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dernière mise à jour : 20/12/2006 par vilas



MF204 : Transferts thermiques et massiques dans les fluides

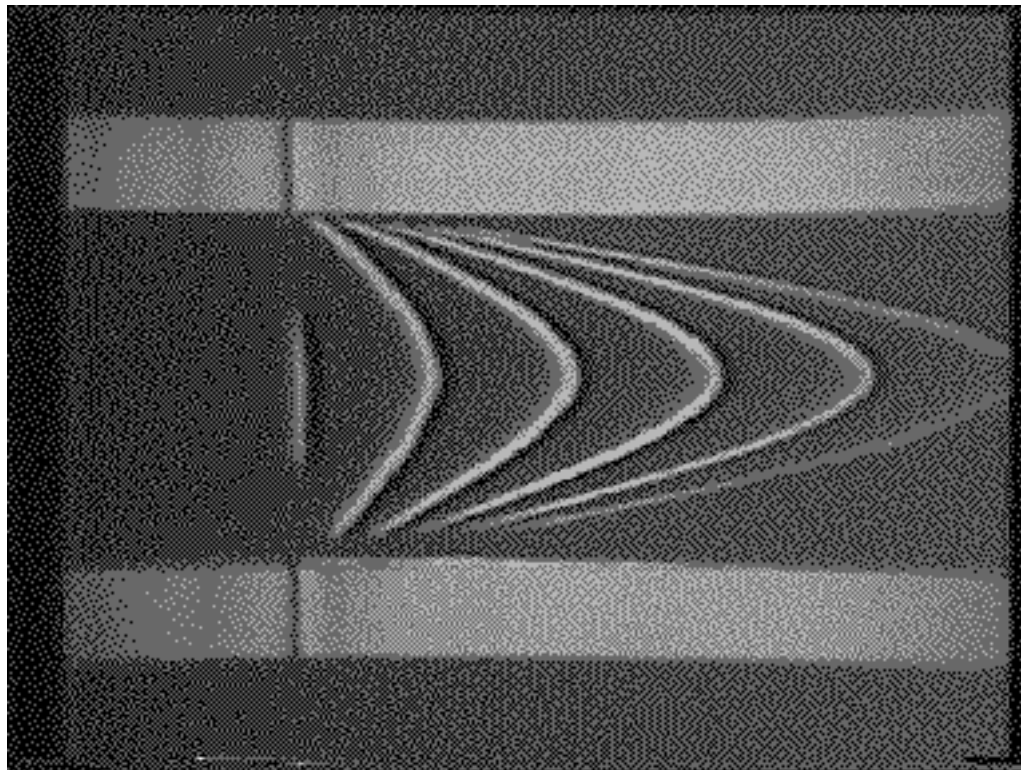
Année : 2A		Module : voie	Crédits ECTS : 2
Professeurs :		Pierre-Yves LAGREE	
Maîtres de conférences :		Pierre-Yves LAGREE ; Michael BAUDOIN	
DATE	HEURES	TYPE SEANCE	PROGRAMME DE LA SEANCE
23 01 2007	08:30 09:30	Cours magistral	Rappels de conduction, coefficient d'échange
23 01 2007	09:45 11:45	Petite classe	
30 01 2007	8:30 9:30	Cours magistral	Convection forcée
30 01 2007	9:45 11:45	Petite classe	
06 02 2007	8:30 9:30	Cours magistral	Convection forcée
06 02 2007	9:45 11:45	Petite classe	
13 02 2007	08:30 09:30	Cours magistral	Convection Naturelle
13 02 2007	09:45 11:45	Petite classe	
20 02 2007	08:30 09:30	Cours magistral	Convection forcée turbulente
20 02 2007	09:45 11:45	Petite classe	
06 03 2007	08:30 09:30	Cours magistral	Transferts de masse
06 03 2007	09:45 11:45	Petite classe	
13 03 2007	08:30 11:30	Contrôle	Contrôle de connaissances
		Petite classe	

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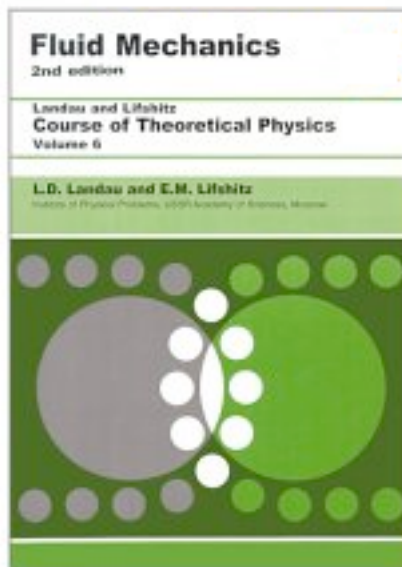
[stats](#) [phpMyVisites](#)

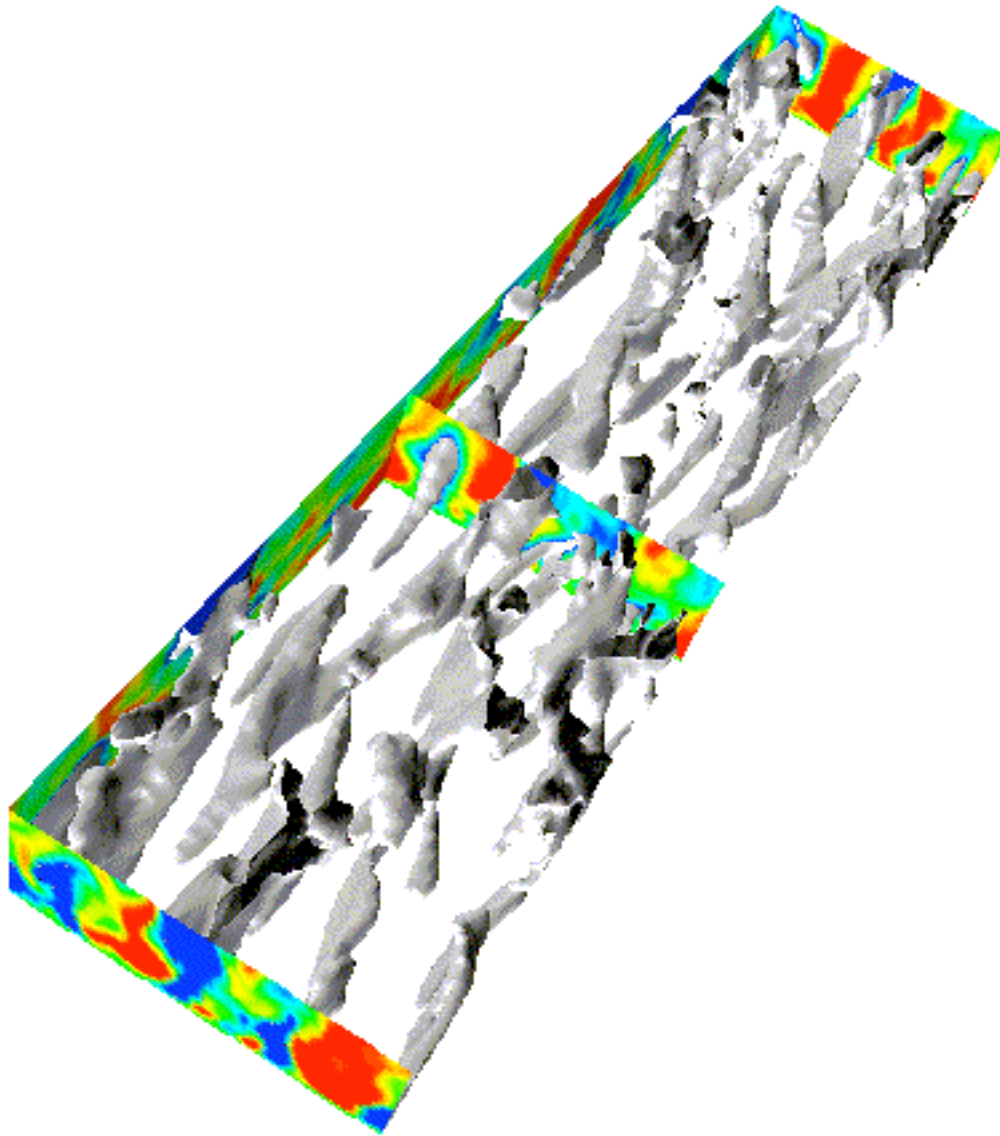
<http://www.imm.jussieu.fr/~lagree/COURS/ENSTA/>

Transferts Turbulents



Landau & Lifshitz (1989) (§26) et citons le: il convient de remarquer que *"tout problème concernant l'écoulement de fluide visqueux dans des conditions stationnaires données doit posséder, ne serait ce qu'en principe, une solution stationnaire exacte des équations de l'hydrodynamique"*(ce qui n'est pas encore prouvé mathématiquement en 3D). *"Or certaines de ces solutions même si elles sont exactes, ne sont pas vérifiées dans la Nature"* (N majuscule). *"Les écoulements"* (sous entendu laminares) *"qui existent dans la Nature doivent être stables: de petites perturbations qui y prennent naissance doivent s'atténuer au cours du temps."*





Isosurface of the streamwise velocity. Low velocity. High velocity. Grey isosurface at the blue level.

http://www2.mech.kth.se/~mskote/html_lib/animations.html

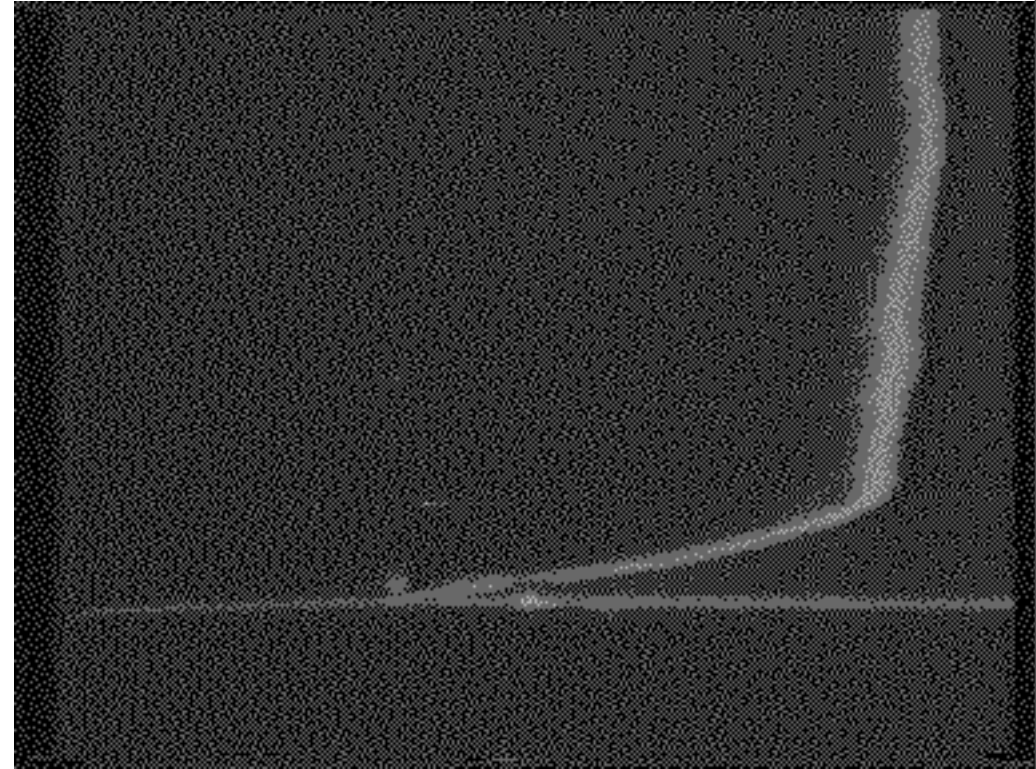
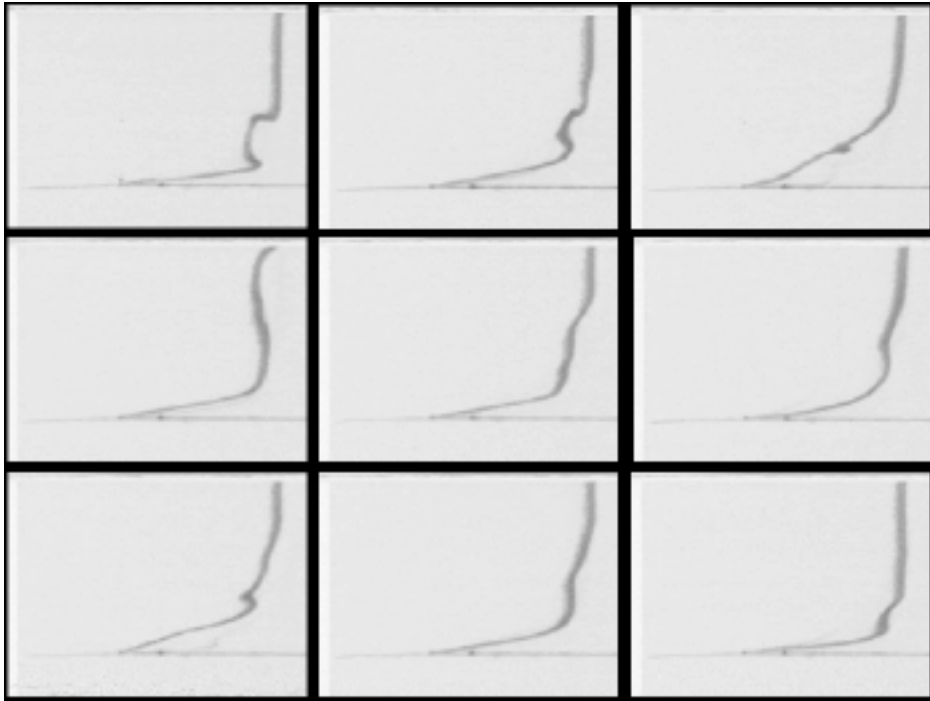
Isosurface of the streamwise vorticity. Downward velocity. Upward velocity.

Définition de la turbulence

Une définition précise de la turbulence est donc difficile à donner (Tennekes & Lumley (1978) p 1), on peut cependant faire un catalogue:

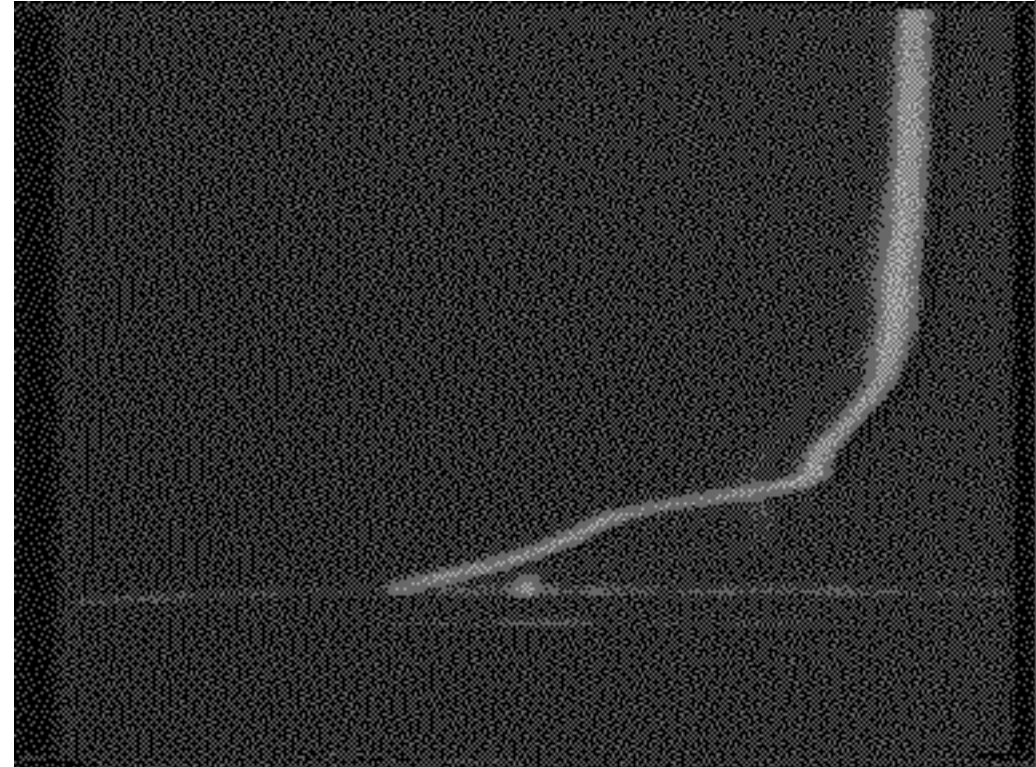
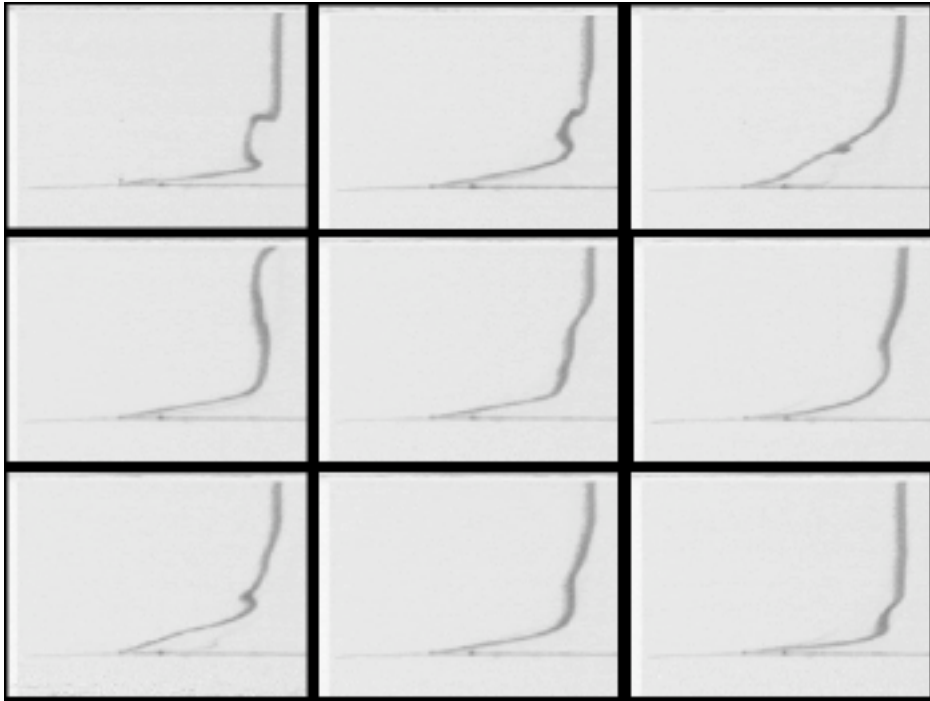
- l'état turbulent est caractérisé par des variations rapides irrégulières et aléatoires de la vitesse
- les mélanges sont importants et plus rapidement faits que par la diffusivité habituelle.
- le nombre de Reynolds est grand
- l'écoulement est 3D, il est rotationnel.
- l'énergie est dégradée: les écoulements turbulents dissipent l'énergie.
- les échelles mises en jeux ne sont pas celles des échanges moléculaire (le cadre de la mécanique des milieux continus reste valable)
- les caractéristiques sont les mêmes pour tous les fluides (gaz ou liquides), la turbulence n'est pas une propriété du fluide mais seulement un régime particulier.

