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*dernière mise à jour : 03/12/2007 par ortiz*



## MF204 : Transferts thermiques et massiques dans les fluides

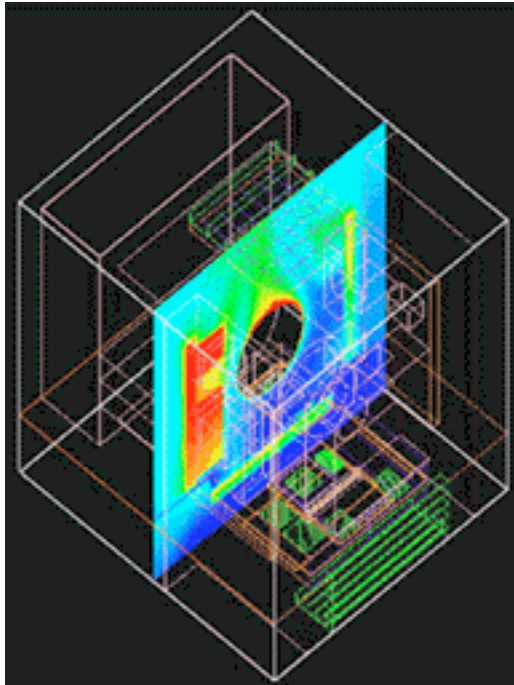
| Année : 2A               |             | Module : voie                        | Crédits ECTS : 2                                                                                                                                                                                                                                        |
|--------------------------|-------------|--------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Professeurs :            |             | Pierre-Yves LAGREE                   |                                                                                                                                                                                                                                                         |
| Maîtres de conférences : |             | Pierre-Yves LAGREE ; ANNE PLANCHETTE |                                                                                                                                                                                                                                                         |
| DATE                     | HEURES      | TYPE SEANCE                          | PROGRAMME DE LA SEANCE                                                                                                                                                                                                                                  |
| 22 01 2008               | 08:30 09:30 | Cours magistral                      | Rappels de conduction, coefficient d'échange<br><a href="#">(équation de la chaleur)</a><br><a href="#">(équation de la chaleur)</a><br><a href="#">thermodynamique des processus irréversibles</a><br>Rappels de conduction. Le coefficient d'échange. |
| 22 01 2008               | 09:45 11:45 | Petite classe                        |                                                                                                                                                                                                                                                         |
| 29 01 2008               | 8:30 9:30   | Cours magistral                      | Convection forcée<br><a href="#">Convection forcée</a>                                                                                                                                                                                                  |
| 29 01 2008               | 9:45 11:45  | Petite classe                        |                                                                                                                                                                                                                                                         |
| 05 02 2008               | 8:30 9:30   | Cours magistral                      | Convection forcée suite et fin                                                                                                                                                                                                                          |
| 05 02 2008               | 9:45 11:45  | Petite classe                        |                                                                                                                                                                                                                                                         |
| 12 02 2008               | 08:30 09:30 | Cours magistral                      | Convection Naturelle<br><a href="#">Convection libre</a>                                                                                                                                                                                                |
| 12 02 2008               | 09:45 11:45 | Petite classe                        |                                                                                                                                                                                                                                                         |
| 19 02 2008               | 08:30 09:30 | Cours magistral                      | Convection forcée turbulente<br><a href="#">Turbulence</a>                                                                                                                                                                                              |
| 19 02 2008               | 09:45 11:45 | Petite classe                        |                                                                                                                                                                                                                                                         |
| 04 03 2008               | 08:30 09:30 | Cours magistral                      | Transferts de masse<br><a href="#">Convection massique</a>                                                                                                                                                                                              |
| 04 03 2008               | 09:45 11:45 | Petite classe                        |                                                                                                                                                                                                                                                         |
| 11 03 2008               | 08:30 11:30 | Contrôle                             | Contrôle de connaissances                                                                                                                                                                                                                               |
|                          |             | Petite classe                        |                                                                                                                                                                                                                                                         |

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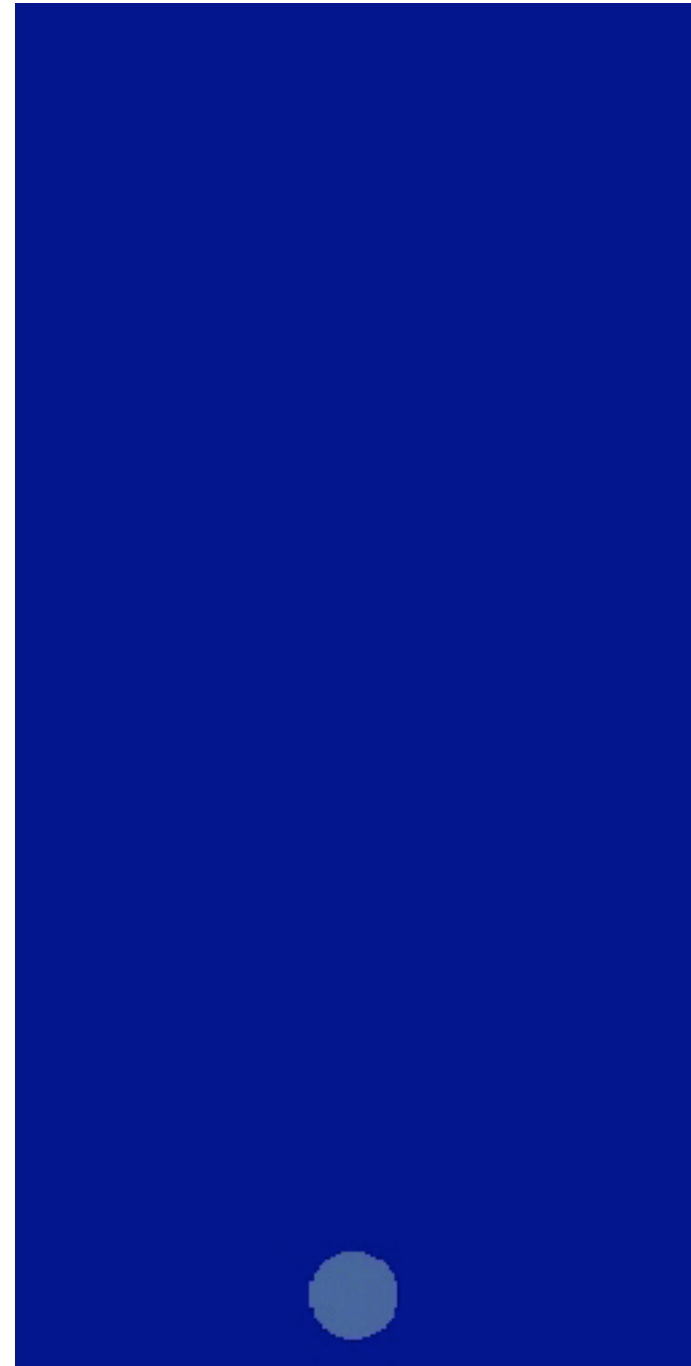
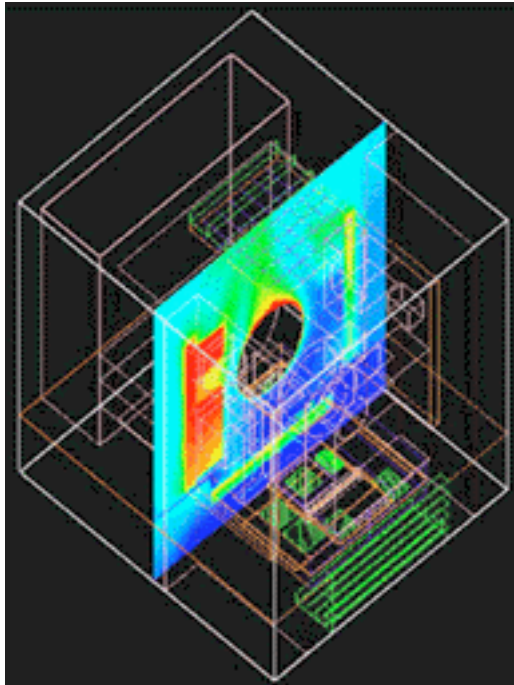
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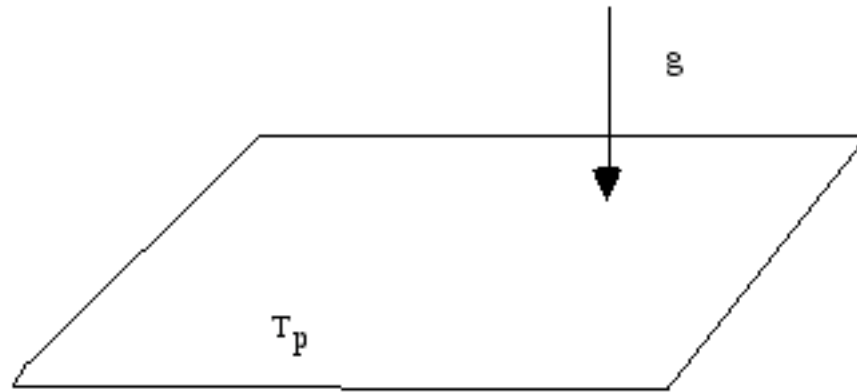
<http://www.dfr.ensta.fr/Cours/index.php?usebdd=ensta&sigle=MF204&afficher=prog>

<http://www.lmm.jussieu.fr/~lagree/COURS/ENSTA/>



<http://www.fluent.com/about/news/newsletters/99v8i2/a15.htm>





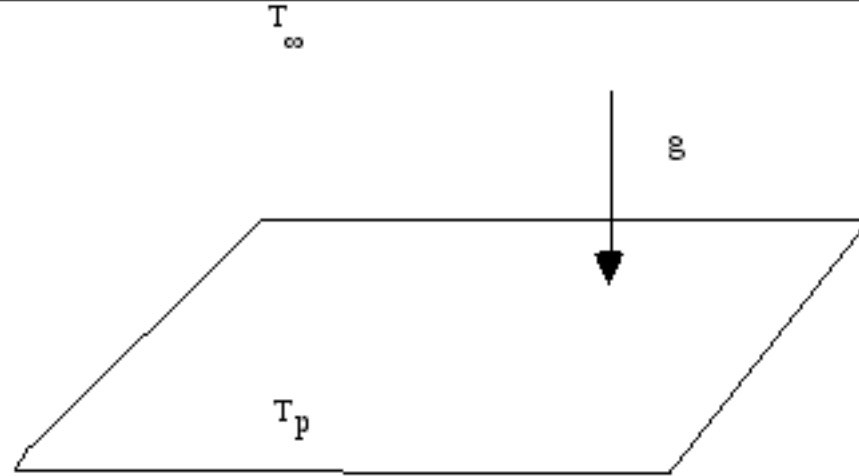
**plaque, dans un champ de pesanteur, soumise à une température différente de son environnement.**

$$\alpha = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_P \quad \chi = \frac{1}{\rho} \left( \frac{\partial \rho}{\partial p} \right)_T$$

Le développement de Taylor de la densité au voisinage de  $T = T_\infty$ , et  $p = P_\infty$  :

$$\rho = \rho_\infty (1 - \alpha(T - T_\infty) + \chi(p - P_\infty) + \dots)$$

|     |                                            |                                                           |
|-----|--------------------------------------------|-----------------------------------------------------------|
| eau | $\alpha = 5.8 \cdot 10^{-4} \text{K}^{-1}$ | $\chi = 5.25 \cdot 10^{-5} \text{Atm}^{-1}$               |
| air | $\alpha = 3.3 \cdot 10^{-4} \text{K}^{-1}$ | $\alpha = T^{-1}$ et $\chi = p^{-1}$ pour un gaz parfait. |



**plaque, dans un champ de pesanteur, soumise à une température différente de son environnement.**

$$\alpha = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_P \quad \chi = \frac{1}{\rho} \left( \frac{\partial \rho}{\partial p} \right)_T$$

$$\rho = \rho_\infty (1 - \alpha(T - T_\infty))$$

# Convection NATURELLE

équation de conservation de la masse :

$$\frac{d\rho}{dt} + \rho \underline{\nabla} \cdot \underline{u} = 0.$$

équation de conservation de la quantité de mouvement :

$$\rho \frac{d\underline{u}}{dt} = \underline{\nabla} \cdot \underline{\underline{\sigma}} + \underline{f}.$$

équation de l'énergie :

$$\rho \frac{de}{dt} = \underline{\underline{\sigma}} : \underline{\underline{D}} - \underline{\nabla} \cdot \underline{q} + r.$$

relations constitutives :

$$\underline{\underline{\sigma}} = -p\underline{I} + \lambda \underline{\nabla} \cdot \underline{u} \underline{I} + 2\mu \underline{\underline{D}}$$

$$\underline{q} = -k \underline{\nabla} T.$$

loi d'état :

$$p(\rho, T)$$

coefficients :

$$c_v(T), \quad c_p(T), \quad \lambda(T), \quad \mu(T), \quad k(T)...$$

conditions aux limites  $T_w$  **OU**  $q_w$  imposés, adhérence à la paroi.

# Convection NATURELLE

équation de conservation de la masse :

$$\frac{d\rho}{dt} + \rho \underline{\nabla} \cdot \underline{u} = 0. \quad \rho = \rho_{\infty} (1 - \varepsilon \bar{T} + \dots) \quad \epsilon = \alpha (T_p - T_{\infty})$$

équation de conservation de la quantité de mouvement :

$$\rho \frac{d\underline{u}}{dt} = \underline{\nabla} \cdot \underline{\underline{\sigma}} + \underline{f}.$$

équation de l'énergie :

$$\rho \frac{de}{dt} = \underline{\underline{\sigma}} : \underline{\underline{D}} - \underline{\nabla} \cdot \underline{q} + r.$$

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# Convection NATURELLE

équation de conservation de la masse :

~~$$\frac{d\rho}{dt} + \rho \nabla \cdot \underline{u} = 0. \quad \nabla \cdot \underline{u} = 0.$$~~

équation de conservation de la quantité de mouvement :

$$\rho_\infty \longrightarrow \rho \frac{d\underline{u}}{dt} = \nabla \cdot \underline{\underline{\sigma}} + \underline{f}. \quad \rho \underline{g} = \rho_\infty (1 - \alpha(T - T_0)) \underline{g}$$

équation de l'énergie :

$$\rho \frac{de}{dt} = \underline{\underline{\sigma}} : \underline{\underline{D}} - \nabla \cdot \underline{q} + r.$$

relations constitutives :

$$\underline{\underline{\sigma}} = -p\underline{I} + \lambda \nabla \cdot \underline{u} \underline{I} + 2\mu \underline{\underline{D}}$$

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# Convection NATURELLE

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$$p = p_0 - \rho_{\infty} g z + (\delta p) \bar{p}$$

# Convection NATURELLE

équation de conservation de la masse :

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$$p = p_0 - \rho_{\infty} g z + (\delta p) \bar{p}$$

$$\rho_{\infty} \left( \frac{\partial}{\partial t} \underline{u} + \underline{u} \cdot \underline{\nabla} \underline{u} \right) = -(\delta p) \underline{\nabla} \bar{p} + \rho_{\infty} g (-\alpha(T - T_0)) \underline{e}_z + \mu \underline{\nabla}^2 \underline{u}$$

# Energie



$$\rho \frac{de}{dt} = \underline{\underline{\sigma}} : \underline{\underline{D}} - \underline{\nabla} \cdot \underline{q}$$

$$\rho \frac{de}{dt} = k \underline{\nabla}^2 T + \lambda (\underline{\nabla} \cdot \underline{u})^2 - p \underline{\nabla} \cdot \underline{u} + 2\mu (\underline{\underline{D}} : \underline{\underline{D}}).$$

# Energie



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Le premier principe :

$$\rho \frac{de}{dt} = \underline{\underline{\sigma}} : \underline{\underline{D}} - \underline{\nabla} \cdot \underline{q}$$

Le second principe :

$$\rho T \frac{ds}{dt} = \rho \frac{de}{dt} + p \underline{\nabla} \cdot \underline{u}$$

# Energie



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$$ds = \left( \frac{\partial s}{\partial p} \right)_T dp + \left( \frac{\partial s}{\partial T} \right)_p dT$$

$$dh = de + d(p/\rho) = (T ds - p d(\rho^{-1}) + d(p/\rho)) = T ds + \rho^{-1} dp$$

# Energie



$$\rho \frac{de}{dt} = \underline{\underline{\sigma}} : \underline{\underline{D}} - \underline{\nabla} \cdot \underline{q}$$

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$$\rho T \frac{ds}{dt} = \rho \frac{de}{dt} + p \underline{\nabla} \cdot \underline{u}$$



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$$dh = de + d(p/\rho) = (Tds - pd(\rho^{-1}) + d(p/\rho)) = Tds + \rho^{-1} dp$$

$$Tds = -\rho^{-1} dp + (dh/dT) dT$$

# Energie



$$\rho \frac{de}{dt} = \underline{\underline{\sigma}} : \underline{\underline{D}} - \underline{\nabla} \cdot \underline{q}$$

