

Convection

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équation de conservation de la masse :

$$\frac{d\rho}{dt} + \rho \underline{\nabla} \cdot \underline{u} = 0.$$

équation de conservation de la quantité de mouvement :

$$\rho \frac{d\underline{u}}{dt} = \underline{\nabla} \cdot \underline{\underline{\sigma}} + \underline{f}.$$

équation de l'énergie :

$$\rho \frac{de}{dt} = \underline{\underline{\sigma}} : \underline{\underline{D}} - \underline{\nabla} \cdot \underline{q} + r.$$

relations constitutives :

$$\underline{\underline{\sigma}} = -p\underline{I} + \lambda \underline{\nabla} \cdot \underline{u} \underline{I} + 2\mu \underline{\underline{D}}$$

$$\underline{q} = -k \underline{\nabla} T.$$

loi d'état :

$$p(\rho, T)$$

coefficients :

$$c_v(T), \quad c_p(T), \quad \lambda(T), \quad \mu(T), \quad k(T)...$$

conditions aux limites T_w **OU** q_w imposés, adhérence à la paroi.

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Adimensionnement

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Adimensionnement

$$S \frac{\partial}{\partial \bar{t}} \bar{T} + (\bar{\underline{u}} \cdot \bar{\underline{\nabla}}) \bar{T} = \frac{1}{Pe} \bar{\underline{\nabla}}^2 \bar{T} + 2 \frac{E}{R_\infty} \bar{\underline{\underline{D}}} : \bar{\underline{\underline{D}}}.$$

Reynolds, Péclet, Prandtl, Eckert & Nusselt...

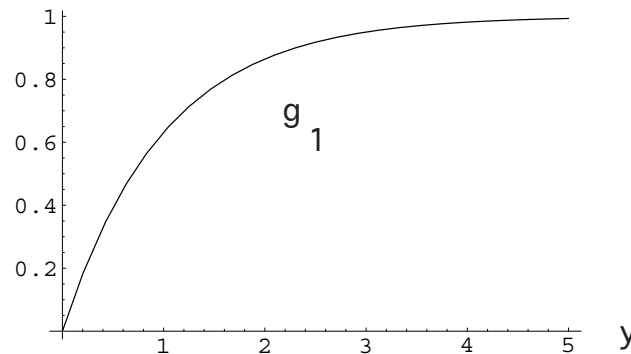
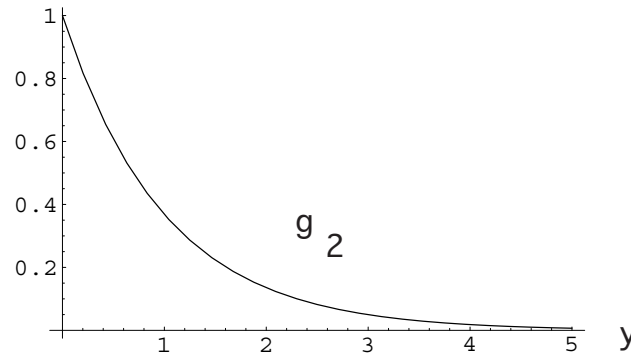
$$Re = \frac{\rho U_0 L}{\mu} \quad Pe = \frac{\rho c_p U_0 L}{k} \quad Pr = \frac{\mu c_p}{k} \quad E = \frac{U_0^2}{c \Delta T} \quad Nu = \frac{\frac{-k \partial T}{\partial x}}{\frac{k \Delta T}{L}}$$

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$$Nu = \frac{\frac{-k\partial T}{\partial n}}{\frac{k\Delta T}{L}}$$

Nombre de Nusselt



$$T_1 = T_w + (T_\infty - T_w)g_1$$

$$T_2 = T_\infty + (T_w - T_\infty)g_2$$

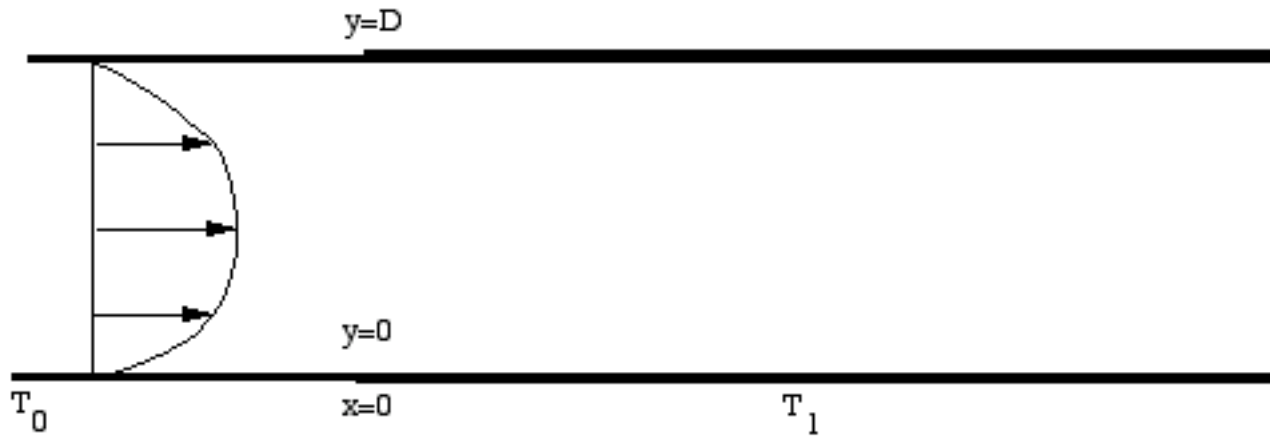
Le Nusselt est

$$-\frac{\partial T_1}{\partial y} / (T_w - T_\infty) = \frac{\partial g_1}{\partial y} > 0$$

$$-\frac{\partial T_2}{\partial y} / (T_w - T_\infty) = -\frac{\partial g_2}{\partial y} > 0$$

il est bien positif dans les deux cas !

Observations d'une couche limite dans un tuyau (calculs FreeFEM++)



écoulement 2D de Poiseuille entre deux plaques planes,
l'écoulement est le même partout, la température de la paroi est discontinue en $x=0$.

résolution numérique

$$u_0 \frac{\partial T}{\partial x} = \frac{1}{Pe} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

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$$\int \int dx dy \phi \left(u_0 \frac{\partial T}{\partial x} - \frac{1}{Pe} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \right) = 0$$

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$$\iint \frac{\partial^2 T}{\partial x^2} \phi dx dy = - \iint \frac{\partial T}{\partial x} \frac{\partial \phi}{\partial x} dx dy + \int \left[\frac{\partial T}{\partial x} \phi \right] dy$$

$$\iint \frac{\partial^2 T}{\partial y^2} \phi dx dy = - \iint \frac{\partial T}{\partial y} \frac{\partial \phi}{\partial y} dx dy + \int \left[\frac{\partial T}{\partial y} \phi \right] dx$$

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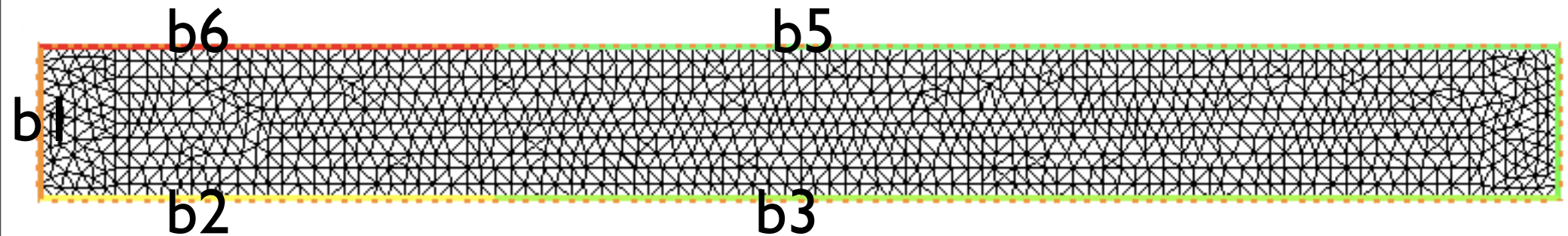
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Éléments Finis

$$-\int \int \frac{1}{Pe} \frac{\partial T}{\partial x} \frac{\partial \phi}{\partial x} dx dy - \int \int \frac{1}{Pe} \frac{\partial T}{\partial y} \frac{\partial \phi}{\partial y} dx dy - \int \int u_0 \frac{\partial T}{\partial x} \phi dx dy = 0$$



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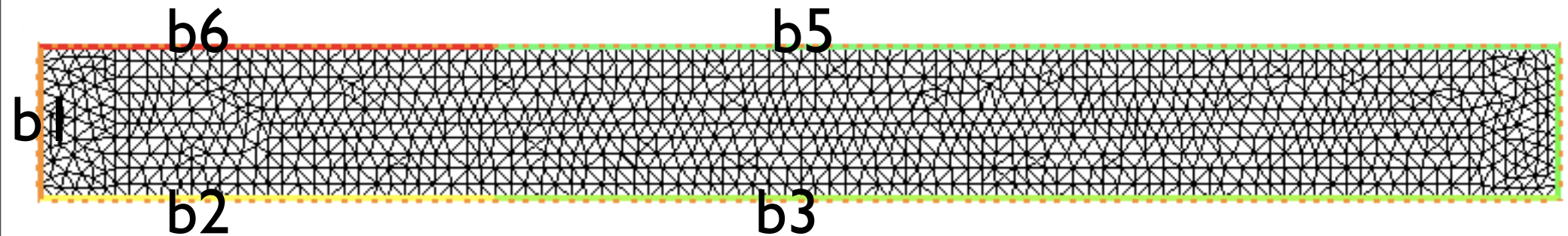
int nmx=10;
int nmy=10;

real L=7.0;
real L1=3;
/* effet d'entree dans un tube */
border b1(t=0,L1) {x=-L1; y=L1-t; };
border b2(t=0,L1) {x=-L1+t; y=0; };
border b3(t=0,L) {x=t; y=0; };
border b4(t=0,L) {x=L; y=t; };
border b5(t=0,L) {x=L-t; y=L; };
border b6(t=0,L1) {x=-t; y=L; };
/* fin de la definition de la geometrie */

