Multiscale Hydrodynamic Phenomena

M2, Fluid mechanics, MU5MEF15 2024/2025
Friday November 30th 2024, 8 :30am - 12 :30pm, Salle : 56.66.103 Part I. : 75 minutes, NO documents

1. Quick Questions In few words and few formula :
1.1 Order of magnitude of drag on a cylinder of radius L at large Re in turbulent flow.
1.2 Order of magnitude of drag on a cylinder of radius L at small Re.
1.3 Order of magnitude of drag on a flat plate of length L at large Re in laminar flow, in turbulent flow ?
1.4 Problem $\nabla^2 p = 0$ in upper half domain ($\forall x$ and $y \geq 0$) with $-\partial p/\partial y|_0 = f'(x)$ and $p(\infty) \to 0$, what is the solution on the line $y = 0$?
1.5 Write 2D Boundary Layer equations (in x, y, u, v) in the case of Blasius problem, what is the scale of y compared to the scale of x ?
1.6 In which one of the 3 decks of Triple Deck is flow separation ?
1.7 Dispersion relation $\omega(k)$ for linear waves on free surface of arbitrary depth (Airy swell problem), what is the small parameter to obtain this relation ?
1.8 What is the KdV equation ? What balance is it ?
1.9 What is the Friedrichs problem ?
1.10 Nobel prizes with references to Asymptotics ?

2. Exercice
Consider the following equation (this is a model of paint brush !) :

$$(E_\varepsilon) \qquad \varepsilon \frac{dH(t)}{dt} = \varepsilon - \frac{H(t)}{H(t) + 1 - t}, \quad H(0) = 0, \qquad \varepsilon \ll 1$$

we want to solve this problem with the Matched Asymptotic Expansion method.
2.1 Why is this problem singular ?
2.2 Looking at the right hand side, show that a first change of scale on H gives that H is $O(\varepsilon)$.
2.3 Say that $H(t) = \varepsilon h(t)$, do not change t, what is the new equation for $h(t)$?
2.4 Why is this new problem on $h(t)$ again singular ?
2.5 What is the outer problem and what is the form of the outer solution for $h(t)$ at order $O(1)$ for $0 \leq t \leq 1$?
2.6 What is the inner problem and what is the inner solution ?
2.7 Suggest the plot of the inner and outer solution.
2.8 The matching is automatic, why ?
2.9 Composite expansion, plot of it ?

3. Exercice
Let us look at the following ordinary differential equation : $(E_\varepsilon) \quad \dfrac{d^2y}{dt^2} + \varepsilon\dfrac{dy}{dt} + \omega^2 y = 0,$ valid for any $t > 0$ with boundary conditions $y(0) = 1$ and $y'(0) = 0$. Of course ε is a given small parameter and ω is a real of order one. We want to solve this problem.
3.1 Solve with Feynman averaging method.
3.2 We want to solve this problem with Multiple Scales Analysis. Introduce two time scales, $t_0 = t$ and t_1, what is the relation between t, t_1 and ε ?
3.3 Compute $\partial/\partial t$ and $\partial^2/\partial t^2$
3.4 Solve the problem.
3.5 Suggest the plot of the solution.
3.6 What is the exact solution for any ε, compare.

4. Exercice
Solve with WKB approximation the problem

$$\varepsilon y''(x) = y(x) \text{ with } y(0) = 0, y(1) = 1$$

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M2, Fluid mechanics 2024/2025
Friday November 30th 2024, 8 :30am - 12 :30pm,
Part II. : 1h 15 min all documents. No Wifi

Multiscale Hydrodynamic Phenomena

Mixed convection

This is a part of "Mixed convection over a cooled horizontal plate : non-uniqueness and numerical instabilities of the boundary-layer equations" By Herbert STEINRÜCK

We consider a 2D steady boundary layer flow (uniform velocity U_∞ along $\overrightarrow{e}_x$) over a flat plate ($y = 0$, $x > 0$) which is at a temperature T_w (maybe function of x) different of the uniform constant temperature T_∞ far away in the flow. Due to this temperature of the plate, the temperature of the flow is modified by convection and diffusion. Furthermore, there is a retroaction of the temperature of the flow on the dynamics of the flow thanks to variations of the density according to Boussinesq approximation $\rho = \rho_0(1-\beta(T-T_\infty))$. This expression is in the weight (gravity is perpendicular to the plate $\overrightarrow{g} = -\overrightarrow{e}_y$).
Note that in this configuration, the pressure P in Navier Stokes Equation is such that

$$P = P_0 - \rho_0 g y + p,$$

where P is indeed the pressure in the $-\overrightarrow{\nabla}P$ of right hand side, where P_0 is a reference pressure constant, where $-\rho_0 g y$ is the hydrostatic variation, and where p is the so called "dynamic pressure" that will be present at the end in the final system.