Observations expérimentales:

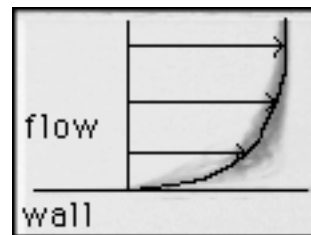
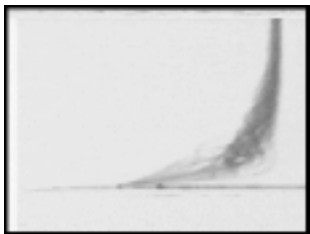


(Encyclopaedia Britannica) des images instantanées d'une couche limite turbulente.

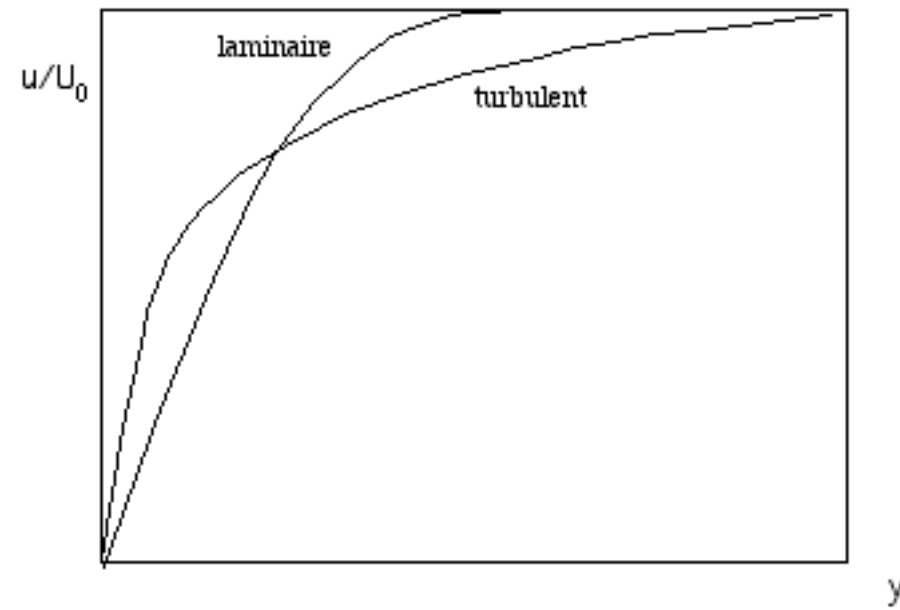
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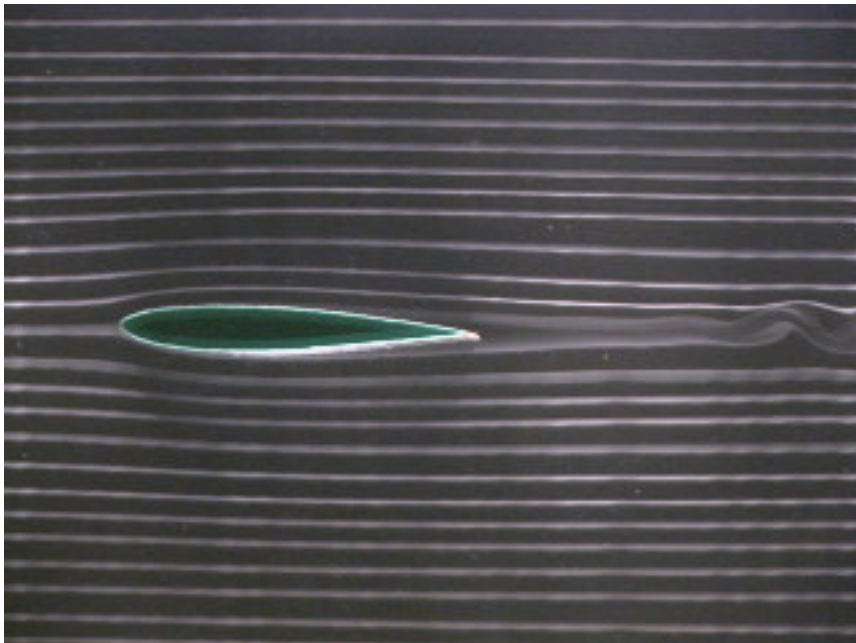


superposition: profil moyen.

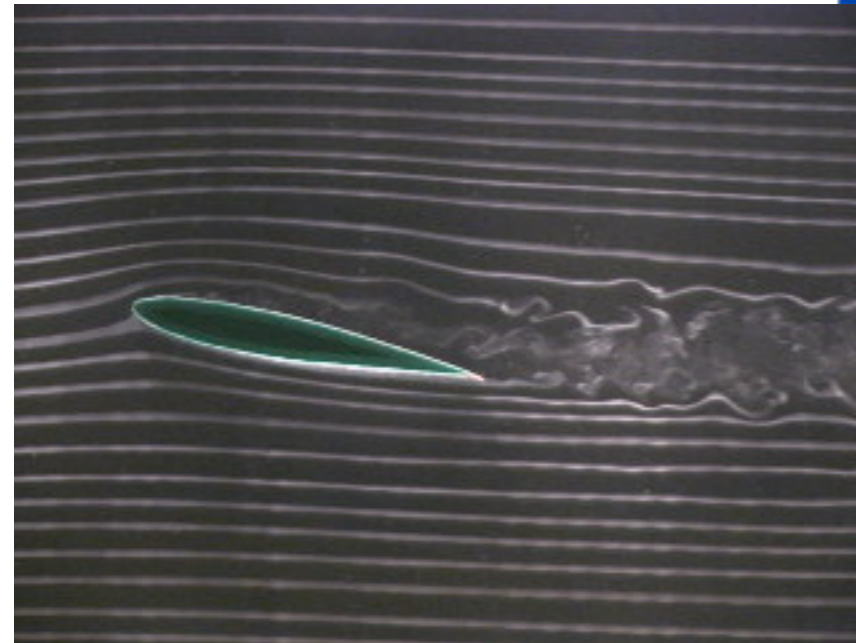


profil laminaire et turbulent en unités de couche limite (0.99).

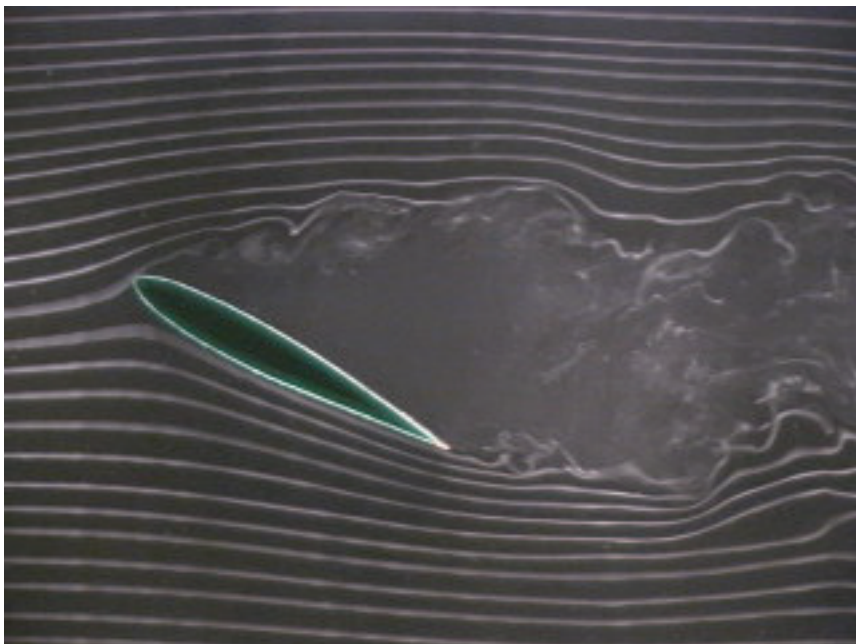
0°



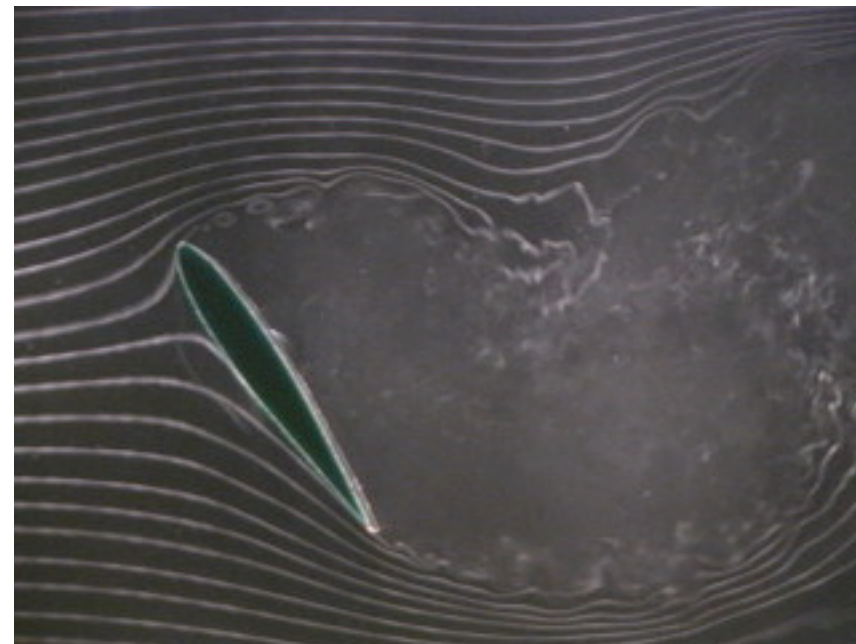
15°



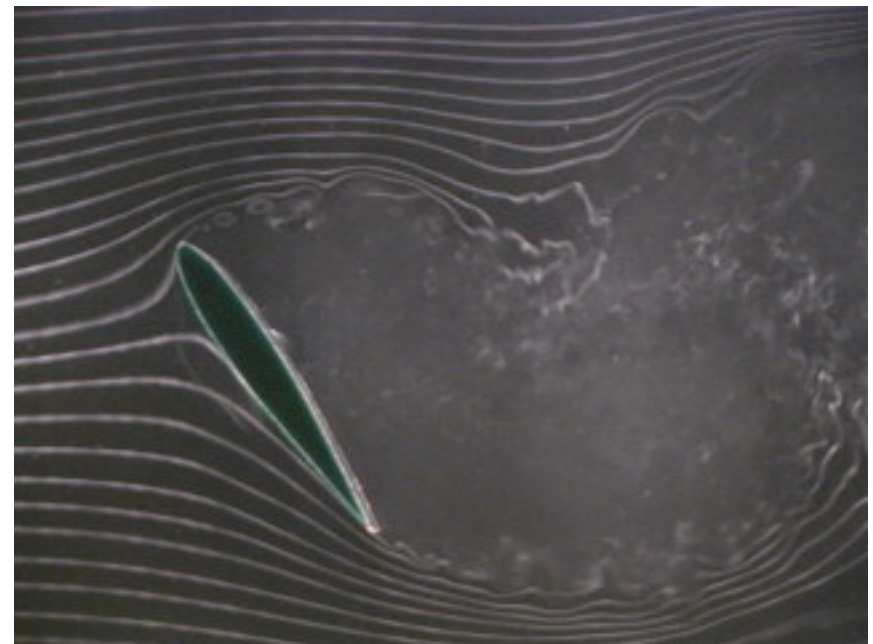
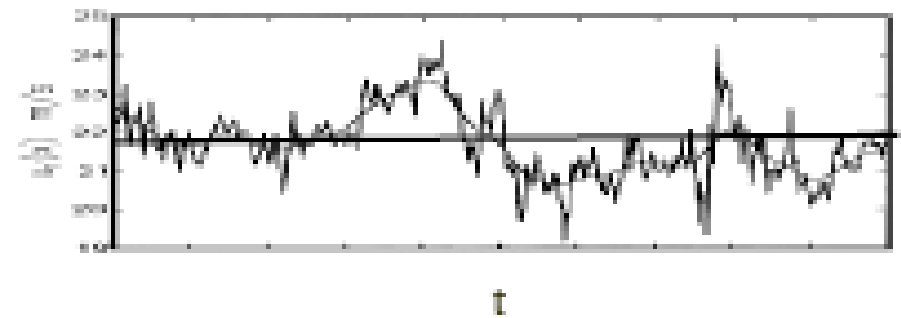
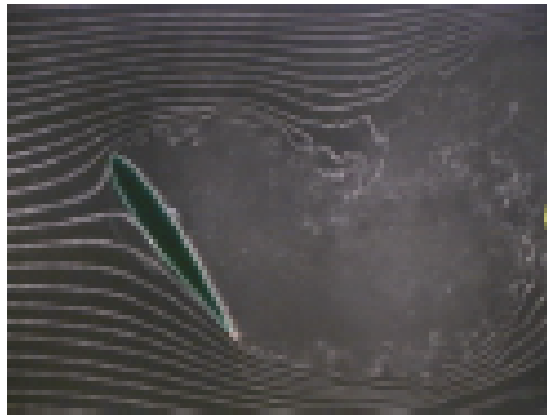
30°



60°



Le problème de la moyenne des Équations de NS: Équations de Reynolds



Moyenne sur l'ensemble des réalisations

$$\langle u \rangle = \frac{1}{n} \sum u_i(r, t).$$

Moyenne sur l'ensemble des réalisations

$$\langle u \rangle = \frac{1}{n} \sum u_i(r, t).$$

$$\langle f \rangle = \int_{-\infty}^{\infty} f(r - r', t - t') G(r', t') dr' dt'.$$

Moyenne simple

$$\langle u \rangle = \frac{1}{T} \int_t^{t+T} u(x, y, t) dt.$$

$$\langle \langle f \rangle \rangle = \langle f \rangle$$

$$\langle \alpha f + \beta g \rangle = \alpha \langle f \rangle + \beta \langle g \rangle \text{ avec } \alpha, \beta \text{ constantes}$$

$$\langle \int f(x) dx \rangle = \int \langle f(x) \rangle dx$$

$$\langle \frac{\partial f}{\partial x} \rangle = \frac{\partial \langle f \rangle}{\partial x}, \langle \frac{\partial f}{\partial t} \rangle = \frac{\partial \langle f \rangle}{\partial t} \text{ etc.}$$

$$\langle \langle f \rangle g \rangle = \langle f \rangle \langle g \rangle$$

$$\langle fg \rangle \neq \langle f \rangle \langle g \rangle.$$

$$\frac{\partial u_i}{\partial x_i} = 0$$

$$\rho \frac{du_i}{dt} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial u_i}{\partial x_j} \right)$$

Moyenne des équations de NS

$$u_i = \langle u_i \rangle + u'_i$$



$$\langle \frac{\partial u_i}{\partial x_i} = 0 \rangle$$

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$$\langle \rho \frac{\partial u_i}{\partial t} \rangle = \rho \frac{\partial \langle u_i \rangle}{\partial t}$$

$$\langle -\frac{\partial p}{\partial x_i} \rangle = -\frac{\partial \langle p \rangle}{\partial x_i}$$

$$\langle \frac{\partial}{\partial x_j} (\mu \frac{\partial u_i}{\partial x_j}) \rangle = \frac{\partial}{\partial x_j} (\mu \frac{\partial \langle u_i \rangle}{\partial x_j})$$

Moyenne des équations de NS

$$u_i = \langle u_i \rangle + u'_i$$



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$$\left\langle \rho \frac{\partial u_i}{\partial t} \right\rangle = \rho \frac{\partial \langle u_i \rangle}{\partial t}$$

$$\left\langle -\frac{\partial p}{\partial x_i} \right\rangle = -\frac{\partial \langle p \rangle}{\partial x_i}$$

$$\left\langle \frac{\partial}{\partial x_j} \left(\mu \frac{\partial u_i}{\partial x_j} \right) \right\rangle = \frac{\partial}{\partial x_j} \left(\mu \frac{\partial \langle u_i \rangle}{\partial x_j} \right)$$

$$\left\langle u_j \frac{\partial u_i}{\partial x_j} \right\rangle = \left\langle \frac{\partial}{\partial x_j} (u_i u_j) \right\rangle$$

$$\left\langle \frac{\partial}{\partial x_j} (\langle u_i \rangle \langle u_j \rangle + \langle u_i \rangle u'_j + \langle u_j \rangle u'_i + u'_j u'_i) \right\rangle =$$

Moyenne des équations de NS

$$u_i = \langle u_i \rangle + u'_i$$



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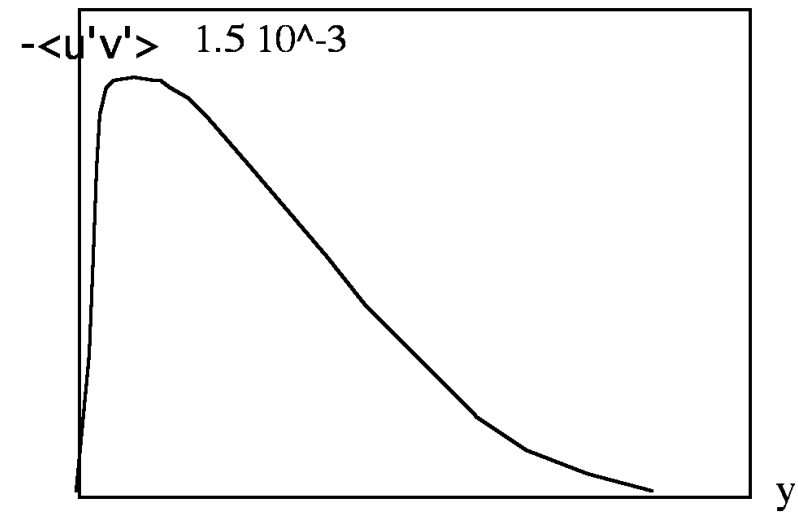
$$\rho \frac{d \langle u_i \rangle}{dt} = -\frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial \langle u_i \rangle}{\partial x_j} - \rho \langle u'_i u'_j \rangle \right).$$

Equations de Navier Stokes moyennées, $\langle u_i \rangle = U_i$

$$\frac{\partial U_i}{\partial x_i} = 0.$$

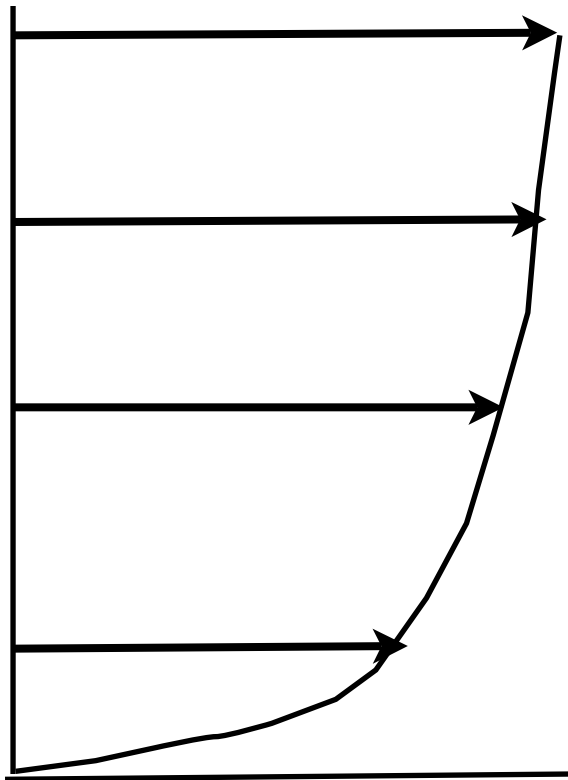
$$\rho U_i \frac{\partial}{\partial x_j} U_i = - \frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial U_i}{\partial x_j} - \rho \langle u_i' u_j' \rangle \right).$$

On a fait apparaître le terme que l'on appelle terme de tension turbulente: $-\rho \langle u_i' u_j' \rangle$.