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$$T \left( \frac{\partial s}{\partial p} \right)_T = \left( \frac{\partial h}{\partial T} \right)_p = c_p$$

$$\left( \frac{\partial s}{\partial p} \right)_T = \left( \frac{\partial \rho^{-1}}{\partial T} \right)_p$$

# Energie

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$$\left( \frac{\partial s}{\partial p} \right)_T = \left( \frac{\partial \rho^{-1}}{\partial T} \right)_p$$

$$\rho T ds = \rho c_p dT + \frac{T}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_p dp.$$

$$\rho \frac{de}{dt} = \underline{\underline{\sigma}} : \underline{\underline{D}} - \underline{\nabla} \cdot \underline{q}$$

$$\rho \frac{de}{dt} = k \underline{\nabla}^2 T + \lambda (\underline{\nabla} \cdot \underline{u})^2 - p \underline{\nabla} \cdot \underline{u} + 2\mu (\underline{\underline{D}} : \underline{\underline{D}}).$$

$$\rho c_p \frac{d}{dt} T + \frac{T}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_p \frac{d}{dt} p = k \underline{\nabla}^2 T + \lambda (\underline{\nabla} \cdot \underline{u})^2 + 2\mu (\underline{\underline{D}} : \underline{\underline{D}})$$

$$\rho c_p \frac{d}{dt} T = k \underline{\nabla}^2 T + 2\mu (\underline{\underline{D}} : \underline{\underline{D}}).$$

système couplé régissant les écoulements de convection libre laminaire:

$$\left\{ \begin{array}{l} \underline{\nabla} \cdot \underline{u} = 0 \\ \rho_{\infty} \left( \frac{\partial}{\partial t} \underline{u} + \underline{u} \cdot \underline{\nabla} \underline{u} \right) = -(\delta p) \underline{\nabla} \bar{p} + \rho_{\infty} g \varepsilon \bar{T} \underline{e}_z + \mu \underline{\nabla}^2 \underline{u}, \\ \rho_{\infty} c_p \left( \frac{\partial}{\partial t} + \underline{u} \cdot \underline{\nabla} \right) T = k \underline{\nabla}^2 T + 2\mu (\underline{D} : \underline{D}), \end{array} \right.$$

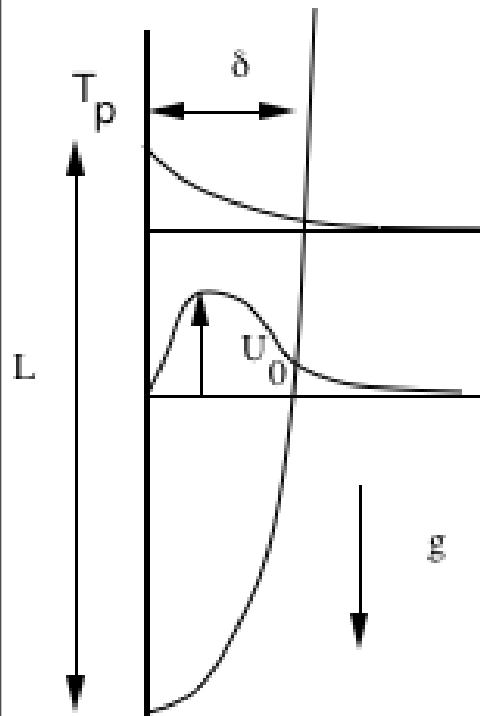
**+ CL**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\rho \partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \alpha (T_p - T_{\infty}) \bar{T}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\rho \partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\nu}{Pr} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$



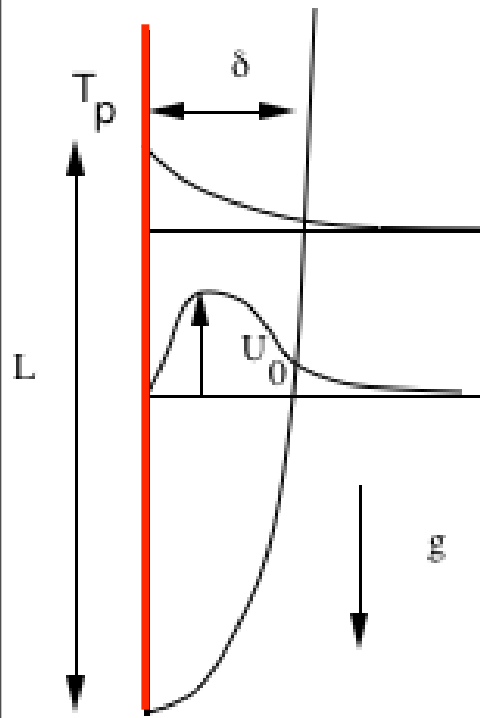
- la plaque verticale chauffée.

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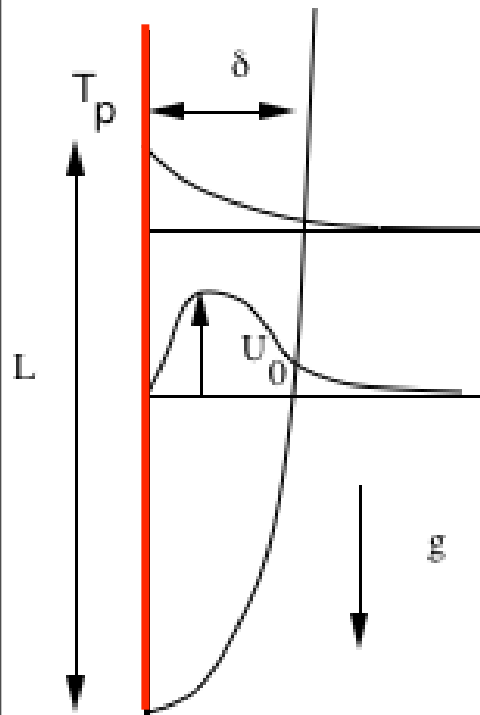
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$$\frac{P}{L} \sim \rho g \alpha \Delta T,$$



- la plaque verticale chauffée.

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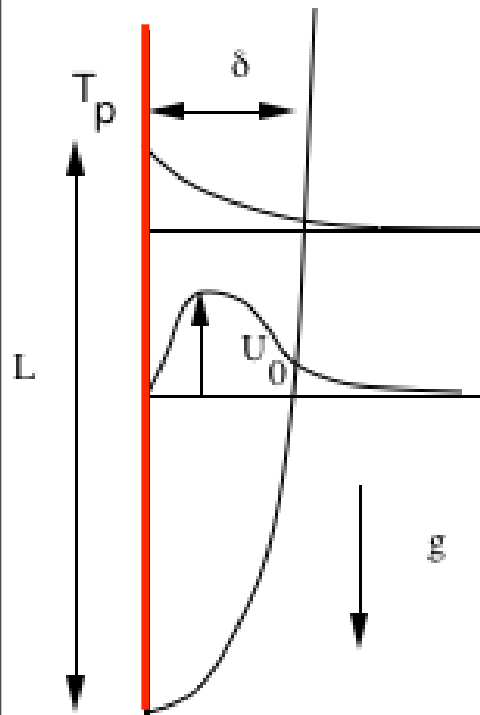
$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\rho \partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \alpha (T_p - T_\infty) \bar{T} g$$

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$$\frac{P}{L} \sim \rho g \alpha \Delta T,$$

$$\rho U_0 \frac{U_0}{L} \sim \frac{P}{L}$$

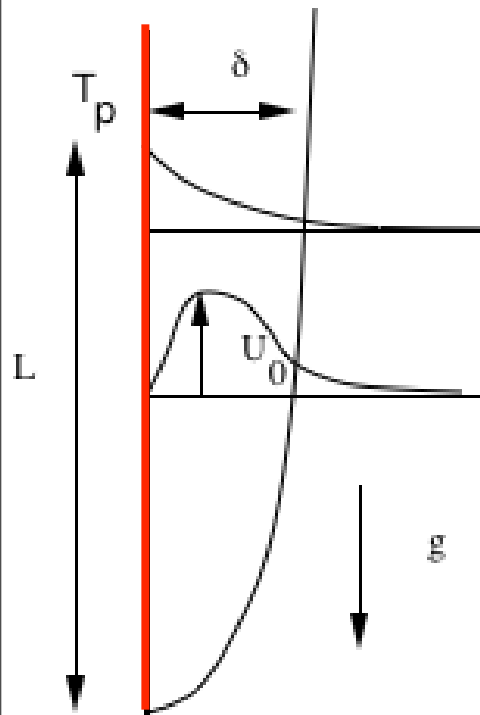


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$$\begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{\partial p}{\rho \partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \alpha (T_p - T_\infty) \bar{T} g \\ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{\partial p}{\rho \partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \\ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \frac{\nu}{Pr} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \end{aligned}$$

$$\frac{P}{L} \sim \rho g \alpha \Delta T,$$

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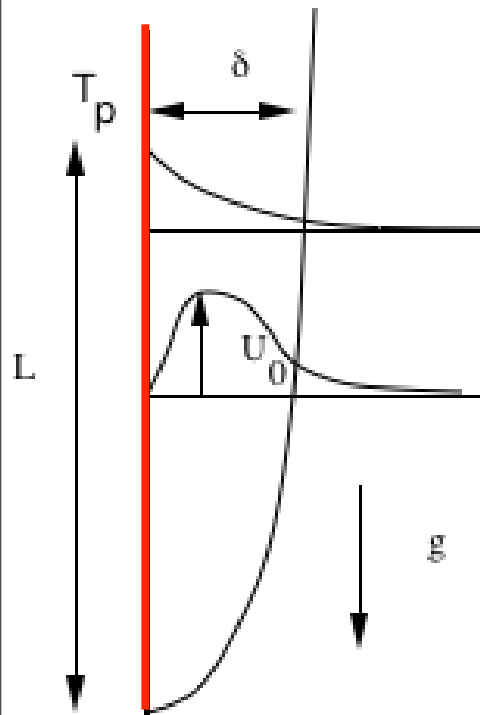
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$$\frac{P}{L} \sim \rho g \alpha \Delta T,$$

$$\rho U_0 \frac{U_0}{L} \sim \frac{P}{L}$$

$$U_0 \sim (L g \alpha \Delta T)^{1/2}.$$



- la plaque verticale chauffée.

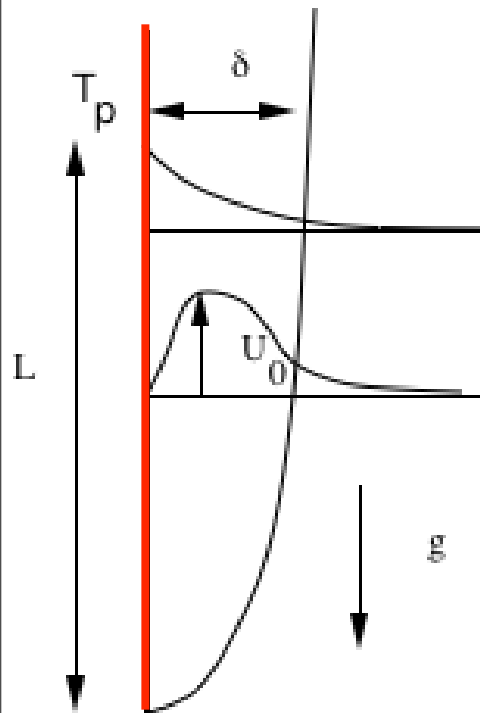
$$\begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{\partial p}{\rho \partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \alpha (T_p - T_\infty) \bar{T} g \\ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{\partial p}{\rho \partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \\ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \frac{\nu}{Pr} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \end{aligned}$$

$$\frac{P}{L} \sim \rho g \alpha \Delta T,$$

$$\rho U_0 \frac{U_0}{L} \sim \frac{P}{L}$$

$$U_0 \sim (L g \alpha \Delta T)^{1/2}.$$

$$U_0 \frac{U_0}{L} \sim \nu \frac{U_0}{\delta^2}$$

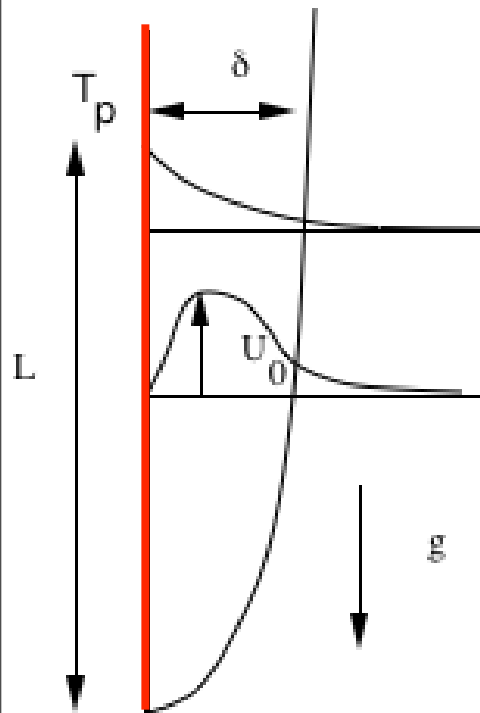


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$$\frac{\delta}{L} = G^{-1/4}$$

$$G = \frac{\alpha g \Delta T L^3}{\nu^2}$$



- la plaque verticale chauffée.

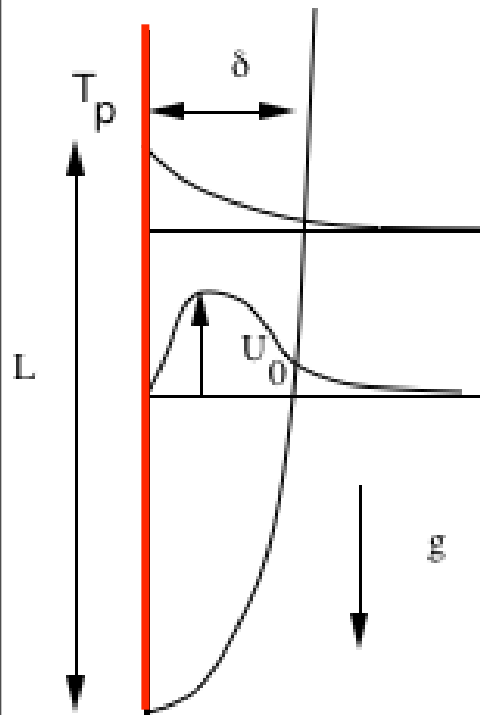
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Grashof

$$\frac{\delta}{L} = G^{-1/4}$$

$$G = \frac{\alpha g \Delta T L^3}{\nu^2}$$

$$x = L\bar{x}, y = LG^{-1/4}\tilde{y}, u = (Lg\alpha\Delta T)^{1/2}\tilde{u} \text{ etc}$$



- la plaque verticale chauffée.

$$\begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{\partial p}{\rho \partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \alpha (T_p - T_\infty) \bar{T} g \\ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{\partial p}{\rho \partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \\ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \frac{\nu}{Pr} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \end{aligned}$$

# Grashof

$$\frac{\delta}{L} = G^{-1/4}$$

$$G = \frac{\alpha g \Delta T L^3}{\nu^2}$$

$$x = L\bar{x}, \quad y = LG^{-1/4}\tilde{y}, \quad u = (Lg\alpha\Delta T)^{1/2}\tilde{u} \text{ etc}$$

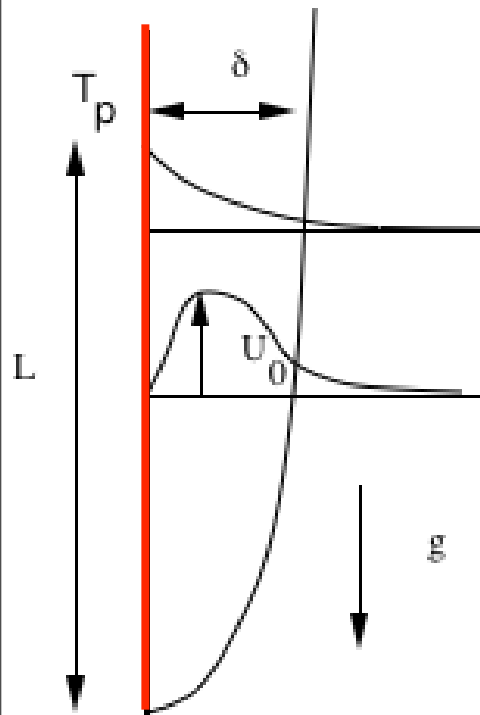
$$\frac{\partial \tilde{u}}{\partial \bar{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0,$$

$$\tilde{u} \frac{\partial \tilde{u}}{\partial \bar{x}} + \tilde{v} \frac{\partial \tilde{u}}{\partial \tilde{y}} = -\frac{\partial \tilde{p}}{\partial \bar{x}} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2} + \tilde{T}, \quad \frac{\partial \tilde{p}}{\partial \tilde{y}} = 0,$$

$$\tilde{u} \frac{\partial \tilde{T}}{\partial \bar{x}} + \tilde{v} \frac{\partial \tilde{T}}{\partial \tilde{y}} = \frac{1}{Pr} \frac{\partial^2 \tilde{T}}{\partial \tilde{y}^2}$$