mesh sh = buildmesh(b1(nmy)+ b2(L1*nmx)+b3(L*nmx)+ b4(nmy)+ b5(L*nmx)+b6(L1*nmx));
plot(sh);
/* construction du maillage */

```

$$-\int \int \frac{1}{Pe} \frac{\partial T}{\partial x} \frac{\partial \phi}{\partial x} dx dy - \int \int \frac{1}{Pe} \frac{\partial T}{\partial y} \frac{\partial \phi}{\partial y} dx dy - \int \int u_0 \frac{\partial T}{\partial x} \phi dx dy = 0$$



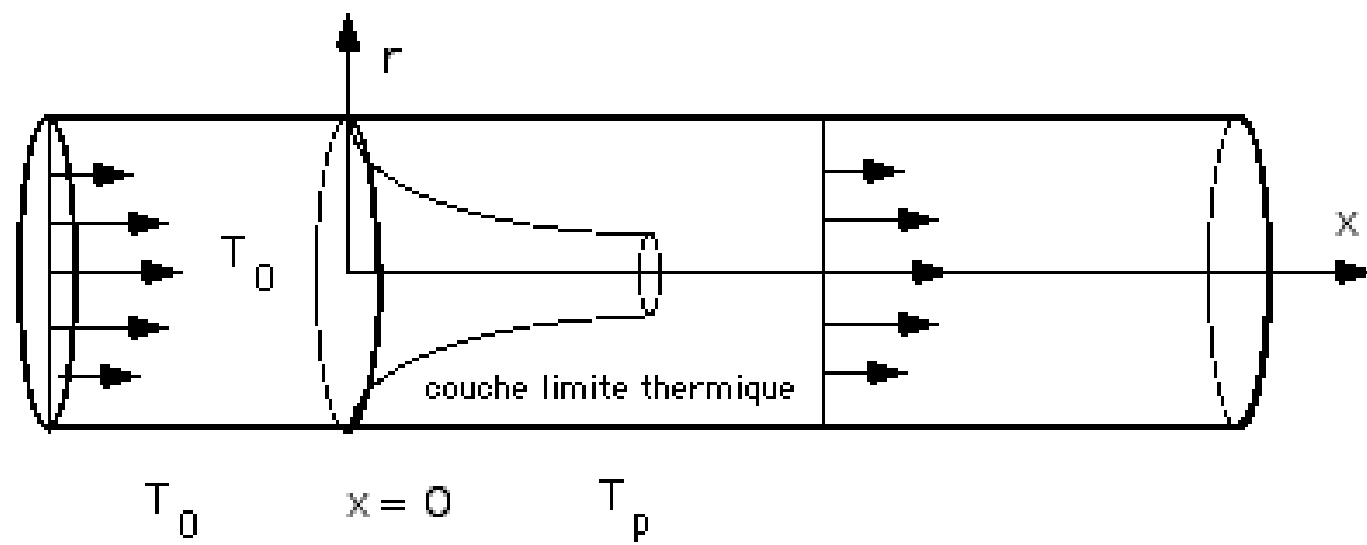
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problem therm(T,Tp) =
  int2d(sh)(dx(T)*dx(Tp)*invP + dy(T)*dy(Tp)*invP)

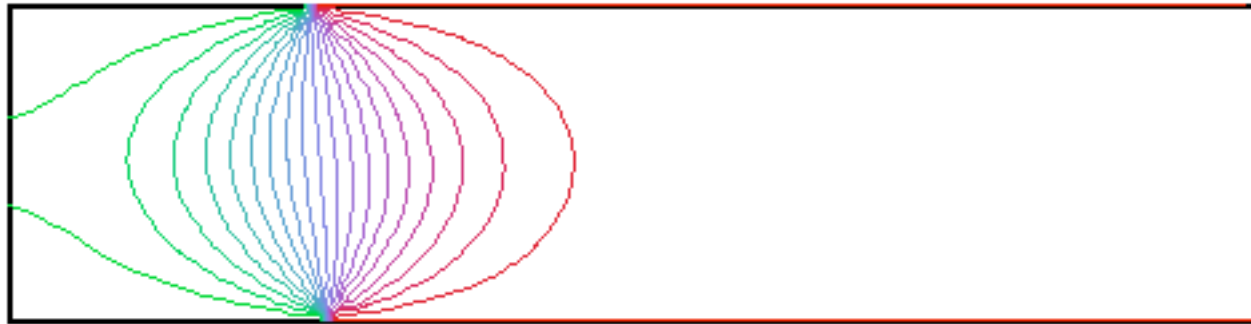
  + int2d(sh)(dx(T)*Tp*u0)

  + on(b1,T=0)
  + on(b2,T=0)
  + on(b3,T=T0)
  + on(b5,T=T0)
  + on(b6,T=0)
  ;

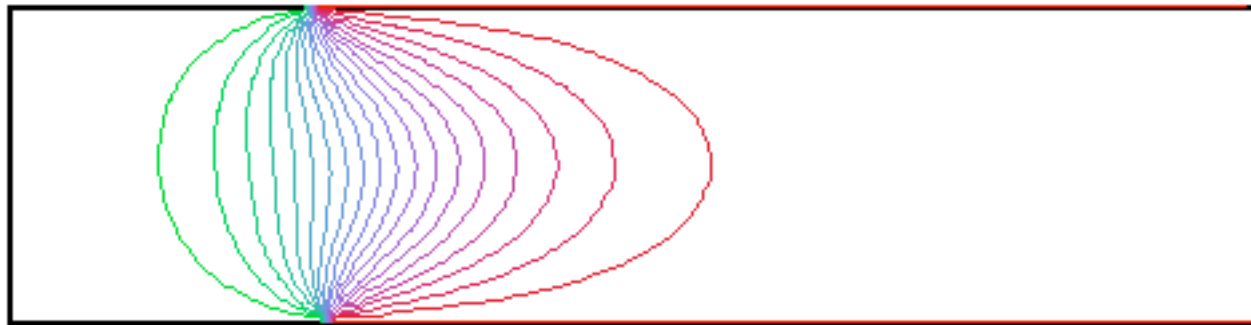
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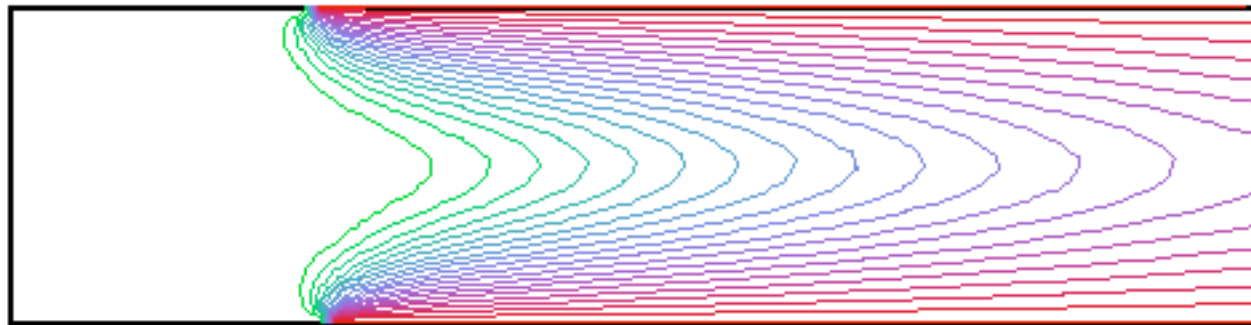
influence du nombre de Péclet



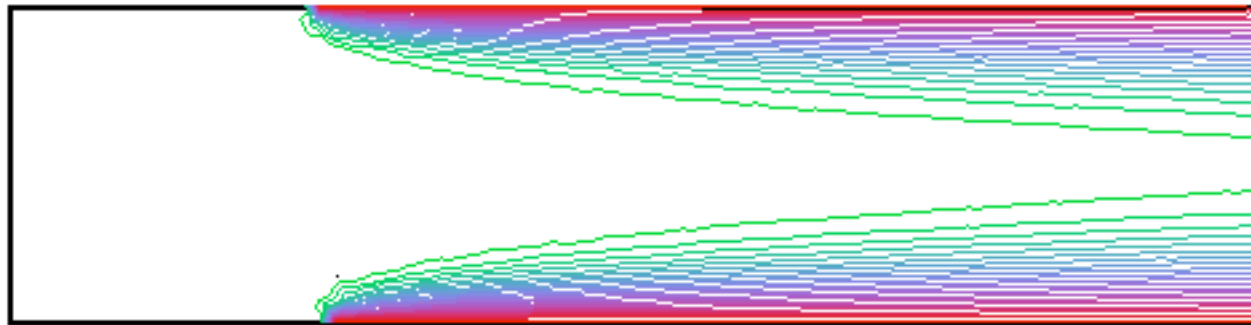
influence du nombre de Péclet



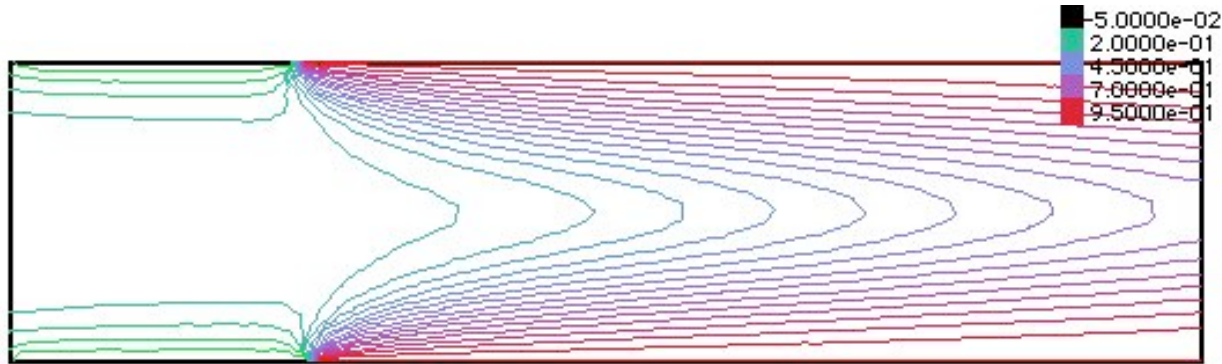