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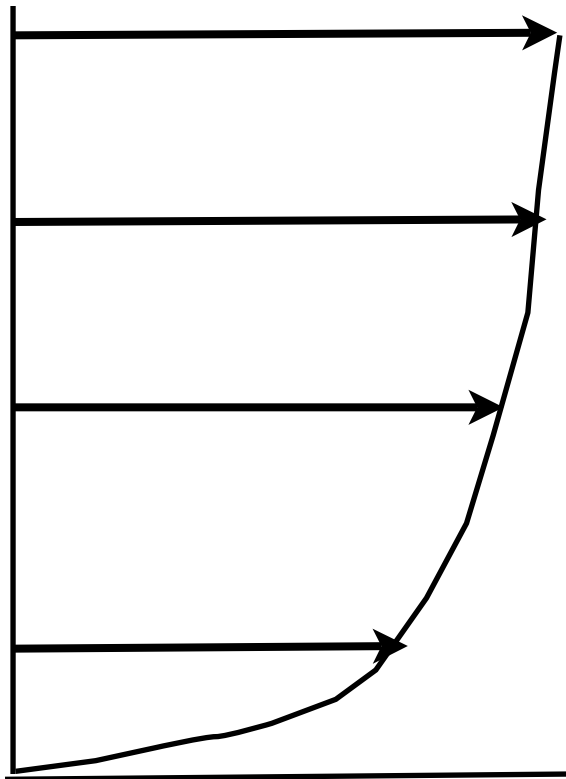
$$\rho \langle u_i \rangle \frac{\partial \langle u_i \rangle}{\partial x} = -\frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial \langle u_i \rangle}{\partial x_j} - \rho \langle u'_i u'_j \rangle \right).$$



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$$\frac{\partial}{\partial y} \left(\nu \frac{\partial \langle u \rangle}{\partial y} - \langle u' v' \rangle \right) \simeq 0$$



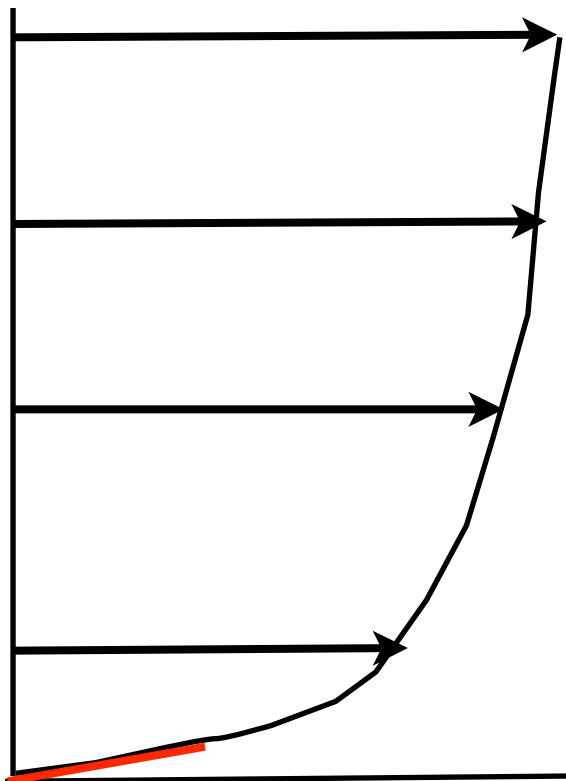
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$$\frac{\partial}{\partial y} \left(\nu \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) \simeq 0$$



$$\left(\nu \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) = \text{constante}.$$



sous couche visqueuse

définition: $u_\tau = \sqrt{(\tau_p / \rho)}$:

$$\tau_p = \mu \frac{\partial \langle u \rangle}{\partial y} :$$

$$\rho \langle u_i \rangle \frac{\partial \langle u_i \rangle}{\partial x} = -\frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial \langle u_i \rangle}{\partial x_j} - \rho \langle u'_i u'_j \rangle \right).$$



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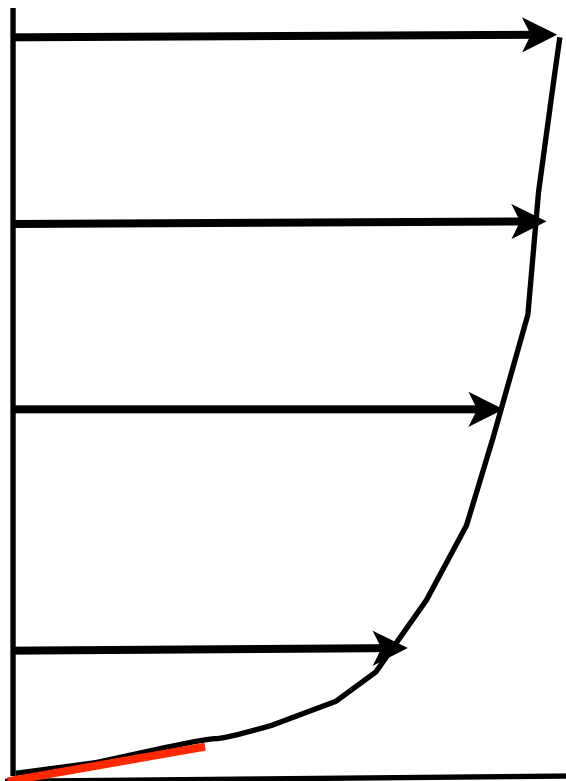


$$\left(\nu \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) = \text{constante}.$$

$$u_\tau = \sqrt{(\tau_p / \rho)} :$$

$$\delta^+ = \frac{\nu}{u_\tau}$$

$$U^+ = \frac{\langle u \rangle}{u_\tau}$$



sous couche visqueuse

$$\tau_p = \mu \frac{\partial \langle u \rangle}{\partial y} :$$

$$y^+ = y u_\tau / \nu = y / \delta^+.$$

$$\langle u \rangle / u_\tau = u_\tau y / \nu, \text{ ou } U^+ = y^+$$

$$\rho \langle u_i \rangle \frac{\partial \langle u_i \rangle}{\partial x} = - \frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial \langle u_i \rangle}{\partial x_j} - \rho \langle u'_i u'_j \rangle \right).$$



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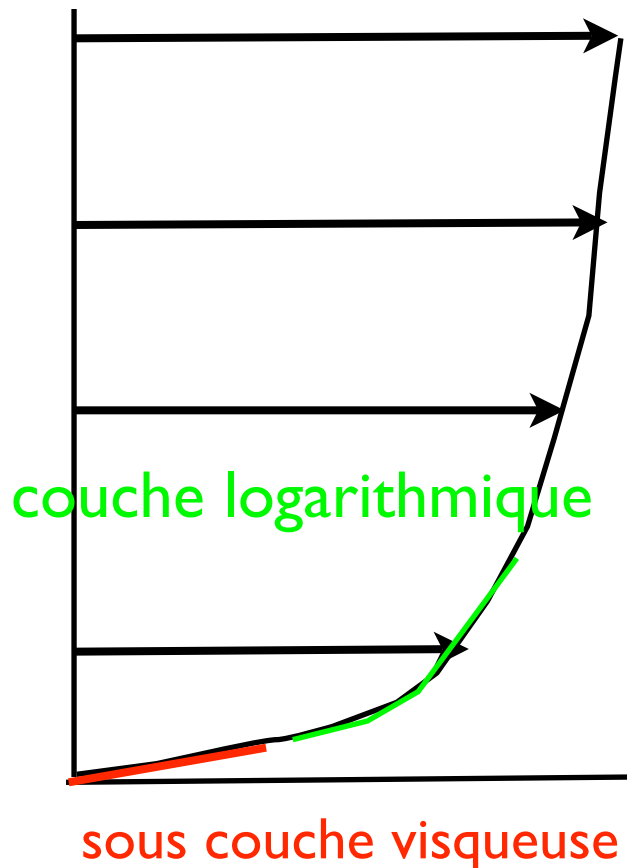


$$\left(\nu \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) = \text{constante.}$$

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$$\frac{\partial \langle u \rangle}{\partial y}$$

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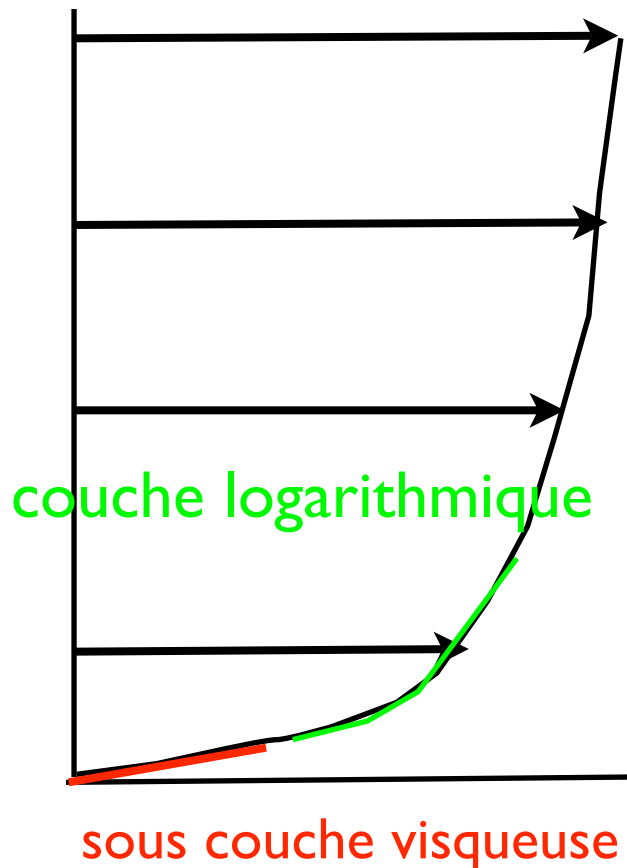


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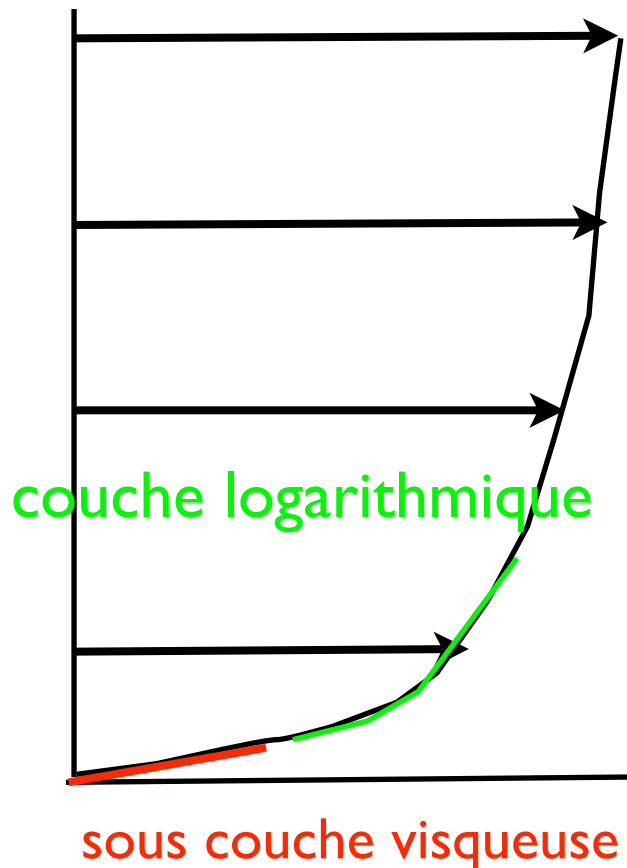


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$$\delta^+ = \frac{\nu}{u_\tau}$$

$$U^+ = \frac{\langle u \rangle}{u_\tau}$$



$$\frac{\partial \langle u \rangle}{\partial y} \sim \frac{u_\tau}{y}$$

$$\langle u \rangle / u_\tau = 2.4 \text{Log}(u_\tau y / \nu) + 5$$

$$\rho \langle u_i \rangle \frac{\partial \langle u_i \rangle}{\partial x} = -\frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial \langle u_i \rangle}{\partial x_j} - \rho \langle u'_i u'_j \rangle \right).$$



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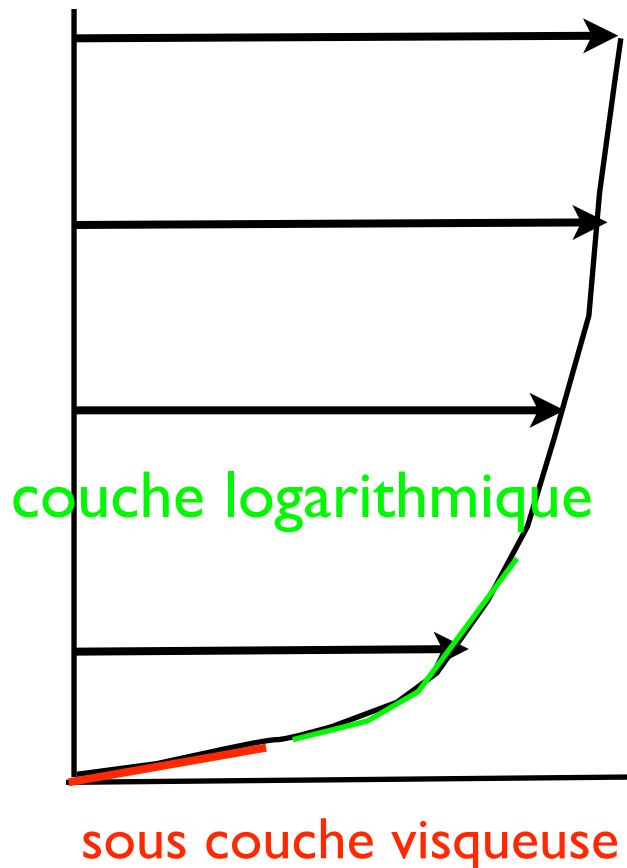


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$$u_\tau = \sqrt{(\tau_p / \rho)} :$$