# Grashof

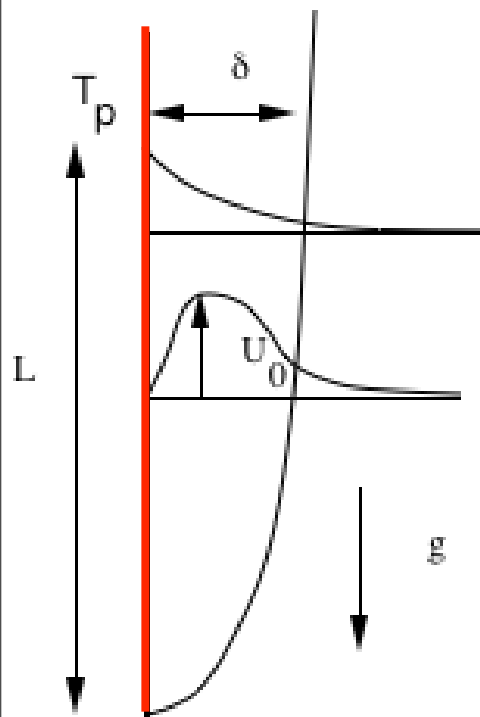
$$\tilde{u}(\bar{x}, 0) = \tilde{v}(\bar{x}, 0) = 0, \quad \tilde{T}(\bar{x}, 0) = 1, \quad \text{et} \quad \tilde{u}(\bar{x}, \infty) = 0, \quad \tilde{T}(\bar{x}, \infty) = 0.$$



- la plaque verticale chauffée.

$$\frac{\delta}{L} = G^{-1/4}$$

$$G = \frac{\alpha g \Delta T L^3}{\nu^2}$$



$$\psi = x^{3/4} f(\eta) \text{ et } T = g(\eta),$$

$$4 f''' + 3 f f' - 2 f^2 + 4 g = 0;$$

$$4 g'' + 3 \text{Pr} f g' = 0;$$

La résolution par une méthode de tir donne

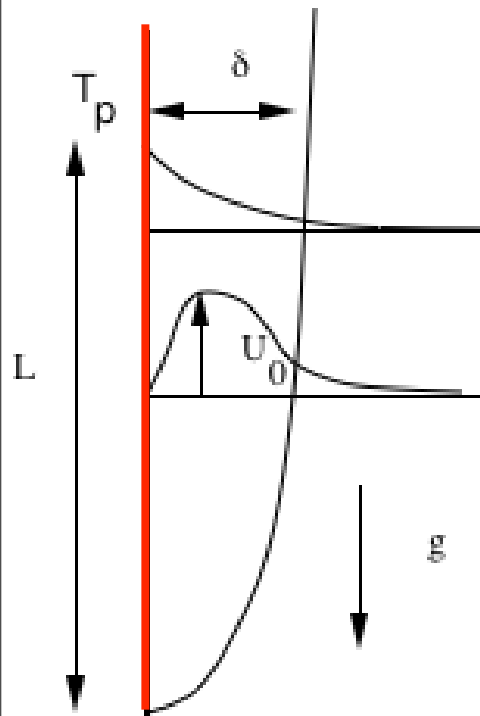
pour  $\text{Pr}=0.7$ ,  $f'(0) = 0.9571$ ,  $g'(0) = -0.3534$

pour  $\text{Pr}=1$ ,  $f'(0) = 0.9069$ ,  $g'(0) = -0.4008$

pour  $\text{Pr}=7$ ,  $f'(0) = 0.6371$ ,  $g'(0) = -0.7450$

- la plaque verticale chauffée.

profils de  $f'$  et de  $g$  en fonction de  $\eta$  pour ces 3 nombres de Prandtl représentatifs.



- la plaque verticale chauffée.

$$\frac{\delta}{L} = G^{-1/4}$$

$$G = \frac{\alpha g \Delta T L^3}{\nu^2}$$

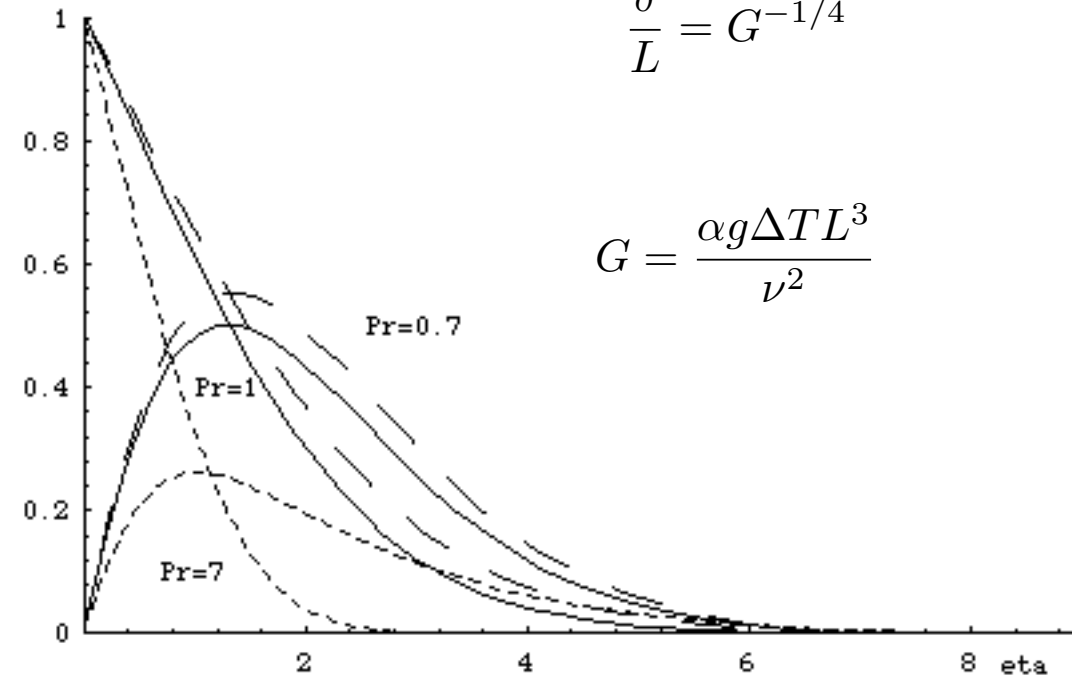
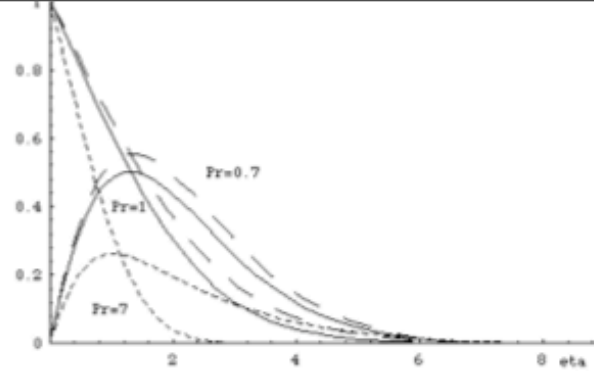


figure trait plein  $Pr=1$ , petit pointillé  $Pr=7$ , gros  $Pr=0.7$

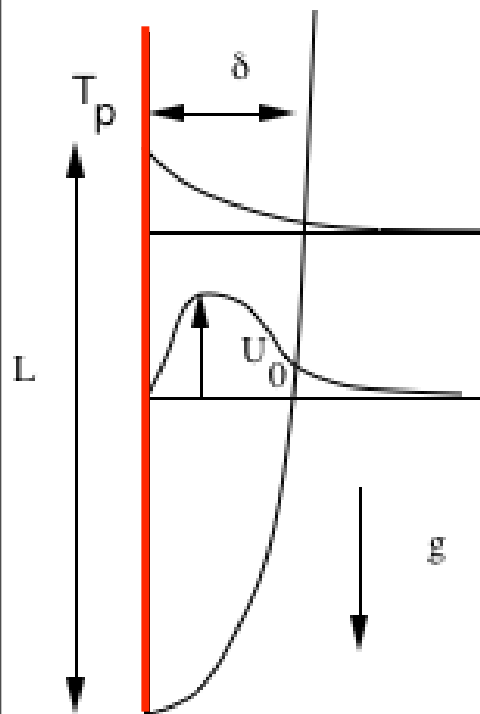
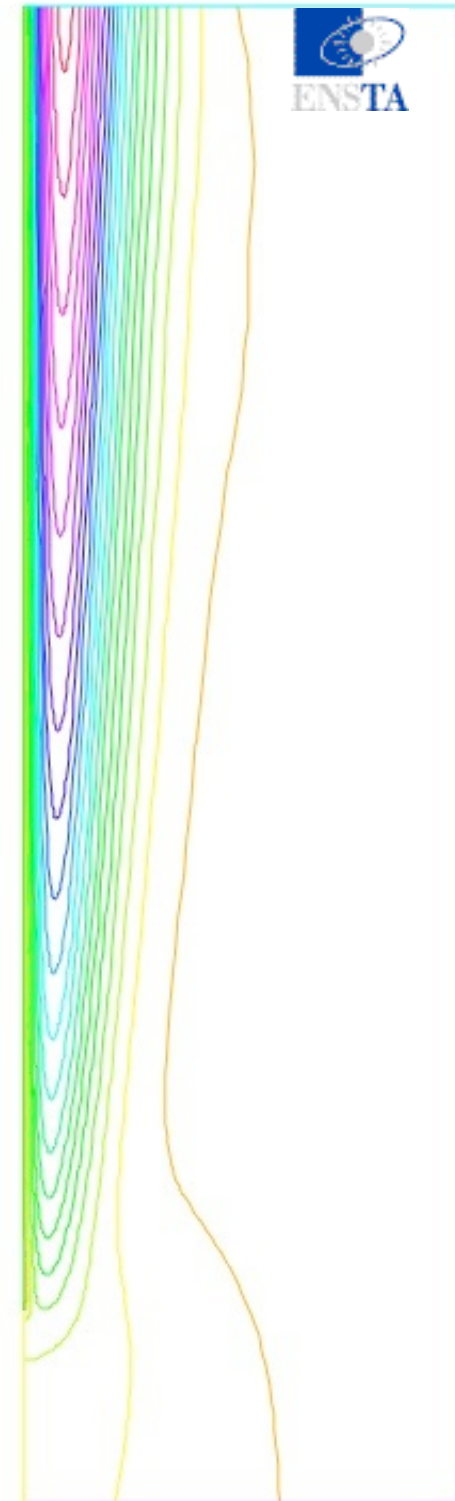
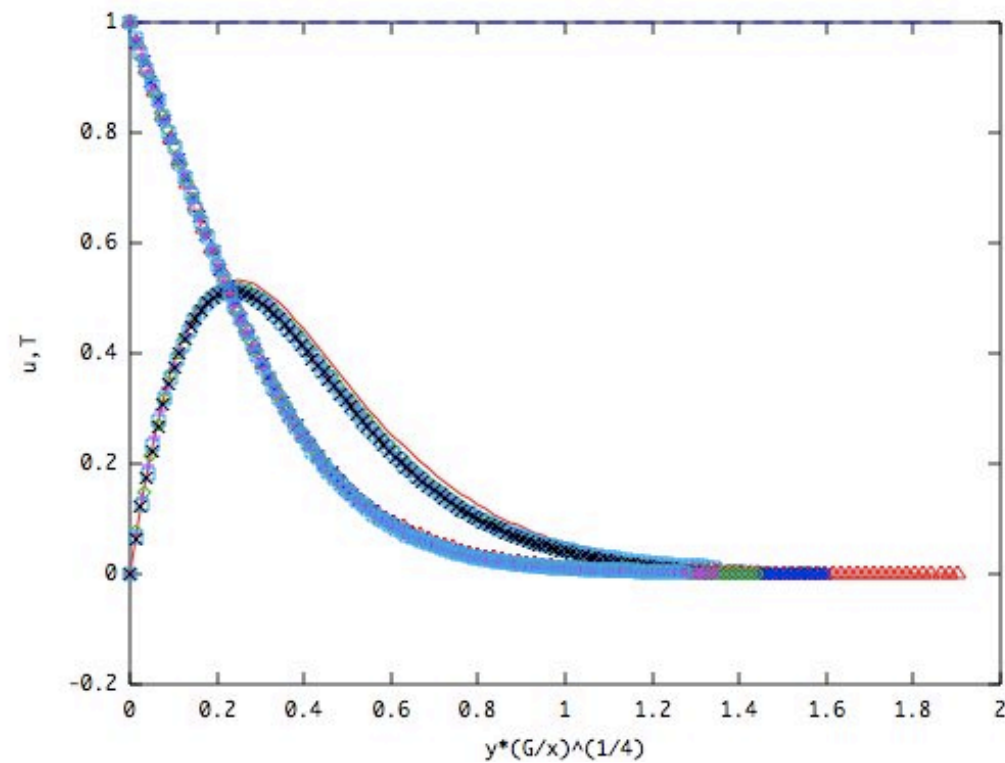
$$Nu_x = 0,359 Gr_x^{1/4}.$$

$$\langle Nu \rangle = 0.478 Gr_L^{1/4}.$$

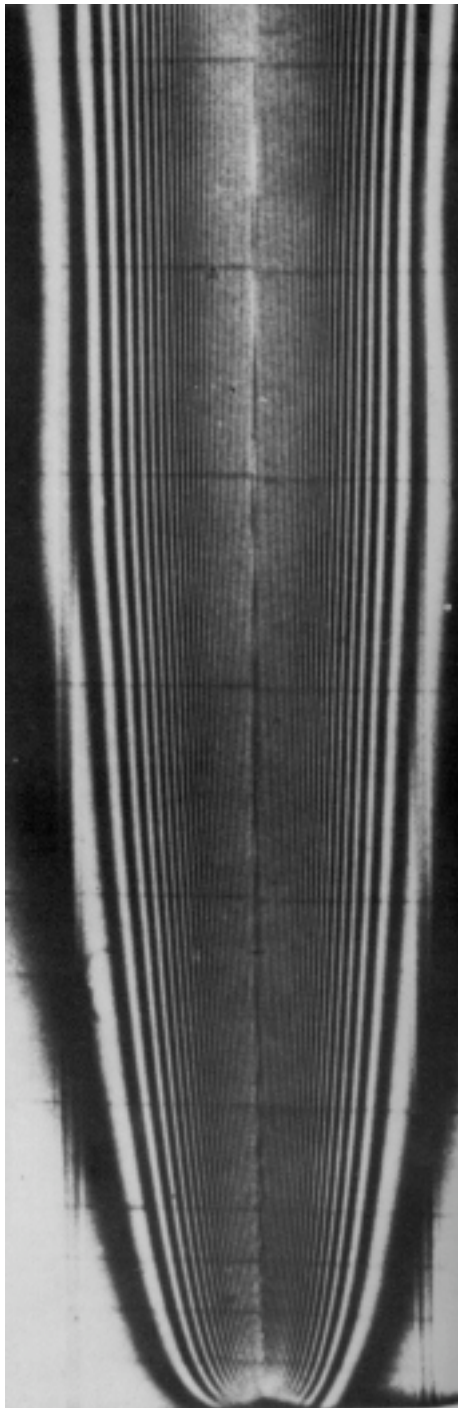


tracé de la solution semblable analytique

Nouvelle version de calcul avec FreeFEM++.  
pour  $Gr = 1000$   
tracé de la solution numérique:

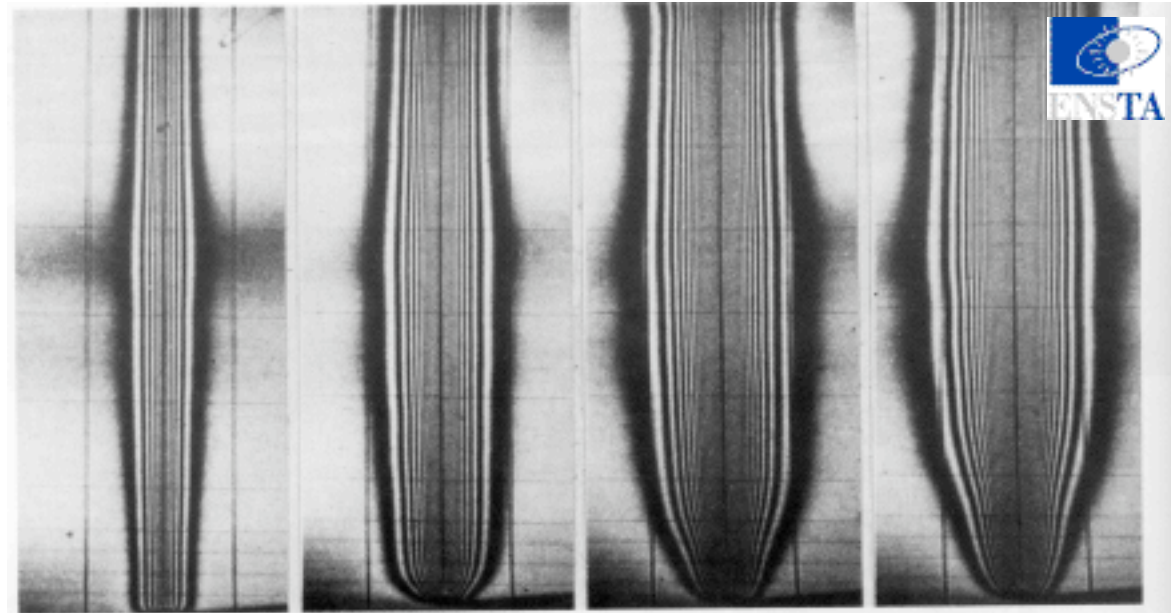
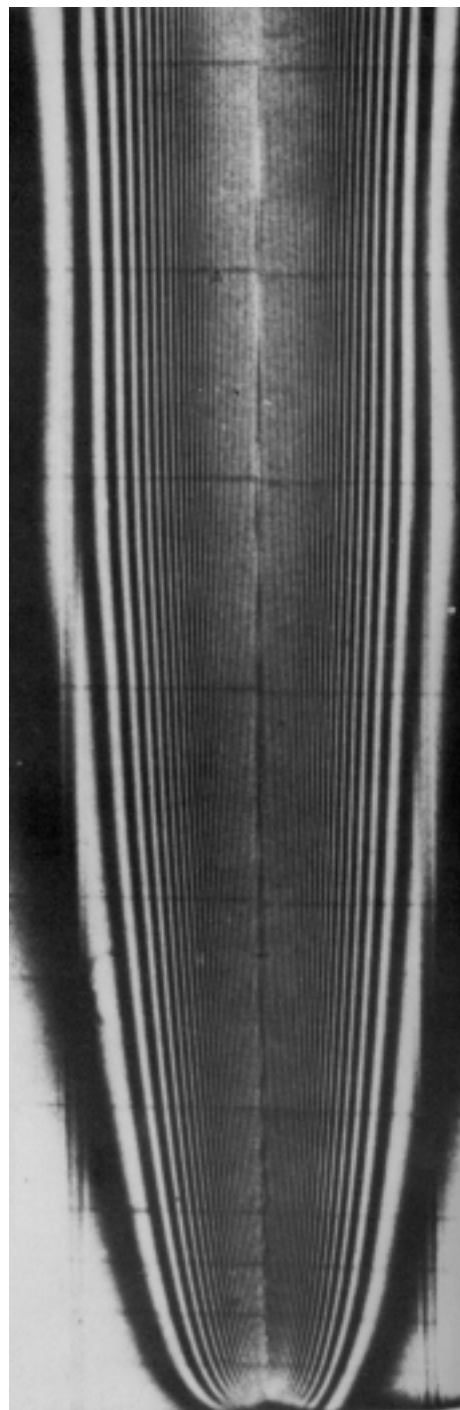


- la plaque verticale chauffée.



Cas vertical, flux imposé: On met maintenant la plaque verticalement, et on la chauffe en imposant le flux

**interférogramme montrant les lignes iso densité**

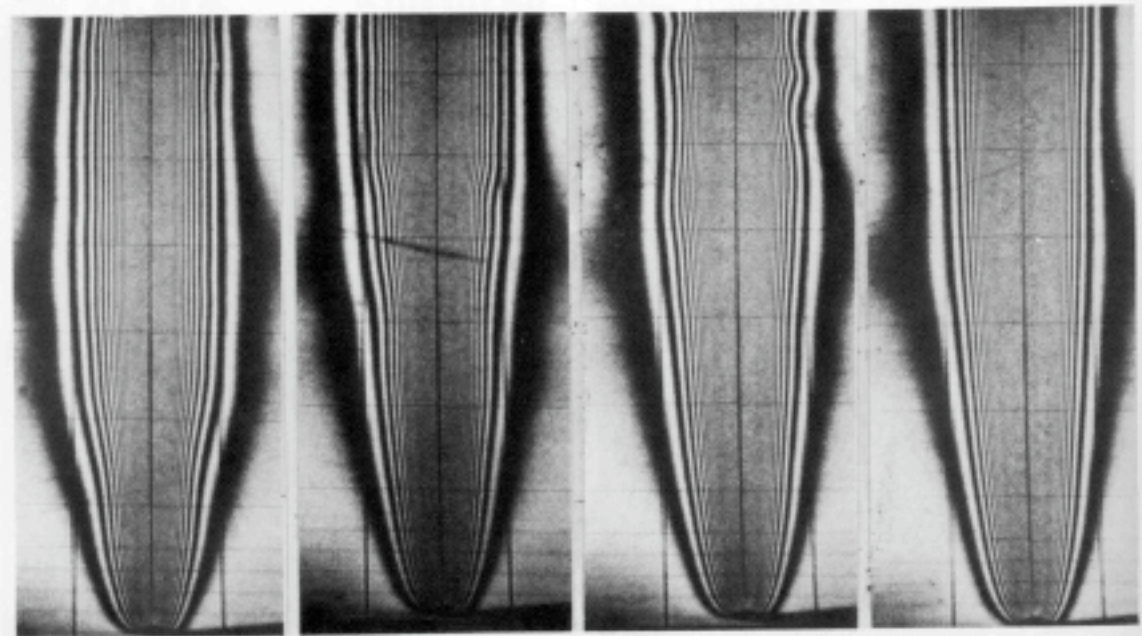


1/2 Sec.

1 Sec.

1-1/2 Sec.

1-3/4 Sec.



2 Sec.

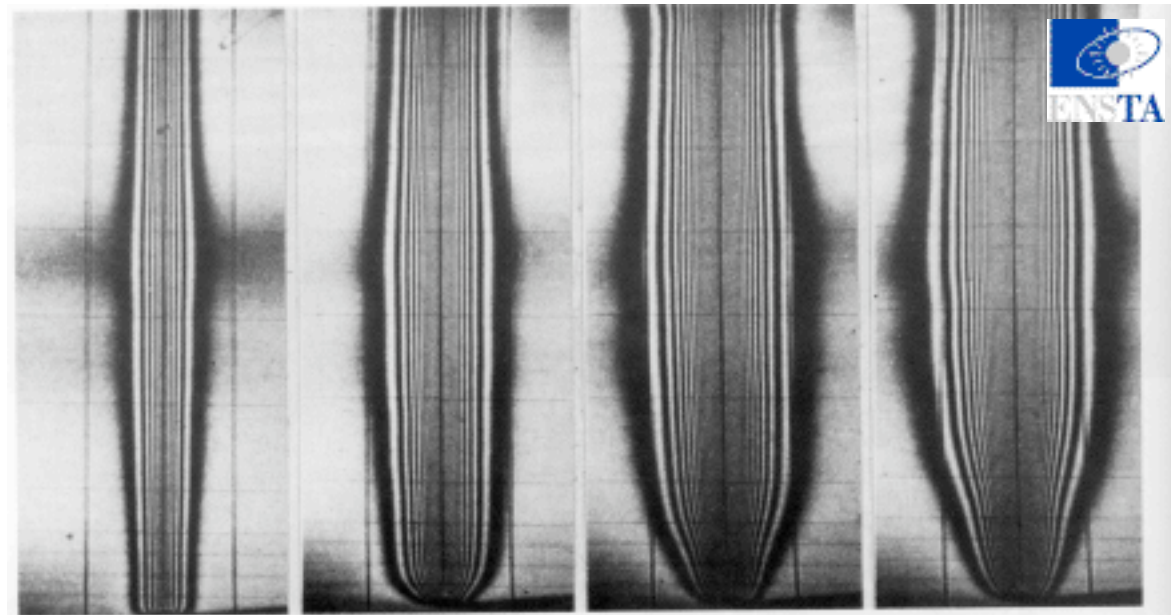
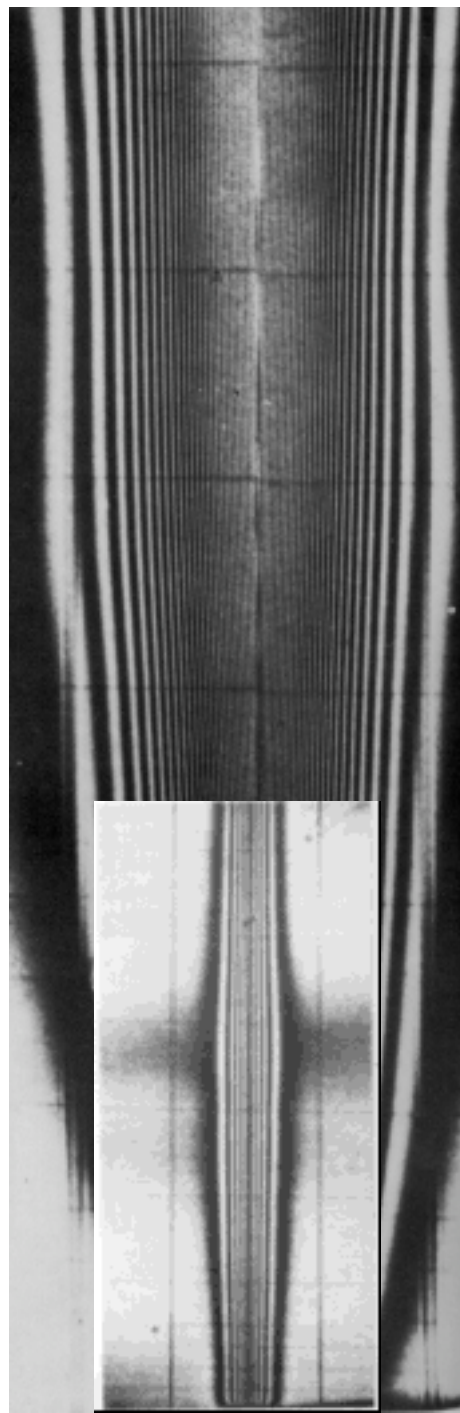
2-1/2 Sec.

3 Sec.

7 Sec.

213. Convection from an impulsively heated vertical foil. A foil 0.013 mm thick is suddenly heated electrically in air at 17 atmospheres. The effect of the leading edge is seen propagating upwards until the steady laminar flow of

figure 205 is established. The vertical spacing of the upper grid wires is 2.5 cm. The fringe field is spread optically by a factor of six in the horizontal direction. Gebhart & Dring 1967

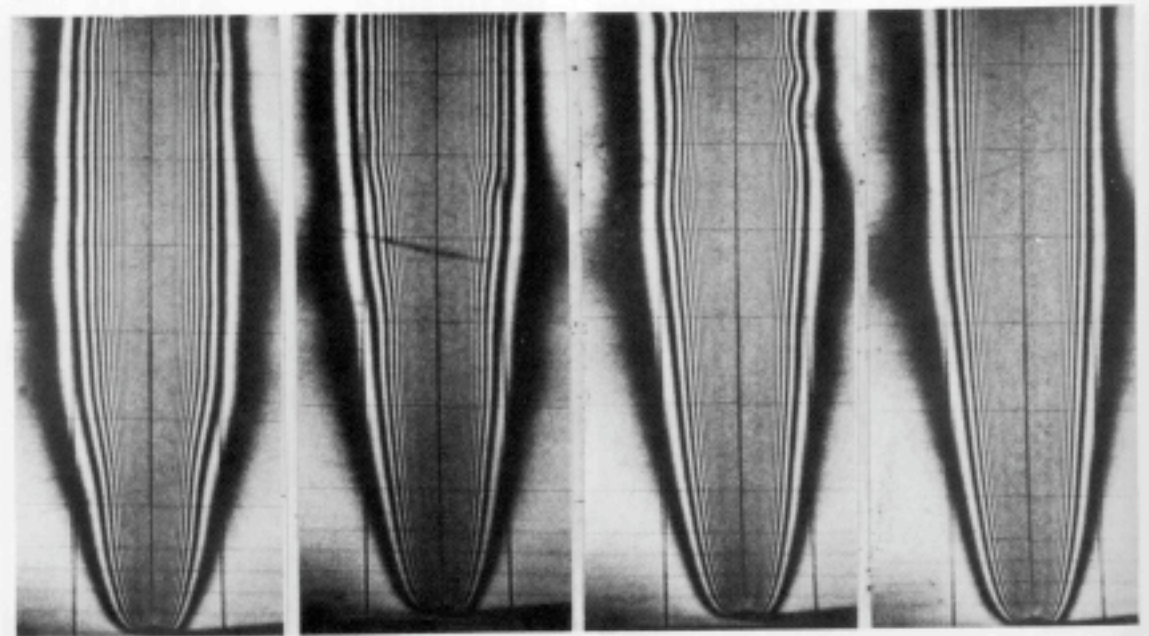


1/2 Sec.

1 Sec.

1-1/2 Sec.

1-3/4 Sec.



2 Sec.

2-1/2 Sec.

3 Sec.

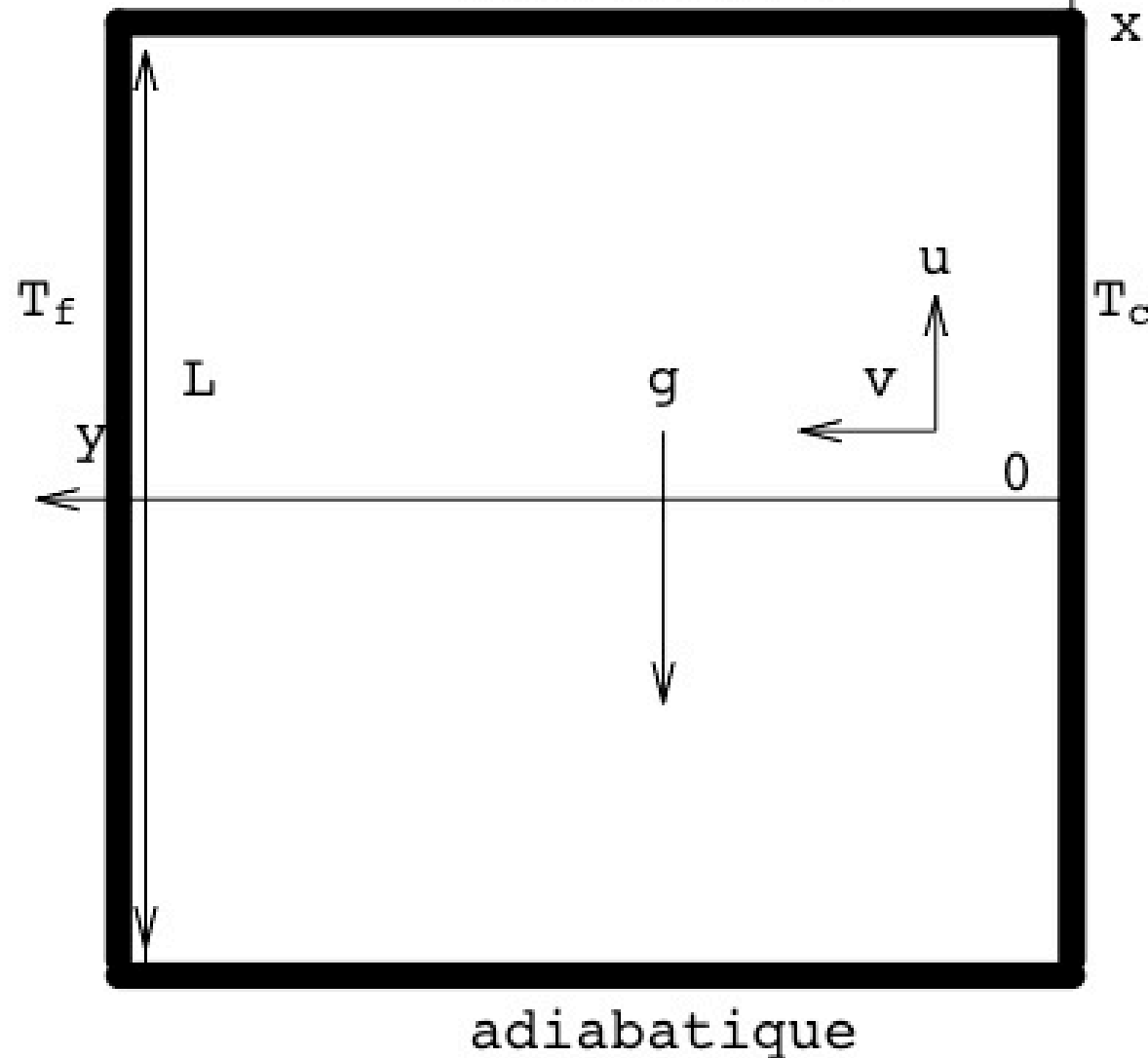
7 Sec.

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# Solution de convection naturelle dans une cavité chauffée adiabatique



La cavité est refroidie à gauche et chauffée à droite.