Two classical configurations exist, first with **no** Boussinesq coupling and a given velocity at infinity, this is "forced convection", second with Boussinesq coupling and **no** given velocity at infinity, this is "free convection". Here we will look at "mixed convection" : with Boussinesq coupling and a given velocity at infinity. This configuration is not in the textbooks of thermal flow...

As all the results are more or less in the paper, be careful and rigorous to prove the results. Numbers refer to equations in the papers (Eq. X.) or questions (Q. X.).

First we suppose no Boussinesq effect, there is **no** temperature effect on the momentum equation. This is "forced convection". There is gravity, but density is constant. We use a length L for space, in question 1.X, this length is given. The scaling $L = U_\infty/(g\beta\Delta T)\nu$ (first line of page 252) will be a result of questions Q2.2. or Q3.2 so do not use it now. The Reynolds number $Re = U_\infty L/\nu$ is large.

1.1 Write incompressible full NS equations and boundary conditions.
1.2 Check that the gravity is nor more present as $P = P_0 - \rho_0 g y + p$. Write momentum equations in x and y with this p.
1.3 Write heat equation and boundary conditions, remember that $\rho c_p \frac{d}{dt}T = k\overrightarrow{\nabla}^2 T + 2\mu D_{ij}D_{ij}$ where strain rate tensor is $D_{ij} = (1/2)(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$, it creates a source of temperature. Thermal diffusivity is $a = k/(\rho c_p)$.
1.4 Write equations and boundary conditions of (Q1.1) without dimension, use L a given length so that $(x, y) = L(\bar{x}, \bar{y})$), and U_∞ so that $(u, v) = U_\infty(\bar{u}, \bar{v})$.
1.5 Write equation and boundary conditions of (Q1.2) without dimension, use ϑ as suggested by first paragraph of page 252. Use Prandtl Pr number, Eckert number $E = U_\infty^2/(c_p\Delta T)$ and Reynolds number in the heat equation without dimension.
1.6 For a given length L, write the boundary layer scaling for u, v and y with dominant balance for incompressibility and momentum. Find indeed the scalings (except the one on the first line of page 252 as L is given) of first paragraph of page 252.
1.7 Check that we obtain (Eq (1.1)) with $\partial_x p = 0$, (Eq(1.3)) and that (Eq(1.2)) reduces indeed to $\partial_y p = 0$. Note that (Eq (1.1)) to (Eq(1.3)) are without dimension (maybe you have tildes or bars in your expressions).

1.8 Without demonstration, remind the Blasius self similar solution. Check that equation (Eq(2.4)) is part of it, boundary conditions for f ? Link between f and stream function and longitudinal and transverse velocity ?
1.9 Write heat equation and boundary conditions without dimension.
1.10 Write heat flux at the wall without dimension.
1.11 As the Reynolds number is large, as $Pr = O(1)$ (example : air $Pr = .7$, water $Pr = 7$) and as E is small (why ?) check that we obtain (Eq (1.4)).
1.12 Using the Blasius self similar variable η check that we can find a self similar solution of the heat equation which reduces to $2\vartheta''/Pr + f\vartheta' = 0$ (Eq (2.5)).

Second We suppose now that there is a Boussinesq effect. The heat equation will retroact on the momentum equation through the coupling of a source term in the transverse momentum equation. We suppose as well a velocity at infinity. This is mixed convection. This corresponds to the second paragraph of page 252 (sentence "In the classical ... in the problem"). The transverse momentum equation will change, we focus on it.
2.1 Write incompressible Navier Stokes dimensionless equation along y with temperature using the scaling $\rho_\infty U_\infty^2$ for dynamic pressure (why ?), the scaling $LRe^{-1/2}$ (why ?) and the scale δT.
2.2 Looking at this Navier Stokes dimensionless equation along y, show by dominant balance that L is as proposed on the first line of page 252. Obtain equation (Eq(1.2)). Verify that the other terms are indeed smaller.
2.3 We have now a full coupling, this is mixed convection. Write all the scaling and write the final system with all boundary condition to sum up.

