiparallel flow $\vec{v} = (\mu(x,y,t),0,0)$ is considered.

B. Inviscid unsteady flow

Strictly speaking, a purely inviscid flow theory would ignore flow separation and thus cannot explain the modulation of the flow by the vocal folds. This corresponds to the so-called paradox of d’Alembert.\textsuperscript{14,15} Indeed, viscous effects induce a flow separation and the formation of a jet. Turbulent dissipation of the kinetic energy within the jet explains the volume flow control. If one assumes a quasisteady behavior of the jet, this implies that the pressure in the jet is equal to the pressure $p_d$ downstream of the glottis. If other effects of
viscosity are neglected, the velocity \( u(0) \) at the inlet of the channel (at \( x=0 \)) can be related to the velocity \( u(L) \) at the channel exit by means of the Bernoulli equation for an incompressible flow

\[
\frac{d}{dt} \left[ \phi(x) - \phi(0) \right] + \frac{1}{2} \rho [u(x)]^2 + p_d
\]

\[
= \frac{1}{2} \rho [u(0)]^2 + p(0),
\]

(4)

where the velocity potential \( \phi \) is given by

\[
\phi(x) - \phi(0) = \int_0^x u \, dx
\]

(5)

which is applied for \( x=L \). It is further assumed that

\[
p_a = \frac{1}{2} \rho [u(x)]^2 + p(0)
\]

(6)

in other words, the unsteadiness of the flow upstream of the inlet \( x=0 \) is neglected.

For an incompressible flow through a channel of uniform height, \( h \), the mass conservation law yields

\[
u(x) - u(0) = -\frac{x}{h} \frac{dh}{dt}.
\]

(7)

Combining the definition (5) for \( \phi \) and the equation of Bernoulli (4) yields a differential equation for the velocity \( u(0) \) at the inlet of the channel,

\[
\frac{L \, du(0)}{dt} = \frac{p_a - p_d}{\rho} + \frac{L^2}{2} \left[ \frac{1}{h} \frac{dh}{dt} \right] - \frac{1}{2} \left[ u(0) - \frac{L}{h} \frac{dh}{dt} \right]^2
\]

(8)

which for given pressure difference, \( p_a-p_d \) and a given \( h \) can be integrated as a function of time. For a harmonically oscillating \( h \), the result of this integration converges to a value which is independent from the initial conditions.

In practice, given \( u(0) \) one can calculate \( u(x) \) and \( \phi(x) \) using the mass conservation law (7) and the definition (5). The pressure \( p_a \) at \( x=x_c \) is then found by applying the equation of Bernoulli (4) between \( x=x_c \) and \( x=L \).

C. Boundary layer solution for steady flows

For a steady flow through a channel of uniform height, \( h \), and driven by a constant pressure difference, \( p_a-p_d \), the Von Kármán integral formulation of the boundary layer equations can be integrated analytically. This solution was already discussed by Ishizaka\(^1\) but only in the case where the boundary layer approximation remains valid over the full length \( L \) of the channel. In such a case, there is always a frictionless core with a uniform velocity \( u_e(x) \) in which the Bernoulli equation can be applied. The frictionless core of the jet at the exit of the channel has a velocity \( u_e(L)=U_B \) [Eq. (1)]. As a more simple alternative to the method of Thwaites,\(^8\) a method of Pohhausen of first order is presented here.\(^12\) In general the method of Pohhausen assumes that the velocity profile \( u(x,y) \) within the viscous boundary layer has a simple shape which can be described by a polynomial of the distance \( y \) from the wall. We use here a polynomial of first order. As shown by Van Zon\(^11\) using a linear velocity profile \( u(x,y) = u_e(x)y/\delta \), where \( \delta \) is the thickness of the boundary layers the corresponding volume flux becomes

\[
\Phi_V = W u_e(h - \delta),
\]

(9)

where \( \delta \) satisfies the nonlinear equation

\[
\frac{4 \delta}{h} + 9 \ln \left( 1 - \frac{\delta}{h} \right) + 5 \frac{\delta}{h - \delta} = \frac{6 \nu x}{h \Phi_V}.
\]

(10)