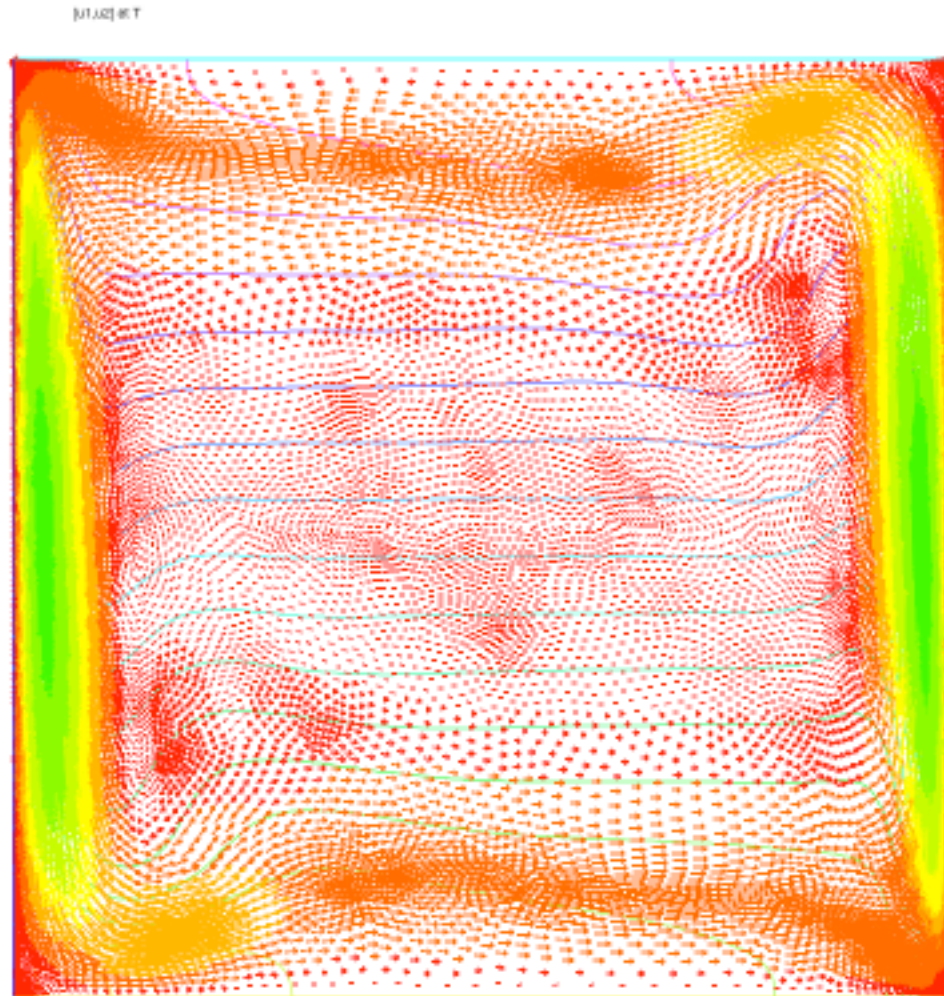
Les murs inférieurs et supérieurs sont adiabatiques.

Le Grashof est de  $10^6$ .

Calcul avec FreeFEM++, pour  $Gr = 1000000$

On observe les deux couches limites montantes et descendantes  
à droite et à gauche, ainsi que la stratification de température induite

## Solution de convection naturelle dans une cavité chauffée



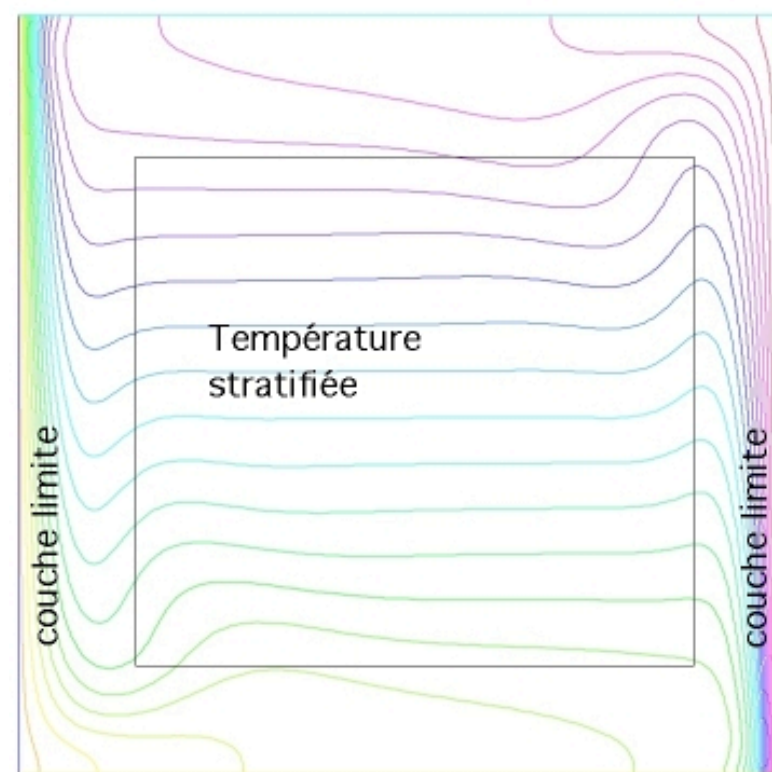
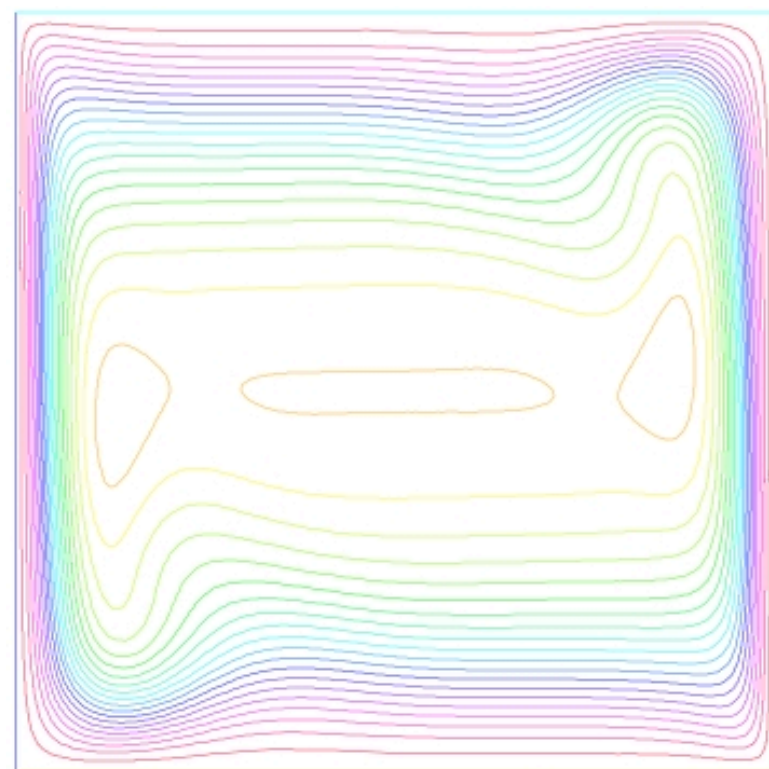
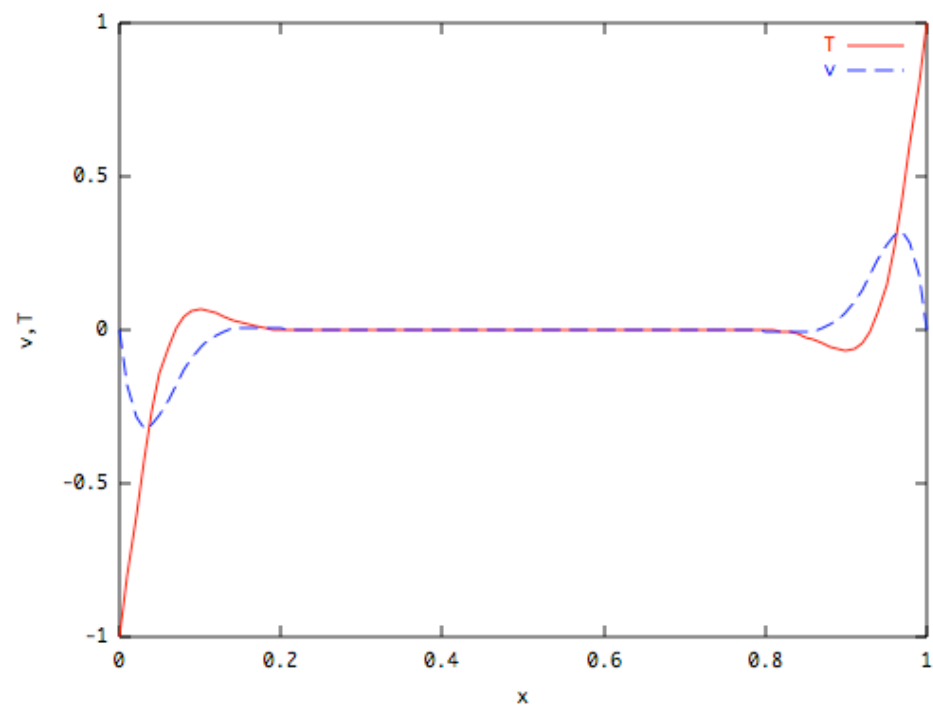
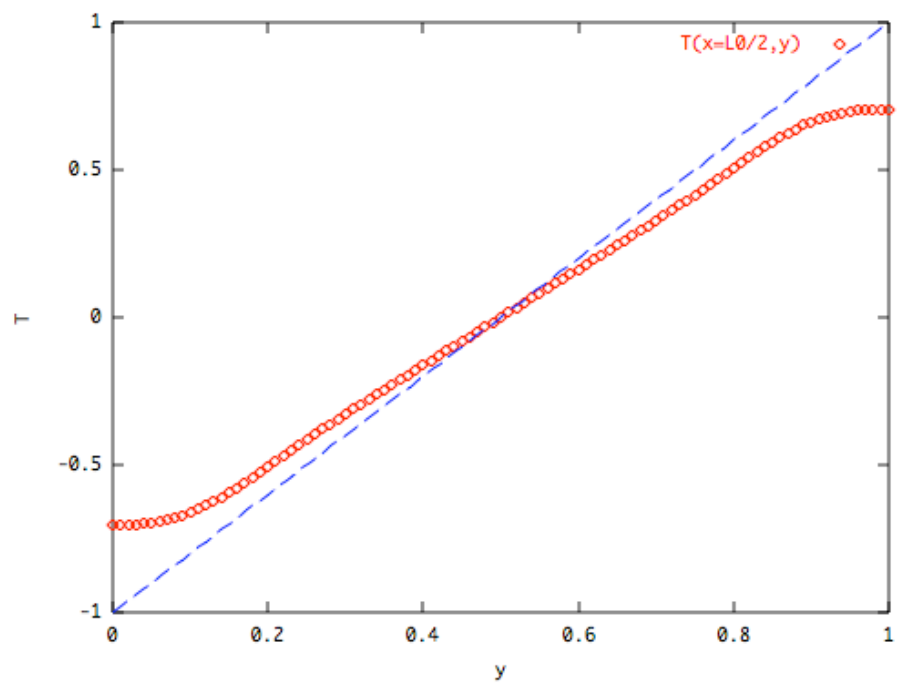
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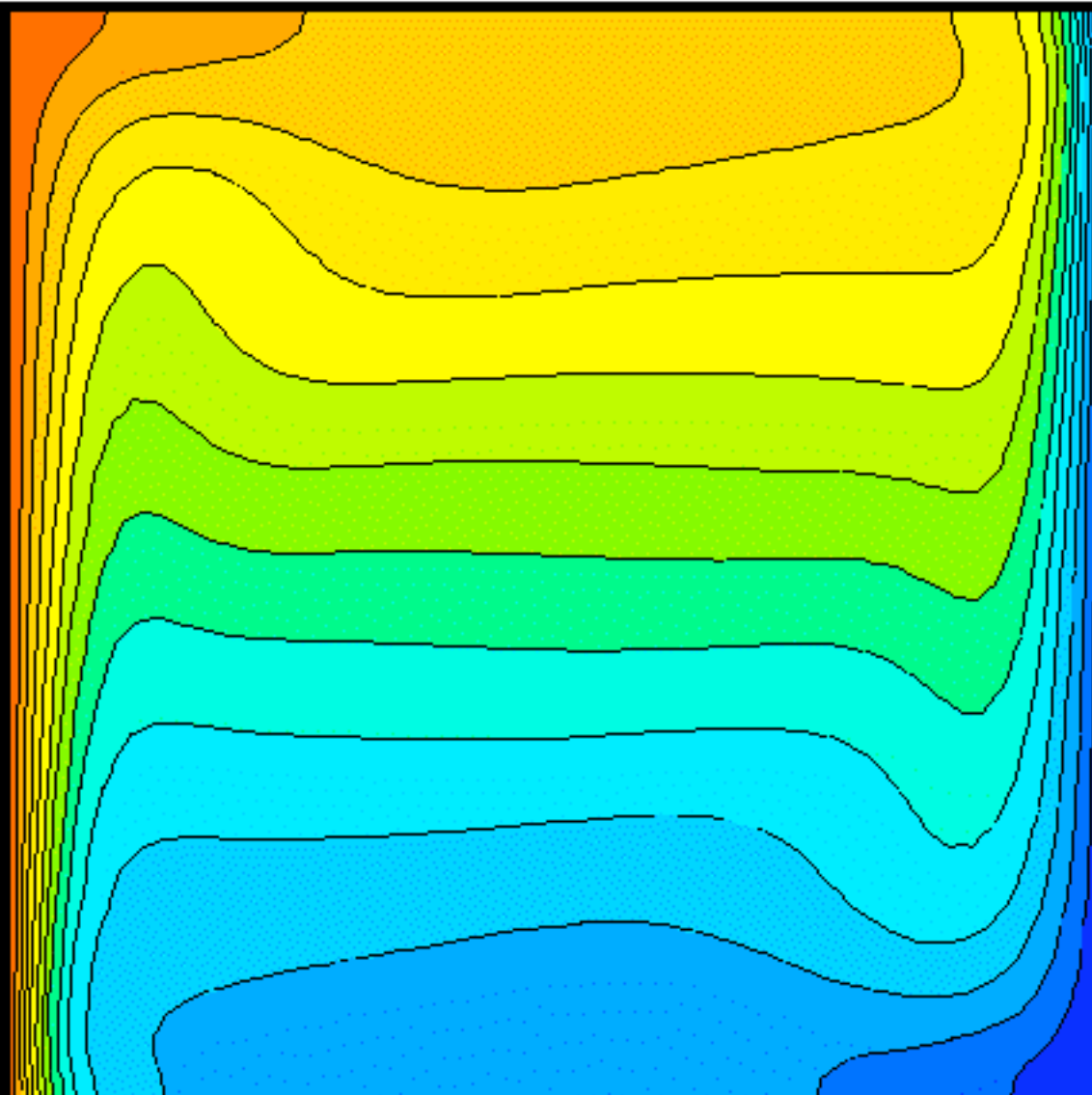


Fin trace

Zoom/Pan

Valeur

Softcopy



VAL - ISO

&gt; 0.00E+00

&lt; 1.00E+00

3.57E-02

0.11

0.18

0.25

0.32

0.39

0.46

0.54

0.61

0.68

0.75

0.82

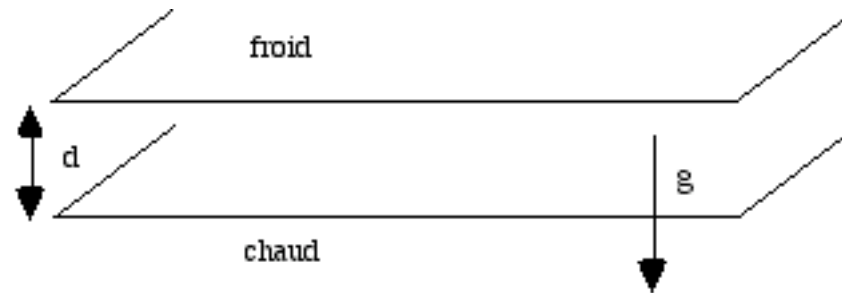
0.89

0.96

GIBI FECIT

CASTEM 2000 - XWindows V1.0

## Convection libre: Le problème de Rayleigh Bénard

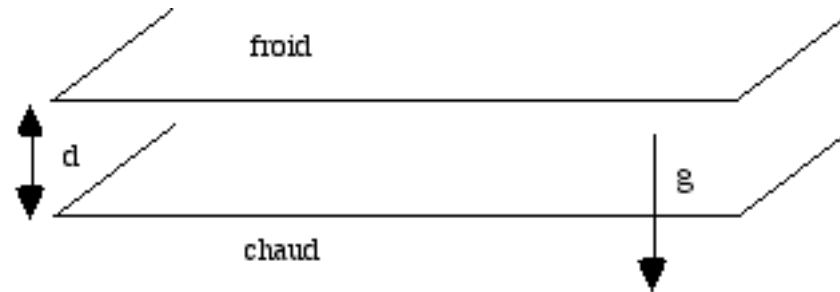


deux plaques limitant un fluide, une est plus chauffée que l'autre.