Free convection Of course if there is no imposed velocity at infinity ($U_\infty = 0$ or $U_\infty \ll U_f$ see Q.3.1) , the problem is a problem of free convection (or natural convection : the flow is generated by changes of density due to the heating of the wall).
3.1 Starting from scratch show with crude balances that for free convection, the velocity is created by the heating and then that the scale of velocity is $U_f = (gL\beta\Delta T\nu)^{1/5}$.
3.2 Check that in the free convection problem all the scalings obtained in Q1.X (in first paragraphe of page 252) are the same, but U_∞ is replaced by U_f.
3.3 In this case of free convection, show that the problem is self similar and that, for an imposed temperature, (without dimension) $u = x^{1/5} f'(y/x^{2/5})$
3.4 Explain why when the imposed velocity U_∞ is of order $U_f = (gL\beta\Delta T\nu)^{1/5}$ we have "mixed convection".

Non self similarity in mixed convection We know that often the selfsimilar solution is a "steady" solution with respect to another variable. We saw that for heat equation. Here we do the same for mixed convection equations.
4.1 In Blasius flow $\psi = \sqrt{x} f(y/\sqrt{y})$ (see Q.1.8), then discuss equations Eq (2.1) and Eq (2.2).
4.2 Check every equation Eq.(2.4), Eq.(2.5), and Eq. (2.6)
4.3 What do you think of (Eq.(2.8)) ?

The paper continues the expansion and shows that there are positives exponents n (figure 3), then in this case there is **no** relaxation to a self similar solution when integrating in marching Eq.(2.5), and Eq. (2.6) with ξ increasing. As visible on figure 1 ; if one solves the equations with ξ increasing, any small perturbation is increased given the exponential branches (1,2...13). In fact, it has been shown latter that this configuration has no self similar solution but creates a kind of hydrolic jump. The exponents can be found with Triple Deck theory. But that is another story....

Biblio
H. Steinrück Mixed convection over a cooled horizontal plate : non-uniqueness and numerical instabilities of the boundary-layer equations J. Fluid Mech. (1994),vol. 278, pp. 251-265

J. Fluid Mech. (1994), *vol.* 278, *pp.* 251–265

Mixed convection over a cooled horizontal plate: non-uniqueness and numerical instabilities of the boundary-layer equations

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The boundary-layer flow over a cooled horizontal plate is considered. It is shown that the real part of the spectrum of the evolution operator of the linearized equations is not bounded uniformly from above which explains the difficulties encounterd by a numerical solution. Furthermore it is shown that near the leading edge an asymptotic expansion of the solution is not unique. A one-parametric family of asymptotic expansions of solutions can be constructed.

1. Introduction

Though there are several papers presenting numerical solutions to the mixed-convection boundary-layer flow above a cooled horizontal plate none of these results is really satisfactory (Schneider, Steinrück & Andre 1994). All solutions agree near the edge of the plate but they differ significantly on where and how a singularity occurs. Schneider & Wasel (1985) were the first to find an unusual behaviour. They found a singularity with a finite wall shear stress. Later Wickern (1991*a*, *b*) claimed that the boundary-layer flow terminates in a Goldstein-type singularity. Daniels (1992) proved analytically the possibility of a singularity with an infinite wall shear stress. Considering that the numerical solution for the case of the boundary-layer flow above a heated horizontal or an inclined heated or cooled plate is straightforward the difficulties in the case of a cooled horizontal plate are surprising. In this paper we investigate the mathematical reason for this controversy.

The modified boundary-layer equations for the mixed-convection flow above a horizontal plate in dimensionless form are

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}, \tag{1.1}$$

$$0=-\frac{\partial p}{\partial y}+\vartheta, \tag{1.2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1.3}$$

$$u\frac{\partial \vartheta}{\partial x}+v\frac{\partial \vartheta}{\partial y}=\frac{1}{Pr}\frac{\partial^2 \vartheta}{\partial y^2}, \tag{1.4}$$

where the dimensionless coordinate x parallel to the plate is made dimensionless

with the reference length $L = U_\infty^5/(g\beta\Delta T)^2\nu$ which depends on the velocity U_∞ of the free stream, the gravity acceleration g, the thermal expansivity β, the kinematic viscosity ν and the difference ΔT between a reference value of the plate temperature T_w and the temperature T_∞ of the undisturbed fluid. The dimensionless coordinate y perpendicular to the plate is scaled with $LRe^{-1/2}$, where $Re = U_\infty L/\nu$ is the Reynolds number. The velocity components u, v parallel and perpendicular to the plate are scaled with U_∞ and $U_\infty Re^{-1/2}$. The difference ϑ between the temperature of the disturbed and undisturbed fluid is scaled with ΔT. Reference values for the dimensionless skin friction τ and the dimensionless heat flux density q are $\rho_\infty U_\infty^2/Re$ and $k\Delta T Re^{-1/2}/L$, where ρ_∞ is the density of the undisturbed fluid and k is the thermal conductivity of the fluid. Note that according to this scaling we have $\tau = (\partial/\partial y)u(x, y = 0)$ and $q = (\partial/\partial y)\vartheta(x, y = 0)$.