Applying Eqs. (9) and (10) at the exit \( x=L \) and using \( u_e(L)=U_B \) one has thus a set of two equations from which \( \delta = \delta(L) \) and \( \Phi_V \) can be obtained. Once \( \Phi_V \) has been calculated the viscous boundary layer \( \delta(x) \) at any arbitrary position \( x \) can be calculated from (10). The corresponding frictionless core velocity \( u_e(x) \) is obtained by application of the mass conservation law (9). Finally, the pressure \( p \) is calculated by using Bernoulli equation

\[
p(x) + \frac{1}{2} \rho (u_e(x))^2 = p_a.
\]

(11)

The generalization of this approach for channel of arbitrary length can be made as proposed by Van Zon.\(^11\) When the critical boundary layer thickness

\[
\frac{\delta_c}{h} = \frac{4}{9} \left( 1 - \sqrt{\frac{5}{32}} \right)
\]

(12)

is reached, the volume flux and momentum flux correspond to those of a fully developed Poiseuille flow. The critical distance \( x=l_c \) at which \( \delta(l_c)=\delta_c \) can be obtained analytically for a given volume flux \( \Phi_V \) using Eq. (10). For \( L > l_c \) and \( x < l_c \) the pressure distribution can be obtained from the Bernoulli equation combined with the equation of mass conservation (9),

\[
p_a = p + \frac{1}{2} \rho \frac{\Phi_V^2}{W^2 (h - \delta)^2}
\]

(13)

while for \( x > l_c \) the equation of Poiseuille\(^12\) is used

\[
p - p_d = \frac{12 \rho \nu \Phi_V}{Wh^4} (L-x).
\]

(14)

Applying Eqs. (13) and (14) at \( x=l_c \) and eliminating \( p(l_c) \) yields a quadratic equation for \( \Phi_V \). Using Eq. (10) in which Eq. (12) is substituted and for \( x=l_c \) a linear relationship between \( \Phi_V \) and \( l_c \), \( \Phi_V = (\nu l_c)/(hc) \) can be obtained. By solving the quadratic equation one finally obtains

\[
l_c = \frac{12 c \left( 1 - \frac{\delta_c}{h} \right)^2}{24 c - 1} \left[ 1 - \left( 1 - \frac{h^4 (24 c - 1)(p_a - p_d)}{72 \rho \nu^2 L^2 \left( 1 - \frac{\delta_c}{h} \right)^2} \right)^{1/2} \right],
\]

(15)

where the constant \( c \) is given by

\[
c = \frac{1}{6} \left[ \frac{4 \delta_c}{h} + 9 \ln \left( 1 - \frac{\delta_c}{h} \right) + 5 \frac{\delta_c}{h - \delta_c} \right].
\]

(16)

Depending on \( x,p(x) \) can be calculated using Eqs. (13) or (14).

The use of Thwaites’s implementation of the integral formulation of Von Kármán equation applied to a channel of arbitrary shape is discussed in detail in Hofman.\(^8\) Similar
results are also discussed by Pelorson² using the approach of Pohlhausen. All these approaches involve a numerical resolution which is, in practice, difficult because of the essential nonlinearity of the problem.¹² A direct solution of the equation of Prandtl is also discussed by Lagrée.¹⁶ A systematic comparison showed that in terms of flow separation point prediction, the method of Pohlhausen and Twaiethes are equivalent.¹⁷,¹⁸ Thwaites method appears more robust numerically and therefore will be used in the following. However, in the case of a straight uniform channel an analytical solution can be obtained as shown previously and there is no need for any numerical resolution. In the following, we will refer to this analytical solution as the solution of van Zon.¹¹

D. Lubrication theory of Reynolds

The lubrication theory of Reynolds combines the assumption of a quasiparallel flow together with the assumption that inertial effects are negligible. As the pressure forces balance the viscous ones, the velocity profile in the channel is given by the Poiseuille formula¹²

\[ u = -\frac{1}{2\rho \nu} \frac{\partial p}{\partial x} (h-y) y. \]  