$$\delta^+ = \frac{\nu}{u_\tau}$$

$$U^+ = \frac{\langle u \rangle}{u_\tau}$$



$$U^+ = 2.4 \text{Log}(y^+) + 5$$

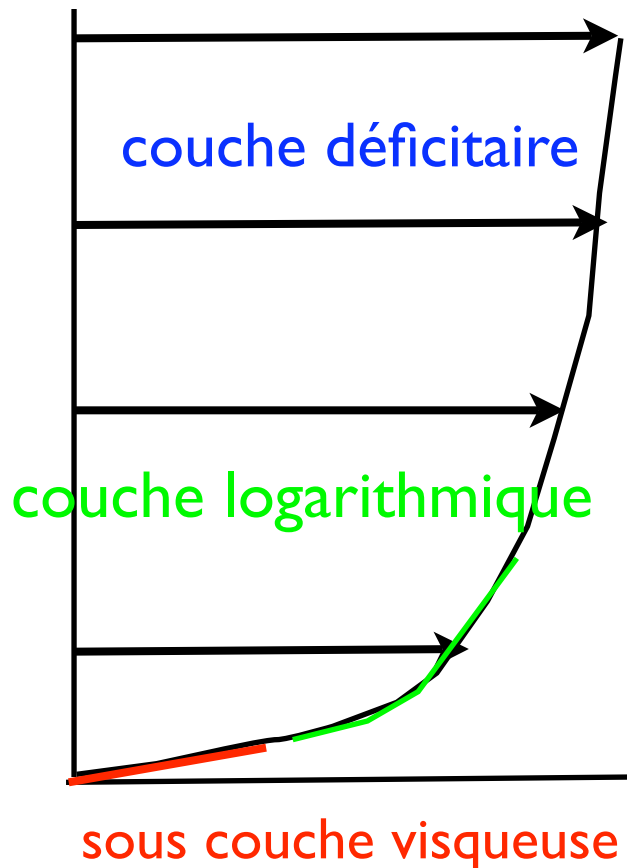
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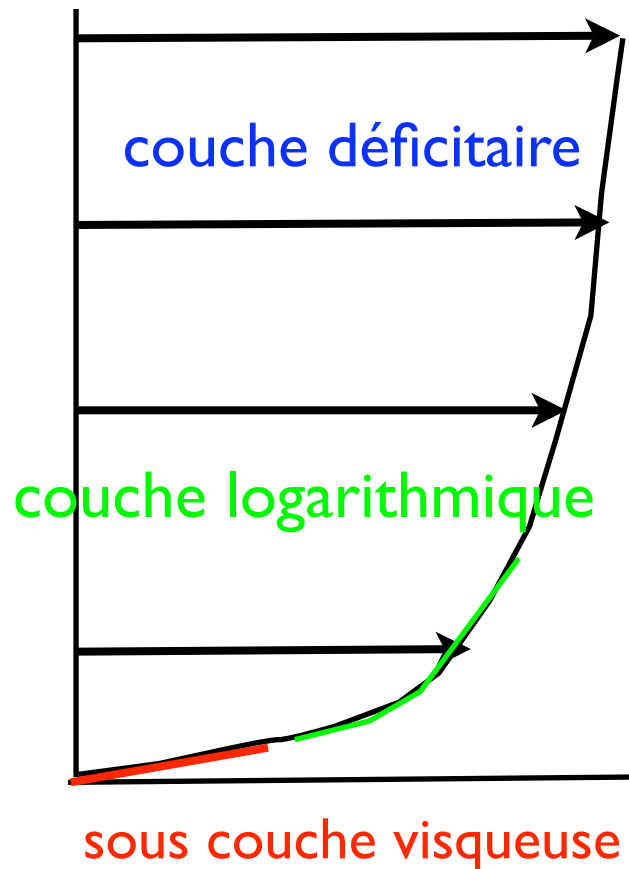
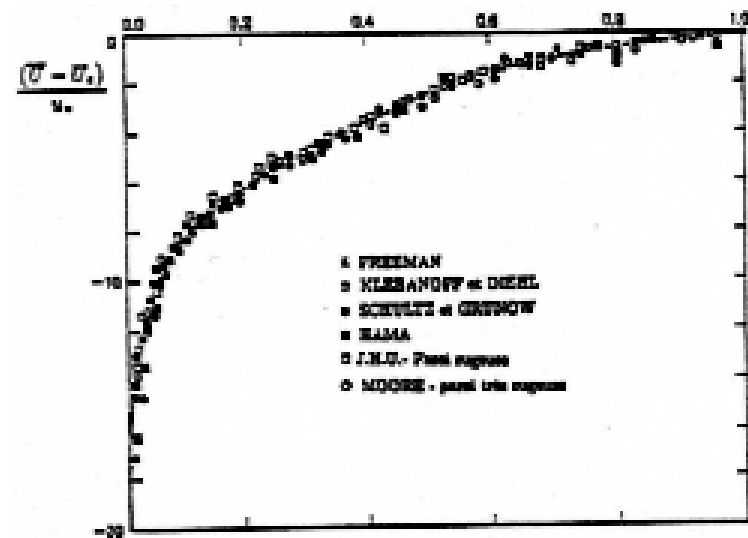
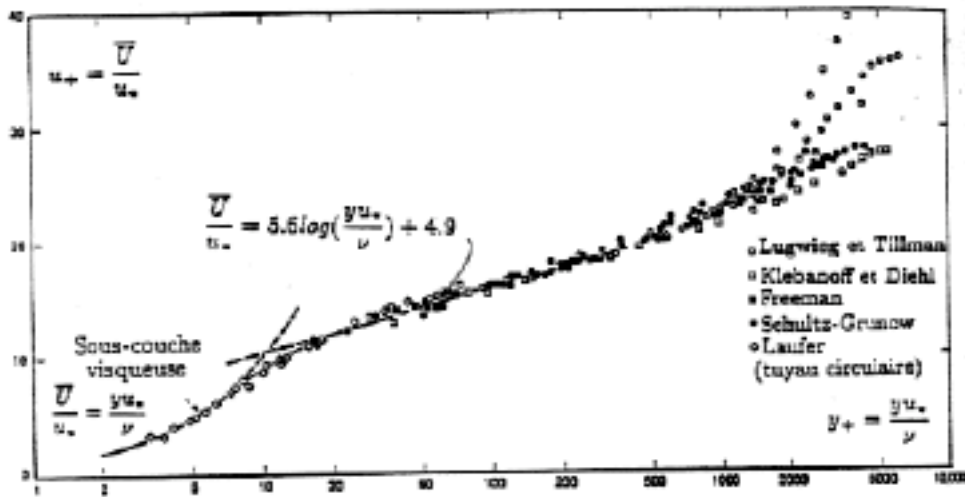
$$u_\tau = \sqrt{(\tau_p / \rho)}$$

$$\delta^+ = \frac{y}{u_\tau}$$

$$U^+ = \frac{\langle u \rangle}{u_\tau}$$

$$U^+ = 2.4 \text{Log}(y^+) + 5$$

$$U^+ = y^+$$



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$$U^+ = 2.4 \text{Log}(y^+) + 5$$

$$U^+ = y^+$$

$$\rho \langle u_i \rangle \frac{\partial \langle u_i \rangle}{\partial x} = -\frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial \langle u_i \rangle}{\partial x_j} - \rho \langle u'_i u'_j \rangle \right).$$

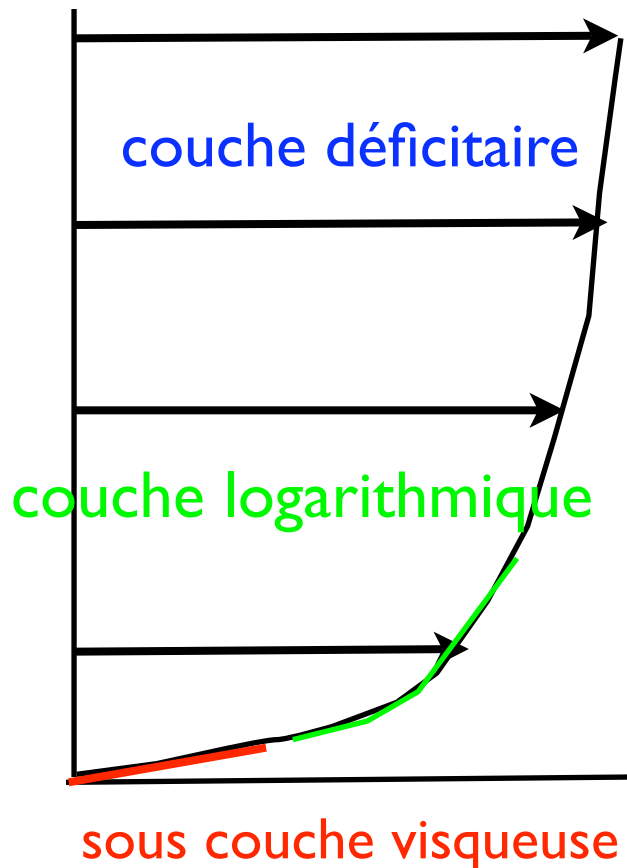


$$\frac{\partial}{\partial y} \left(\nu \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) \simeq 0$$



$$\left(\nu \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) = \text{constante}.$$

fermeture pour $-\langle u'v' \rangle$



$$\rho \langle u_i \rangle \frac{\partial \langle u_i \rangle}{\partial x} = -\frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial \langle u_i \rangle}{\partial x_j} - \rho \langle u'_i u'_j \rangle \right).$$

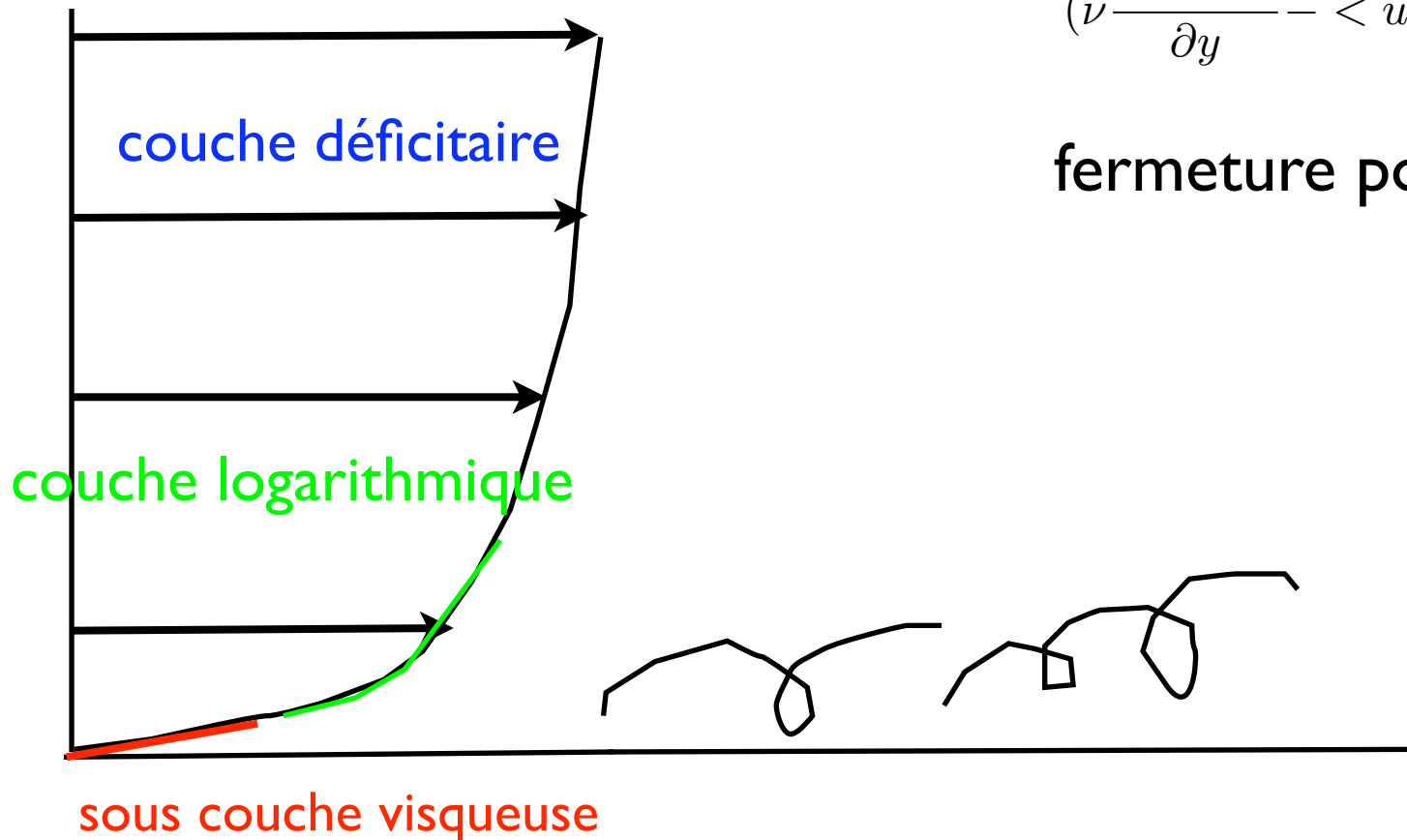


$$\frac{\partial}{\partial y} \left(\nu \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) \simeq 0$$



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$$\rho \langle u_i \rangle \frac{\partial \langle u_i \rangle}{\partial x} = -\frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial \langle u_i \rangle}{\partial x_j} - \rho \langle u'_i u'_j \rangle \right).$$



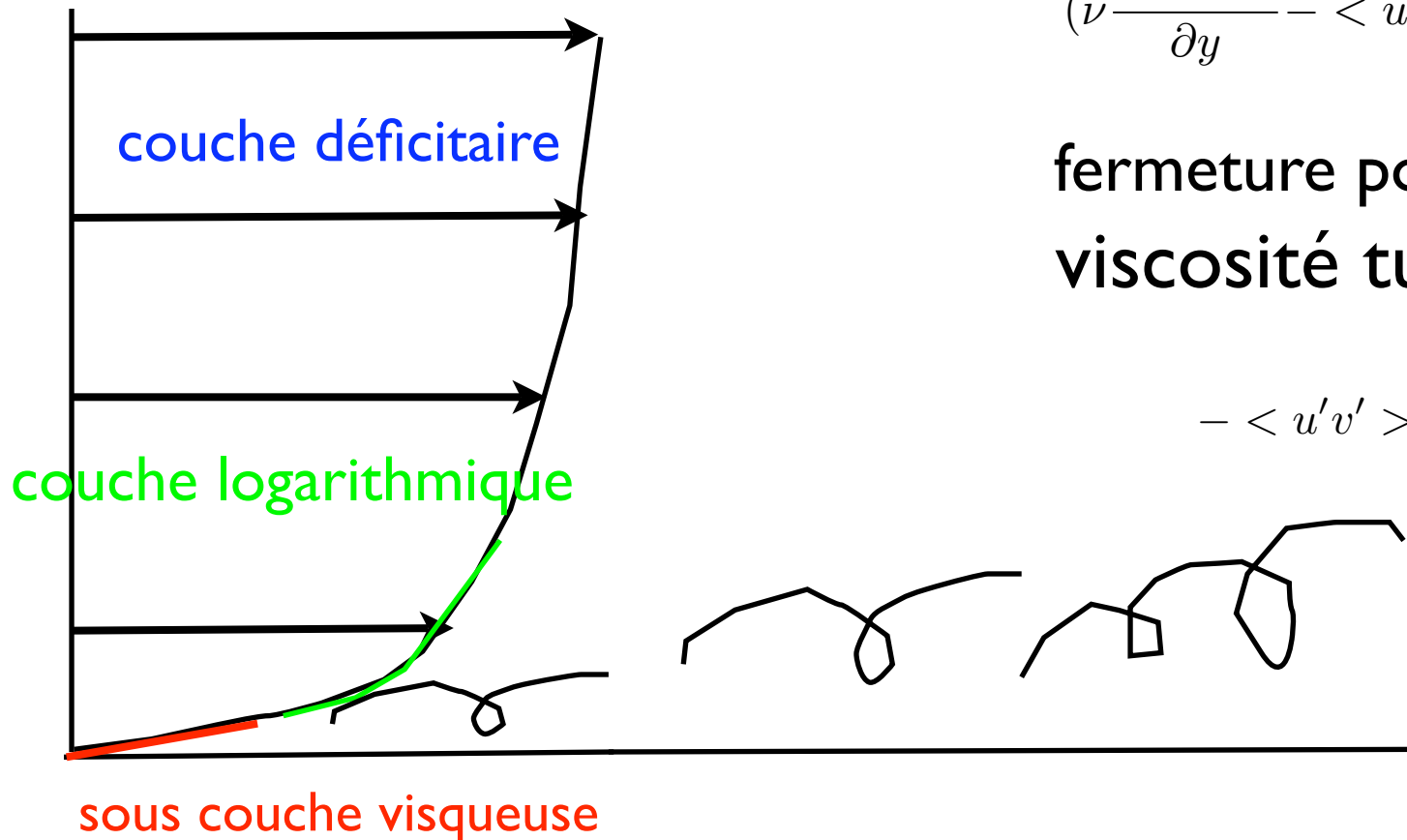
$$\frac{\partial}{\partial y} \left(\nu \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) \simeq 0$$



$$\left(\nu \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) = \text{constante.}$$

fermeture pour $-\langle u'v' \rangle$
viscosité turbulente

$$-\langle u'v' \rangle = \nu_t \frac{\partial \langle u \rangle}{\partial y}.$$



$$\rho \langle u_i \rangle \frac{\partial \langle u_i \rangle}{\partial x} = - \frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial \langle u_i \rangle}{\partial x_j} - \rho \langle u'_i u'_j \rangle \right).$$

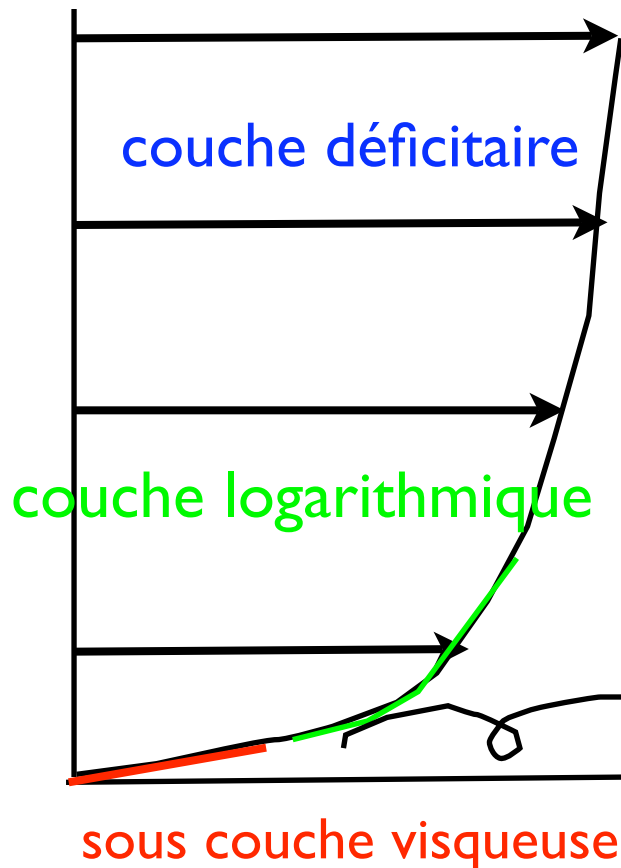


$$\frac{\partial}{\partial y} \left(\nu \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) \simeq 0$$

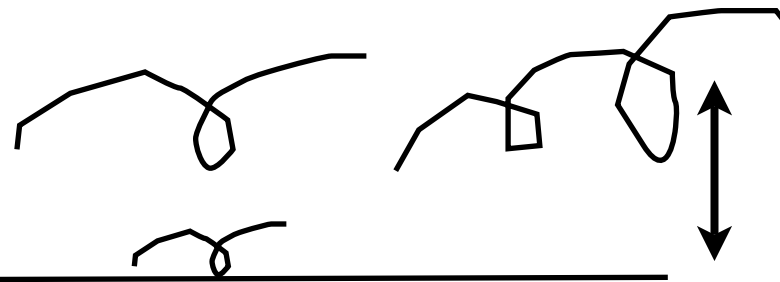


$$\left(\nu \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) = \text{constante}.$$

fermeture pour $-\langle u'v' \rangle$
viscosité turbulente



$$-\langle u'v' \rangle = \nu_t \frac{\partial \langle u \rangle}{\partial y}.$$



$$\rho \langle u_i \rangle \frac{\partial \langle u_i \rangle}{\partial x} = - \frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial \langle u_i \rangle}{\partial x_j} - \rho \langle u'_i u'_j \rangle \right).$$