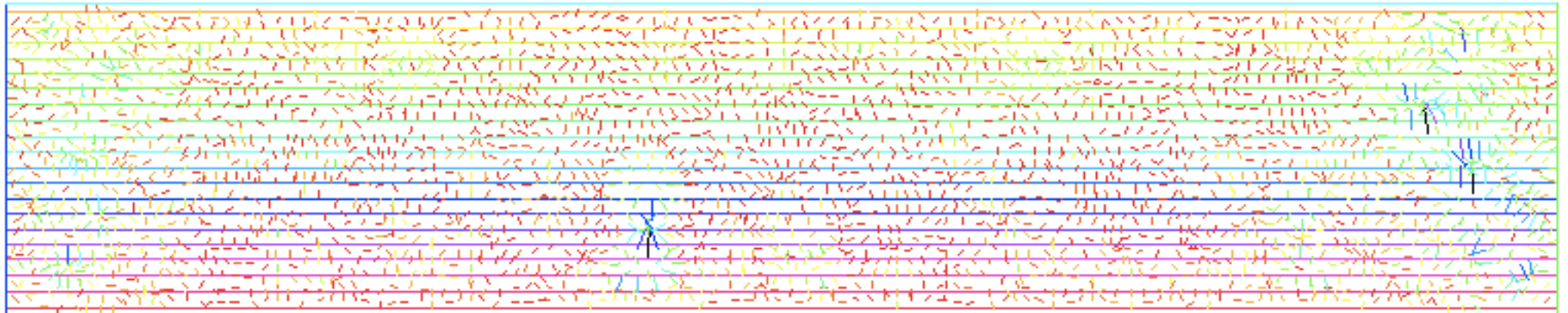
rouleaux vus de côté (plaque en haut et en bas)

## Convection libre: Le problème de Rayleigh Bénard

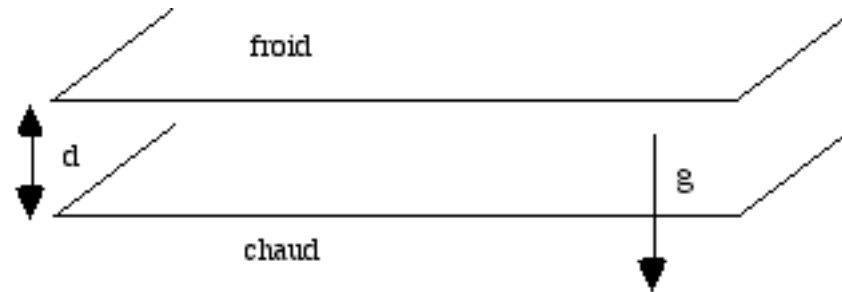


deux plaques limitant un fluide, une est plus chauffée que l'autre.

$$Ra = \frac{\alpha g \Delta T P r d^3}{\nu^2}$$



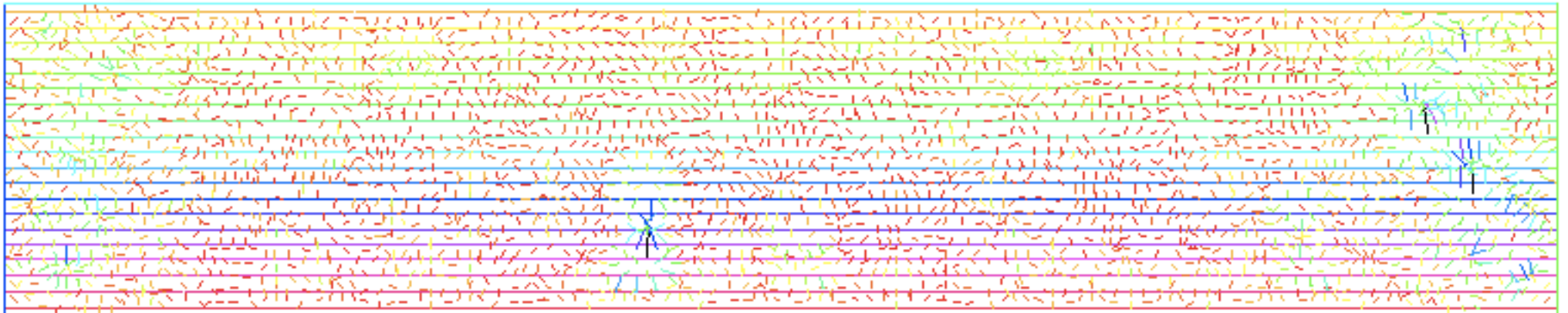
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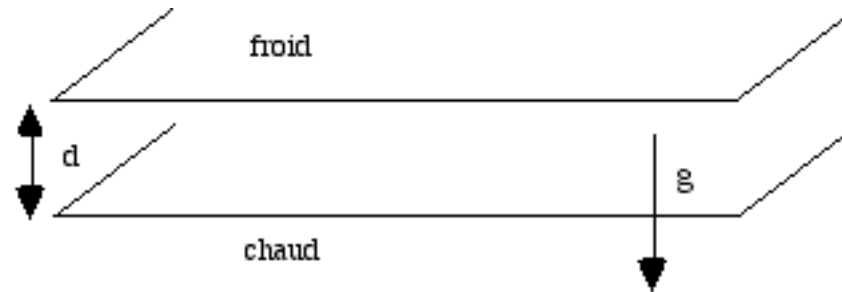
deux plaques limitant un fluide, une est plus chauffée que l'autre.

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$$R < 1800$$



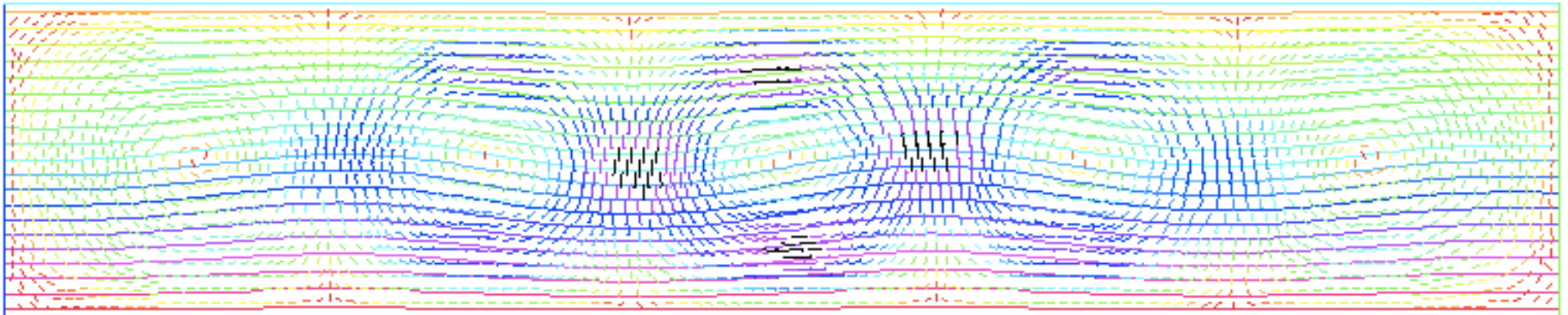
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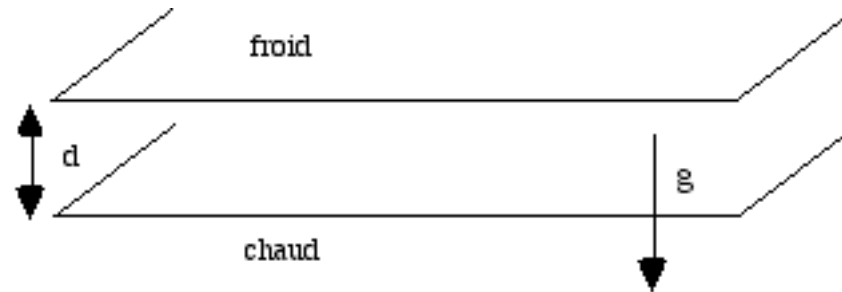
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**R=1800**



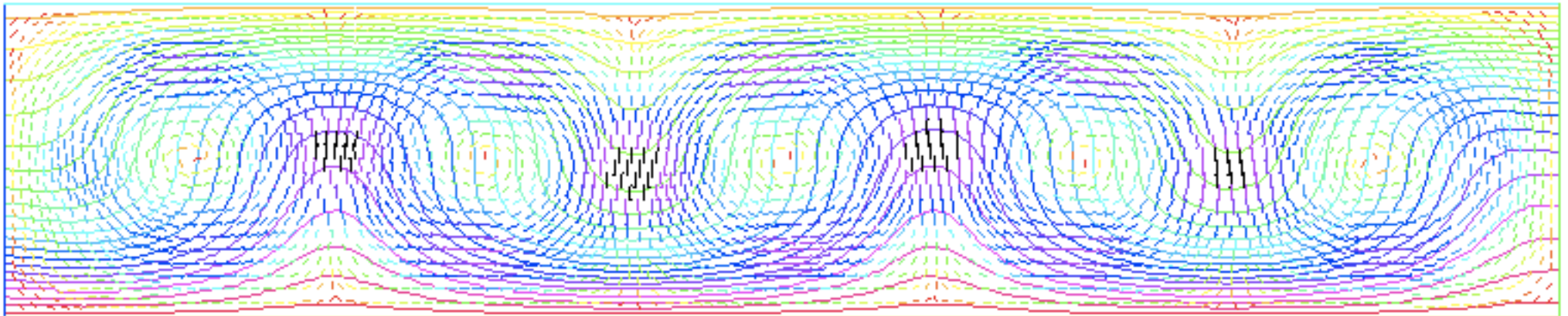
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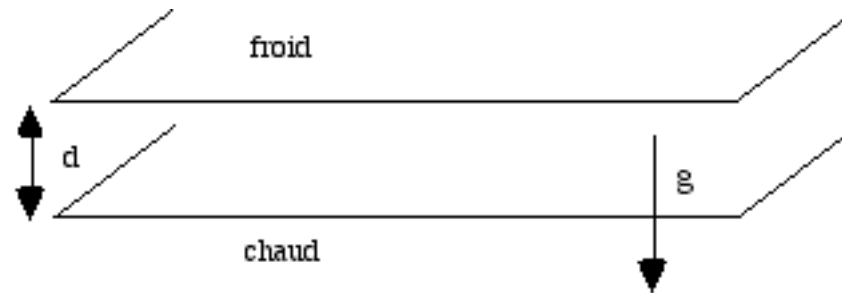
deux plaques limitant un fluide, une est plus chauffée que l'autre.

$$Ra = \frac{\alpha g \Delta T P d^3}{\nu^2}$$

**R=4000**



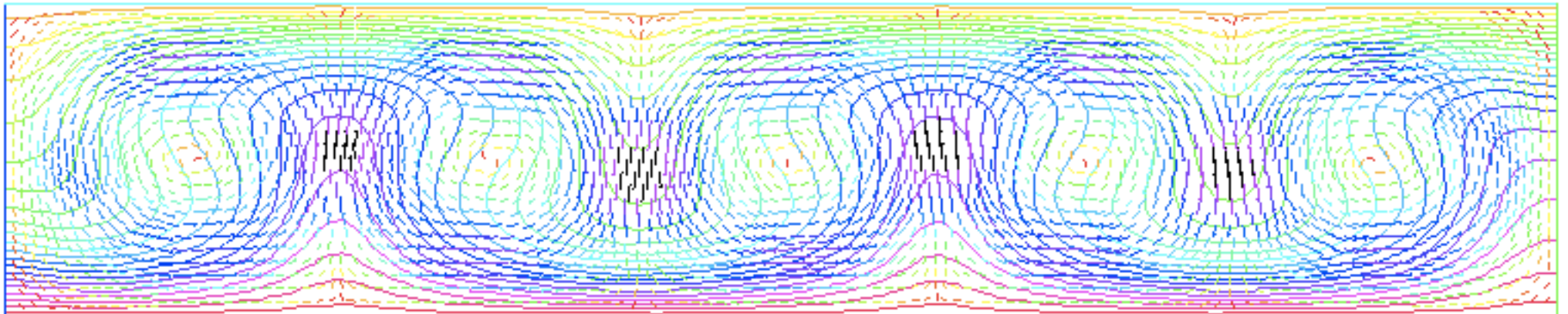
## Convection libre: Le problème de Rayleigh Bénard



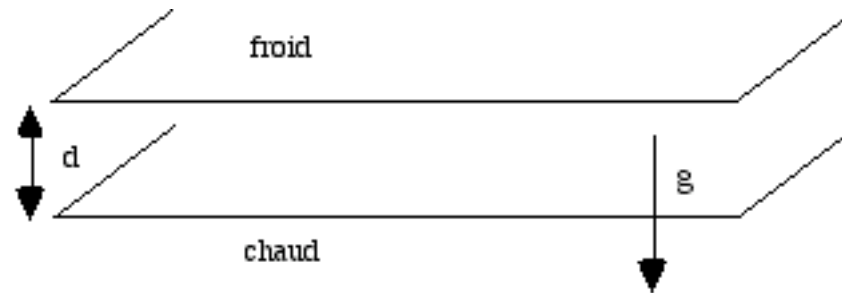
deux plaques limitant un fluide, une est plus chauffée que l'autre.

$$Ra = \frac{\alpha g \Delta T P r d^3}{\nu^2}$$

**R=8000**



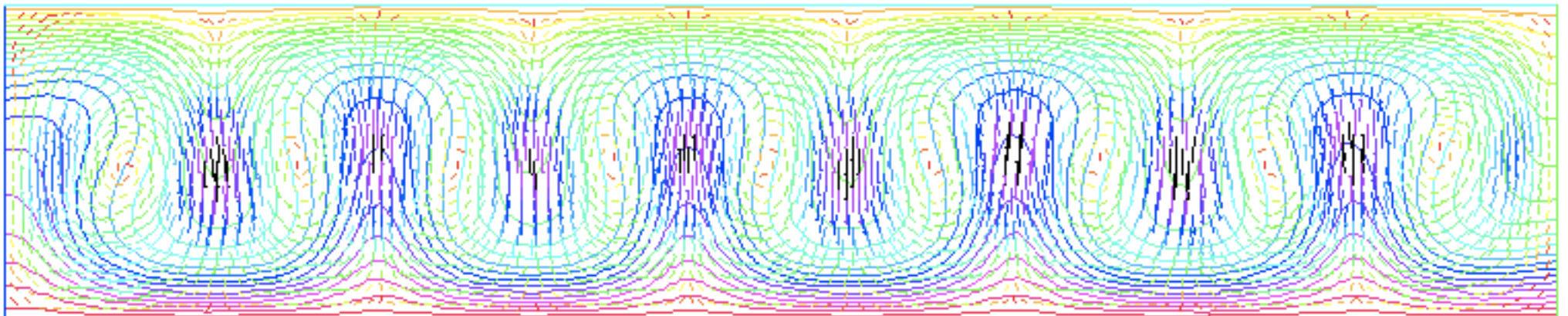
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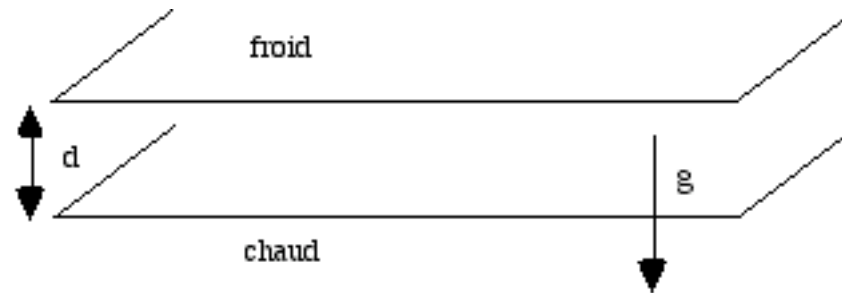
deux plaques limitant un fluide, une est plus chauffée que l'autre.

$$Ra = \frac{\alpha g \Delta T P r d^3}{\nu^2}$$

**R=16000**

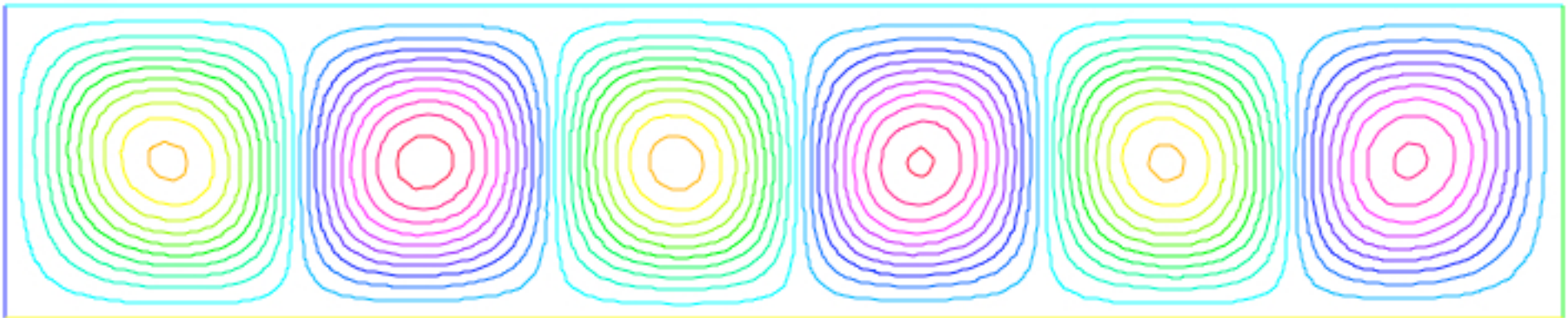


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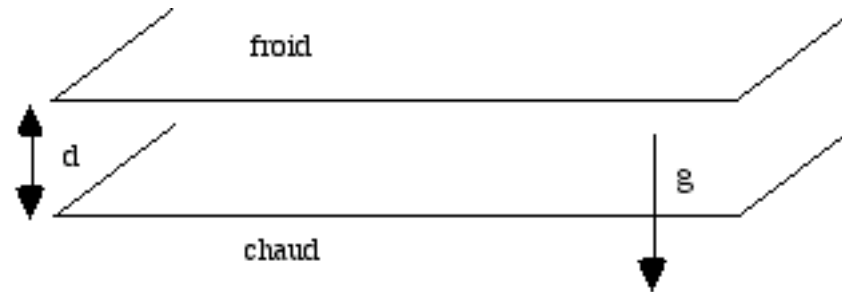