This velocity profile combined with the mass conservation law

\[ W \frac{\partial h}{\partial t} = -\frac{\partial \Phi_V}{\partial x}, \]  

where

\[ \Phi_V = W \int_0^h u \, dy, \]  

yields the equation

\[ \frac{1}{12\rho \nu} \frac{\partial}{\partial x} \left( h^3 \frac{\partial p}{\partial x} \right) = \frac{\partial h}{\partial t} \]  

which for a channel of uniform height can be integrated to give

\[ p - p_a = \left( \frac{p_d - p_a}{L} \right) x + \frac{12\rho \nu}{h^3} \left( \frac{\partial h}{\partial t} \right) x (x-L)/2. \]  

E. Steady Bernoulli corrected for friction

Often one seeks for a simple correction the inviscid theory by adding an extra term to account for viscous pressure losses. This corresponds to the original approach of Ishizaka.¹ A similar approach is proposed by Antunes.¹⁹ For the sake of simplicity, and because it will be shown that the unsteadiness seem to be less important than viscous effects, only the steady flow case will be developed here. This choice is further supported by the fact that if one neglects the effect of the wall movement on the mass conservation law, then the unsteady term in Bernoulli equation should certainly be neglected as well. The wall movement is, indeed, the main cause of unsteadiness. In this sense, the flow model of Ishizaka¹ is not consistent. Such an inconsistent approach might result in some poor behavior of the model as observed by Lous.⁵

The Bernoulli equation corrected for viscous pressure losses \( \Delta p_v \) becomes

\[ \frac{1}{2} \rho \left( \frac{\Phi_V}{Wh} \right)^2 + p_d + \Delta p_v = p_a. \]  

Using the lubrication theory of Reynolds we find

\[ \Delta p_v(x) = \frac{12\rho \nu \Phi_V}{W} \int_0^x h^2 \, dx. \]  

Combined with the modified Bernoulli Equation (22) this equation, when applied at the separation point \( x_s \), yields a quadratic equation for \( \Phi_V \) which is easily solved for given \( x_s \). In a straight uniform channel one has simply \( x_s = L \) and \( \Delta p_v = (12\rho \nu \Phi_V L)/(Wh^3) \).

The extension of the above theory to the case of arbitrary shapes can, of course, be done by numerically integrating Eq. (23). However, as the friction losses scale with \( h^{-3} \) they depend strongly on the channel height. It can therefore be assumed that the losses are determined locally in a small region close to the throat, \( x_g \) of the channel. Using a Taylor expansion, one can therefore use for the channel height \( h \) the approximation

\[ h = h_g + \frac{(x-x_g)^2}{R}, \]  

where \( R \) is the (local) radius of curvature of the wall, we find by integrating (23) from \( x = -\infty \) to \( x = \infty \),

\[ \Delta p_v \approx \frac{9\pi \nu \Phi_V}{2Wh_g^2} \left( \frac{R}{h_g} \right)^{1/2}. \]  

In order to solve for \( \Phi_V \) one now needs an estimation for the height of the channel \( h(x_s) \) at the separation point \( x_s \) where it is assumed in the first approximation that \( p(x_s) = p_d \). The relevance of this later assumption has been discussed in detail by Hofmans.⁵ In view of its simplicity the semiempirical criterium of Liljencrantz,¹³

\[ \frac{h(x_s)}{h_g} = 1 + \epsilon \]  

with \( \epsilon = 0.1 \) seems a reasonable order of magnitude.⁸

In summary, four kinds of theoretical predictions will be considered in the following.

1. The boundary solution which refers as the (analytical) van Zon solution in the case of a uniform straight channel and as the Thwaites (numerical) solution in the case of a nonuniform channel.
2. The lubrication theory of Reynolds as developed in Sec. III D.
3. The steady Bernoulli theory corrected for pressure losses described in Sec. III E.
4. The unsteady Bernoulli solution presented in Sec. III B.

