

Conduction

Conduction

équation de conservation de la masse :

$$\frac{d\rho}{dt} + \rho \underline{\nabla} \cdot \underline{u} = 0.$$

équation de conservation de la quantité de mouvement :

$$\rho \frac{d\underline{u}}{dt} = \underline{\nabla} \cdot \underline{\underline{\sigma}} + \underline{f}.$$

équation de l'énergie :

$$\rho \frac{de}{dt} = \underline{\underline{\sigma}} : \underline{\underline{D}} - \underline{\nabla} \cdot \underline{q} + r.$$

relations constitutives :

$$\underline{\underline{\sigma}} = -p\underline{I} + \lambda \underline{\nabla} \cdot \underline{u} \underline{I} + 2\mu \underline{\underline{D}}$$

$$\underline{q} = -k \underline{\nabla} T.$$

loi d'état :

$$p(\rho, T)$$

coefficients :

$$c_v(T), \quad c_p(T), \quad \lambda(T), \quad \mu(T), \quad k(T)...$$

conditions aux limites T_w **OU** q_w imposés, adhérence à la paroi.

Conduction

équation de conservation de la masse :

$$\frac{d\rho}{dt} + \rho \underline{\nabla} \cdot \underline{u} = 0.$$

équation de conservation de la quantité de mouvement :

~~$$\rho \frac{d\underline{u}}{dt} = \underline{\nabla} \cdot \underline{\underline{\sigma}} + \underline{f}.$$~~

équation de l'énergie :

$$\rho \frac{de}{dt} = \underline{\underline{\sigma}} : \underline{\underline{D}} - \underline{\nabla} \cdot \underline{q} + r.$$

relations constitutives :

$$\underline{\underline{\sigma}} = -p\underline{I} + \lambda \underline{\nabla} \cdot \underline{u} \underline{I} + 2\mu \underline{\underline{D}}$$

$$\underline{q} = -k \underline{\nabla} T.$$

loi d'état :

$$p(\rho, T)$$

coefficients :

$$c_v(T), \quad c_p(T), \quad \lambda(T), \quad \mu(T), \quad k(T)...$$

conditions aux limites T_w **OU** q_w imposés, adhérence à la paroi.

Conduction

équation de conservation de la masse :

$$\frac{d\rho}{dt} + \rho \underline{\nabla} \cdot \underline{u} = 0.$$

équation de conservation de la quantité de mouvement :

~~$$\rho \frac{d\underline{u}}{dt} = \underline{\nabla} \cdot \underline{\underline{\sigma}} + \underline{f}.$$~~

équation de l'énergie :

$$\rho \frac{de}{dt} = \underline{\underline{\sigma}} : \underline{\underline{D}} - \underline{\nabla} \cdot \underline{q} + r.$$

relations constitutives :

~~$$\underline{\underline{\sigma}} = p \underline{\underline{I}} + \lambda \underline{\nabla} \cdot \underline{\underline{u}} \underline{\underline{I}} + 2\mu \underline{\underline{D}}$$~~

$$\underline{q} = -k \underline{\nabla} T.$$

loi d'état :

$$p(\rho, T)$$

coefficients :

$$c_v(T), \quad c_p(T), \quad \lambda(T), \quad \mu(T), \quad k(T)...$$

conditions aux limites T_w **OU** q_w imposés, adhérence à la paroi.

Conduction

équation de conservation de la masse :

~~$$\frac{d\rho}{dt} + \rho \nabla \cdot \underline{u} = 0.$$~~

équation de conservation de la quantité de mouvement :

~~$$\rho \frac{d\underline{u}}{dt} = \underline{\nabla} \cdot \underline{\underline{\sigma}} + \underline{f}.$$~~

équation de l'énergie :

$$\rho \frac{de}{dt} = \underline{\underline{\sigma}} : \underline{\underline{D}} - \underline{\nabla} \cdot \underline{q} + r.$$

relations constitutives :

~~$$\underline{\underline{\sigma}} = p \underline{\underline{I}} + \lambda \underline{\underline{\nabla}} \cdot \underline{\underline{u}} \underline{\underline{I}} + 2\mu \underline{\underline{D}}$$~~

$$\underline{q} = -k \underline{\nabla} T.$$

loi d'état :

~~$$p(\rho, T)$$~~

coefficients :

$$c_v(T), \quad c_p(T), \quad \lambda(T), \quad \mu(T), \quad k(T)...$$

conditions aux limites T_w **OU** q_w imposés, adhérence à la paroi.