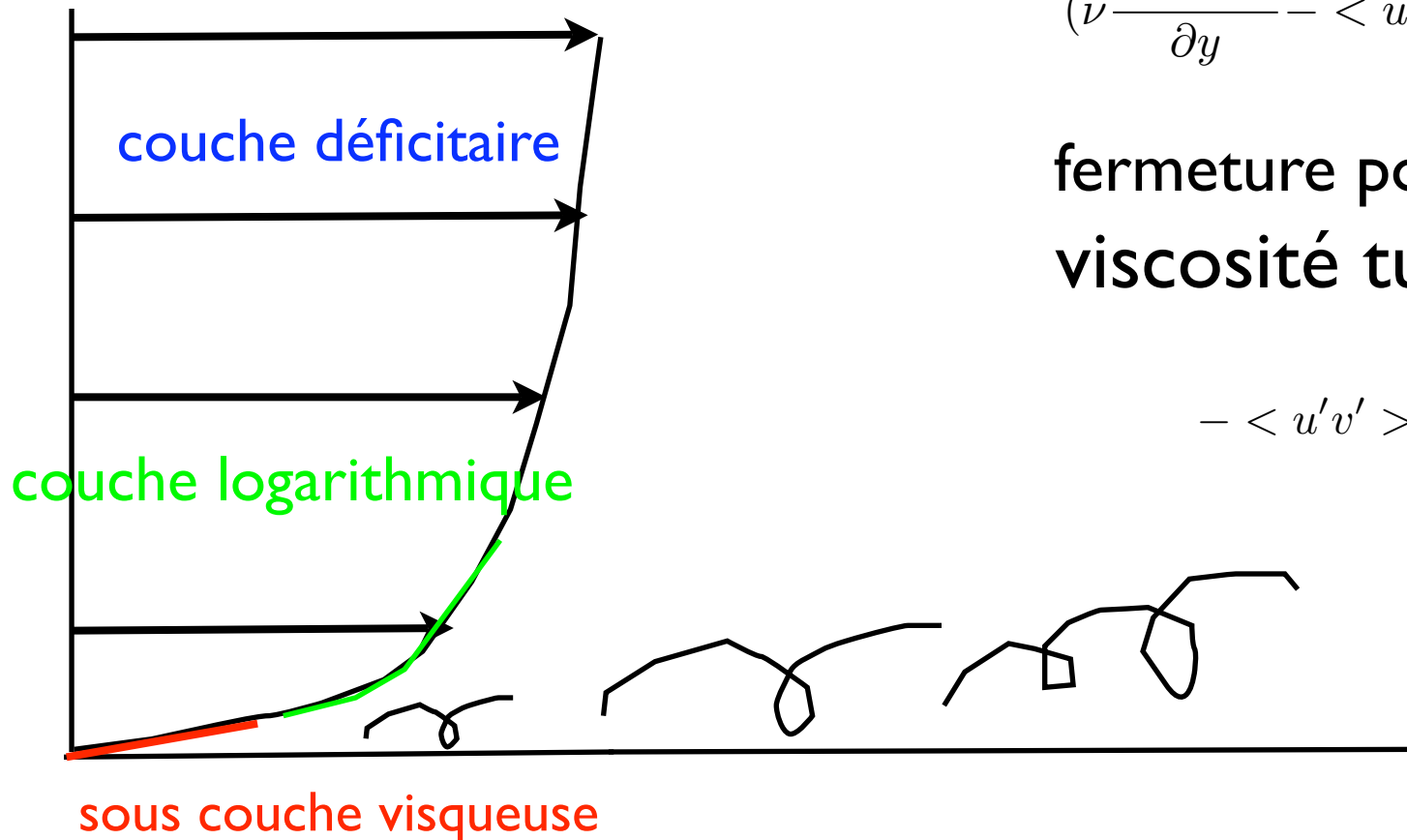
$$\frac{\partial}{\partial y} \left(\nu \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) \simeq 0$$



$$\left(\nu \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) = \text{constante}.$$

fermeture pour $-\langle u'v' \rangle$
viscosité turbulente

$$-\langle u'v' \rangle = \nu_t \frac{\partial \langle u \rangle}{\partial y}.$$

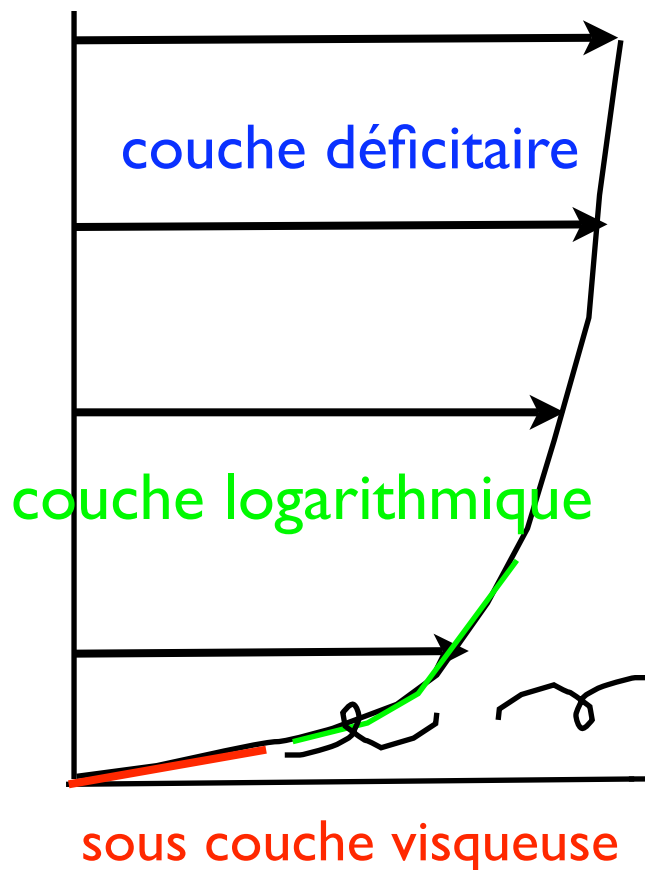


$$\nu_t = l^2 \left| \frac{\partial \langle u \rangle}{\partial y} \right|$$

$$\left(\nu \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) = \text{constante.}$$

viscosité turbulente

$$- \langle u'v' \rangle = \nu_t \frac{\partial \langle u \rangle}{\partial y}.$$

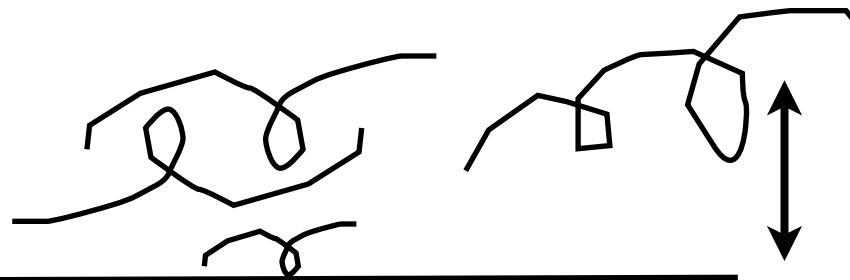
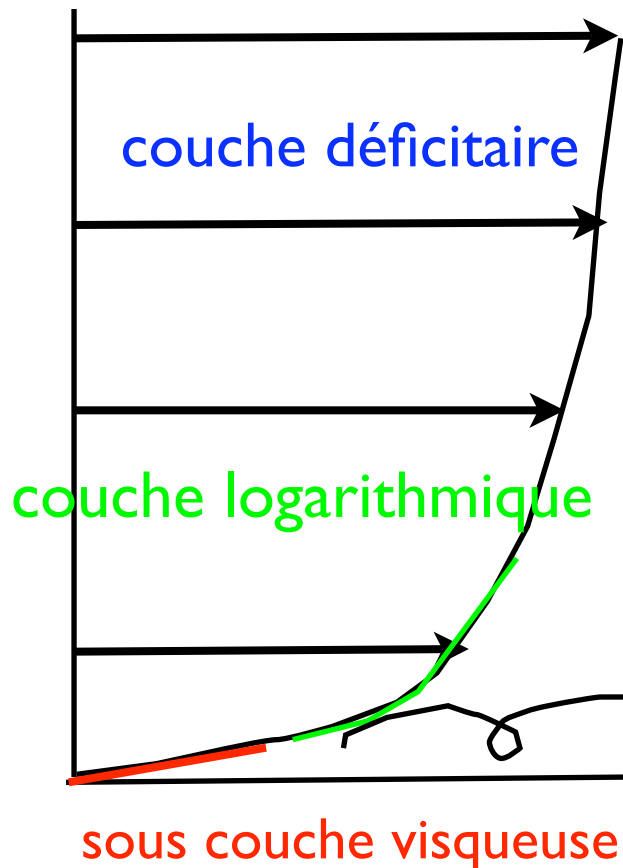


$$\nu_t = l^2 \left| \frac{\partial \langle u \rangle}{\partial y} \right| \quad l = \kappa y.$$

$$\left(\nu \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) = \text{constante}.$$

viscosité turbulente

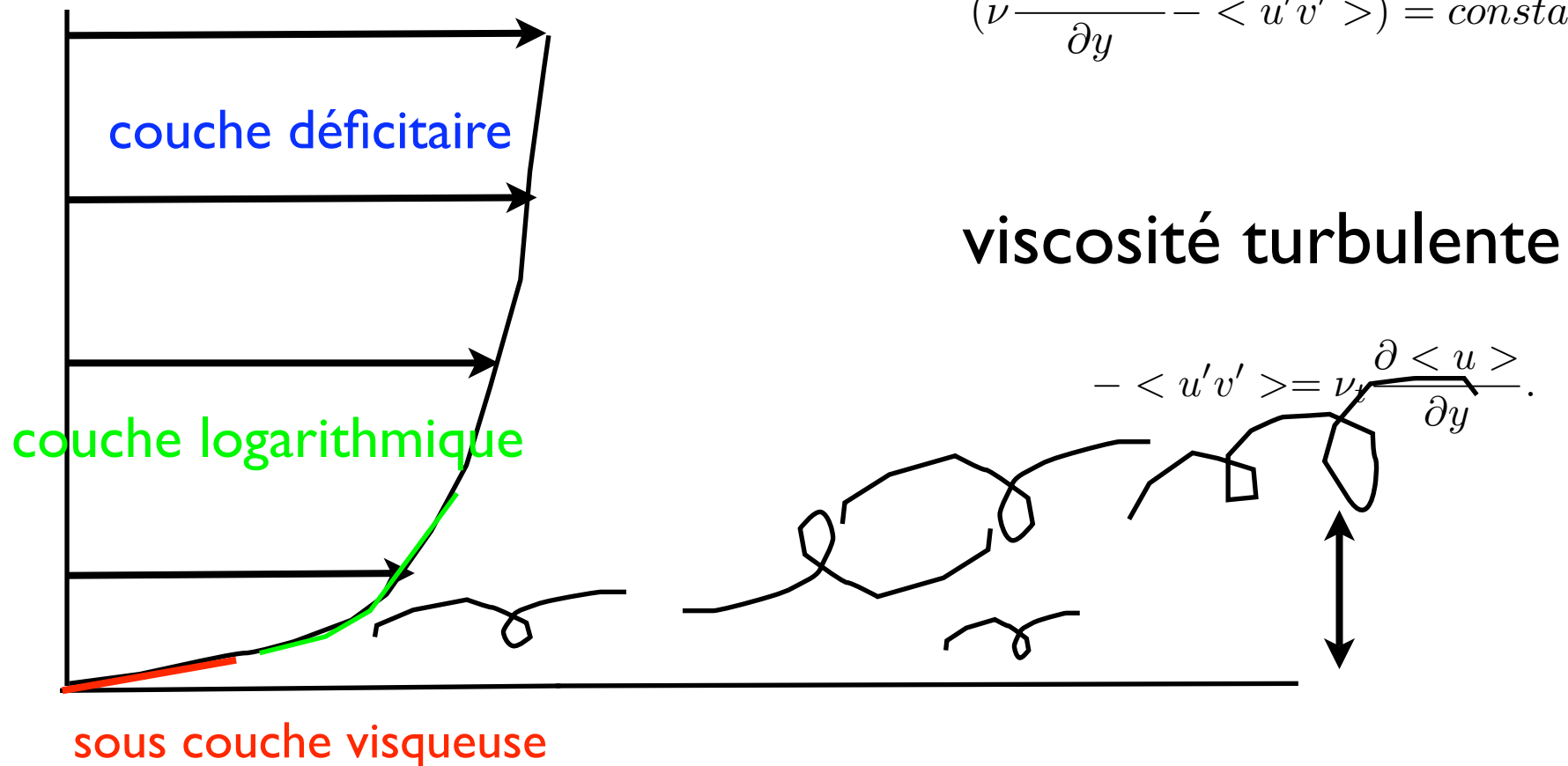
$$- \langle u'v' \rangle = \nu_t \frac{\partial \langle u \rangle}{\partial y}.$$



On retrouve la couche visqueuse et la loi log

$$\nu_t = l^2 \left| \frac{\partial \langle u \rangle}{\partial y} \right| \quad l = \kappa y.$$

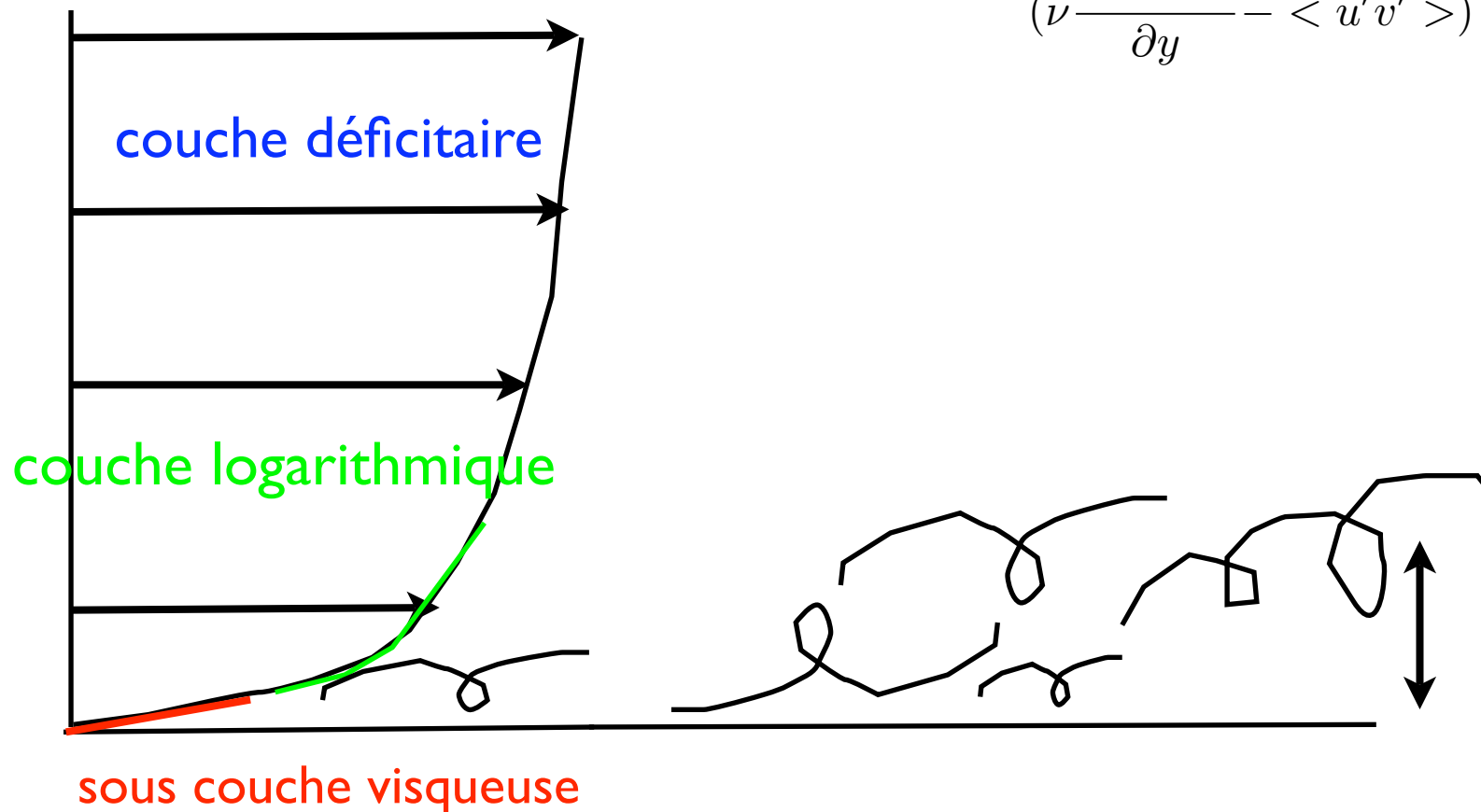
$$\left(\nu \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) = \text{constante}.$$



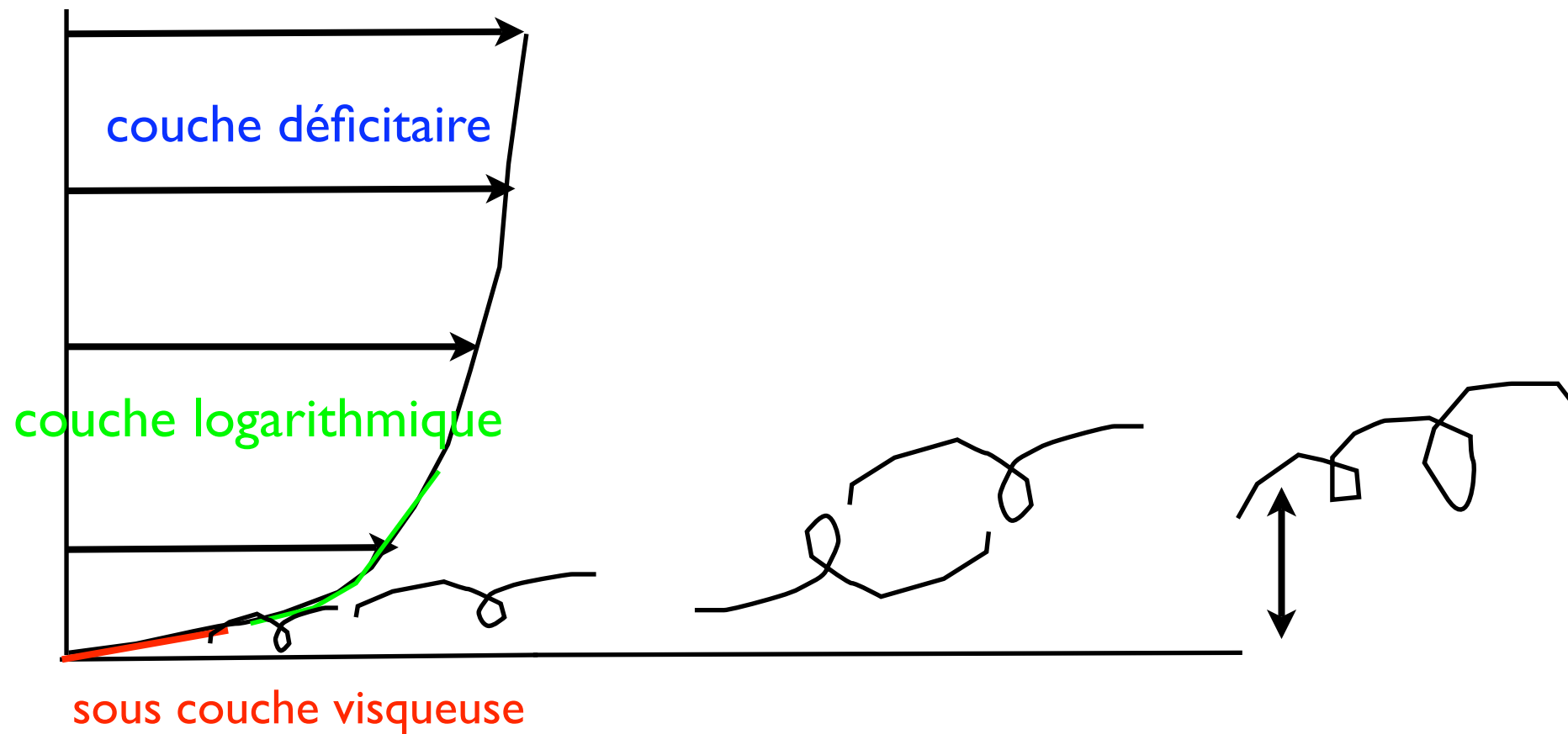
On retrouve la couche visqueuse et la loi log

$$\nu_t = l^2 \left| \frac{\partial \langle u \rangle}{\partial y} \right| \quad l = \kappa y.$$

$$\left(\nu \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) = \text{constante}.$$