IV. EXPERIMENTAL SETUP AND PROCEDURE

In order to validate the theoretical models presented in the preceding sections, an experimental setup with oscillating vocal-folds replica is used. The main interest of this approach lies in the better control of the experimental condi-
tions compared to in-vivo measurements. While most in-vitro experiments in the literature deal with steady replicas of vocal folds (e.g., Refs. 22–25), or with numerical simulations on steady replicas (e.g., Refs. 26 and 27) very few attempts have been made using moving vocal folds. Much more realistic flow conditions were obtained recently using setups including moving replicas of vocal folds. These experiments were intended either to focus on the onset of phonation or to mimic self-sustained oscillations using forced motion of the vocal-folds replica. The experimental setup used in this study is in the tradition of these latter studies.

A. Vocal-fold replicas and sensors

Figure 1 shows the three different vocal folds mechanical models (or replicas) used. All these mechanical models have an overall length of 2 cm in the flow direction and a width $W = 3$ cm.

Compared with human size, the mechanical model appears thus up-scaled by a factor of 3. To keep the Reynolds number constant, this implies that the velocities in the mechanical model are a factor of 3 times smaller than those expected for humans. At constant Strouhal and Reynolds numbers, the frequency of oscillation of the mechanical replica must be smaller than the one expected for human phonation by a factor of 9.

A pressure tap with a radius of 0.4 mm and 1 mm length is placed 1 cm upstream from the downstream end. This allows measurements of the wall glottal pressure $p_g - p_d$ by means of a Kulite pressure transducer (type XCS-093, diameter 1.6 mm) placed in a cavity below the pressure tap. The pressure gauge was calibrated by using a Betz water micromanometer with a precision of 1 Pa. The response of the gauge was found to be linear within the accuracy of the measurement. The calibration was repeated after each series of measurements and appeared to be stable.

The first replica is the straight uniform channel. On the upstream side, the edges have been rounded with a radius of curvature of 2 mm. On the opposite, the downstream edges are made sharp. The transducer position is located at $x_g = 1$ cm.

The second replica, denoted as the rounded vocal folds mechanical model has a length of $L = 2$ cm. The walls are half cylinders with a radius of $R = 1$ cm. The pressure tap is placed at the throat of the replica $x_g = 1$ cm.

The third replica, the Gaussian vocal folds, has a more complex shape. In the region $2 \text{ mm} < x < 18 \text{ mm}$ the vocal folds have a Gaussian shape described by the equation $y$=

\[ A \exp\left[-\left(x - (L/2)\right)^2/\alpha^2\right] \]

with $A = 2$ mm and $\alpha = 9$ mm. The edges of both the inlet and outlet are rounded with a radius of curvature of 2 mm. The pressure tap is located at $x_g = 1$ cm.

In the case of unsteady flow measurements, the lower fold with the pressure tap is maintained fixed while the upper fold is driven by a piston using an electrical motor and an eccentric wheel. This allowed to simulate self-sustained oscillations with a fundamental frequency of oscillation ranging from 5 Hz up to 35 Hz. Only results concerning the highest frequency are presented here as they correspond to the highest Strouhal numbers achievable using this setup. During the collision the mechanical folds were prevented from bouncing thanks to both to the backlash of the driving mechanism and the strong damping of the piston. The channel height $h_g$ at $x_g$ is measured by means of an optical sensor (type OPB700). The sensor was calibrated by placing gauges of known thickness at the throat of the glottis. The calibration was performed before and after each measurement. The estimated uncertainty in the measurement of the channel height $h_g$ is $10^{-2}$ mm. The range of variation for $h_g$ was chosen to keep a fairly linear behavior of the sensor.

The presence of an asymmetrical flow (Coanda effect) through rigid nonmoving mechanical replicas of the glottis has been observed in past experiments. Although the most direct way to observe such an asymmetry was to use pressure measurement at both sides of the replica, this phenomenon could be observed in dynamic experiments even using measurements of a single side of the replica due to the presence of an abrupt transition in the pressure signal. This transition corresponds to the time needed by the flow to establish an asymmetrical behavior. As such behavior was never observed here, there is no evidence for the presence of any asymmetry in the flow.