Conduction

équation de conservation de la masse :

~~$$\frac{d\rho}{dt} + \rho \nabla \cdot \underline{u} = 0.$$~~

équation de conservation de la quantité de mouvement :

~~$$\rho \frac{d\underline{u}}{dt} = \underline{\nabla} \cdot \underline{\underline{\sigma}} + \underline{f}.$$~~

équation de l'énergie :

~~$$\rho \frac{de}{dt} = \underline{\underline{\sigma}} : \underline{\underline{D}} - \underline{\nabla} \cdot \underline{q} + r.$$~~

relations constitutives :

~~$$\underline{\underline{\sigma}} = p \underline{\underline{I}} + \lambda \underline{\underline{\nabla}} \cdot \underline{\underline{u}} \underline{\underline{I}} + 2\mu \underline{\underline{D}}$$~~

$$\underline{q} = -k \underline{\nabla} T.$$

loi d'état :

~~$$p(\rho, T)$$~~

coefficients :

$$c_v(T), \quad c_p(T), \quad \lambda(T), \quad \mu(T), \quad k(T)...$$

conditions aux limites T_w **OU** q_w imposés, adhérence à la paroi.

Conduction

équation de conservation de la masse :

~~$$\frac{d\rho}{dt} + \rho \nabla \cdot \underline{u} = 0.$$~~

équation de conservation de la quantité de mouvement :

~~$$\rho \frac{d\underline{u}}{dt} = \underline{\nabla} \cdot \underline{\underline{\sigma}} + \underline{f}.$$~~

équation de l'énergie :

~~$$\rho \frac{de}{dt} = \underline{\underline{\sigma}} : \underline{\underline{D}} - \underline{\nabla} \cdot \underline{q} + r.$$~~

relations constitutives :

$\frac{\partial}{\partial t}$

~~$$\underline{\underline{\sigma}} = p \underline{\underline{I}} + \lambda \underline{\underline{\nabla}} \cdot \underline{\underline{u}} \underline{\underline{I}} + 2\mu \underline{\underline{D}}$$~~

$$\underline{q} = -k \underline{\nabla} T.$$

loi d'état :

~~$$p(\rho, T)$$~~

coefficients :

$$c_v(T), \quad c_p(T), \quad \lambda(T), \quad \mu(T), \quad k(T)...$$

conditions aux limites T_w **OU** q_w imposés, adhérence à la paroi.

Conduction

équation de conservation de la masse :

~~$$\frac{d\rho}{dt} + \rho \nabla \cdot \underline{u} = 0.$$~~

équation de conservation de la quantité de mouvement :

~~$$\rho \frac{d\underline{u}}{dt} = \nabla \cdot \underline{\underline{\sigma}} + \underline{f}.$$~~

équation de l'énergie :

~~$$\rho \frac{de}{dt} = \underline{\underline{\sigma}} : \underline{\underline{D}} - \nabla \cdot \underline{q} + r.$$~~

relations constitutives :

~~$$\underline{\underline{\sigma}} = p \underline{\underline{I}} + \lambda \nabla \cdot \underline{\underline{u}} \underline{\underline{I}} + 2\mu \underline{\underline{D}}$$~~

~~$$\underline{q} = -k \nabla T.$$~~

loi d'état :

~~$$p(\rho, T)$$~~

coefficients :

$$c_v(T), \quad c_p(T), \quad \lambda(T), \quad \mu(T), \quad k(T)...$$

conditions aux limites T_w **OU** q_w imposés, adhérence à la paroi.

$$de = cdT$$

$\frac{\partial}{\partial t}$

Conduction

équation de conservation de la masse :

~~$$\frac{d\rho}{dt} + \rho \nabla \cdot \underline{u} = 0.$$~~

équation de conservation de la quantité de mouvement :

~~$$\rho \frac{d\underline{u}}{dt} = \nabla \cdot \underline{\underline{\sigma}} + \underline{f}.$$~~

équation de l'énergie :

~~$$\rho \frac{de}{dt} = \underline{\underline{\sigma}} : \underline{\underline{D}} - \nabla \cdot \underline{q} + r.$$~~

relations constitutives :

~~$$\underline{\underline{\sigma}} = p \underline{\underline{I}} + \lambda \nabla \cdot \underline{\underline{u}} \underline{\underline{I}} + 2\mu \underline{\underline{D}}$$~~

$$\underline{q} = -k \nabla T.$$

$\frac{\partial}{\partial t}$

loi d'état :

~~$$p(\rho, T)$$~~

$$de = cdT$$

coefficients :

$$c_v(T), \quad c_p(T), \quad \lambda(T), \quad \mu(T), \quad k(T)...$$

conditions aux limites T_w **OU** q_w imposés, adhérence à la paroi.