influence du nombre de Péclet



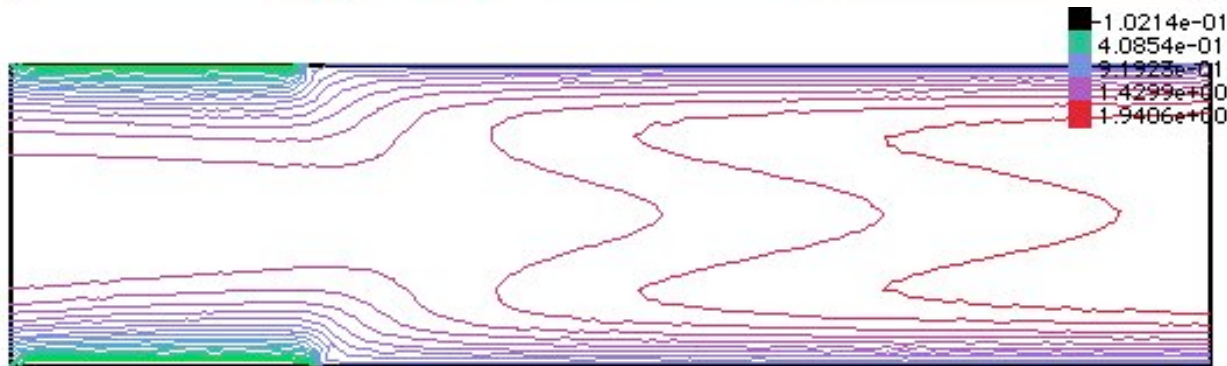
influence du nombre de Péclet



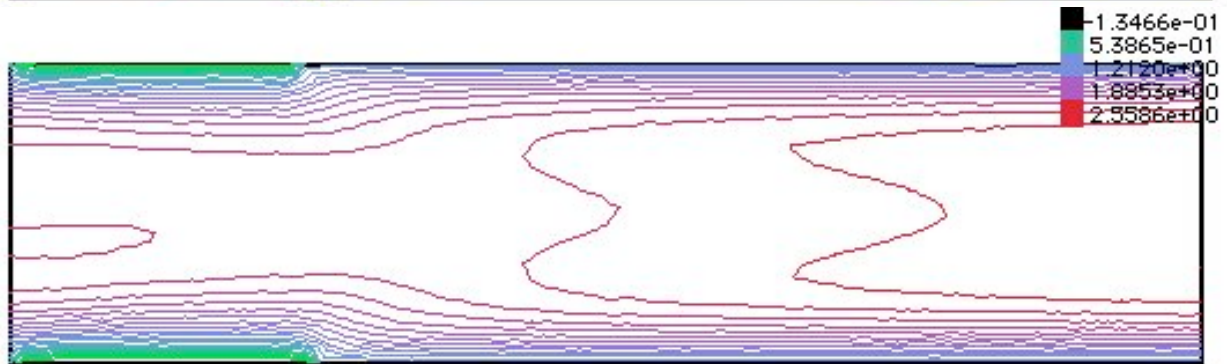
Influence du nombre d'Eckert



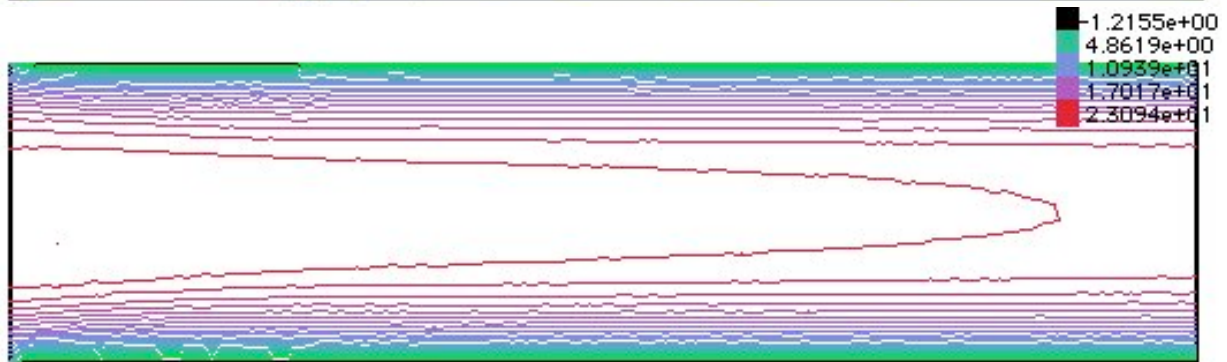
E=1 $1/Pe=0.005$



E=10 $1/Pe=0.005$



E=70 $1/Pe=0.005$



E=100 $1/Pe=0.005$

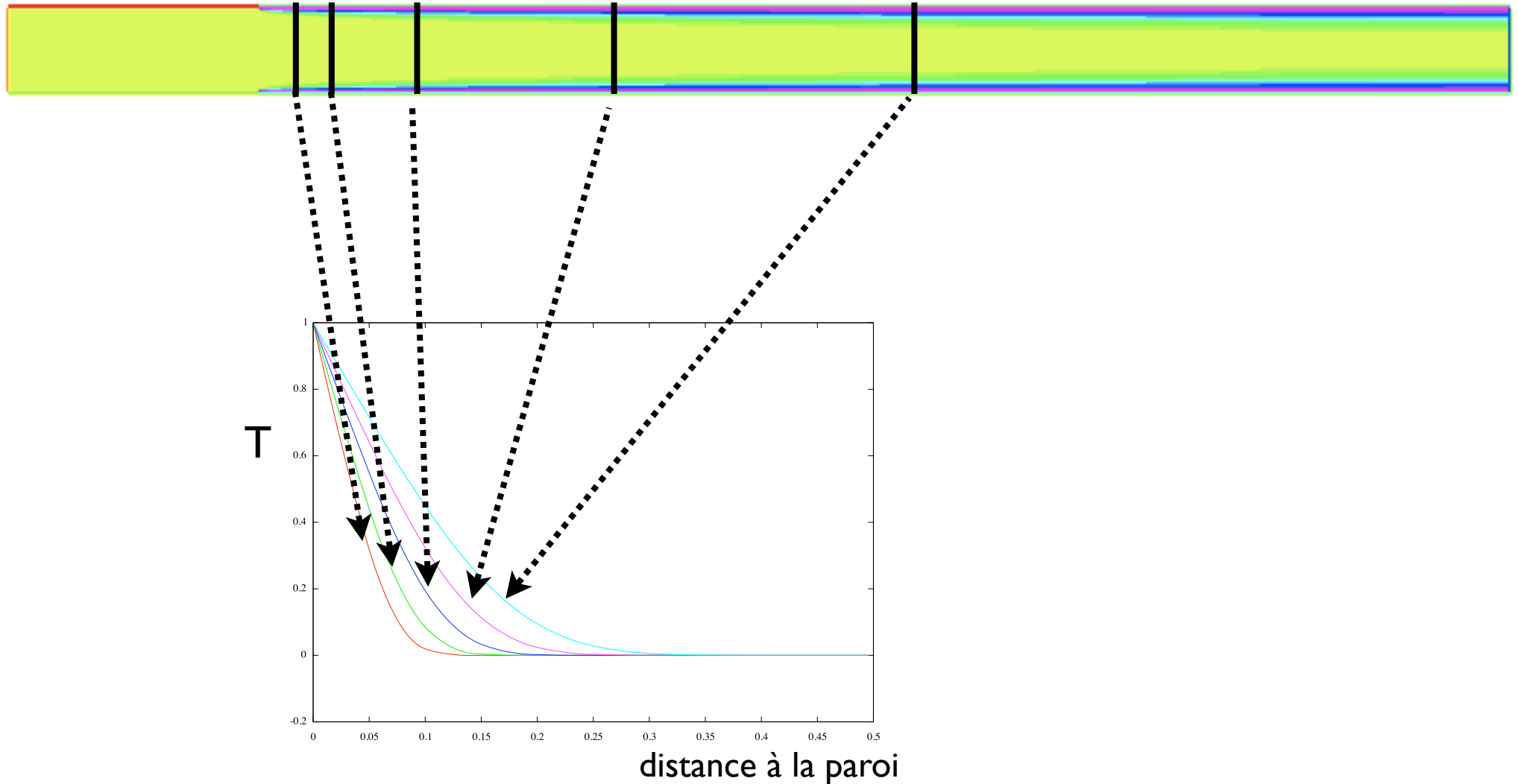
solution de Lévêque



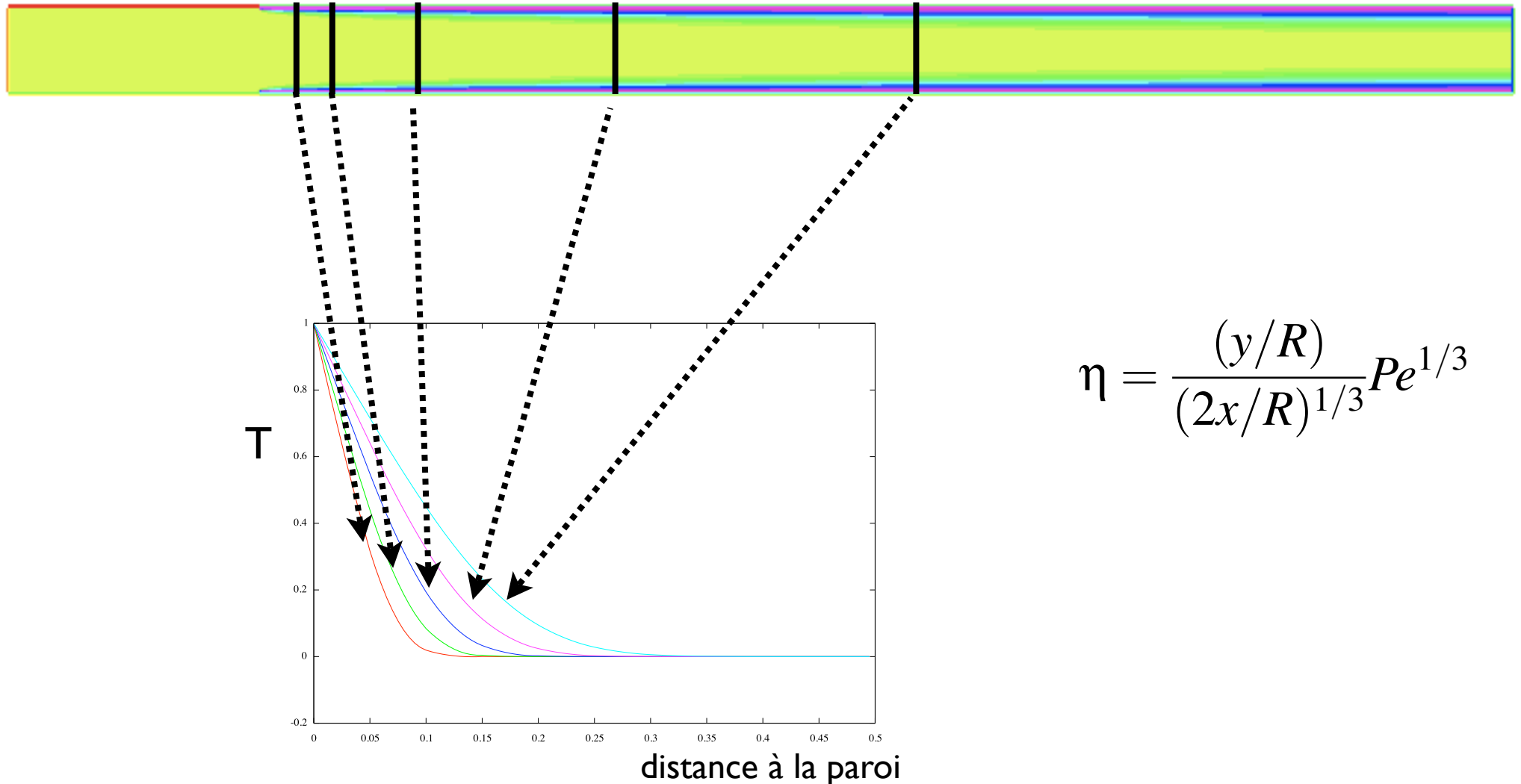
solution de Lévêque



solution de L  v  que

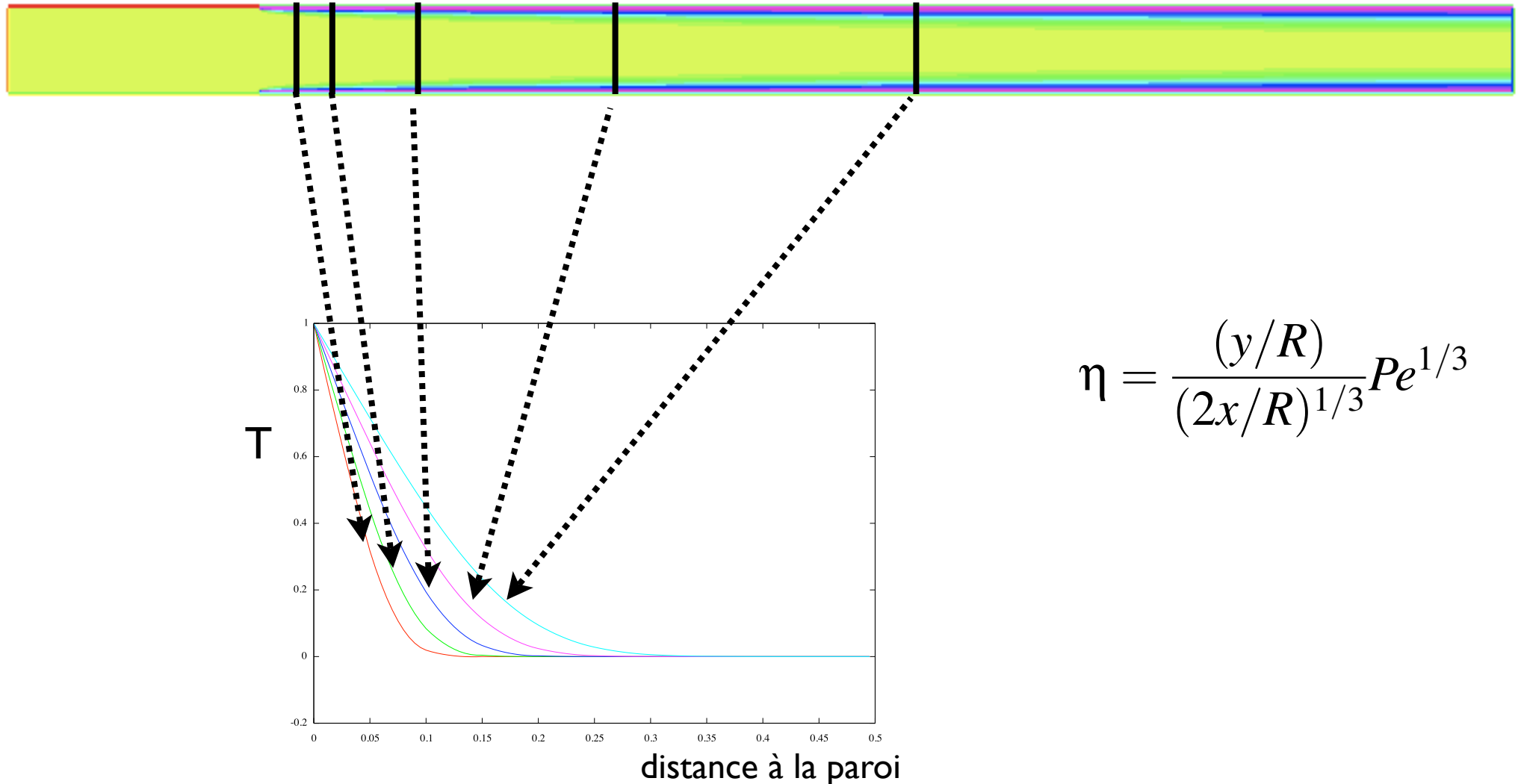


solution de L  v  que



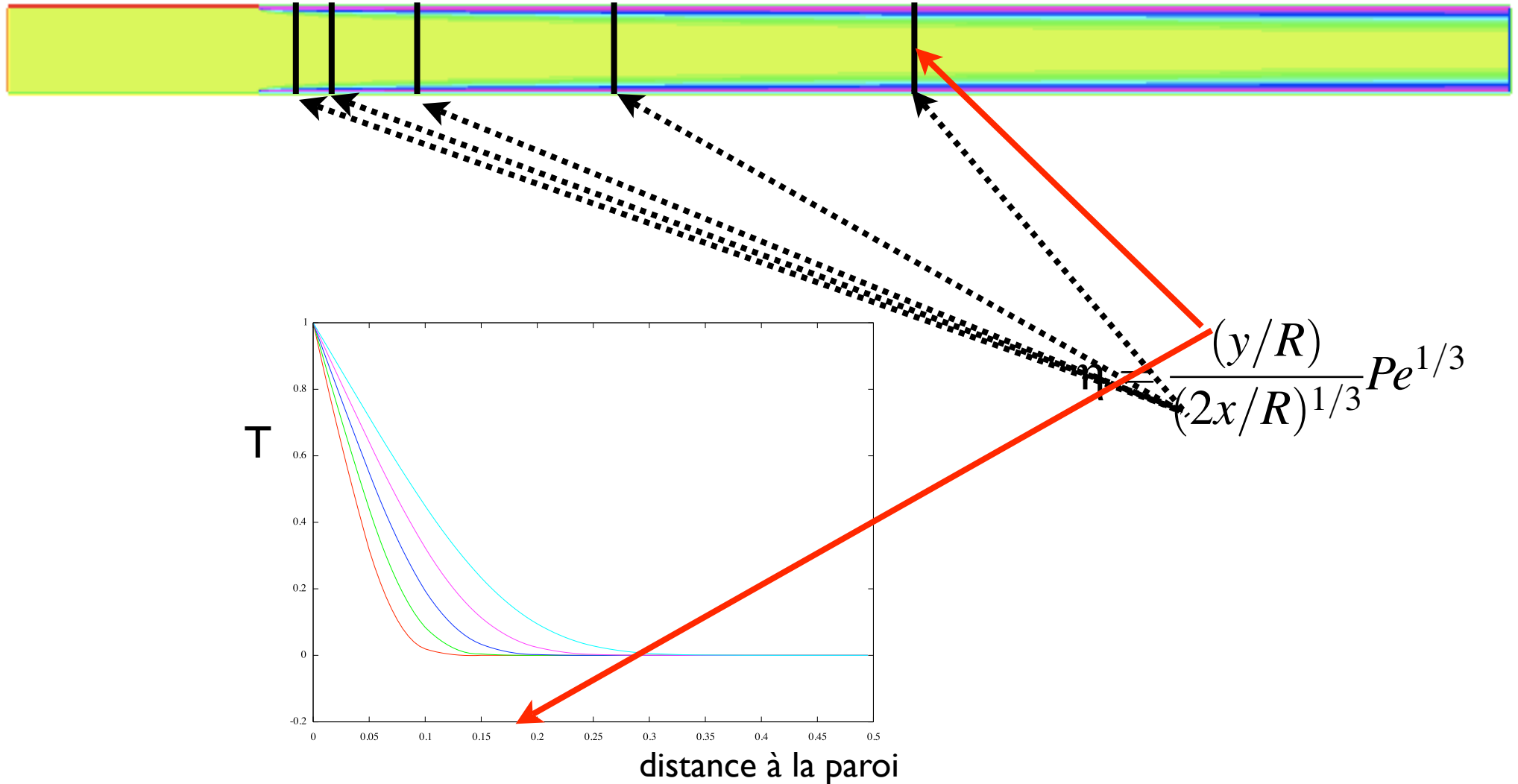
$$\eta = \frac{(y/R)}{(2x/R)^{1/3}} Pe^{1/3}$$

solution de L  v  que

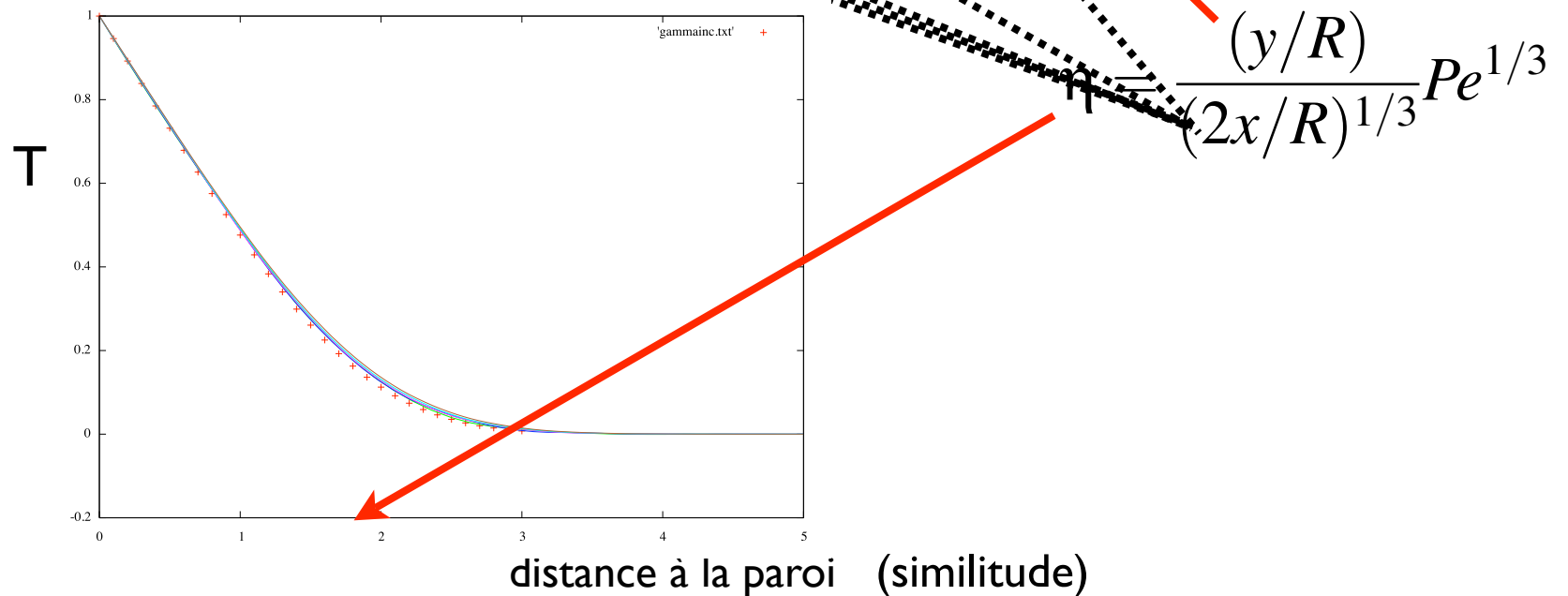


$$\eta = \frac{(y/R)}{(2x/R)^{1/3}} Pe^{1/3}$$

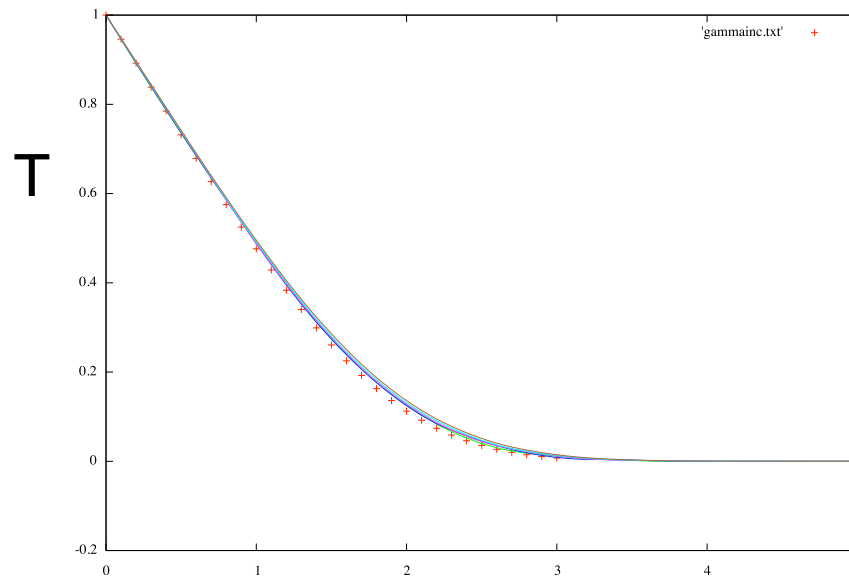
solution de L  v  que



solution de L  v  que



solution de Lévêque

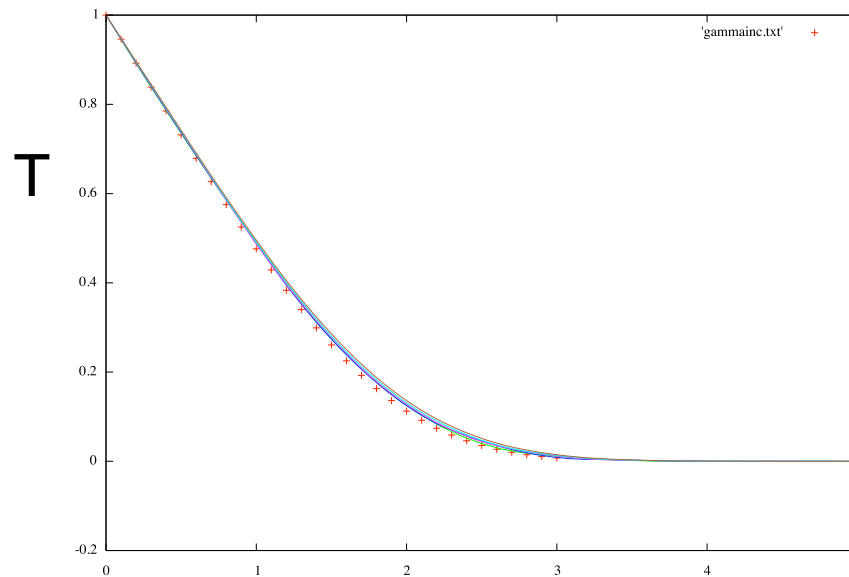


distance à la paroi (similitude)

$$\eta = \frac{(y/R)}{(2x/R)^{1/3}} Pe^{1/3}$$