B. Global description of the setup

A global view of the setup is shown in Fig. 2. The vocal folds are mounted at the end of a pipe of 30 cm length and 3 cm diameter connected to a pressure reservoir with a volume of 0.68 m$^3$ filled with acoustical foam in order to prevent acoustical resonances. The pressure $p_u - p_d$ is measured 2 cm upstream from the inlet of the replica by means of a Kulite pressure transducer (type XCS-093) mounted flush with the pipe wall. The pressure reservoir is filled with

![FIG. 1. Mechanical models of the vocal folds: (a) straight uniform channel, (b) rounded vocal folds, and (c) Gaussian vocal folds.](Image)

![FIG. 2. Global sketch of the setup.](Image)
acoustical foam to avoid acoustical resonances. The air flows from a 8 bar pressure supply through a choked valve into the reservoir. Downstream of the replica the flow exits into a large room. The pressure $p_d$ in this room is used as a reference by the pressure gauges. The signals are recorded with a sample frequency of 1 kHz by means of a data acquisition card (NI PCI-MIO 16XE10) in a PC.

V. RESULTS

A. Steady flow measurements

In Fig. 3 the measured dimensionless pressure $(p_g - p_d)/(p_u - p_d)$ inside the straight uniform replica is plotted as a function of $Re_g(hL)$. The measured data are compared with the predictions from the lubrication theory of Reynolds (Sec. III C), the boundary layer theory (Sec. III B) and the Bernoulli theory corrected for friction (Sec. III D). These measurements have been carried out at a fixed pressure of $p_u - p_d = 1$ kPa. It can be observed that the boundary layer approximation provides quite reasonable agreement while the theory of Reynolds overestimates $(p_g - p_d)/(p_u - p_d)$ for $Re_g(hL)>10$. The inviscid approximation based on Bernoulli (Sec. III B) would just predict $(p_g - p_d)/(p_u - p_d) = 0$. However, the addition of a correction based on a Poiseuille flow profile allows for a reasonable agreement although the predictions are systematically higher than the measured data.

In Fig. 4 are presented the measured $(p_g - p_d)/(p_u - p_d)$ for a steady flow within the rounded replica. These data are compared with the predictions obtained by the boundary layer approximation of Thwaites, the approximation of Reynolds and the Bernoulli theory corrected for friction. It can be observed that the theory of Reynolds (Sec. III D) always predicts $(p_g - p_d)/(p_u - p_d) = 1/2$ due to the symmetry of the replica. This theory cannot predict the negative values of $p_g - p_d$ which are essentially due to inertial effects. The Thwaites theory does predict the order of magnitude of those negative values for $Re_g(h/L)>10$ mm. When the glottis is more and more closed $h_g \rightarrow 0$ one measures a pressure which is approaching the value predicted by the theory of Reynolds. The use of the Bernoulli theory corrected for friction explains the measured data quite well. The separation criterion chosen in Eq. (26) thus seems relevant for this geometry.

The corresponding results for the Gaussian replica are shown in Fig. 5. The conclusions that can be drawn are similar to those expressed for the rounded geometry.

We conclude from those data that for $Re_g(h/L)>10$ the boundary layer theory seems reasonable while the data approach the prediction of the lubrication theory of Reynolds for smaller values.

B. Unsteady flow measurements

In Fig. 6(a) are presented the measurements of the glottal pressure $p_g - p_d$, the upstream pressure $p_u - p_d$ and the channel height $h$. In this experiment, the vocal folds were not allowed to collide, the minimum channel throat was fixed as $h_{min}=0.10$ mm. The measured glottal pressure is compared with the prediction obtained by means of the boundary layer theory (Sec. III C), the lubrication theory of Reynolds (Sec. III D) and the inviscid unsteady solution (Sec. III B). The

![FIG. 3. Steady flow measurements for a straight channel ($p_u - p_d = 1$ kPa).](image)

![FIG. 4. Steady flow measurements for the rounded replica ($p_u - p_d = 1$ kPa).](image)

![FIG. 5. Steady flow measurements for the Gaussian replica ($p_u - p_d = 1$ kPa).](image)
good agreement between the boundary layer prediction and the measured data tends to show that unsteady flow effects
are not very important. The comparison with the lubrication
theory of Reynolds (Sec. III E) shows that inertial effects at
the channel inlet are important except for $h_g = h_{\text{min}}$. For a
short time interval (when $h_g = h_{\text{min}}$) the theory of Reynolds
predicts the experimental data. We further observe that the
inviscid unsteady approximation (Sec. III B) provides quite
poor results.