Conduction

équation de conservation de la masse :

~~$$\frac{d\rho}{dt} + \rho \nabla \cdot \underline{u} = 0.$$~~

équation de conservation de la quantité de mouvement :

~~$$\rho \frac{d\underline{u}}{dt} = \nabla \cdot \underline{\underline{\sigma}} + \underline{f}.$$~~

équation de l'énergie :

$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T$$

$$de = cdT$$

relations constitutives :

~~$$\underline{\underline{\sigma}} = p \underline{\underline{I}} + \lambda \nabla \cdot \underline{\underline{u}} \underline{\underline{I}} + 2\mu \underline{\underline{D}}$$~~

$$\underline{q} = -k \nabla T.$$

$\frac{\partial}{\partial t}$

loi d'état :

~~$$p(\rho, T)$$~~

coefficients :

$$c_v(T), \quad c_p(T), \quad \lambda(T), \quad \mu(T), \quad k(T)...$$

conditions aux limites T_w **OU** q_w imposés, adhérence à la paroi.

$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T$$

$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T$$

$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T$$

$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T$$

$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T$$

$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T$$

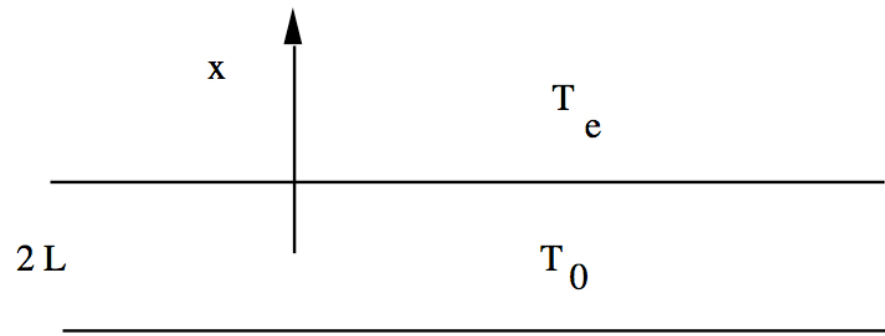
Conduction

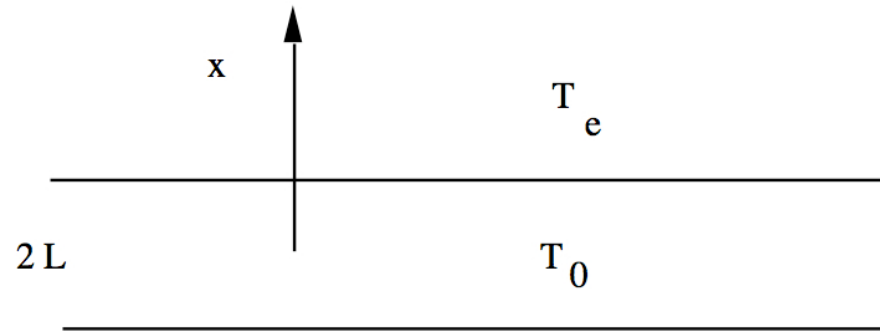
$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T$$

Conduction

en PC:

refroidissement d'une lamelle infinie





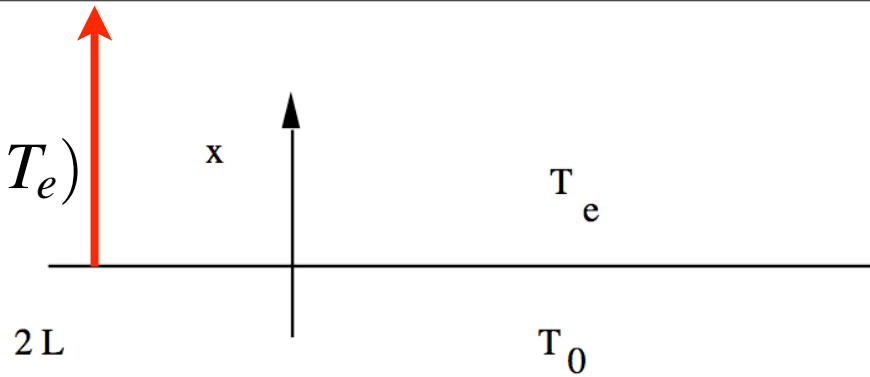
Echange

$$\underline{q}_w = h(T_w - T_f)\underline{n}.$$

$$\underline{q}_w = -k\left[\frac{\partial T}{\partial n}\right]_w \underline{n},$$

$$\left(k\left[\frac{\partial T}{\partial n}\right]_w \underline{n} + h(T_w - T_f)\right)\underline{n} = 0.$$

$$-k \frac{\partial T}{\partial x} = h(T - T_e)$$



Echange

$$k \frac{\partial T}{\partial x} = h(T - T_e)$$

$$\underline{q}_w = h(T_w - T_f)\underline{n}.$$

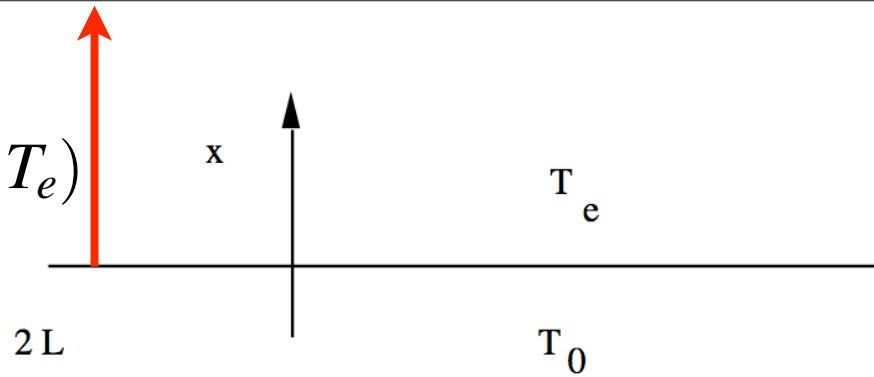
$$\underline{q}_w = -k \left[\frac{\partial T}{\partial n} \right]_w \underline{n},$$

$$\left(k \left[\frac{\partial T}{\partial n} \right]_w \underline{n} + h(T_w - T_f) \right) \underline{n} = 0.$$

$$Bi = \frac{hL}{k}$$

Echange

$$-k \frac{\partial T}{\partial x} = h(T - T_e)$$



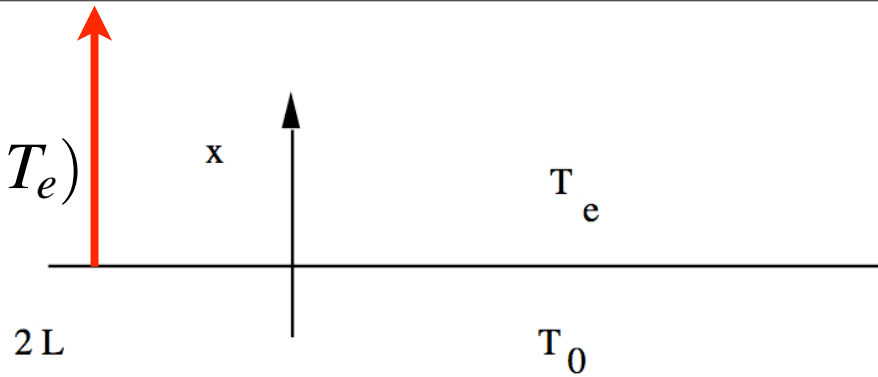
$$k \frac{\partial T}{\partial x} = h(T - T_e)$$