$$\left(\frac{\partial \theta}{\partial \eta}\right)_0 = -3^{1/3} / \Gamma(1/3) = -0.538366$$

solution de Lévêque

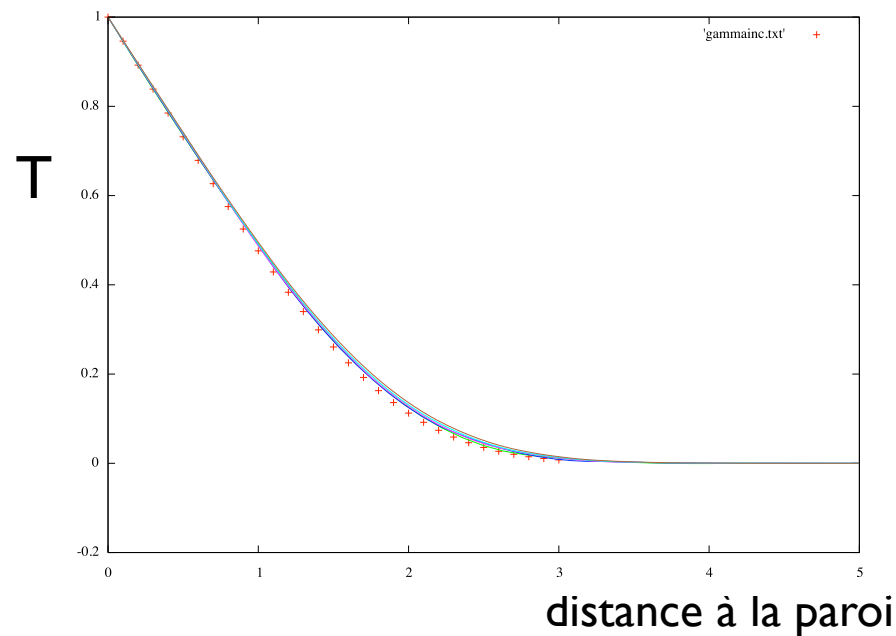
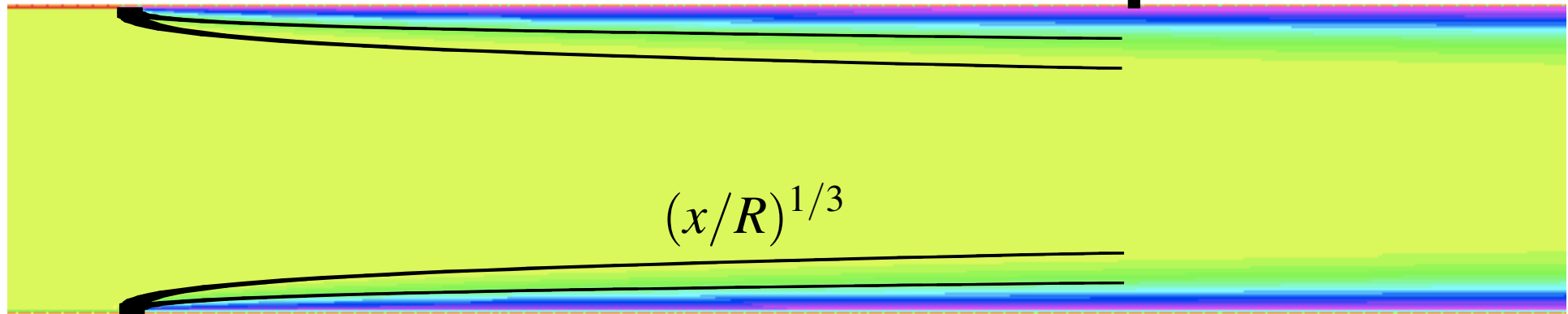


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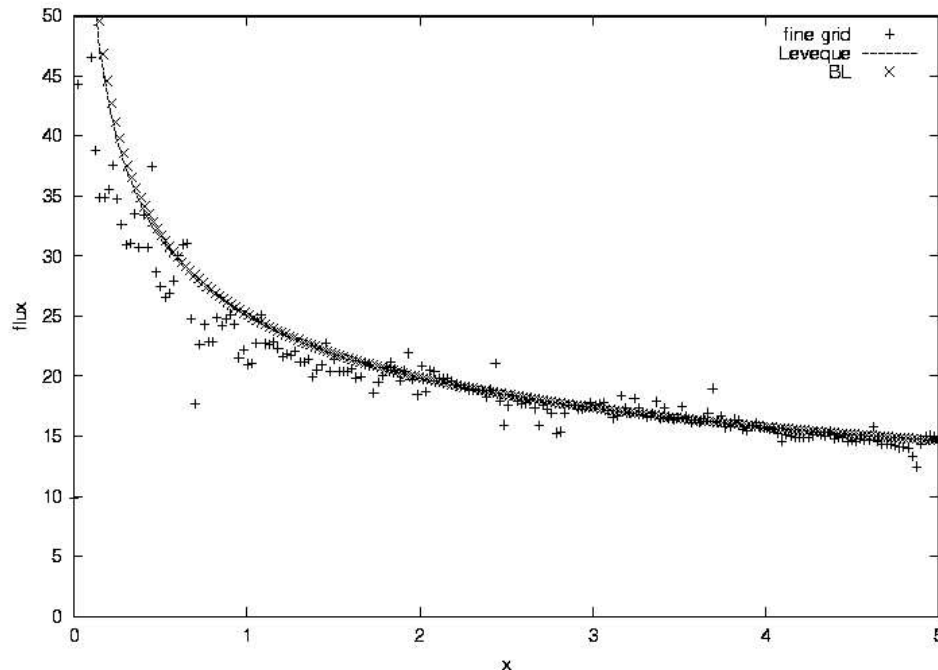
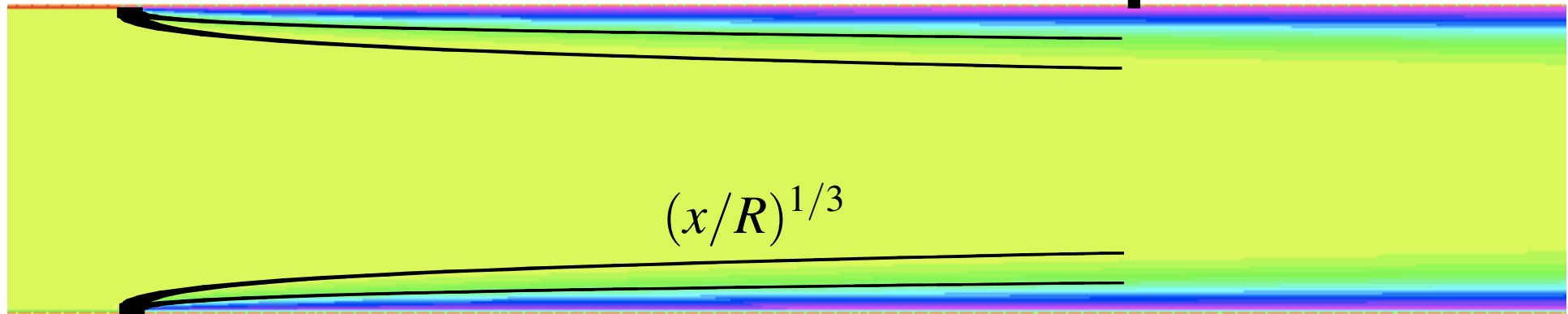
solution de L  v  que



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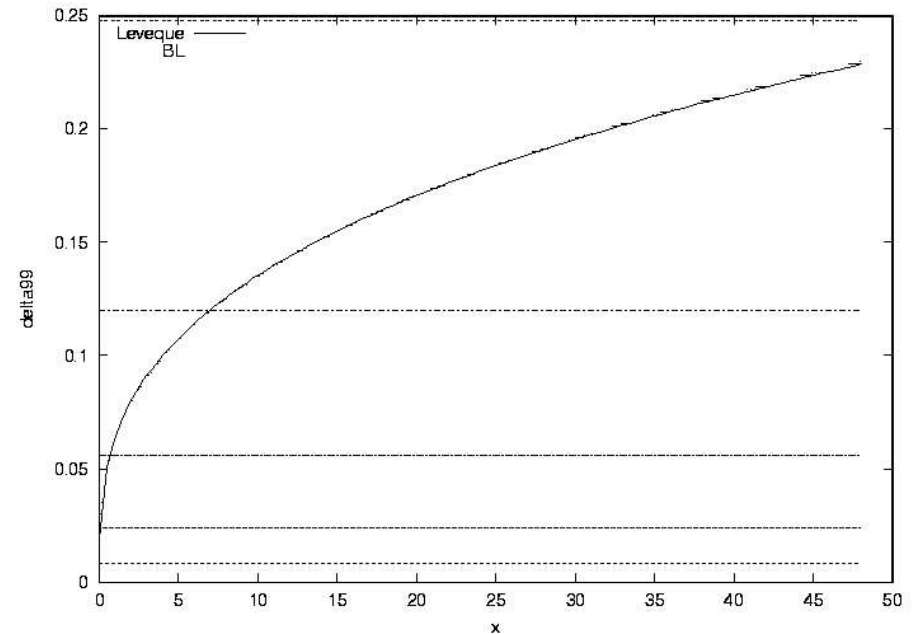
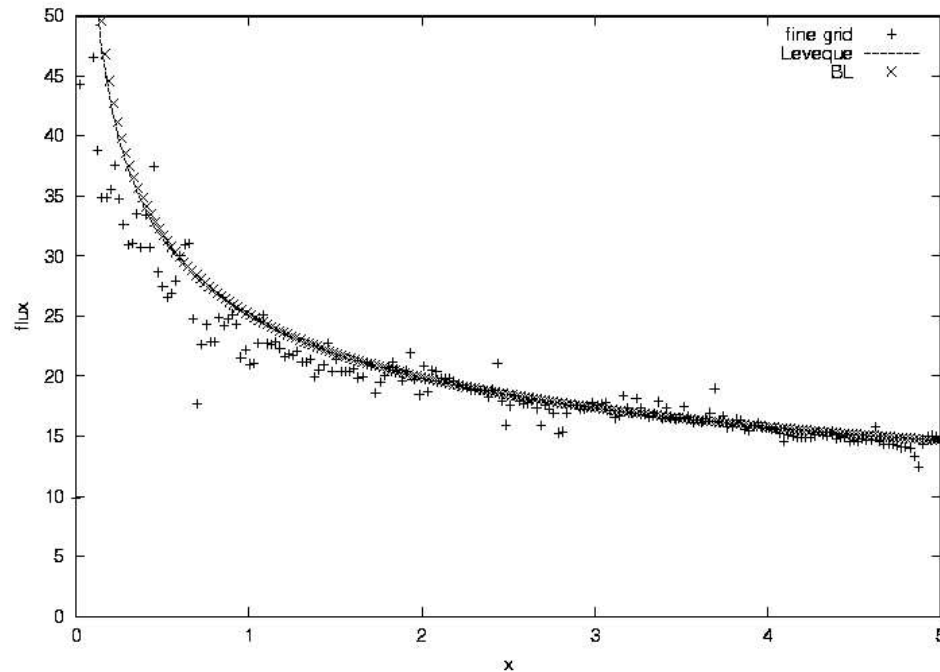
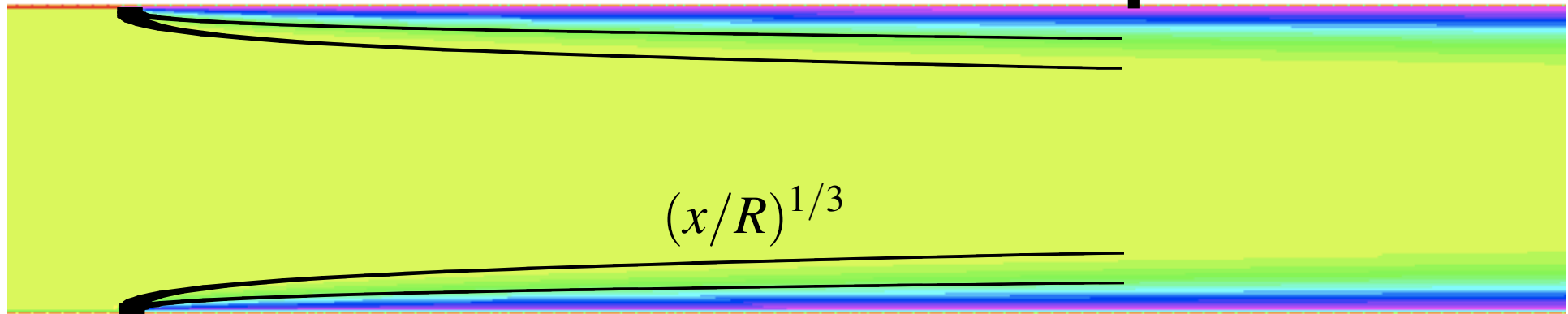
solution de L  v  que

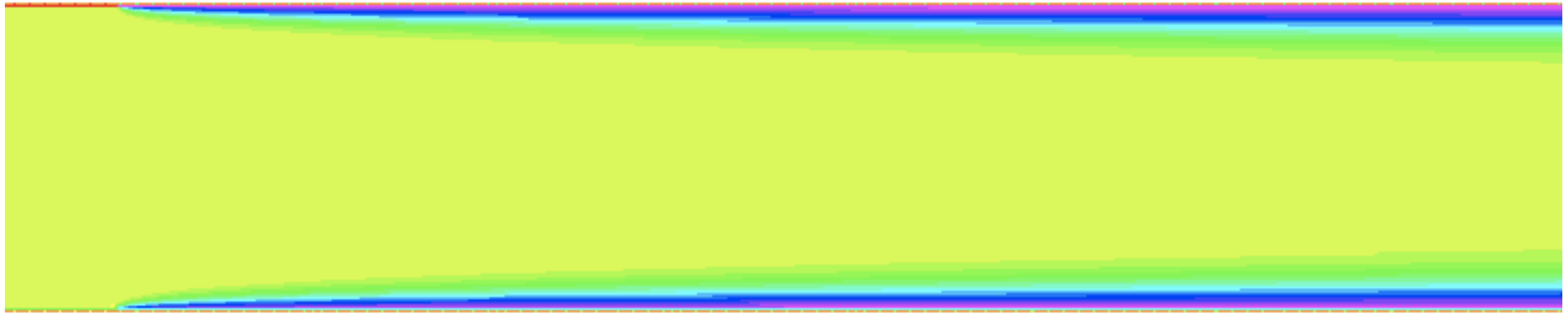


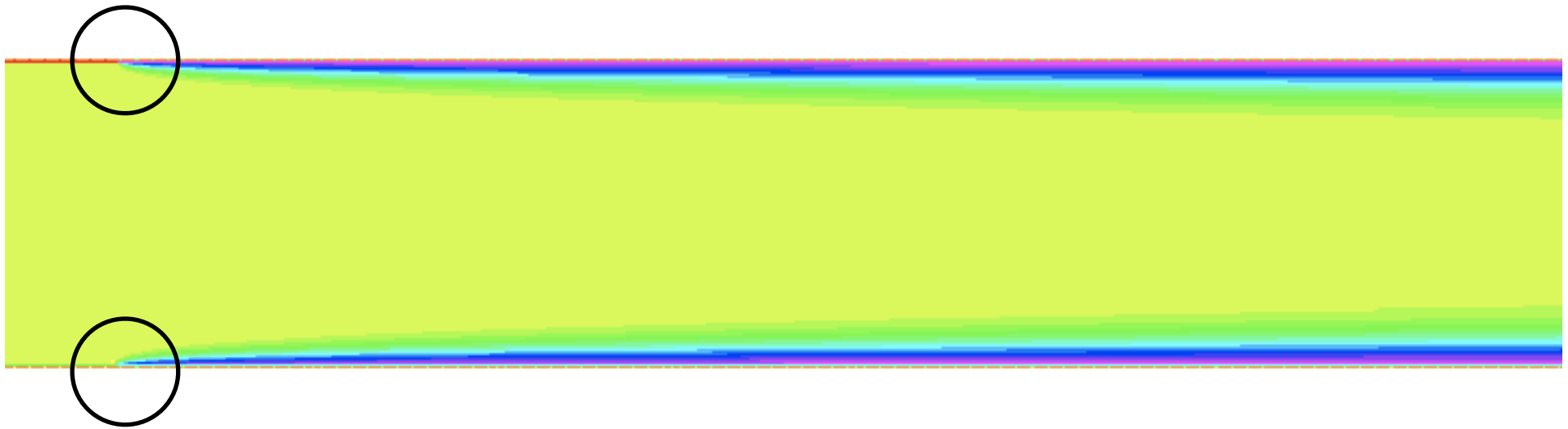
Nusselt

$$Nu = 0.538 \frac{(Pe_D^{1/3})}{(x/D)^{1/3}}$$

solution de L  v  que







$$\bar{x} = \varepsilon \tilde{\xi} \text{ et } \bar{r} = 1 - \varepsilon \tilde{\zeta}$$

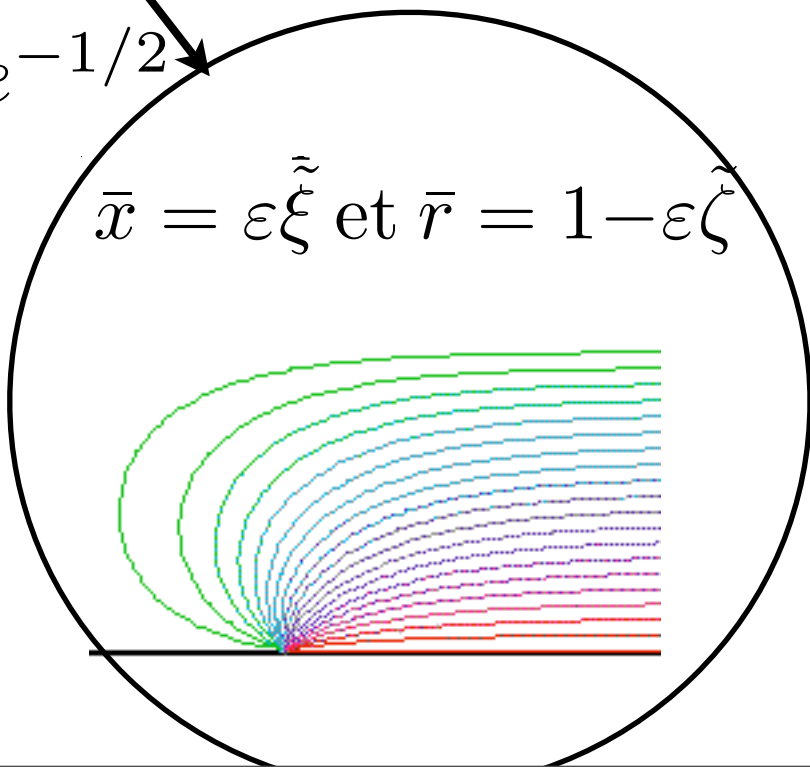
$$\tilde{\xi} \frac{\partial \tilde{\theta}}{\partial \tilde{\xi}} = \frac{\partial^2 \tilde{\theta}}{\partial \tilde{\xi}^2} + \frac{\partial^2 \tilde{\theta}}{\partial \tilde{\zeta}^2}$$