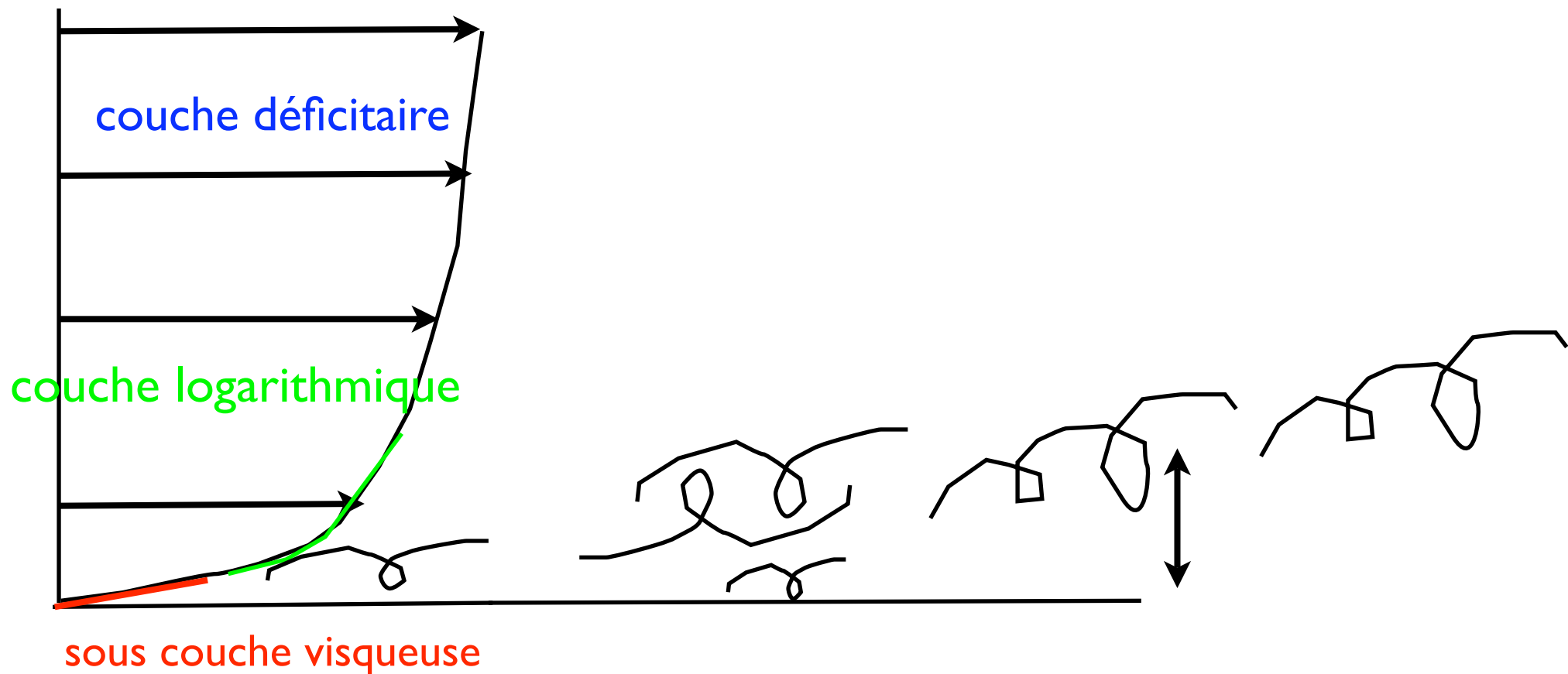


$$\frac{\partial}{\partial y} \left(\nu \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) = 0$$



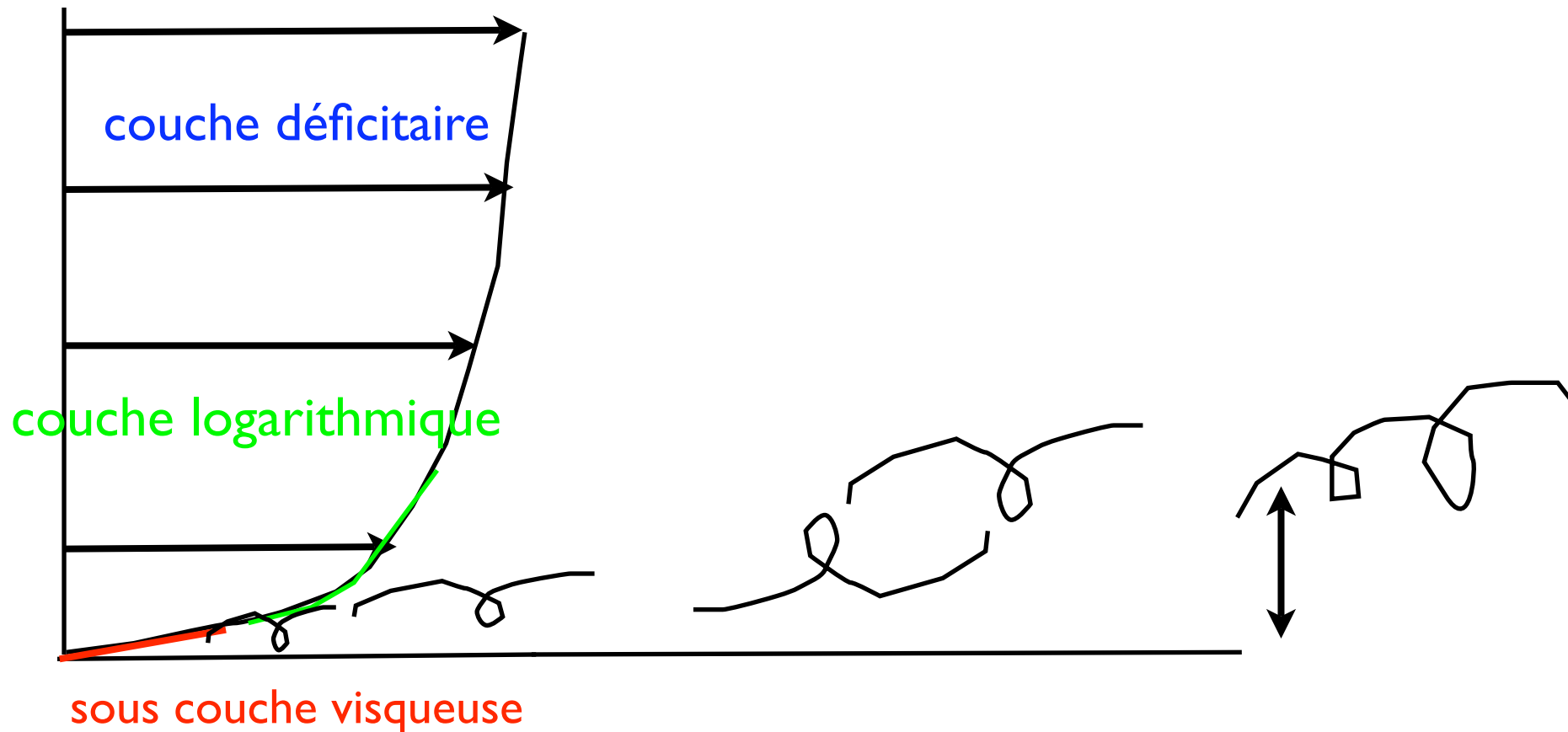
$$\frac{\partial}{\partial y} \left(\nu_t \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) = 0$$

$$\frac{\partial}{\partial y} \left(\nu \frac{\partial \langle u \rangle}{\partial y} + \kappa^2 y^2 \left(\frac{\partial \langle u \rangle}{\partial y} \right)^2 \right) = 0$$



$$\frac{\partial}{\partial y} \left(\nu_t \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) = 0$$

$$\frac{\partial}{\partial y} \left(\nu \frac{\partial \langle u \rangle}{\partial y} + \kappa^2 y^2 \left(\frac{\partial \langle u \rangle}{\partial y} \right)^2 \right) = 0$$

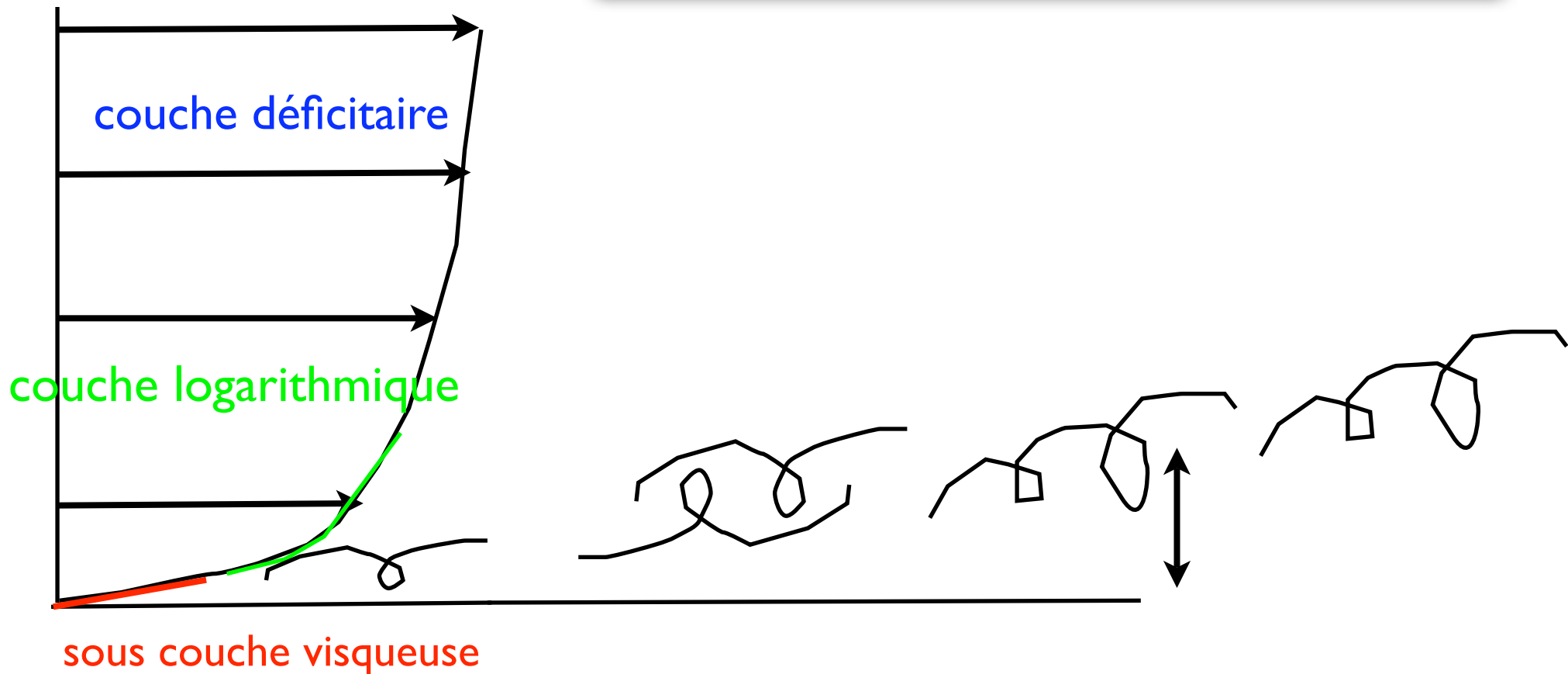


$$\frac{\partial}{\partial y} \left(\nu_t \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) = 0$$

$$\frac{\partial}{\partial y} \left(\nu \frac{\partial \langle u \rangle}{\partial y} + \kappa^2 y^2 \left(\frac{\partial \langle u \rangle}{\partial y} \right)^2 \right) = 0$$

$$\tau_p = \mu \frac{\partial \langle u \rangle}{\partial y} : \quad y^+ = y u_\tau / \nu = y / \delta^+.$$

$$\langle u \rangle / u_\tau = u_\tau y / \nu, \text{ ou } U^+ = y^+$$

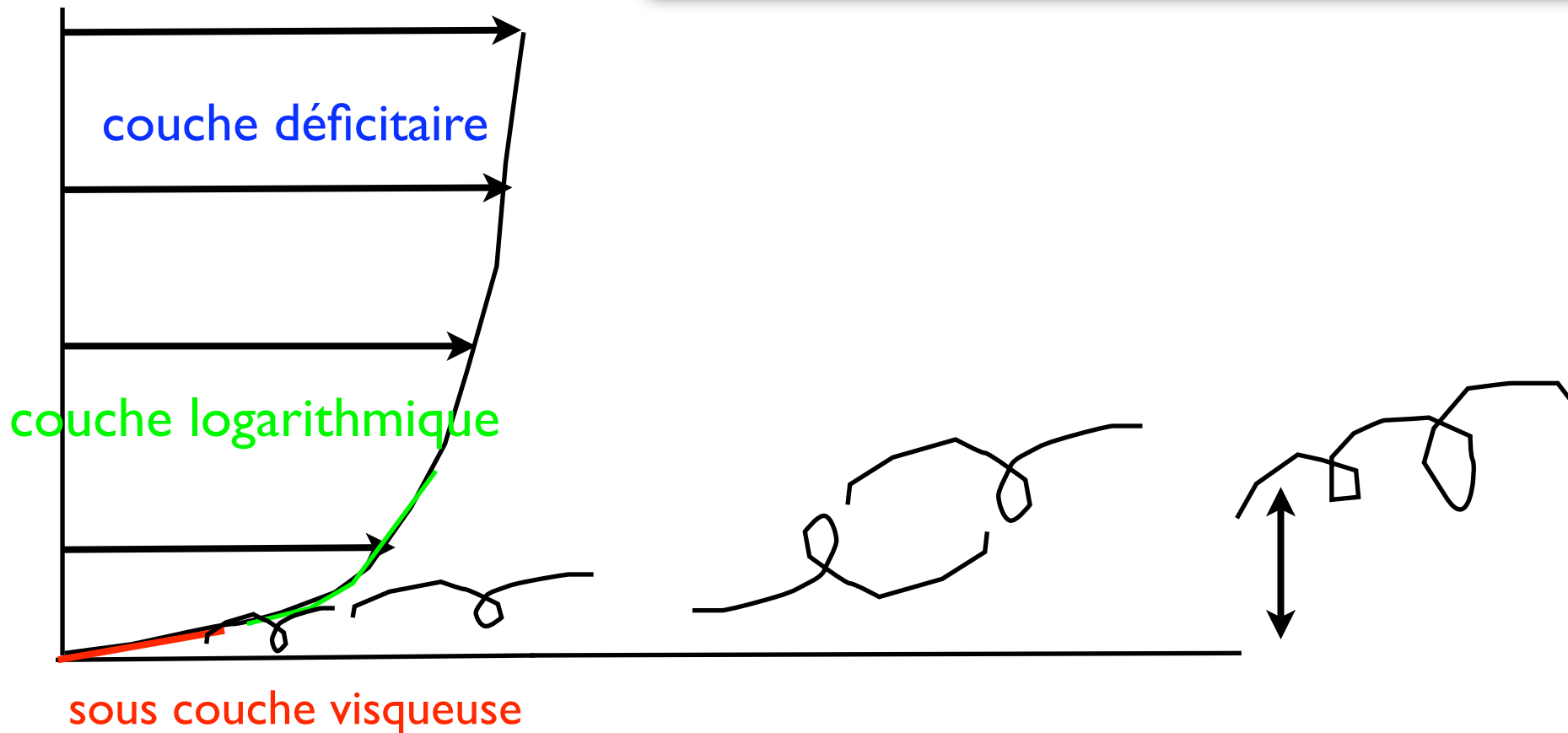


$$\frac{\partial}{\partial y} \left(\nu_t \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) = 0$$

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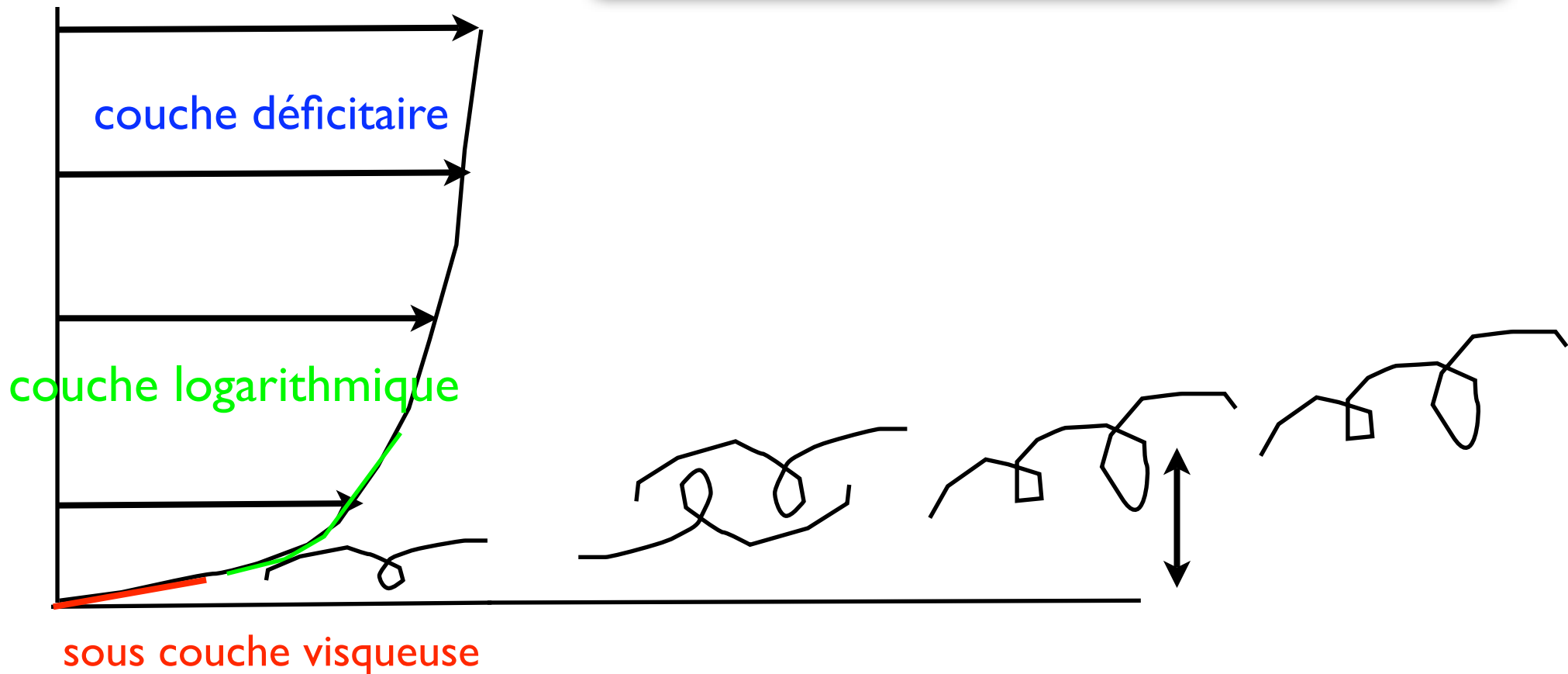
$$\frac{\partial}{\partial y} \left(\nu_t \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) = 0$$