$$T = T_e + (T_0 - T_e)\bar{T}$$

$$Bi = \frac{hL}{k}$$

Echange

$$-k \frac{\partial T}{\partial x} = h(T - T_e)$$



$$k \frac{\partial T}{\partial x} = h(T - T_e)$$

$$T = T_e + (T_0 - T_e)\bar{T}$$

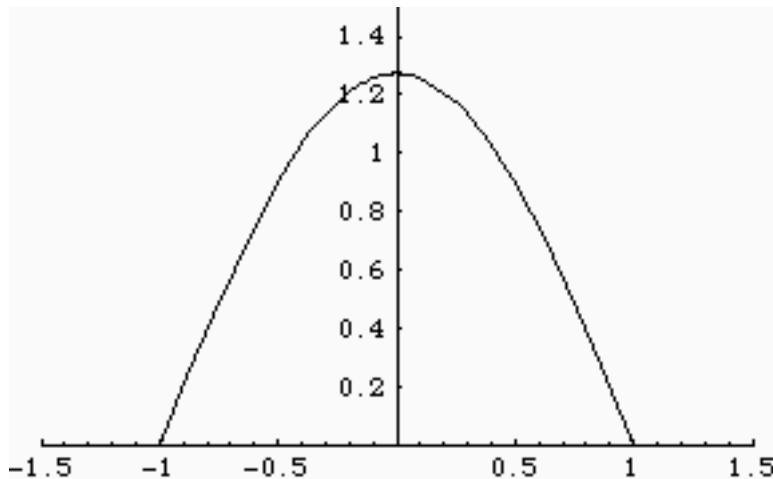
$$Bi = \frac{hL}{k}$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} = \left(\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} \right)$$

$$\bar{T} = 1 \text{ en } \bar{t} = 0 \text{ et } -\left(\frac{\partial \bar{T}}{\partial \bar{x}} \right) = \pm Bi \bar{T} \text{ en } \bar{t} > 0 \text{ et } \bar{x} = \pm 1.$$

Solution en Séries de Fourier

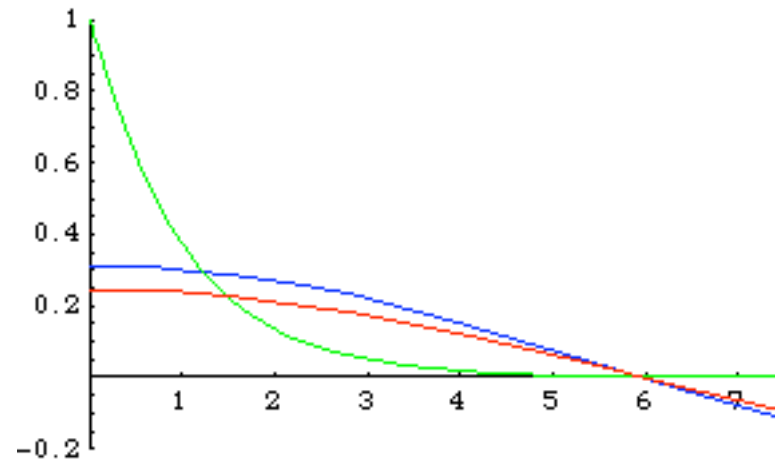
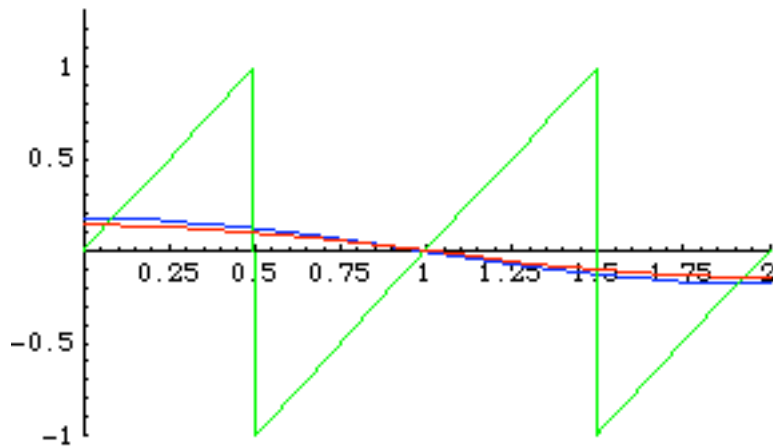
$$\sum_{i>0} A_i \exp(-k_i^2 \bar{t}) \cos(k_i \bar{x})$$