$$\varepsilon = Pe^{-1/2}$$



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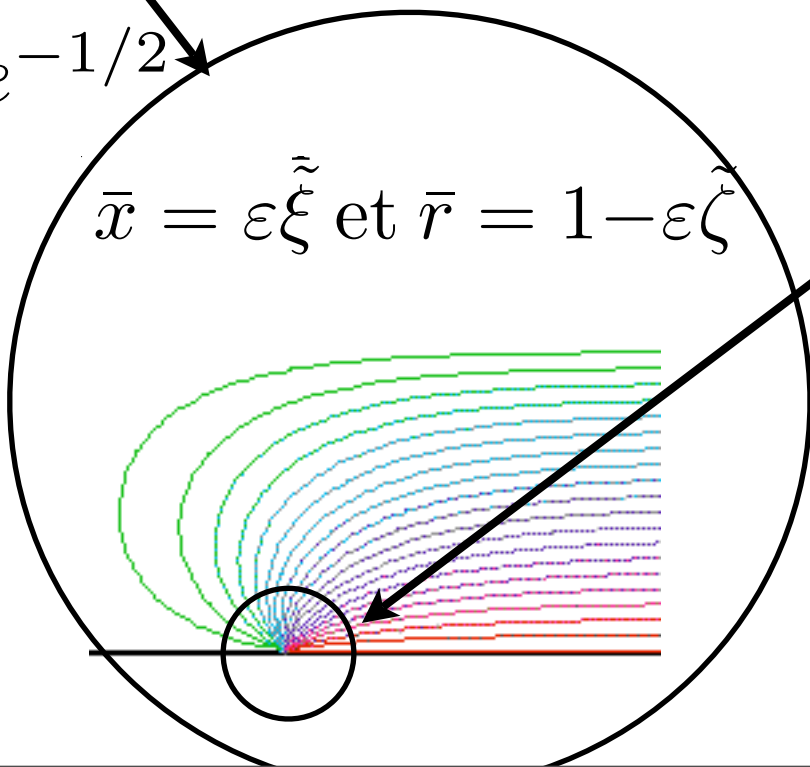


$$\varepsilon = Pe^{-1/2}$$

$$\bar{x} = \varepsilon \tilde{\xi} \text{ et } \bar{r} = 1 - \varepsilon \tilde{\zeta}$$

Diffusion Pure

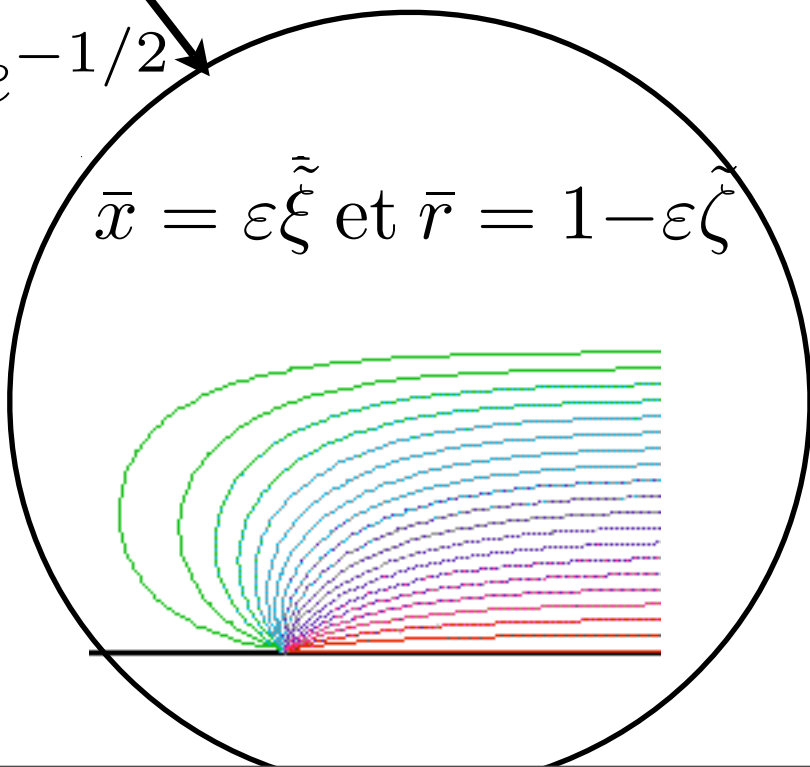
$$0 = \frac{\partial^2 \tilde{\theta}}{\partial \tilde{\xi}^2} + \frac{\partial^2 \tilde{\theta}}{\partial \tilde{\zeta}^2}$$



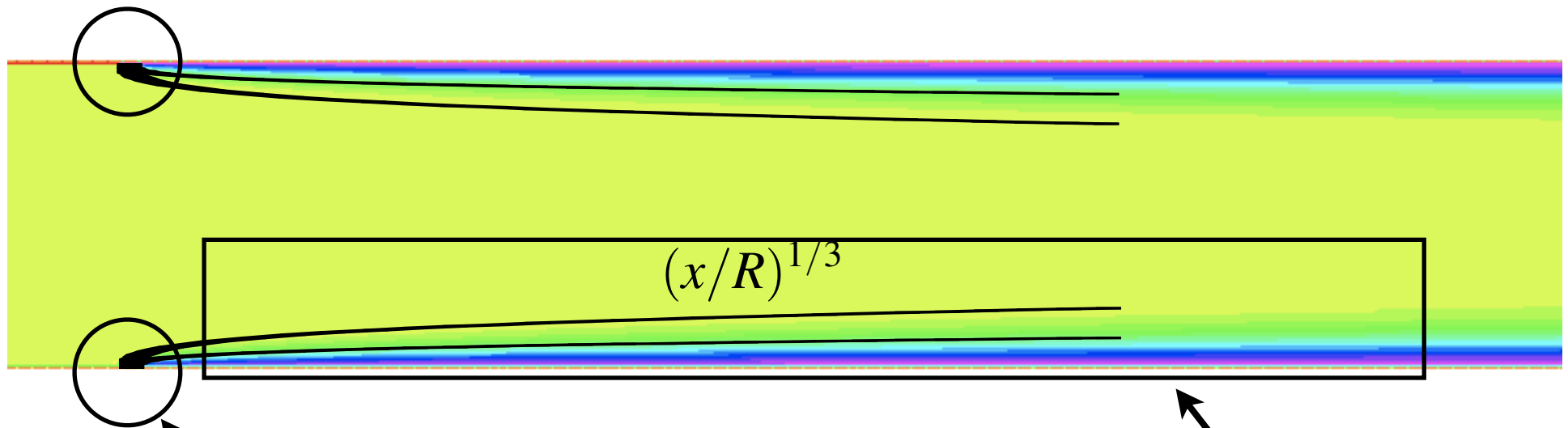


$$\varepsilon = Pe^{-1/2}$$

$$\bar{x} = \varepsilon \tilde{\xi} \text{ et } \bar{r} = 1 - \varepsilon \tilde{\zeta}$$

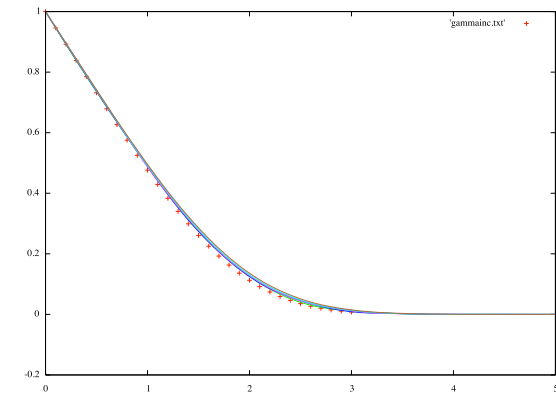
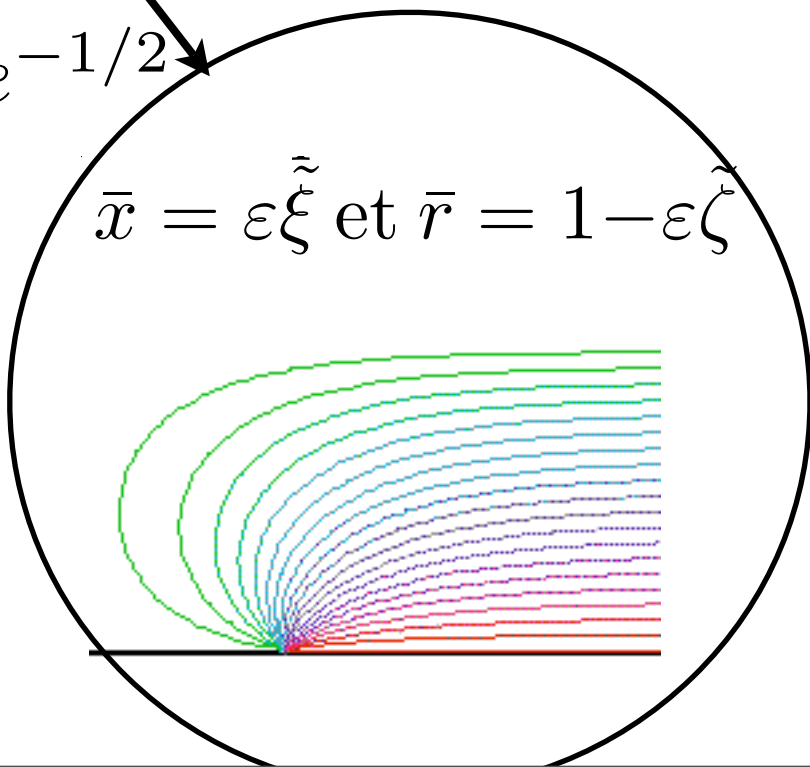


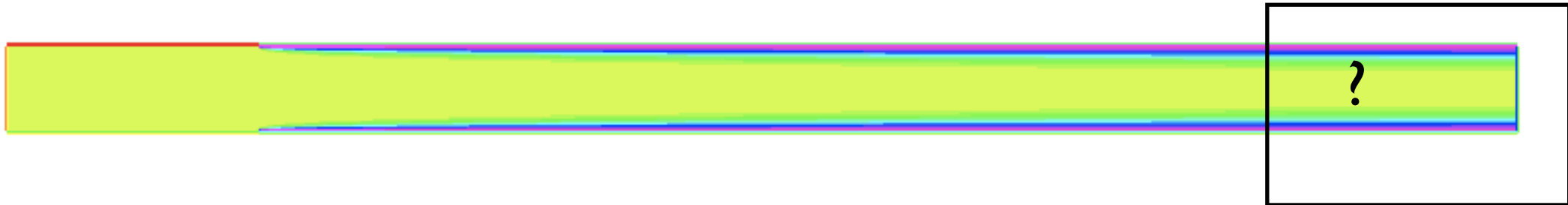
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$$\varepsilon = Pe^{-1/2}$$

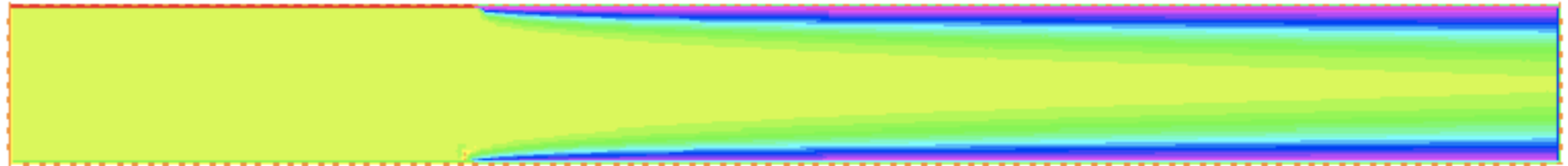
$$\bar{x} = \varepsilon \tilde{\xi} \text{ et } \bar{r} = 1 - \varepsilon \tilde{\zeta}$$