In the classical boundary-layer equations the pressure p (which is scaled with $\rho_\infty U_\infty^2$) is determined by the outer flow and does not depend on the perpendicular coordinate y. Here, using Boussinesq's approximation, the boundary-layer equations are modified so that the hydrostatic pressure depends on buoyancy effects induced by the temperature difference ϑ with the unperturbed fluid. The Prandtl number $Pr = \nu/a$, with a the thermal diffusivity, is the only non-dimensional parameter in the problem.

The boundary conditions at the plate are given by the no-slip conditions and a prescribed temperature difference with the unperturbed fluid:

$$u(x,0)=0,\quad v(x,0)=0,\quad \vartheta(x,0)=\vartheta_w(x),\quad x>0. \tag{1.5}$$

Since the solution of the boundary-layer equation has to match with the outer flow the asymptotic boundary conditions for $y\to\infty$ must hold:

$$u(x,\infty)=1,\quad \vartheta(x,\infty)=0,\quad p(x,\infty)=0. \tag{1.6}$$

At the leading edge the flow is unperturbed, thus the initial conditions

$$u(0,y)=1,\quad \vartheta(0,y)=0,\quad y>0, \tag{1.7}$$

hold. It is easy to verify that the modified boundary-layer equations (1.1)–(1.4) are of parabolic type in the sense defined by Courant & Hilbert (1968). Thus one might assume that (1.1)–(1.4) together with the boundary and initial conditions (1.5)–(1.7) is a well-posed problem. Indeed it is observed that (1.1)–(1.4) is well posed in case of a heated plate ($\vartheta_w > 0$). But this is not the case for a cooled plate.

A similarity solution exists for a plate temperature distribution of the form $\vartheta_w(x) = kx^{-1/2}$ for $k \geqslant k_0 < 0$ (Schneider 1979). In particular for $k \geqslant 0$ (heated plate) a unique similarity solution exists, while for $k_0 < k < 0$ (cooled plate) two similarity solutions exist. For $k < k_0$ no similarity solution exits at all. It turns out that the plate is adiabatic and that the heat transfer is concentrated at the leading edge of the plate. In this paper we will consider the case of a constant wall temperature $\vartheta_w = -1$ and point out the mathematical difficulties.

A necessary condition for the well posedness of a linear evolution problem

$$u_t = A(t)u,\quad u(0)=u_0, \tag{1.8}$$

is that the real part of the spectrum of the evolution operator A is bounded uniformly from above (Pazy 1983). This is the case for the heat equation or the wave equation on a bounded interval with Dirichlet boundary conditions and appropriate initial

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conditions. But it is not the case for the Laplace equation $u_{tt} = -u_{xx}$, with $u(-1,t) = u(1,t) = 0$, and the initial conditions $u(x,0) = f(x)$, $u_t(x,0) = g(x)$.

Since the boundary-layer equations are solved like an evolution system starting from the leading edge of the plate, we try to verify the above condition. Since (1.1)–(1.4) form a nonlinear system and the x-derivatives are not given explicitly we have to linearize (1.1)–(1.4) at a given solution and study locally the linearization which yields a generalized eigenvalue problem which will be analysed and the consequences of the results will be discussed.

2. Eigenvalues near the leading edge

To analyse the boundary-layer equations near the leading edge it is convenient to introduce the coordinate transform

$$\xi = (x/Pr)^{1/2}, \quad \eta = y/x^{1/2}. \tag{2.1}$$

Let $\psi(x,y)$ be a streamfunction; then we indroduce a transformed stream function f by

$$\psi(x,y) = x^{1/2} f(x^{1/2}/Pr^{1/2}, y/x^{1/2}). \tag{2.2}$$

The dependence of the coordinate transform (2.1) on the Prandtl number turns out to be useful when considering the limiting case of a small Prandtl number. From now on we will denote derivatives with respect to η with a prime and with respect to ξ with a subscript. The dimensionless skin friction τ and the dimensonless heat flux density q are given by

$$\tau = \frac{\partial^2}{\partial y^2}\psi = \frac{1}{Pr^{1/2}\xi} f'', \quad q = \frac{\partial}{\partial y}\vartheta = \frac{1}{Pr^{1/2}\xi}\vartheta'. \tag{2.3}$$