$$= \sum_{i>0} A_i \cos(k_i \bar{x}),$$

Solution en Séries de Fourier

$$\sum_{i>0} A_i \exp(-k_i^2 \bar{t}) \cos(k_i \bar{x})$$



$$\sum_{i>0} A_i \exp(-k_i^2 \bar{t}) \cos(k_i \bar{x})$$

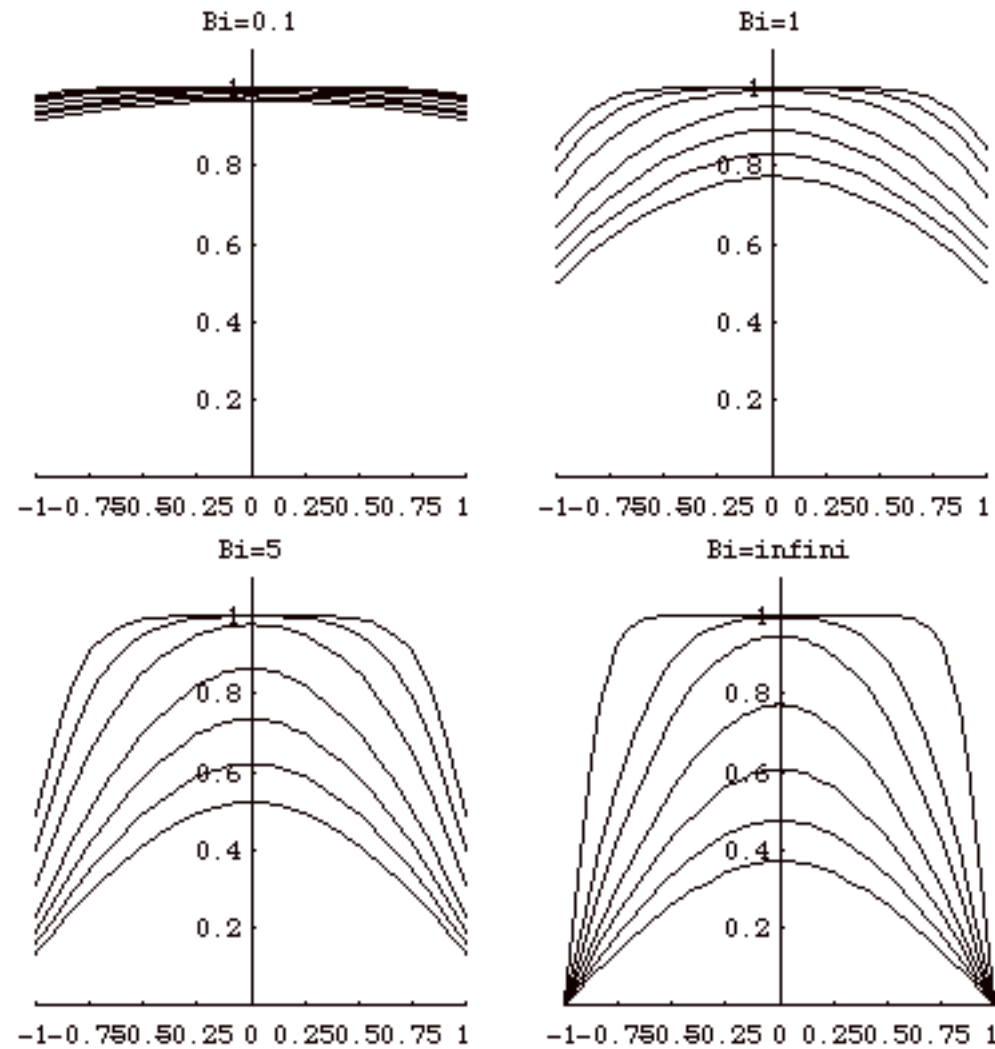
$$A_i = \frac{2 \sin(k_i)}{k_i + \sin(k_i) \cos(k_i)}$$

Si $Bi = 0$, $k_i = (i - 1)\pi$.