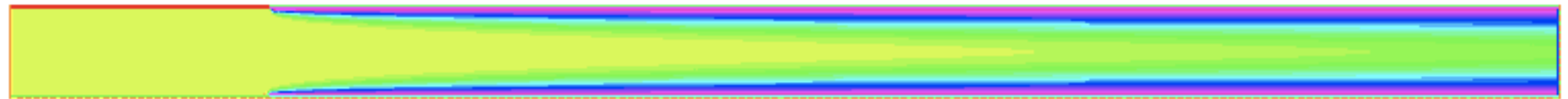


Pe=2000;

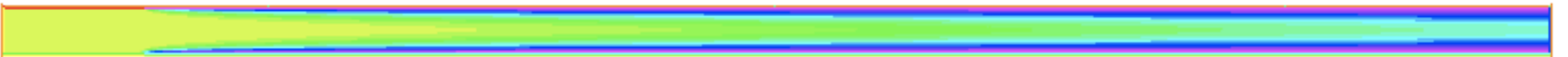
L=7.5



15



30



60



Solution de Graetz

$$(1 - \bar{r}^2) \frac{\partial \bar{T}}{\partial \bar{x}} = \frac{1}{Pe} \left(\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial}{\partial \bar{r}} \left(\bar{r} \frac{\partial \bar{T}}{\partial \bar{r}} \right) \right).$$

$$T = T_p + (T_0 - T_p)\theta.$$

Solution de Graetz

$$(1 - \bar{r}^2) \frac{\partial \bar{T}}{\partial \bar{x}} = \frac{1}{Pe} \left(\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial}{\partial \bar{r}} \left(\bar{r} \frac{\partial \bar{T}}{\partial \bar{r}} \right) \right).$$

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Solution de Graetz

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Solution de Graetz

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Solution de Graetz

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$$\varepsilon = 1/Pe$$

Solution de Graetz