~~$$\frac{\partial}{\partial y} \left(\nu \frac{\partial \langle u \rangle}{\partial y} + \kappa^2 y^2 \left(\frac{\partial \langle u \rangle}{\partial y} \right)^2 \right) = 0$$~~

$$\tau_p = \mu \frac{\partial \langle u \rangle}{\partial y} :$$

$$y^+ = y u_\tau / \nu = y / \delta^+.$$

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$$\frac{\partial}{\partial y} \left(\nu_t \frac{\partial \langle u \rangle}{\partial y} - \langle u'v' \rangle \right) = 0$$

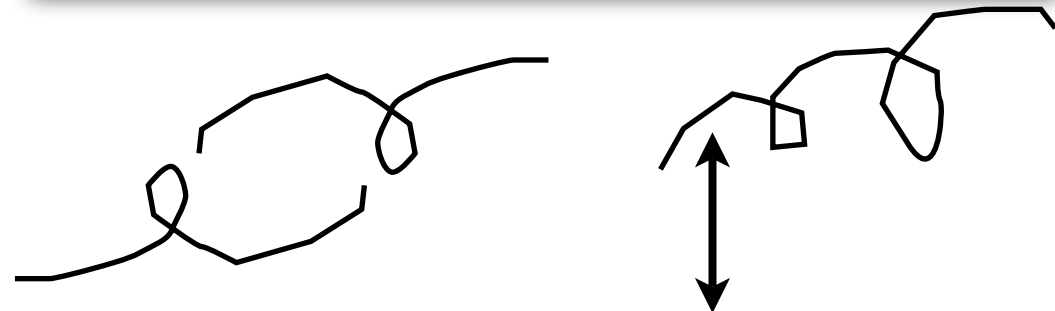
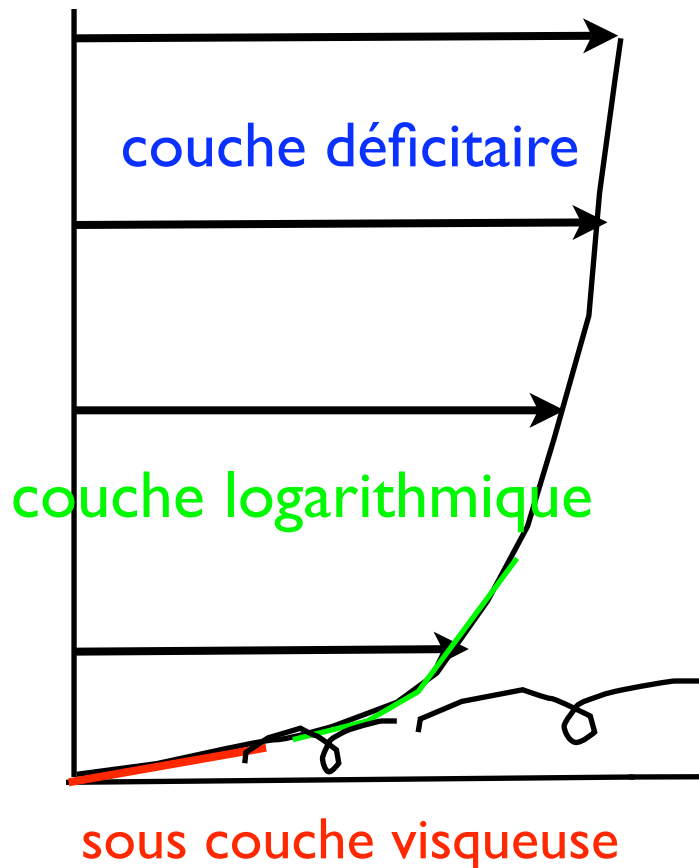
~~$$\frac{\partial}{\partial y} \left(\nu \frac{\partial \langle u \rangle}{\partial y} + \kappa^2 y^2 \left(\frac{\partial \langle u \rangle}{\partial y} \right)^2 \right) = 0$$~~

$$\tau_p = \mu \frac{\partial \langle u \rangle}{\partial y} : \quad y^+ = y u_\tau / \nu = y / \delta^+.$$

$$\langle u \rangle / u_\tau = u_\tau y / \nu, \text{ ou } U^+ = y^+$$

$$\frac{\partial \langle u \rangle}{\partial y} \sim \frac{u_\tau}{y}$$

$$\langle u \rangle / u_\tau = 2.4 \text{Log}(u_\tau y / \nu) + 5$$



Equation de la chaleur

$$\langle u \rangle \frac{\partial \langle T \rangle}{\partial x} + \langle v \rangle \frac{\partial \langle T \rangle}{\partial y} = \frac{\partial}{\partial y} \left(\frac{k}{\rho c_p} \frac{\partial \langle T \rangle}{\partial y} - \langle v' T' \rangle \right) + etc$$

$$- \langle v' T' \rangle = \frac{\nu_t}{Pr_t} \frac{\partial}{\partial y} \langle T \rangle$$

$$T_\tau = - \langle q_w \rangle / (\rho c_p u_\tau)$$

Equation de la chaleur

$$\langle u \rangle \frac{\partial \langle T \rangle}{\partial x} + \langle v \rangle \frac{\partial \langle T \rangle}{\partial y} = \frac{\partial}{\partial y} \left(\frac{k}{\rho c_p} \frac{\partial \langle T \rangle}{\partial y} - \langle v' T' \rangle \right) + etc$$

$$- \langle v' T' \rangle = \frac{\nu_t}{Pr_t} \frac{\partial \langle T \rangle}{\partial y}$$

$$T_\tau = - \langle q_w \rangle / (\rho c_p u_\tau)$$

loi lin.

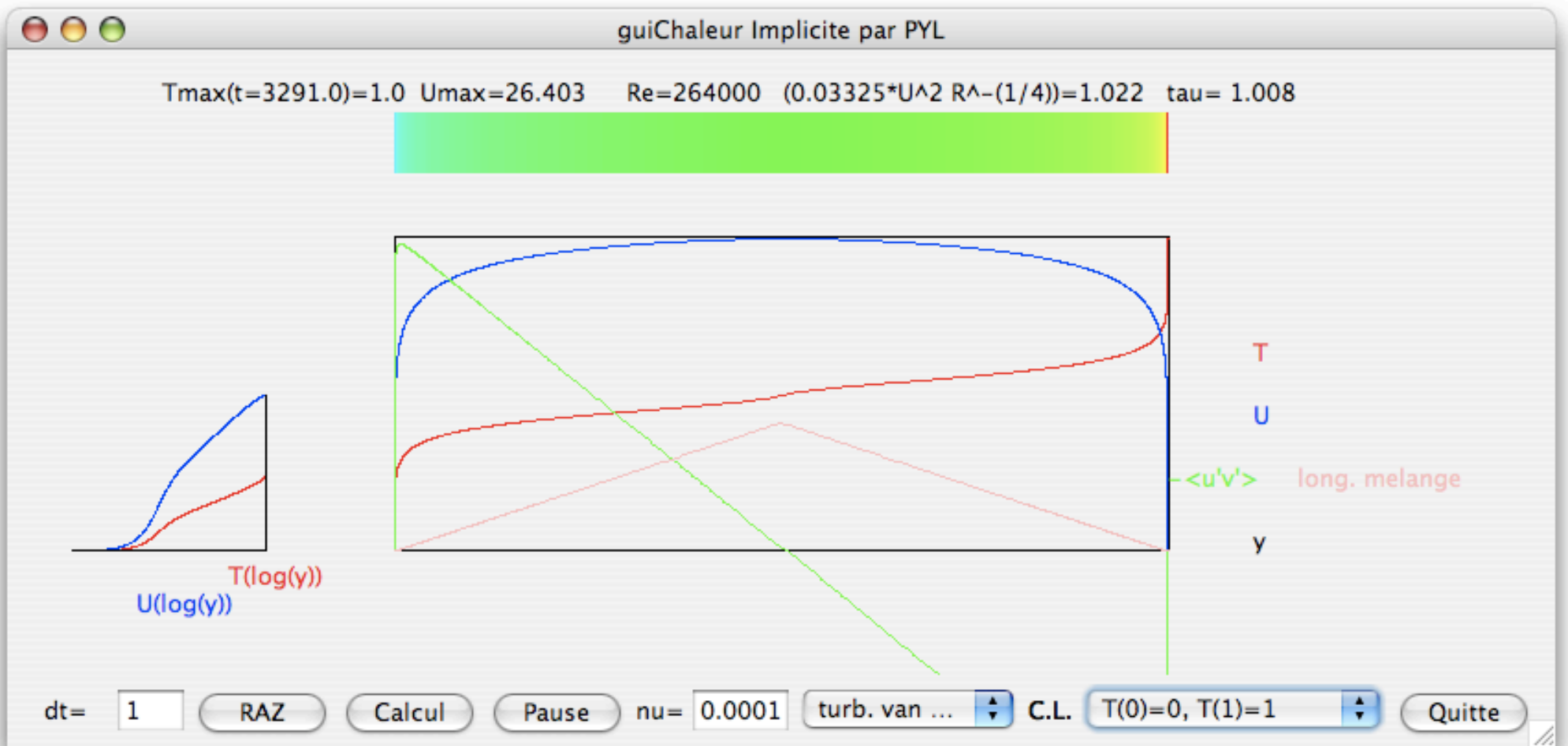
$$\frac{\langle T \rangle - T_w}{T_\tau} \sim \frac{y \nu Pr_t}{u_\tau}$$

loi log.

$$\frac{\langle T \rangle - T_w}{T_\tau} \sim \frac{Pr_t}{\kappa} \text{Log}\left(\frac{y \nu}{u_\tau}\right) + C(Pr),$$

$$Nu_L \sim 0.03 Re_L^{4/5}.$$

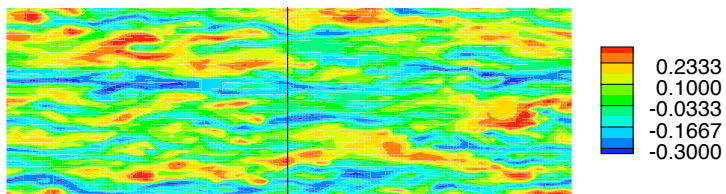
Rappel: la PC Poiseuille Turbulent



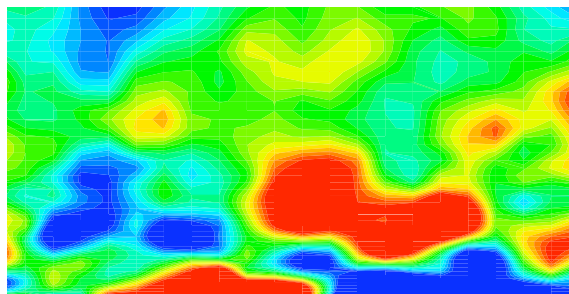
TAB. 4.2 – Valeurs des paramètres de discrétisation

L_x	L_y	L_z	N_x	N_y	N_z	z_0	CFL
3π	π	1	192	64	129	$5 \cdot 10^{-3}$	0.5

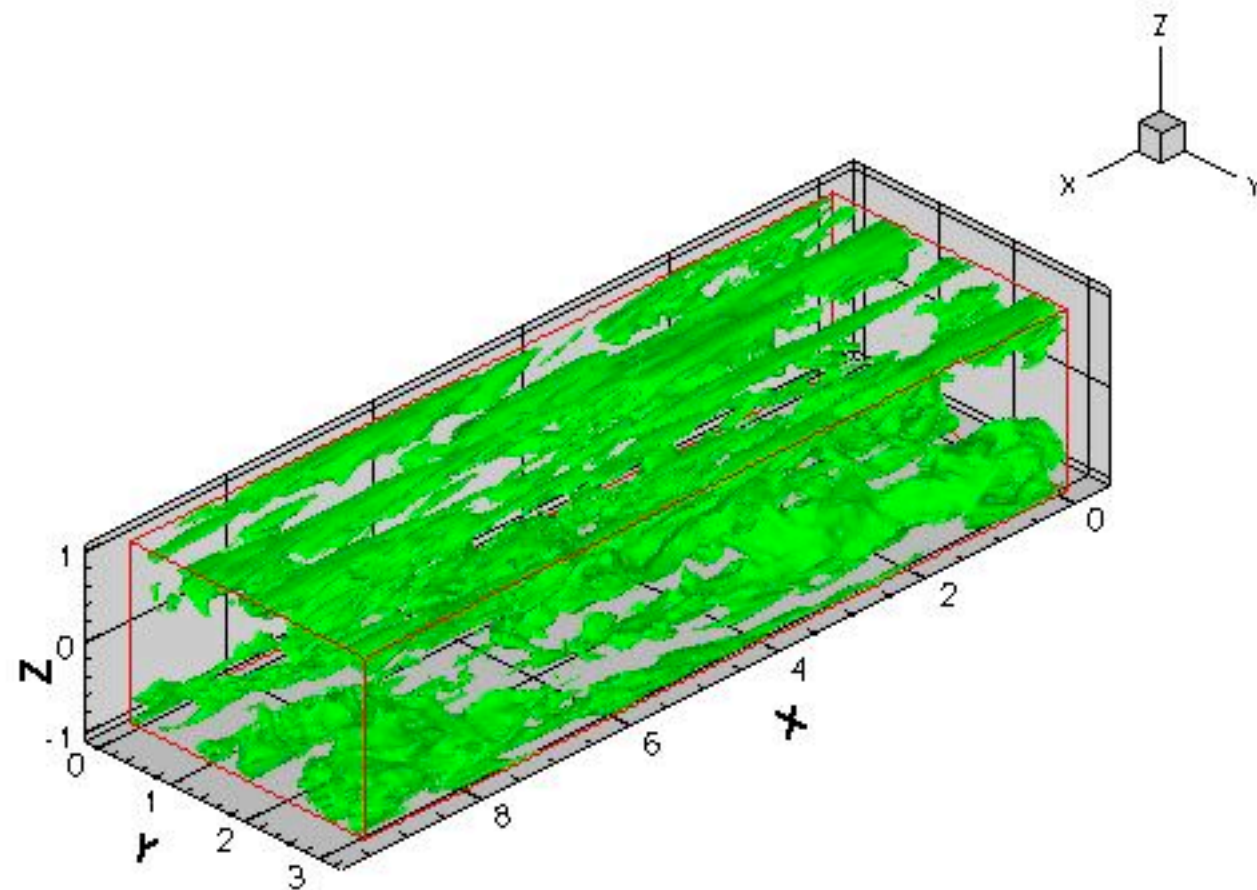
Re	Pr	γ	T_0
3121.044	0.72	1.2	293.5

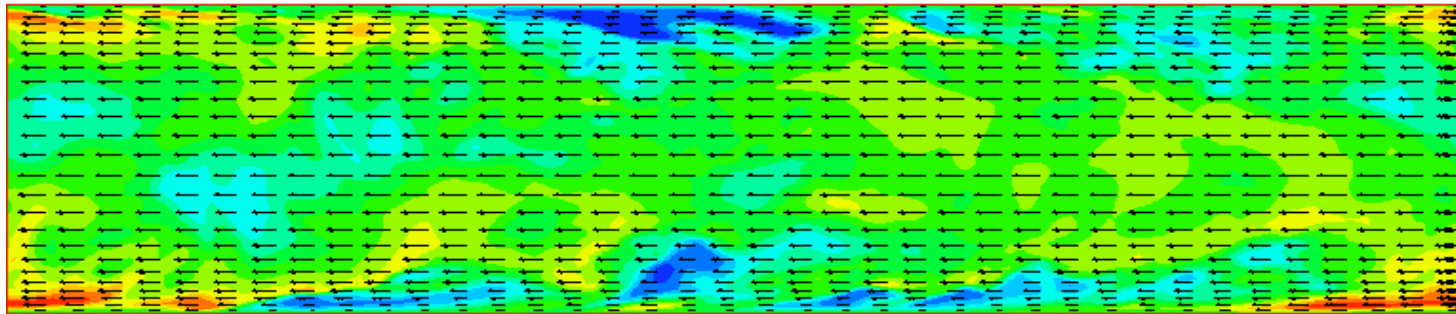


(a) Streaks haute et basse vitesses

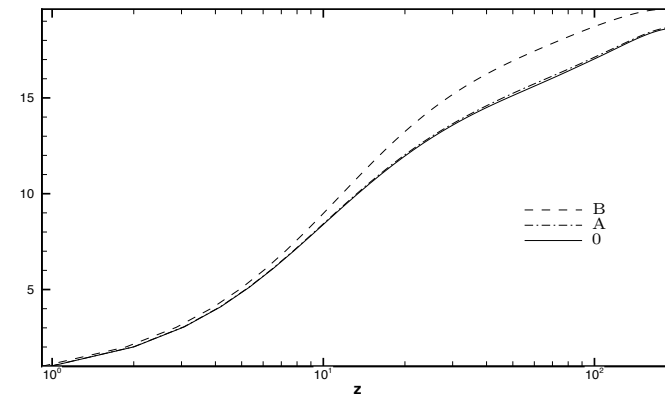
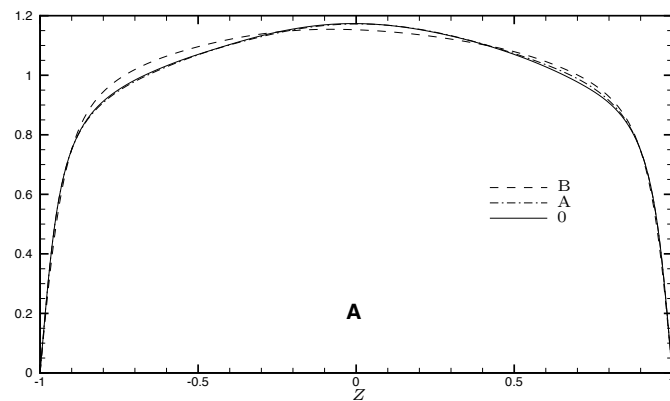


(b) Une paire de tourbillons contrarotatifs.