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## Convection libre: Le problème de Rayleigh Bénard

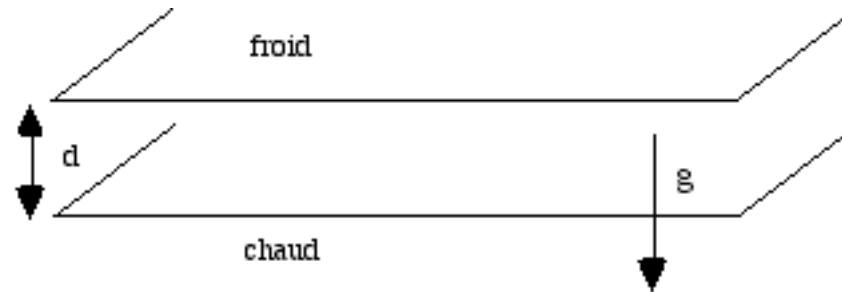


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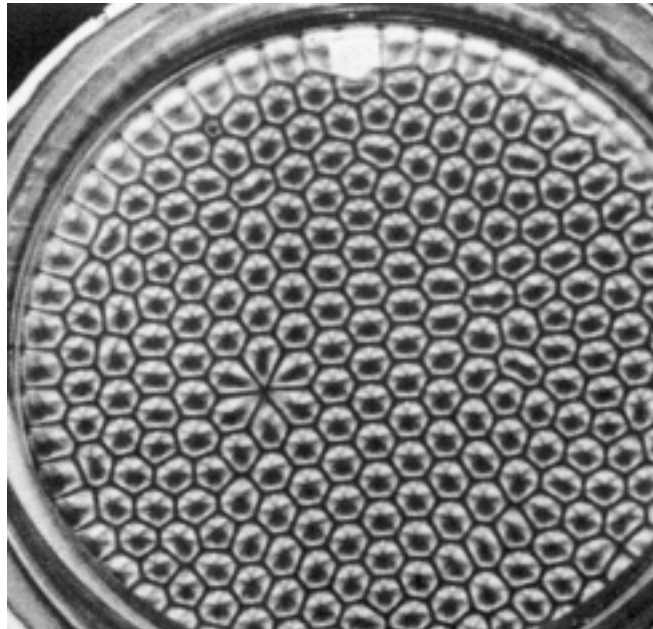


rouleaux vus de côté (plaque en haut et en bas)

## Convection libre: Le problème de Rayleigh Bénard



deux plaques limitant un fluide, une est plus chauffée que l'autre.



ans le cas où la surface est libre et où le fond est chauffé: on voit (en vue de dessus) des hexagones

$$\underline{\nabla} \cdot \underline{u} = 0$$

$$\rho_{\infty}(\frac{\partial}{\partial t}\underline{u} + \underline{u} \cdot \underline{\nabla} \underline{u}) = -\underline{\nabla} p + \rho_{\infty}g(1 - \alpha(T - T_0))\underline{e}_z + \mu_{\infty}\underline{\nabla}^2 \underline{u}.$$

$$\rho_{\infty}c_p(\frac{\partial}{\partial t} + \underline{u} \cdot \underline{\nabla})T = k\underline{\nabla}^2 T$$

# Etat de Base

$$\frac{\partial^2 T}{\partial x^2}=0;$$

$$u=v=w=0,\quad T=T_0-\Delta z/d,$$

:

$$p=p_0-\rho_0gd(z/d+1/2\alpha\Delta Tz^2/d^2).$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{\partial p}{\rho \partial x} + Pr(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}),$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{\partial p}{\rho \partial y} + Pr(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2})$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{\partial p}{\rho \partial z} + Pr(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}) - PrRa\Theta$$

$$\frac{\partial \Theta}{\partial t} + u \frac{\partial \Theta}{\partial x} + v \frac{\partial \Theta}{\partial y} - w = \frac{\nu}{Pr}(\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} + \frac{\partial^2 \Theta}{\partial z^2})$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{\partial p}{\rho \partial x} + Pr \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right),$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{\partial p}{\rho \partial y} + Pr \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{\partial p}{\rho \partial z} + Pr \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - Pr Ra \Theta$$

$$\frac{\partial \Theta}{\partial t} + u \frac{\partial \Theta}{\partial x} + v \frac{\partial \Theta}{\partial y} - w = \frac{\nu}{Pr} \left( \frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} + \frac{\partial^2 \Theta}{\partial z^2} \right)$$

$$Ra = \frac{\alpha g \Delta T Pr d^3}{\nu^2}$$

$$Ra = \frac{\alpha g \Delta T Pr d^3}{\nu^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{\partial p}{\rho \partial x} + Pr \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right),$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{\partial p}{\rho \partial y} + Pr \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{\partial p}{\rho \partial z} + Pr \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - Pr Ra \Theta$$

$$\frac{\partial \Theta}{\partial t} + u \frac{\partial \Theta}{\partial x} + v \frac{\partial \Theta}{\partial y} - w = \frac{\nu}{Pr} \left( \frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} + \frac{\partial^2 \Theta}{\partial z^2} \right)$$

$$Ra = \frac{\alpha g \Delta T Pr d^3}{\nu^2}$$

$$\frac{\partial}{\partial t} (\nabla^2 w) = Ra Pr \left( (\nabla^2 - \frac{\partial^2}{\partial z^2}) \Theta + Pr \nabla^2 \nabla^2 w \right)$$

$$\frac{\partial}{\partial t} \Theta - w = \nabla^2 \Theta$$

$$w = 0, \partial w / \partial z = 0, \Theta = 0 \text{ sur } z = 0 \text{ et } z = 1$$

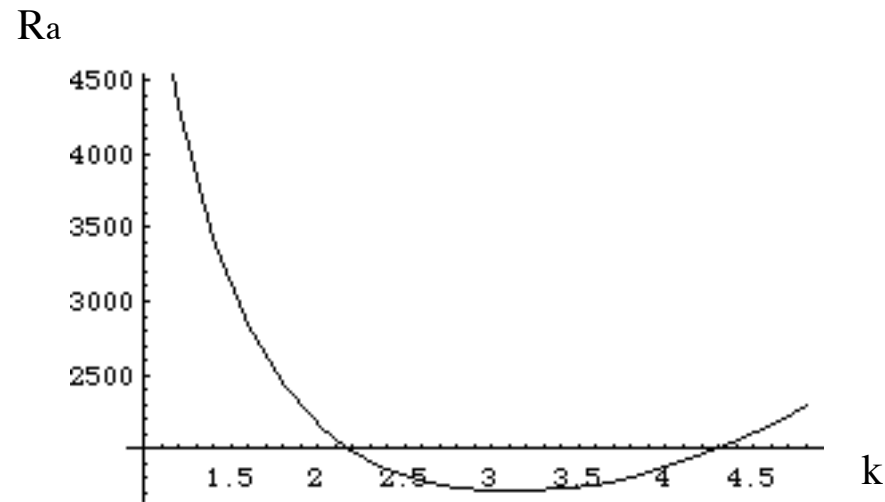
$$(w, \Theta) = (W(z), T(z)) e^{imx + ily} e^{\sigma t}.$$

$$k = \sqrt{(m^2 + l^2)}$$

# Relation de dispersion

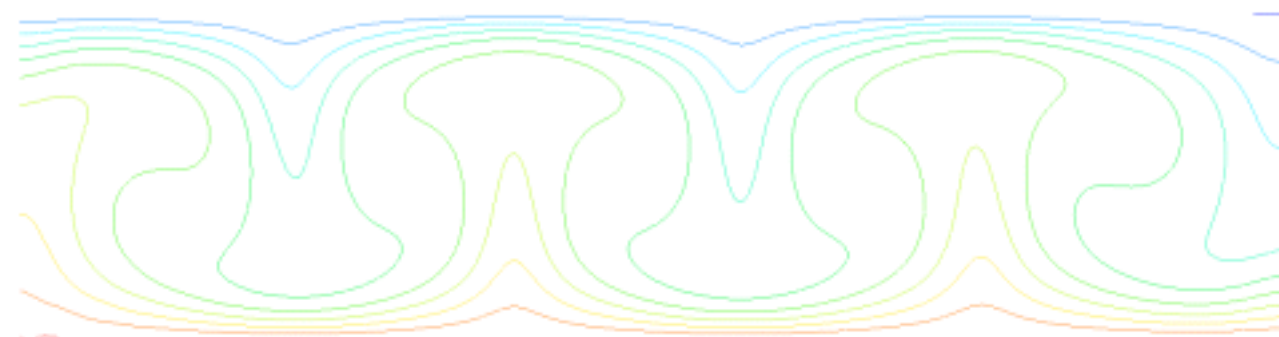
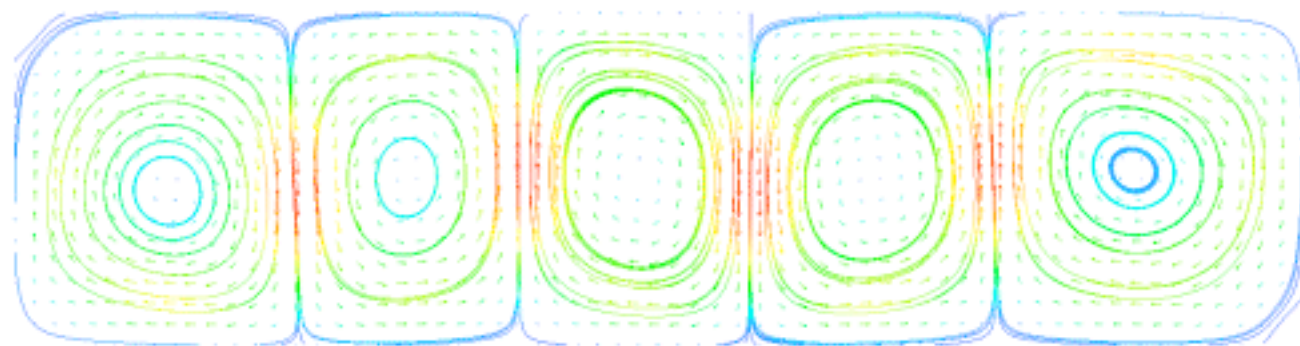
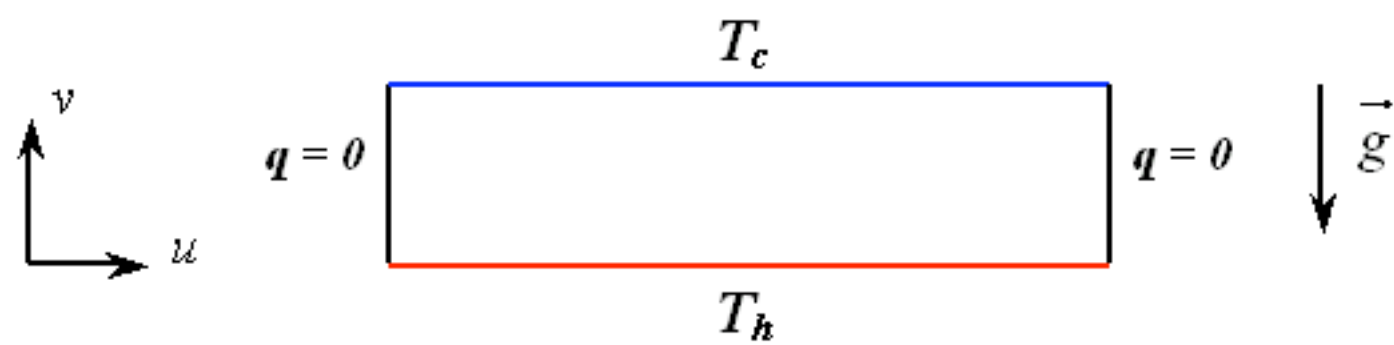
$$(D^2 - k^2)^2 W = k^2 \text{Ra} T; (D^2 - k^2) T = -W;$$

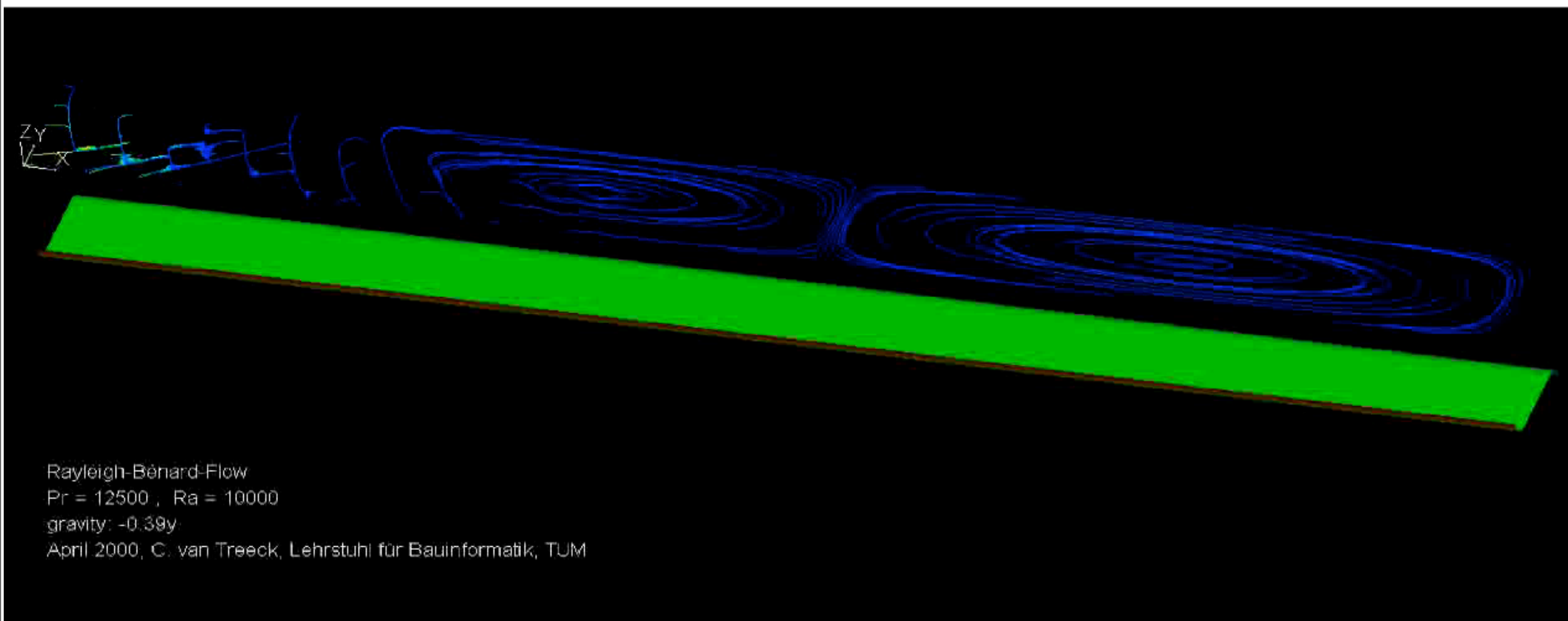
$$\text{soit } (D^2 - k^2)^3 T = -k^2 \text{Ra} T;$$