Thus (1.1)–(1.4) are equivalent to

$$2f''' + f f'' = \xi(f' f'_\xi - f'' f_\xi + g), \tag{2.4}$$

$$\frac{2}{Pr}\vartheta'' + f\vartheta' = \xi(f'\vartheta_\xi - \vartheta' f_\xi), \tag{2.5}$$

$$Pr^{-1/2} g' + \eta\vartheta' = \xi\vartheta_\xi, \tag{2.6}$$

with the boundary conditions

$$f(\xi,0) = f'(\xi,0) = \vartheta(\xi,0) + 1 = f'(\xi,\infty) - 1 = \vartheta(\xi,\infty) = g(\xi,\infty) = 0. \tag{2.7}$$

Note that the function g is the transformed pressure gradient parallel to the plate. Assuming a regular behaviour of f, ϑ and g near the leading edge we obtain ordinary differential equations for the initial values of f, ϑ and g where $f(0,\eta)$ satisfies the Blasius equation. We can construct formally a regular expansion of a solution of (2.4)–(2.7) by a power series expansion with respect to ξ (Afzal & Hussain 1984):

$$f_r^{(N)}(\xi,\eta) = \sum_{n=0}^{N} \xi^n f_n(\eta), \quad \vartheta_r^{(N)}(\xi,\eta) = \sum_{n=0}^{N} \xi^n \vartheta_n(\eta), \quad g_r^{(N)}(\xi,\eta) = \sum_{n=0}^{N} \xi^n g_n(\eta). \tag{2.8}$$

We denote by f_r, ϑ_r, g_r a solution of (2.4)–(2.7) with a regular expansion (2.8).

Let us assume we perturb a given solution at ξ_0 and let ΔF, $\Delta\vartheta$, ΔG denote the perturbation of f, ϑ, g. We are interested in whether the perturbation grows or decays locally. Thus we linearize (2.4)–(2.7), freeze the ξ-dependence of the coefficient

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functions and insert

$$\Delta F(\xi,\eta) = F(\eta)e^{\lambda\xi}, \quad \Delta\vartheta(\xi,\eta) = D(\eta)e^{\lambda\xi}, \quad \Delta G(\xi,\eta) = G(\eta)e^{\lambda\xi}, \tag{2.9}$$

into the linearized equations. This yields a generalized eigenvalue problem:

$$2F''' - \lambda\xi_0(f'F' - f''F) - \xi_0 G = -fF'' - f''F + \xi_0(f'_\xi F' - f_\xi F''), \tag{2.10}$$

$$\frac{2}{Pr}D'' - \lambda\xi_0(f'D - \vartheta'F) = -fD' - \vartheta'F + \xi_0(\vartheta_\xi F' - f_\xi D'), \tag{2.11}$$

$$Pr^{-1/2}G' - \lambda\xi_0 D = -\eta D', \tag{2.12}$$

$$F(0) = F'(0) = D(0) = F'(\infty) = D(\infty) = G(\infty) = 0. \tag{2.13}$$

Let us first consider the linearization near the leading edge of the plate. We solve the eigenvalue problem by an asymptotic expansion with respect to small values of ξ_0. We introduce the expansion

$$\lambda = \frac{\nu_0}{\xi_0} + \nu_1 + \nu_2\xi_0 + \ldots, \tag{2.14}$$

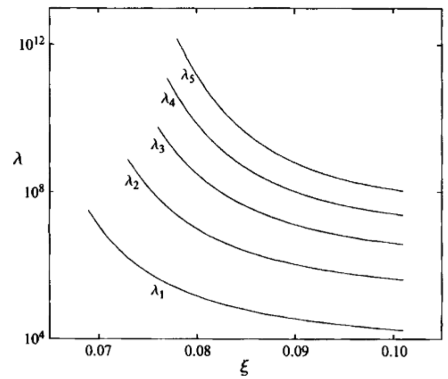

FIGURE 3. Positive eigenvalues $\lambda_1 \cdots \lambda_5$ of (2.10)–(2.13) along the solution of the initial value problem with inital data $\xi_0 = 0.05$, $\sigma = -0.001438$ as functions of ξ.