(a) $R_b Re_\sigma = 10^{-3}$



100h de calcul par DNS

Equations de Navier Stokes moyennées, $\langle u_i \rangle = U_i$

$$\frac{\partial U_i}{\partial x_i} = 0.$$

$$\rho U_i \frac{\partial}{\partial x_j} U_i = - \frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial U_i}{\partial x_j} - \rho \langle u_i' u_j' \rangle \right).$$

On a fait apparaître le terme que l'on appelle terme de tension turbulente: $-\rho \langle u_i' u_j' \rangle$.

k ε k epsilon

Equation du transport de k , $k = \frac{1}{2} \langle u_i'^2 \rangle$

$$U_i \frac{\partial k}{\partial x_i} = - \frac{\partial}{\partial x_j} \left(\frac{1}{\rho} \langle u_j' p' \rangle + \frac{1}{2} \langle u_i' u_i' u_j' \rangle - 2 \nu \langle u_i' s'_{ij} \rangle \right) + P - \varepsilon$$

$$\varepsilon = 2\nu \langle s'_{ij} s'_{ij} \rangle, P = - \langle u_i' u_j' \rangle S_{ij}.$$

$$S_{ij} = (U_{i,j}' + U_{j,i}')/2 \text{ et } s'_{ij} = (u_{i,j}' + u_{j,i}')/2.$$

$k \varepsilon$ k epsilon

Equation du transport de ε $\varepsilon = 2\nu \langle s'_{ij} s'_{ij} \rangle$

$$\begin{aligned} \rho U_i \frac{\partial \varepsilon}{\partial x_i} = & - 2\mu U_{i,k} (\langle u'_{k,j} u'_{i,k} \rangle + \langle u'_{j,i} u'_{j,k} \rangle) \\ & - 2\mu \langle u'_{i,j} u'_{i,k} u'_{k,j} \rangle - \mu \partial_k \langle u'_k u'_{i,j} u'_{i,j} \rangle \\ & - 2\nu \partial_i \langle p'_{,j} u'_{i,j} \rangle - 2\rho \nu^2 \langle u'_{i,k} u'_{i,jk} \rangle \\ & - 2\mu U_{i,kj} \langle u'_k u'_{i,j} \rangle + \mu \varepsilon_{,kk}. \end{aligned}$$

$k \epsilon$ k epsilon

Fermeture

$$-(\langle u'_i u'_j \rangle - \frac{2}{3} \delta_{ij} k) = \nu_t \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial U_j}{\partial x_j} \right)$$

$$\nu_t = c_\mu \frac{k^2}{\epsilon}.$$

k ε k epsilon

Equation du transport de k , $k = \frac{1}{2} \langle u_i'^2 \rangle$

$$U_i \frac{\partial k}{\partial x_i} = - \frac{\partial}{\partial x_j} \left(\frac{1}{\rho} \langle u_j' p' \rangle + \frac{1}{2} \langle u_i' u_i' u_j' \rangle - 2 \nu \langle u_i' s'_{ij} \rangle \right) + P - \varepsilon$$

$$\varepsilon = 2\nu \langle s'_{ij} s'_{ij} \rangle, P = - \langle u_i' u_j' \rangle S_{ij}.$$

$$S_{ij} = (U_{i,j}' + U_{j,i}')/2 \text{ et } s'_{ij} = (u_{i,j}' + u_{j,i}')/2.$$

k ϵ k epsilon

Equation du transport de k , $k = \frac{1}{2} \langle u_i'^2 \rangle$

$$U_i \frac{\partial k}{\partial x_i} = - \frac{\partial}{\partial x_j} \left(\frac{1}{\rho} \langle u_j' p' \rangle + \frac{1}{2} \langle u_i' u_i' u_j' \rangle - 2 \nu \langle u_i' s'_{ij} \rangle \right) + P - \epsilon$$

$$\epsilon = 2\nu \langle s'_{ij} s'_{ij} \rangle, P = - \langle u_i' u_j' \rangle S_{ij}.$$

$$S_{ij} = (U_{i,j}' + U_{j,i}')/2 \text{ et } s'_{ij} = (u_{i,j}' + u_{j,i}')/2.$$

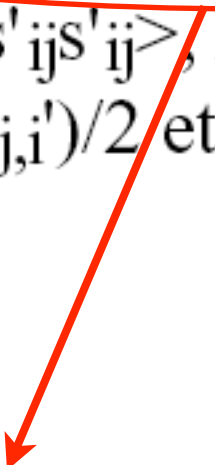
$k \ \epsilon$ **k epsilon**

Equation du transport de k, $k = \frac{1}{2} \langle u_i'^2 \rangle$

$$U_i \frac{\partial k}{\partial x_i} = - \frac{\partial}{\partial x_j} \left(\frac{1}{\rho} \langle u_j' p' \rangle + \frac{1}{2} \langle u_i' u_i' u_j' \rangle - 2 \nu \langle u_i' s'_{ij} \rangle \right) + P - \epsilon$$

$$\epsilon = 2\nu \langle s'_{ij} s'_{ij} \rangle, \quad P = - \langle u_i' u_j' \rangle S_{ij}.$$

$$S_{ij} = (U_{i,j}' + U_{j,i}')/2 \text{ et } s'_{ij} = (u_{i,j}' + u_{j,i}')/2.$$



$$U_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_j} \left((\nu + \sigma k^{-1} \nu_t) \frac{\partial k}{\partial x_j} \right) - \langle u_i' u_j' \rangle \frac{\partial U_i}{\partial x_j} - \epsilon.$$

$k \varepsilon$ k epsilon

Equation du transport de ε $\varepsilon = 2\nu \langle s'_{ij} s'_{ij} \rangle$

$$\begin{aligned} \rho U_i \frac{\partial \varepsilon}{\partial x_i} = & - 2\mu U_{i,k} (\langle u'_{k,j} u'_{i,k} \rangle + \langle u'_{j,i} u'_{j,k} \rangle) \\ & - 2\mu \langle u'_{i,j} u'_{i,k} u'_{k,j} \rangle - \mu \partial_k \langle u'_k u'_{i,j} u'_{i,j} \rangle \\ & - 2\nu \partial_i \langle p'_{,j} u'_{i,j} \rangle - 2\rho \nu^2 \langle u'_{i,k} u'_{i,jk} \rangle \\ & - 2\mu U_{i,kj} \langle u'_k u'_{i,j} \rangle + \mu \varepsilon_{,kk}. \end{aligned}$$

$k \ \epsilon$ k epsilon

Equation du transport de ϵ $\epsilon = 2\nu \langle s'_{ij} s'_{ij} \rangle$

$$\begin{aligned} \rho U_i \frac{\partial \epsilon}{\partial x_i} = & - 2\mu U_{i,k} (\langle u'_{k,j} u'_{i,k} \rangle + \langle u'_{j,j} u'_{i,k} \rangle) \\ & - 2\mu \langle u'_{i,j} u'_{i,k} u'_{k,j} \rangle - \mu \partial_k \langle u'_k u'_{i,j} u'_{i,j} \rangle \\ & - 2\nu \partial_i \langle p'_{,j} u'_{i,j} \rangle - 2\rho \nu^2 \langle u'_{i,kj} u'_{i,jk} \rangle \\ & - 2\mu U_{i,kj} \langle u'_k u'_{i,j} \rangle + \mu \epsilon_{,kk}. \end{aligned}$$

$$U_i \frac{\partial \epsilon}{\partial x_i} = \frac{\partial}{\partial x_j} \left((v + \sigma_\epsilon^{-1} \nu_t) \frac{\partial \epsilon}{\partial x_j} \right) - c_1 \frac{\nu_t \epsilon}{k} \frac{\partial U_i}{\partial x_j} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - c_2 \frac{\epsilon^2}{k}.$$

$k \epsilon$ k epsilon

Equation du transport de ϵ $\epsilon = 2\nu \langle s'_{ij} s'_{ij} \rangle$

$$\rho U_i \frac{\partial \epsilon}{\partial x_i} = -2\mu U_{i,k} (\langle u'_{k,j} u'_{i,k} \rangle + \langle u'_{j,i} u'_{j,k} \rangle) - 2\mu \langle u'_{i,j} u'_{i,k} u'_{k,j} \rangle - \mu \partial_k \langle u'_k u'_{i,j} u'_{i,j} \rangle - 2\nu \partial_i \langle p'_j u'_{i,j} \rangle - 2\rho \nu^2 \langle u'_{i,kj} u'_{i,jk} \rangle - 2\mu U_{i,kj} \langle u'_k u'_{i,j} \rangle + \mu \epsilon_{,kk}.$$

$$U_i \frac{\partial \epsilon}{\partial x_i} = \frac{\partial}{\partial x_j} \left((v + \sigma_\epsilon^{-1} v_t) \frac{\partial \epsilon}{\partial x_j} \right) - c_1 \frac{v_t \epsilon}{k} \frac{\partial U_i}{\partial x_j} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - c_2 \frac{\epsilon^2}{k}.$$

Equation du transport de ϵ $\epsilon = 2\nu \langle s'_{ij} s'_{ij} \rangle$

$$\rho U_i \frac{\partial \epsilon}{\partial x_i} = -2\mu U_{i,k} (\langle u'_{k,j} u'_{i,k} \rangle + \langle u'_{j,j} u'_{i,k} \rangle) - 2\mu \langle u'_{i,j} u'_{i,k} u'_{k,j} \rangle - \mu \partial_k \langle u'_k u'_{i,j} u'_{i,j} \rangle - 2\nu \partial_i \langle p'_j u'_{i,j} \rangle - 2\rho \nu^2 \langle u'_{i,kj} u'_{i,jk} \rangle - 2\mu U_{i,kj} \langle u'_k u'_{i,j} \rangle + \mu \epsilon_{,kk}$$

$$U_i \frac{\partial \epsilon}{\partial x_i} = \frac{\partial}{\partial x_j} \left((v + \sigma_\epsilon^{-1} v_t) \frac{\partial \epsilon}{\partial x_j} \right) - c_1 \frac{v_t \epsilon}{k} \frac{\partial U_i}{\partial x_j} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - c_2 \frac{\epsilon^2}{k}$$

$k \ \varepsilon$ **k epsilon**

Modèle "k epsilon"

$$\frac{\partial U_i}{\partial x_i} = 0,$$
$$\rho U_i \frac{\partial}{\partial x_j} U_i = - \frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial U_i}{\partial x_j} - \rho \langle u_i' u_j' \rangle \right),$$

$$- \langle u_i' u_j' \rangle = \nu_t \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} k$$

$$\nu_t = c_\mu \frac{k^2}{\varepsilon}.$$

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$$\nu_t = c_\mu \frac{k^2}{\varepsilon}.$$

$$U_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_j} \left((v + \sigma_k^{-1} \nu_t) \frac{\partial k}{\partial x_j} \right) - \langle u_i' u_j' \rangle \frac{\partial U_i}{\partial x_j} - \varepsilon.$$

$$U_i \frac{\partial \varepsilon}{\partial x_i} = \frac{\partial}{\partial x_j} \left((v + \sigma_\varepsilon^{-1} \nu_t) \frac{\partial \varepsilon}{\partial x_j} \right) - c_1 \frac{\nu_t \varepsilon}{k} \frac{\partial U_i}{\partial x_j} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - c_2 \frac{\varepsilon^2}{k}.$$

$$c_\mu = 0.09, c_1 = 1.44, c_2 = 1.92, \sigma_k = 1, \sigma_\varepsilon = 1.3$$

Modèle "k epsilon"

$$\frac{\partial U_i}{\partial x_i} = 0,$$

$$\rho U_i \frac{\partial}{\partial x_j} U_i = - \frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_j} (\mu \frac{\partial U_i}{\partial x_j} - \rho \langle u_i' u_j' \rangle),$$

$$- \langle u_i' u_j' \rangle = \nu_t \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} k$$

$$\nu_t = c_\mu \frac{k^2}{\varepsilon}.$$

$$U_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_j} \left((v + \sigma_k^{-1} \nu_t) \frac{\partial k}{\partial x_j} \right) - \langle u_i' u_j' \rangle \frac{\partial U_i}{\partial x_j} - \varepsilon.$$

$$U_i \frac{\partial \varepsilon}{\partial x_i} = \frac{\partial}{\partial x_j} \left((v + \sigma_\varepsilon^{-1} \nu_t) \frac{\partial \varepsilon}{\partial x_j} \right) - c_1 \frac{\nu_t \varepsilon}{k} \frac{\partial U_i}{\partial x_j} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - c_2 \frac{\varepsilon^2}{k}.$$

$$c_\mu = 0.09, c_1 = 1.44, c_2 = 1.92, \sigma_k = 1, \sigma_\varepsilon = 1.3$$

Avec sur les parois engraissées d'une épaisseur y_p
limitant le fluide:

$$U_n = 0, \quad \frac{U_t}{(v \partial U_t / \partial n)^{1/2}} = \kappa^{-1} \text{Log} \left(\frac{y_p}{(v \partial U_t / \partial n)^{1/2}} \right) + 2.4$$

$$\text{où } 20 < \frac{y_p}{\sqrt{(v \partial U_t / \partial n)}} < 100.$$

Modèle "k epsilon"

$$\frac{\partial U_i}{\partial x_i} = 0,$$

$$\rho U_i \frac{\partial}{\partial x_j} U_i = - \frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_j} (\mu \frac{\partial U_i}{\partial x_j} - \rho \langle u_i' u_j' \rangle),$$

$$- \langle u_i' u_j' \rangle = \nu_t \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} k$$

$$\nu_t = c_\mu \frac{k^2}{\varepsilon}.$$

$$U_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_j} \left((v + \sigma_k^{-1} \nu_t) \frac{\partial k}{\partial x_j} \right) - \langle u_i' u_j' \rangle \frac{\partial U_i}{\partial x_j} - \varepsilon.$$

$$U_i \frac{\partial \varepsilon}{\partial x_i} = \frac{\partial}{\partial x_j} \left((v + \sigma_\varepsilon^{-1} \nu_t) \frac{\partial \varepsilon}{\partial x_j} \right) - c_1 \frac{\nu_t \varepsilon}{k} \frac{\partial U_i}{\partial x_j} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - c_2 \frac{\varepsilon^2}{k}.$$

$$c_\mu = 0.09, c_1 = 1.44, c_2 = 1.92, \sigma_k = 1, \sigma_\varepsilon = 1.3$$

Avec sur les parois engraissées d'une épaisseur y_p limitant le fluide:

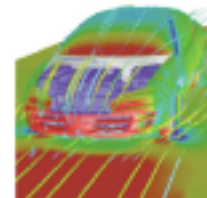
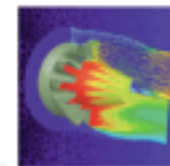
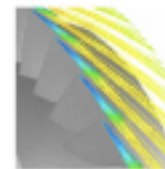
$$U_n = 0, \quad \frac{U_t}{(v \partial U_t / \partial n)^{1/2}} = \kappa^{-1} \text{Log} \left(\frac{y_p}{(v \partial U_t / \partial n)^{1/2}} \right) + 2.4$$

$$\text{où } 20 < \frac{y_p}{\sqrt{(v \partial U_t / \partial n)}} < 100.$$

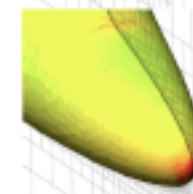
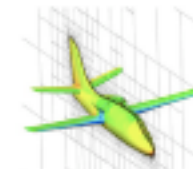
$$\rho c_p U_i \frac{\partial (T)}{\partial x_i} = \frac{\partial}{\partial x_j} \left((k_f (1 + \text{Pr}_t^{-1}) \frac{\partial T}{\partial x_j} \right)$$

Commercial CFD, Steady RANS -> Standard

FLUENT™

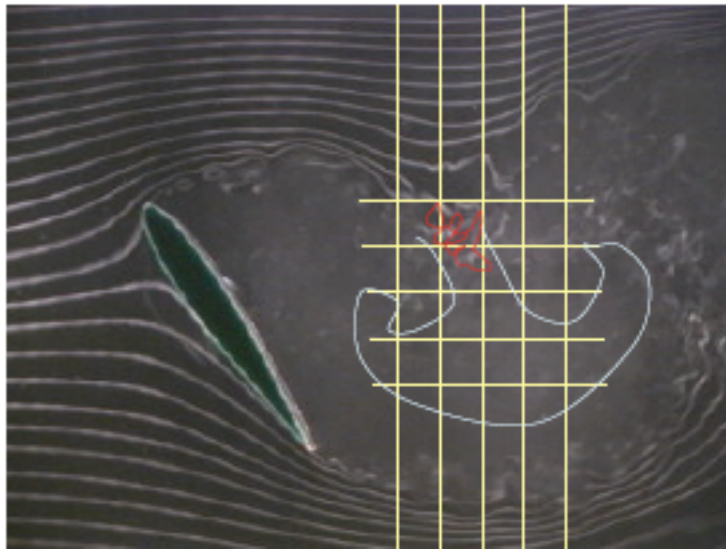


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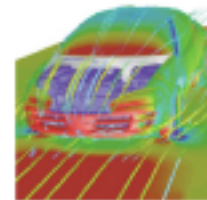
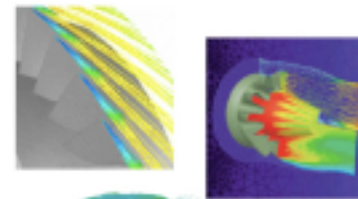
Applications:

- Aerospace (external)
- Chemical (mixing)
- Propulsion (internal flow)
- Biomedical
- HVAC
- Pollution, dispersion
- Etc...

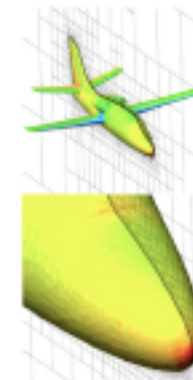


Commercial CFD,
Steady RANS -> Standard

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CFDRC™



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- Aerospace (external)
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