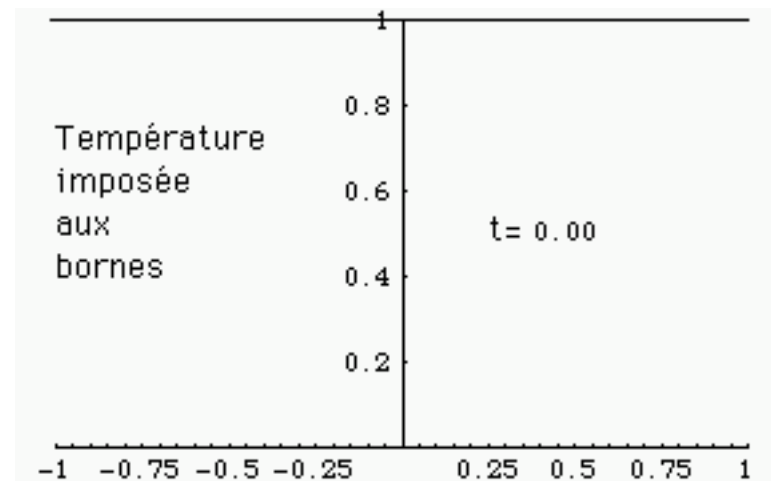
Si $Bi = \infty$, $k_i = (2i - 1)\pi/2$.

Si $Bi = 1$, $k_1 = 0.863$, $k_2 = 3.4256$, $k_3 = 6.4373$, $k_4 = 9.5293 \dots$

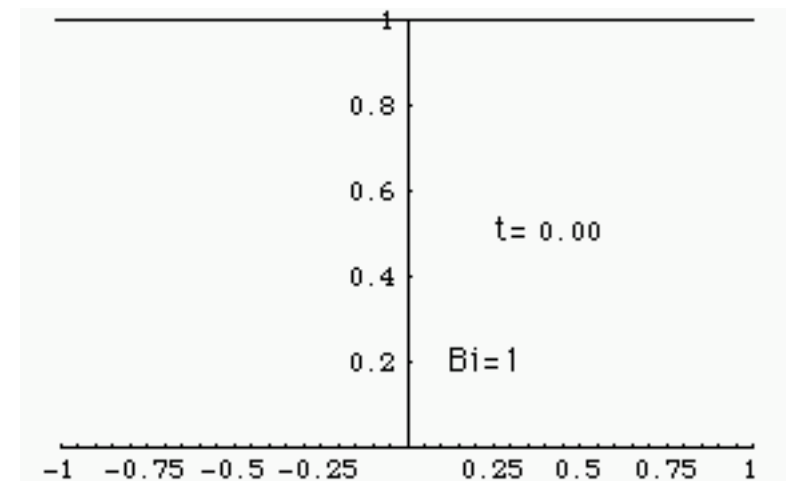
$$ktg(k) = Bi.$$



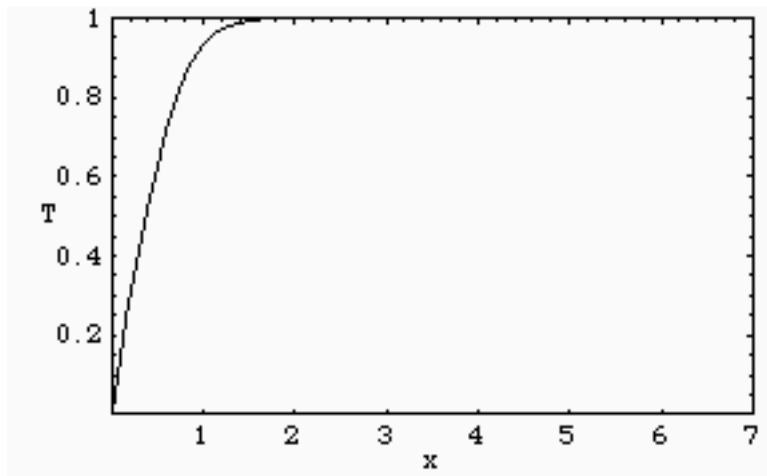
figures pour $t=0.025$ 0.05 0.1 0.2 0.3 0.4 et 0.5 à différents nombres de Biot



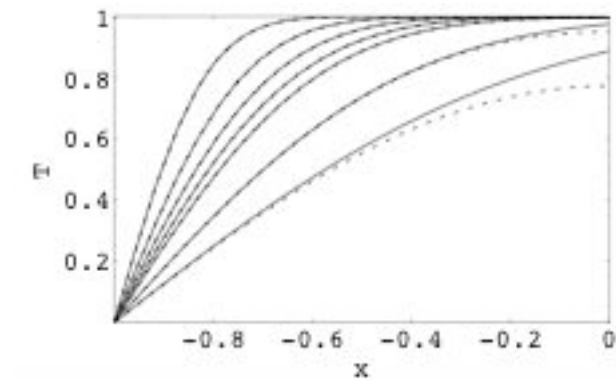
Bi infini



Bi=1



erf



Solution Erf et solution exacte

tracé de $\text{Erf}[(x + 1)/2/\text{Sqrt}[t]]$; et de la solution pour $t=0.01$, $t=0.02$, $t=0.03$, $t=0.04$, $t=0.05$, puis $t=0.1$ et enfin $t=0.2$