$$(1 - \bar{r}^2) \frac{\partial \theta}{\partial X} = \frac{\partial}{\partial \bar{r}} \frac{\bar{r} \partial \theta}{\partial \bar{r}}.$$

$$T = T_p + (T_0 - T_p)\theta.$$

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$$\epsilon = 1/Pe$$

Solution de Graetz

$$(1 - \bar{r}^2) \frac{\partial \theta}{\partial X} = \frac{\partial}{\partial \bar{r}} \frac{\bar{r} \partial \theta}{\partial \bar{r}}.$$

$$\theta_m = -4 \sum_{n=0}^{\infty} c_n T'_n \lambda_n^{-2} \exp(-\lambda_n^2 X/2).$$

n	λ_n	c_n	T'_n
0	2.704	0.75	-1.01
1	6.779	-0.81	1.35
2	10.67	0.59	-1.57
3	14.67	-0.48	1.74
4	18.67	0.41	-1.90
...	...	...	...

$$Nu_X = -2 \frac{\sum_{n=0}^{\infty} c_n T'_n \exp(-\lambda_n^2 X/2)}{\sum_{n=0}^{\infty} c_n T'_n \lambda_n^{-2} \exp(-\lambda_n^2 X/2)}$$

Solution de Graetz

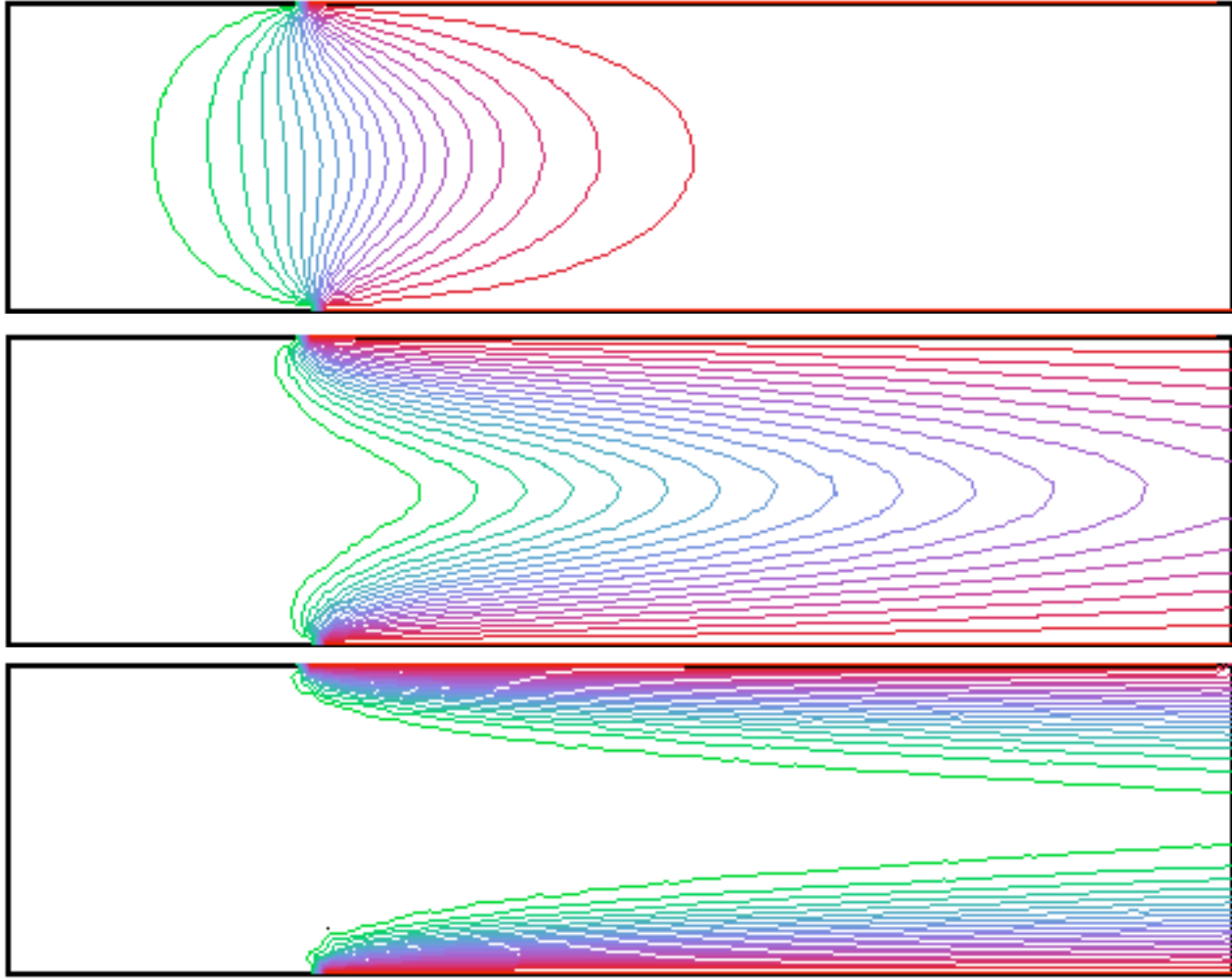
$$(1 - \bar{r}^2) \frac{\partial \theta}{\partial X} = \frac{\partial}{\partial \bar{r}} \frac{\bar{r} \partial \theta}{\partial \bar{r}}.$$