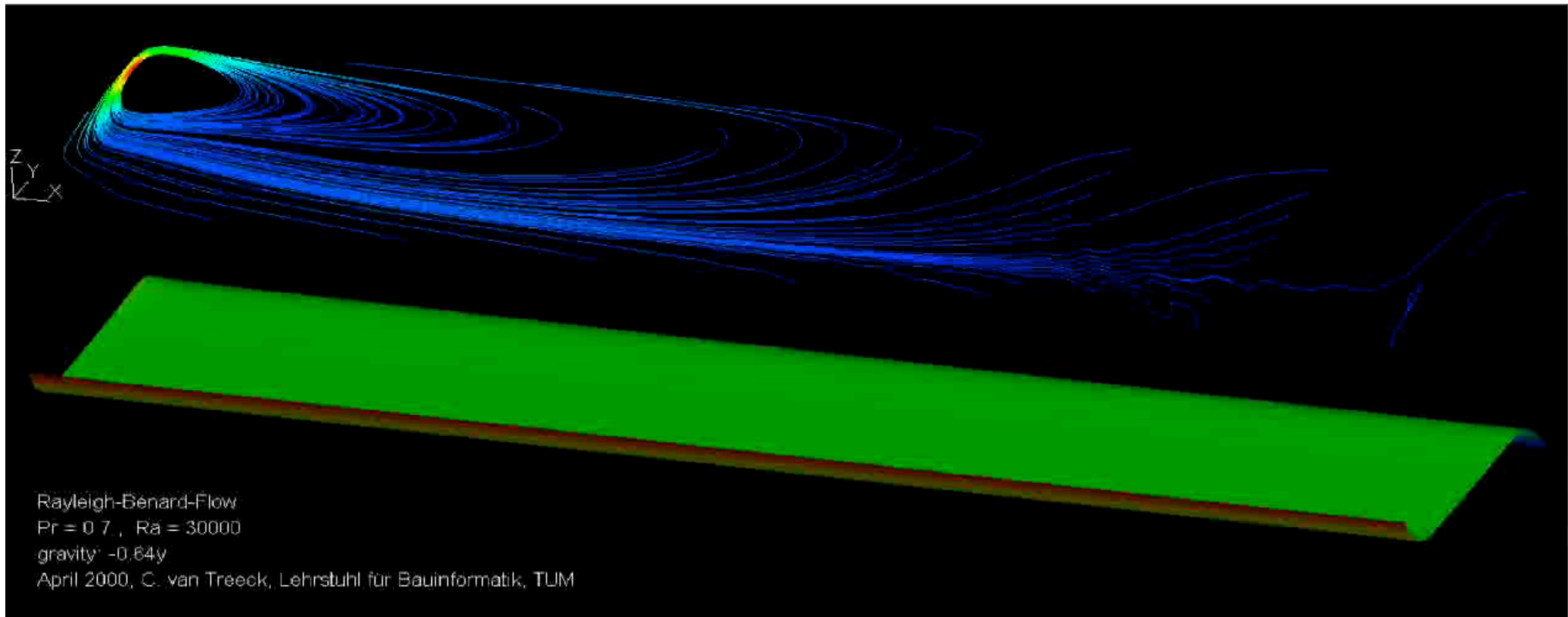


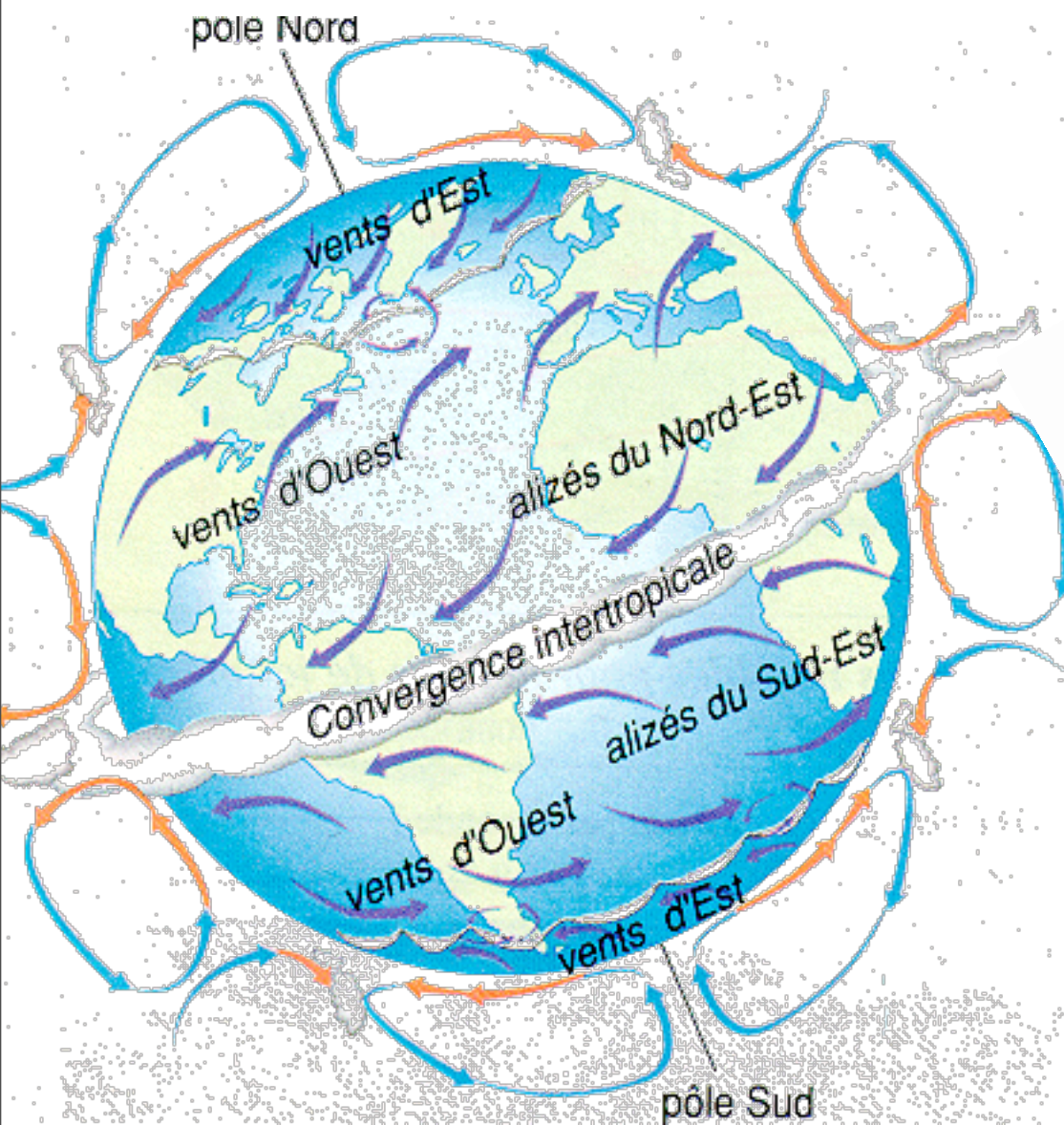
La valeur minimale est  $\text{Ra}_c \sim 1707,76$ . Le nombre d'onde critique est  $k_c \sim 3,1$

$$(w, \Theta) = (W(z), T(z)) e^{imx + ily} e^{\sigma t}.$$

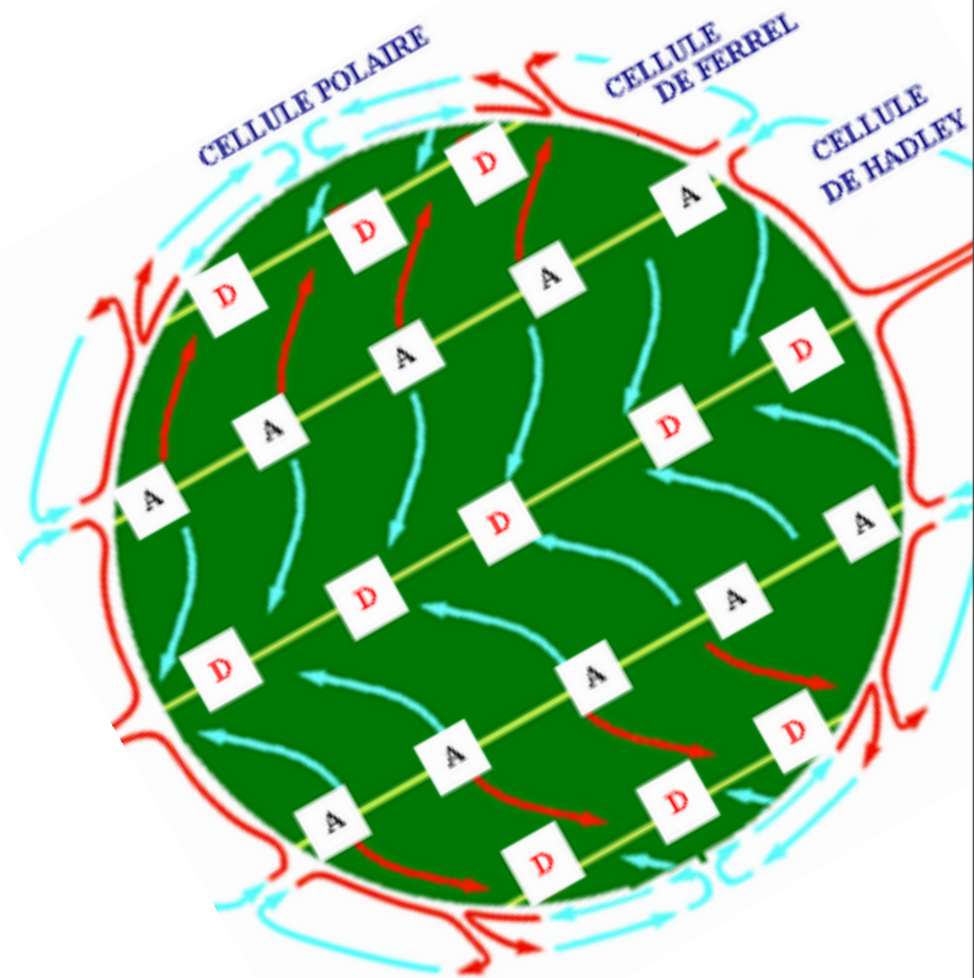




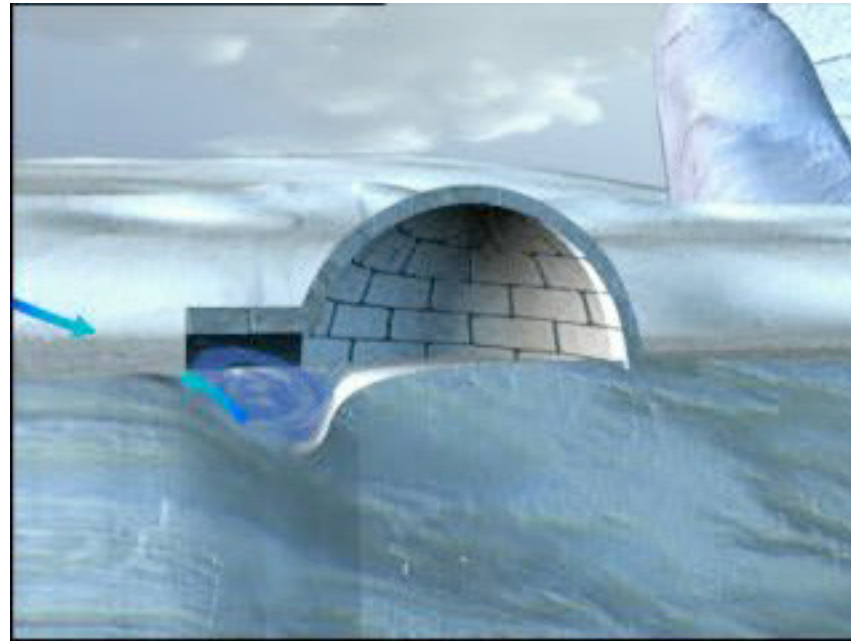




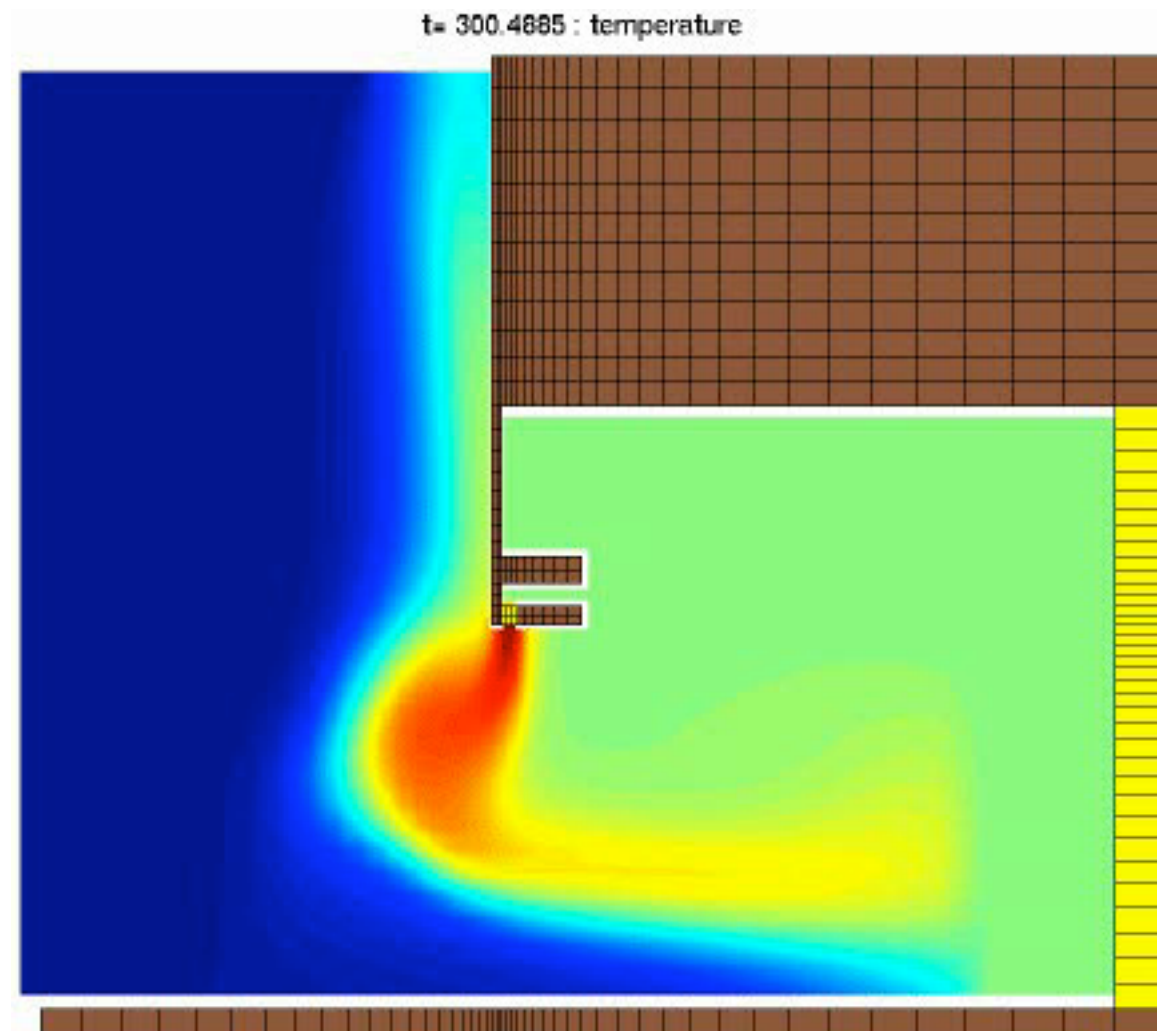
<http://www.ifremer.fr/lpo/cours/mouvement/sab17.html#>



[http://fr.wikipedia.org/wiki/Circulation\\_atmosphérique](http://fr.wikipedia.org/wiki/Circulation_atmosphérique)

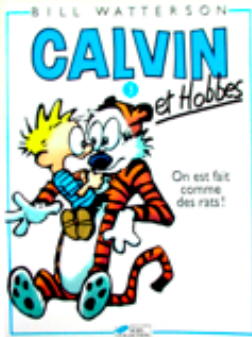












PARCE QUE L'AIR CHAUD MONTE. LE SOLEIL EST CHAUD EN MI-JOURNÉE, ALORS IL MONTE HAUT DANS LE CIEL.







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*dernière mise à jour : 20/12/2006 par vilas*



## MF204 : Transferts thermiques et massiques dans les fluides

| Année : 2A               |             | Module : voie                        | Crédits ECTS : 2                             |
|--------------------------|-------------|--------------------------------------|----------------------------------------------|
| Professeurs :            |             | Pierre-Yves LAGREE                   |                                              |
| Maîtres de conférences : |             | Pierre-Yves LAGREE ; Michael BAUDOIN |                                              |
| DATE                     | HEURES      | TYPE SEANCE                          | PROGRAMME DE LA SEANCE                       |
| 23 01 2007               | 08:30 09:30 | Cours magistral                      | Rappels de conduction, coefficient d'échange |
| 23 01 2007               | 09:45 11:45 | Petite classe                        |                                              |
| 30 01 2007               | 8:30 9:30   | Cours magistral                      | Convection forcée                            |
| 30 01 2007               | 9:45 11:45  | Petite classe                        |                                              |
| 06 02 2007               | 8:30 9:30   | Cours magistral                      | Convection forcée                            |
| 06 02 2007               | 9:45 11:45  | Petite classe                        |                                              |
| 13 02 2007               | 08:30 09:30 | Cours magistral                      | Convection Naturelle                         |
| 13 02 2007               | 09:45 11:45 | Petite classe                        |                                              |
| 20 02 2007               | 08:30 09:30 | Cours magistral                      | Convection forcée turbulente                 |
| 20 02 2007               | 09:45 11:45 | Petite classe                        |                                              |
| 06 03 2007               | 08:30 09:30 | Cours magistral                      | Transferts de masse                          |
| 06 03 2007               | 09:45 11:45 | Petite classe                        |                                              |
| 13 03 2007               | 08:30 11:30 | Contrôle                             | Contrôle de connaissances                    |
|                          |             | Petite classe                        |                                              |

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