Mixed convection over a cooled horizontal plate 261

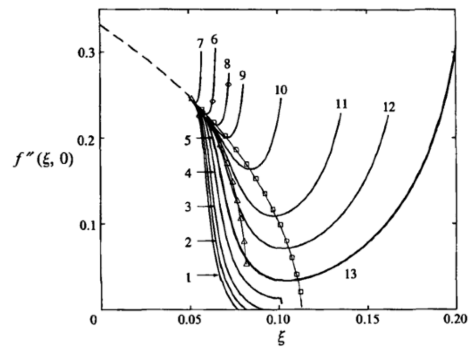

FIGURE 1. Wall shear stress $f''(\xi,0) = \tau/Pr^{1/2}\xi$ for $Pr = 0.72$. Regular expansion with 8 terms: dashed line. Numbered curves show solution of inital value problem with initial data $f(\xi_0,\eta) = f_r^{(8)}(\xi_0,\eta) + \sigma F_+(\eta)$, $\vartheta(\xi_0,\eta) = \vartheta_r^{(8)}(\xi_0,\eta) + \sigma D_+(\eta)$ for various values of ξ_0 and σ (see table 1). ◇, $\delta^2 = \frac{1}{4} - q/\tau^2 = 0$; △ – △, Schneider & Wasel (1985); □ – □, Wickern (1991*a*, *b*).

Kapitza
Landau 1962 Feynman 1965 HAroche 2012 Wilson De Gennes
singular as we are loosing the derivative

$$0 = \varepsilon - \frac{H(t)}{H(t) + 1 - t}, \quad H(0) = 0, \qquad \varepsilon \ll 1$$

I confess that singularity is not so clear, $\varepsilon = 0$ gives $H = 0$ which is OK
seems that H is small, we test $H = \delta(\varepsilon)h$
by dominant balance the right hand side gives $\delta = \varepsilon$ and the RHS is $\varepsilon(1 - \frac{h(t)}{\varepsilon h(t)+1-t})$
The equation is now

$$\varepsilon h' = (1 - \frac{h(t)}{\varepsilon h(t) + 1 - t})$$

again singular
Change of scale $t = \delta(\varepsilon)\tilde{t}$, by dominant balance $\delta = \varepsilon$, ad has the denominator is $1 + O(\varepsilon)$

$$\frac{d\tilde{h}}{d\tilde{t}} = (1 - \frac{\tilde{h}(\tilde{t})}{1})$$

solution $\tilde{h} = 1 - e^{-\tilde{t}}$
automatic as only first order derivative and the BC is OK
$H = \varepsilon(1 - t - e^{-t/\varepsilon})$

```
e = .01;
sol = NDSolve[{e H'[t] == e - H[t] /(H[t] + 1 - t), H[0] == 0},
  H[t], {t, 0, 1}][[]]


   Plot[{0, (H[t] /. sol[[1]]), e (1 - t)}, {t, 0, 1}]
```

correction Ex 3
$y = A\cos(\omega t)$ and $y \simeq \omega A \sin(\omega t)$
mean value, $T = \omega/(2\pi)$

$$\frac{1}{T}\int_0^T \sin^2 \omega t dt = \frac{\omega}{2\pi}[\omega/(2\pi) + \frac{1}{2}\sin 2\omega t]_0^T = \frac{1}{2}$$

$< y^2 >$ is $\frac{A^2}{2}$, $< (y')^2 >$ is $\omega^2\frac{(A)^2}{2}$,

$$< \frac{d}{dt}(\bar{y}'^2/2 + \omega^2 y^2/2) >= -\varepsilon < y'^2 > \text{ so } \frac{d}{dt}(\omega^2 A^2/4 + \omega^2 A^2/4) >= -\varepsilon\omega^2 A^2/2.$$

$$\frac{d}{dt}(A^2) = -\varepsilon A^2.$$

correction Ex 4

• with $\delta = \sqrt{\varepsilon}$, the eikonal $(S_0')^2 = 1$ then $S_0 = \pm x$
and $S_1 = cst$ hence the solution is the sum of $e^{\pm x/\sqrt{\varepsilon}}$:

$$y(x) = \frac{e^{x/\sqrt{\varepsilon}} - e^{-x/\sqrt{\varepsilon}}}{e^{1/\sqrt{\varepsilon}} - e^{-1/\sqrt{\varepsilon}}}$$

c'est exactement la solution exacte !

$$y(x) = \frac{\sinh(x/\sqrt{\varepsilon})}{\sinh(1/\sqrt{\varepsilon})}$$