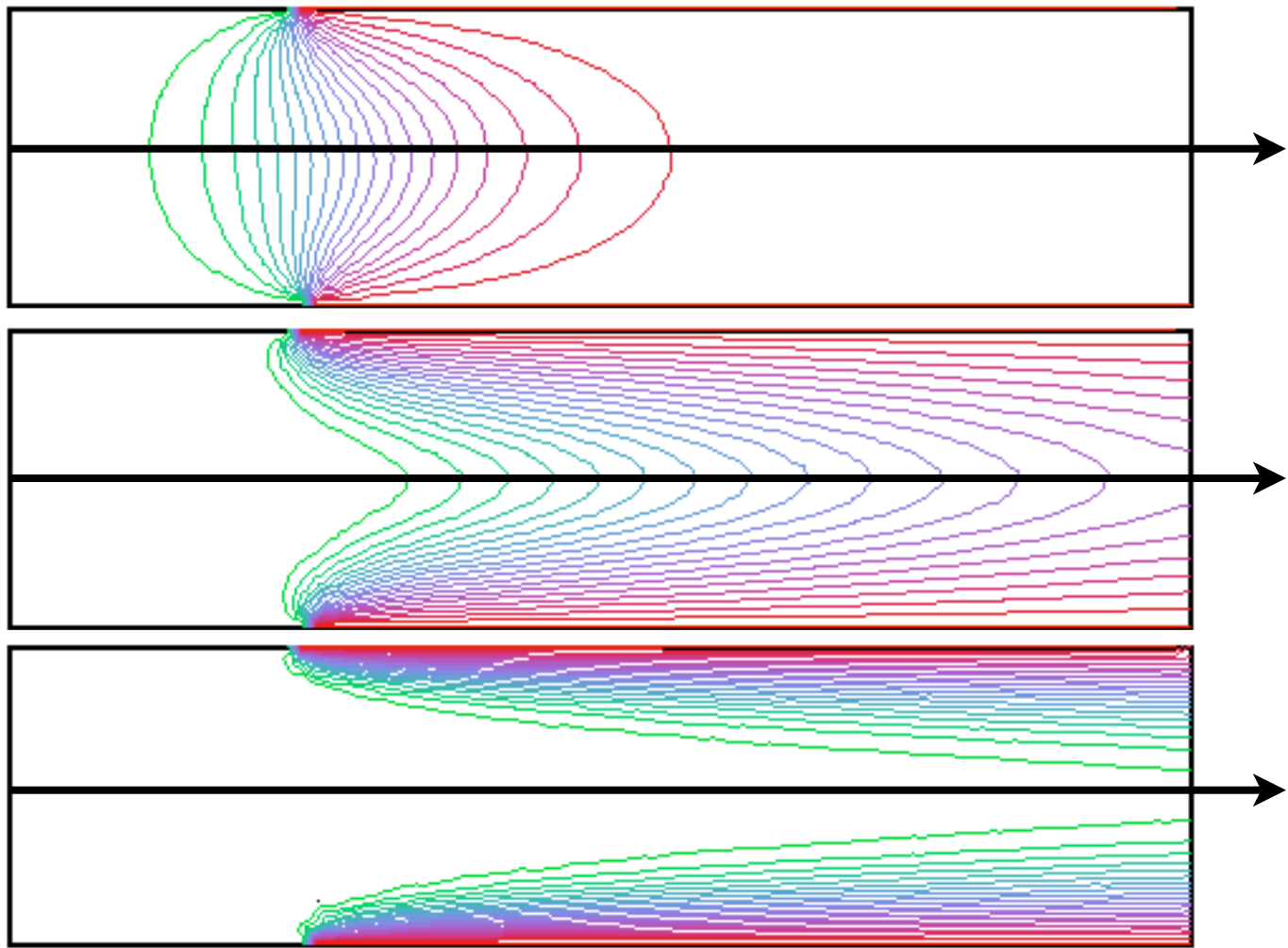
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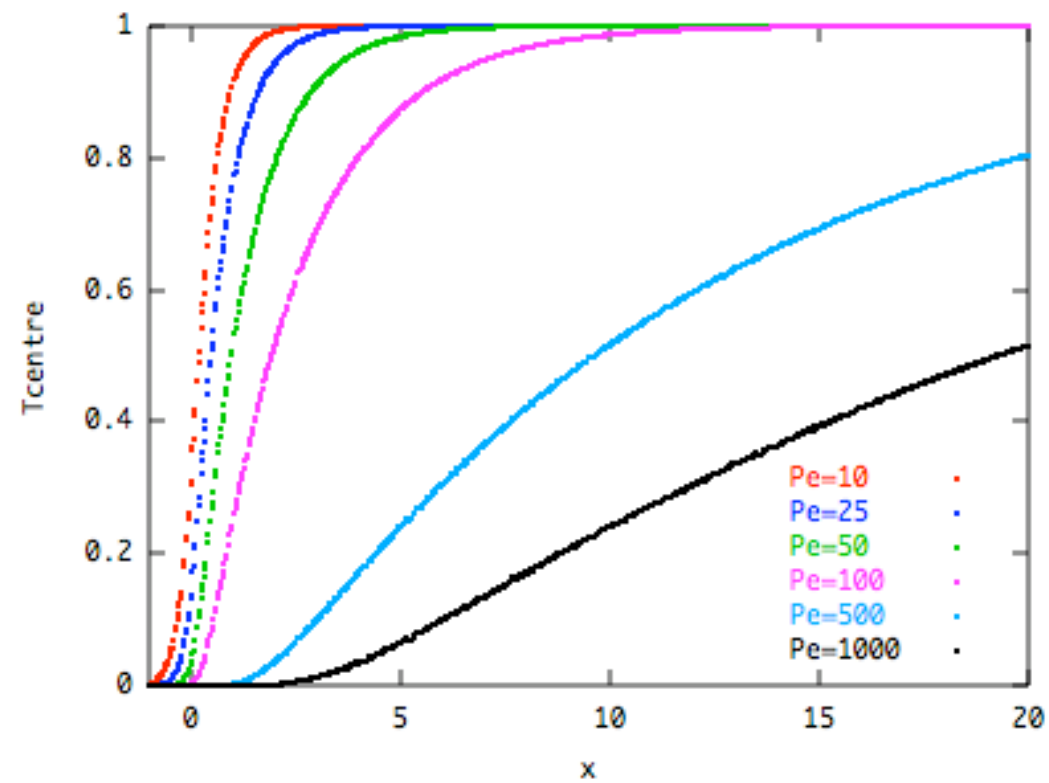
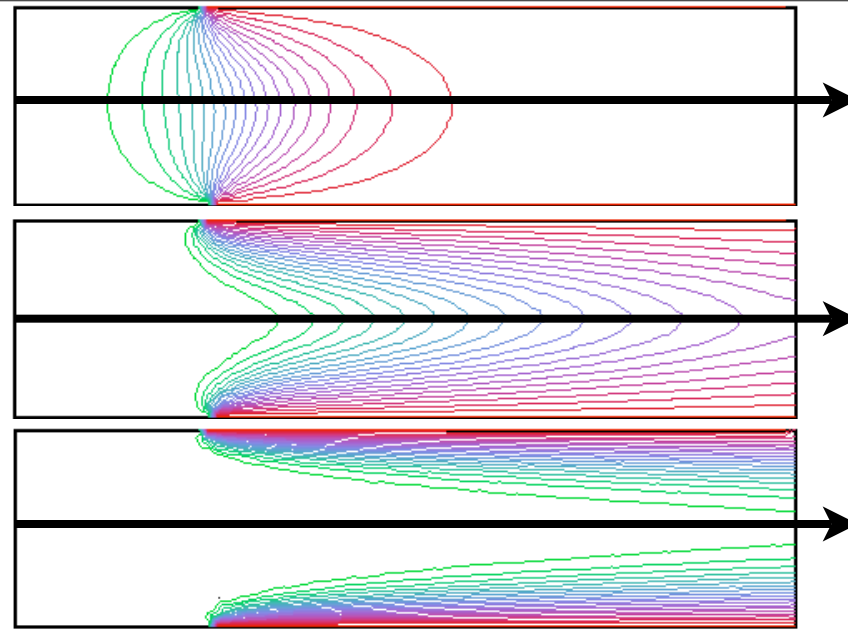
Nusselt calculé avec la valeur moyenne de la température comme valeur de référence

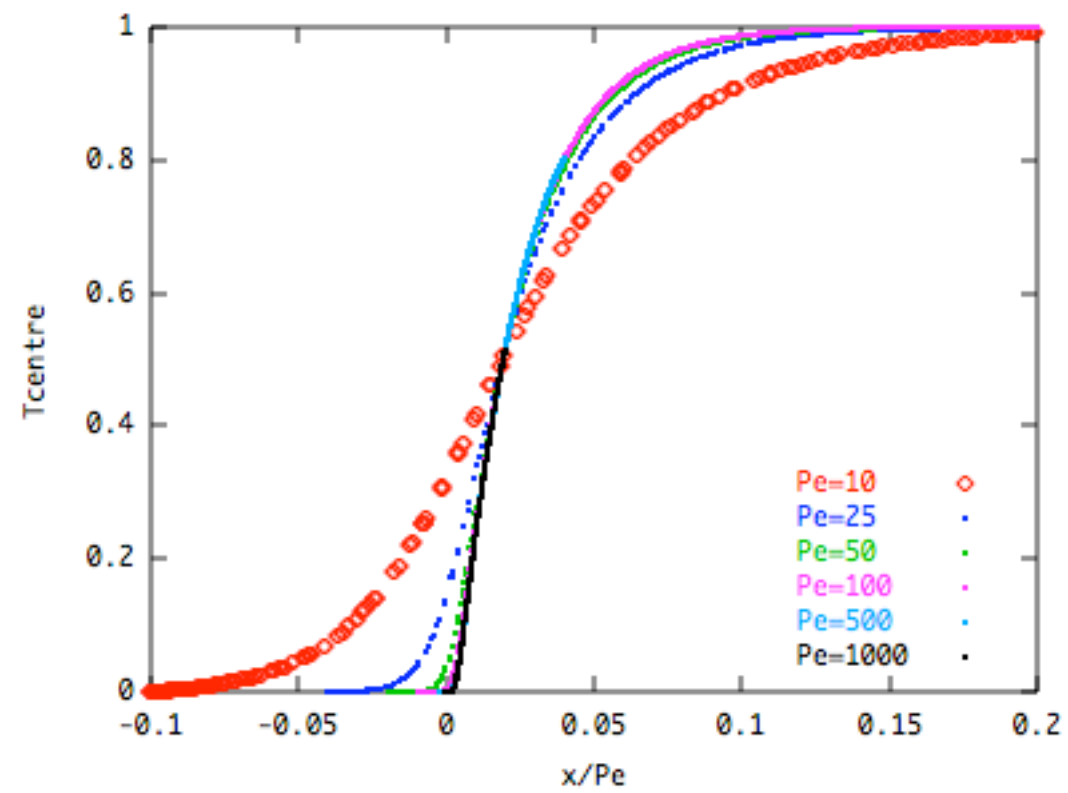
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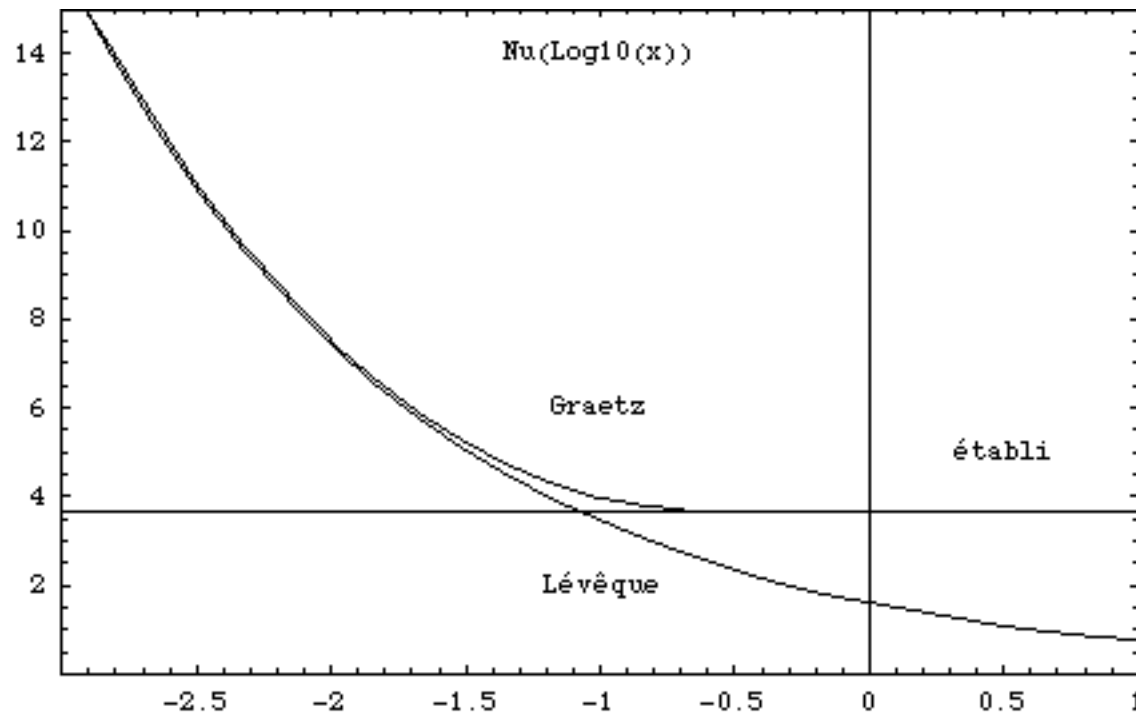






Solution de Graetz

Nusselt dans le tuyau



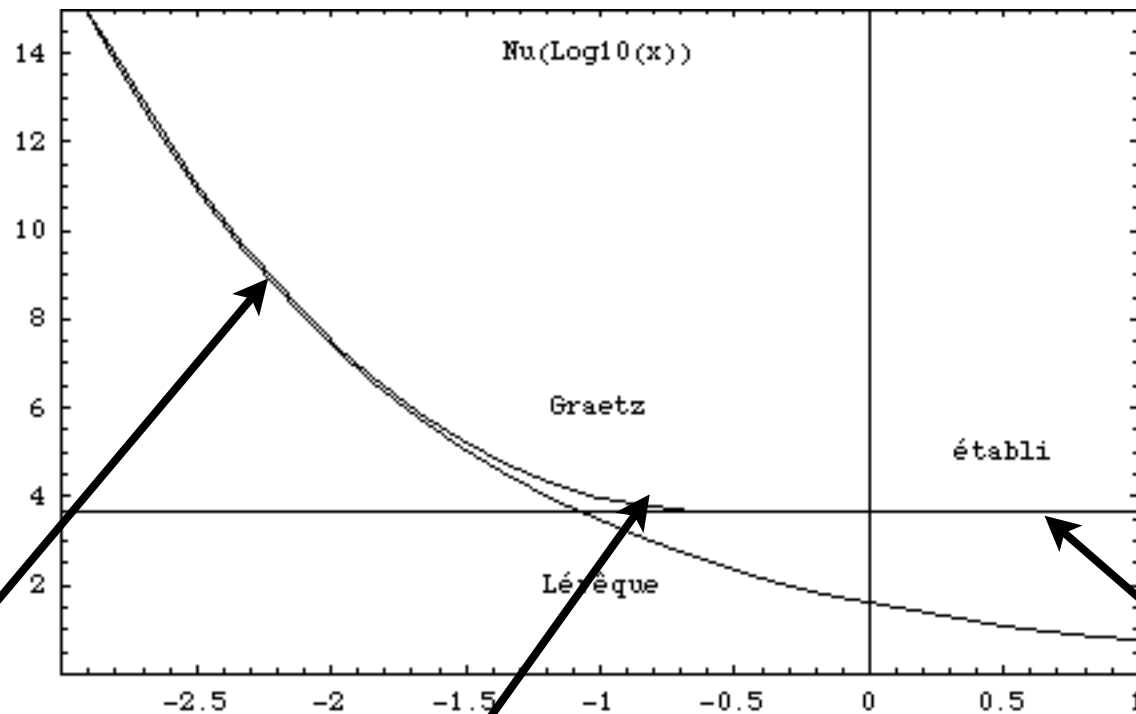
$\log(2xPe_D)$

$$Nu_x = 1.71 \frac{1}{(2xPe_D)^{1/3}} \cdot \quad Nu_{approx} = 3.66 + \frac{0.127}{2L/(Pe_D D) + 0.0635(2L/(Pe_D D))^{1/3}}$$



Solution de Graetz

Nusselt dans le tuyau

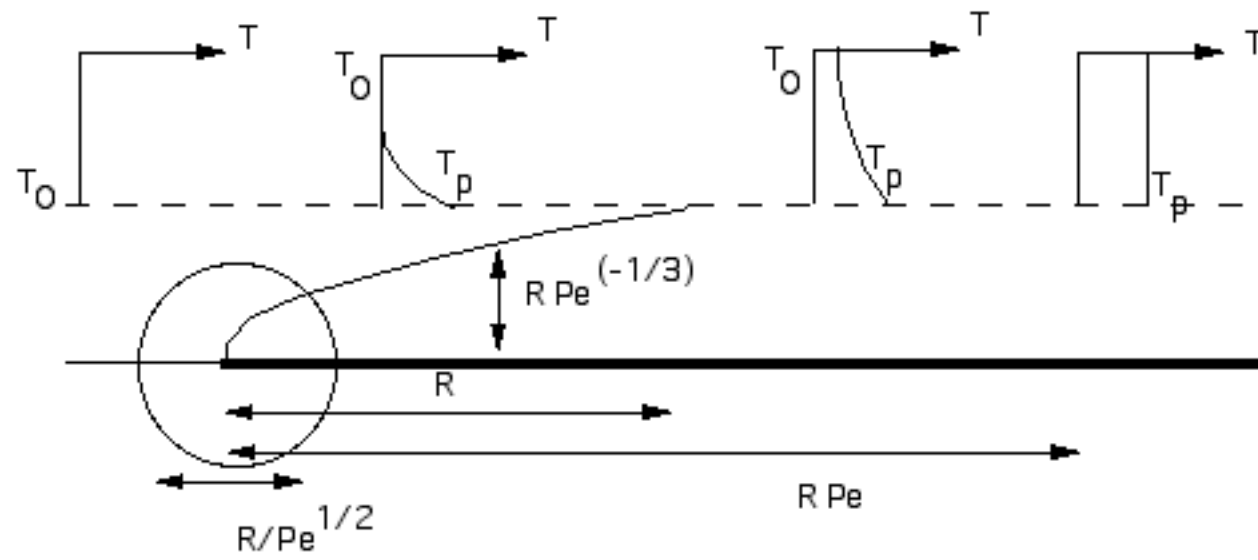


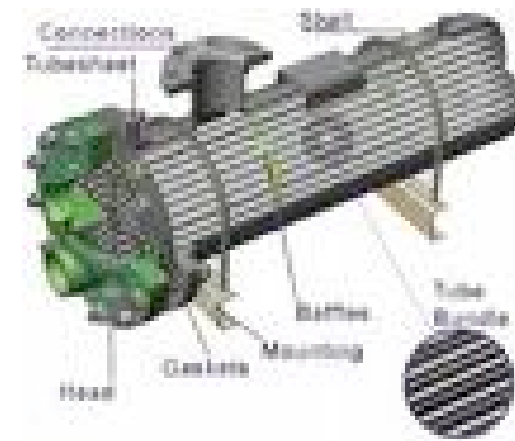
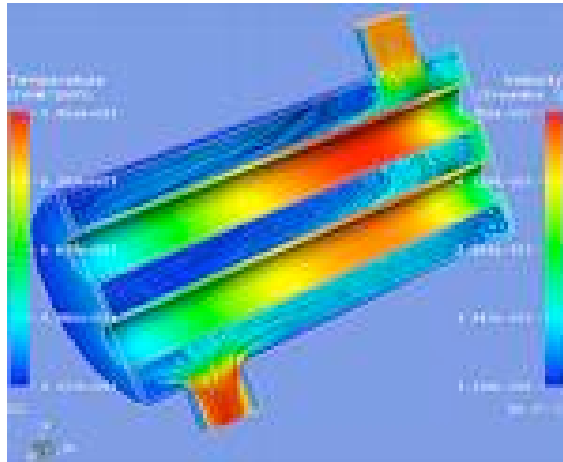
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$$Nu_{approx} = 3.66 + \frac{0.127}{2L/(Pe_D D) + 0.0635(2L/(Pe_D D))^{1/3}}$$

$$Nu_{approx} = 3.66$$







Interne... externe