In the case where a collision is allowed [Fig. 6(b)], it can
be observed that a finite glottal pressure could be measured
even when the glottis is closed. This surprising result is due
to the fact that owing to the surface roughness of the me-
chanical folds (of order of $10^{-6}$ m) a complete closure can-
not be achieved.

It can also be observed that the glottal pressure $p_g - p_d$
can become larger than the transglottal pressure $p_u - p_d$
which can only be explained by the flow unsteadiness due to
the movement of the walls. This effect is indeed predicted by
the lubrication theory of Reynolds. Deviation between
theory and experiments could be partially due to the effect of
errors in the measurement of the channel height $h_g$ when
$h_g \rightarrow 0$. The results of the lubrication theory are indeed very
sensitive to such errors, because the viscous losses are pro-
portional to $h^{-3}$. Last, the inviscid unsteady approximation,
which is not shown here, fails to explain the measured data.

In Figs. 7(a) and 7(b) are presented results for the
rounded replica for $h_{\text{min}} = 0.08$ mm and $h_{\text{min}} = 0$ mm,
respectively. It can be observed that as $h$ approaches $h_{\text{min}}$ the glottal
pressure changes from a negative pressure predicted by the
theory of Thwaites towards a positive pressure which ap-
proaches $(p_g - p_d)/(p_u - p_d) = 0.5$ as predicted by the steady
lubrication theory of Reynolds. When there is a collision, as
shown in Fig. 7(b) this value is indeed reached. The fact that

FIG. 6. Experimental and theoretical results for the
straight uniform replica (a) $h_{\text{min}} = 0.10$ mm and (b) $h_{\text{min}} = 0$.

FIG. 7. Experimental and theoretical results for the
rounded replica (a) $h_{\text{min}} = 0.08$ mm and (b) $h_{\text{min}} = 0$. 
we do not find glottal pressures significantly higher than this limit tends to indicate that, for the rounded replica, even in the case of a collision the unsteady flow induced by the wall movement remains negligible compared to the viscous effects.

In Fig. 8 it can be seen that the results obtained using a Gaussian replica are very similar to those obtained with the rounded one. The main difference is that the prediction of the steady boundary layer theory of Thwaites as well as the Bernoulli theory modified for friction appear less accurate than for rounded replica experiments.

VI. CONCLUSION

From this study it was observed that upon collision the flow unsteadiness due to the wall movement appears to be only significant in the case of the straight uniform replica. In such a case the unsteady theory of Reynolds (Sec. III E) predicts qualitatively the measurements. In the case of more physiological vocal folds shapes, a transition between a boundary-layer behavior towards a friction dominated behavior was clearly observed without significant effect of the flow unsteadiness.

It was shown that the boundary layer solution is quite reasonable for $Re(h/L)>10$. In practice such a theory is still quite complex and one may prefer a more simple model to predict flow separation. The models of Liljencrants\textsuperscript{13} or the model of Pelorson\textsuperscript{2} are easy to implement. When such a simple model is used, the Bernoulli equation corrected for a pressure loss term calculated on the basis of the theory of Reynolds provides a surprisingly accurate approximation. Addition of unsteady terms in the equation of Bernoulli will not improve the model. Such an unsteady term should certainly not be used if the mass conservation law does account for wall movement.

It must be noted that in our experiments a complete closure of the glottis could not be simulated due to the surface roughness of the replicas. In the case of human phonation one could expect a complete closure due to both the elastic deformation of the tissues and the presence of moisture or of mucus on the folds. While the dynamics of the tissues during collision can be studied by means of finite element method as shown recently by Gunter,\textsuperscript{32} the possible presence of moisture seems another potentially important aspect. The movement of water driven by the colliding folds is expected to involve strong inertial effects due to the relatively large density of water. Therefore a complete and accurate physical model for the closure of the vocal folds might involve more than an interaction between an airflow and dry elastic tissues.

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FIG. 8. Experimental and theoretical results for the Gaussian replica (a) $h_{\text{min}}=0.05$ mm and (b) $h_{\text{min}}=0$. 


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