Bi petit

$$\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} = \frac{\partial \bar{T}}{\partial \bar{t}}$$

Bi petit

$$\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} = \frac{\partial \bar{T}}{\partial \bar{t}}$$

$$\int \frac{\partial^2 \bar{T}}{\partial \bar{x}^2} dx = \int \frac{\partial \bar{T}}{\partial \bar{t}} dx \quad \text{moyenne de la température}$$

Bi petit

$$\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} = \frac{\partial \bar{T}}{\partial \bar{t}}$$

$$\int \frac{\partial^2 \bar{T}}{\partial \bar{x}^2} d\bar{x} = \int \frac{\partial \bar{T}}{\partial \bar{t}} d\bar{x} \quad \text{moyenne de la température}$$

$$-2Bi(\bar{T}) \sim \frac{\partial}{\partial \bar{t}} \int \bar{T} d\bar{x} \qquad -Bi\bar{T}_m = \frac{\partial}{\partial \bar{t}} \bar{T}_m$$

Bi petit

$$\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} = \frac{\partial \bar{T}}{\partial \bar{t}}$$

$$\int \frac{\partial^2 \bar{T}}{\partial \bar{x}^2} dx = \int \frac{\partial \bar{T}}{\partial \bar{t}} dx \quad \text{moyenne de la température}$$

$$-2Bi(\bar{T}) \sim \frac{\partial}{\partial \bar{t}} \int \bar{T} d\bar{x} \quad -Bi\bar{T}_m = \frac{\partial}{\partial \bar{t}} \bar{T}_m$$

temps lent $Bi\bar{t} = \tilde{t}$

Bi petit

moyenne de la température

Systèmes Minces $Bi < 0.1$

$$-Bi\bar{T}_m = \frac{\partial}{\partial \bar{t}}\bar{T}_m$$

temps lent $Bi\bar{t} = \tilde{t}$

Bi quelconque

Résolution numérique

$$\frac{\partial T}{\partial t} = \frac{\partial^2}{\partial x^2} T$$

discrétisation

$$\frac{\partial T}{\partial t} = \frac{\partial^2}{\partial x^2} T$$

discrétisation

$$\frac{T(t + \Delta t) - T}{\Delta t}$$

$$\frac{\partial T}{\partial t} = \frac{\partial^2}{\partial x^2} T$$

discrétisation

$$\frac{T(t + \Delta t) - T}{\Delta t}$$

$$\frac{(T(t, x + \Delta x) - 2T(t, x) + T(t, x - \Delta x))}{\Delta x^2}$$

Explicite

$$\frac{\partial T}{\partial t} = \frac{\partial^2}{\partial x^2} T$$

discrétisation

$$\frac{T(t + \Delta t) - T}{\Delta t} = \frac{(T(t, x + \Delta x) - 2T(t, x) + T(t, x - \Delta x))}{\Delta x^2}$$

Explicite

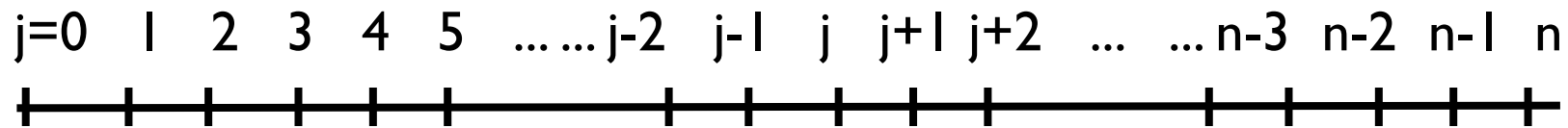
$$\frac{(T(t + \Delta t, x + \Delta x) - 2T(t + \Delta t, x) + T(t + \Delta t, x - \Delta x))}{\Delta x^2}$$

Implicite

Implicite

$$T[j] = T_0[j] + dt (T[j-1] - 2 T[j] + T[j+1]) / dx / dx$$

$$diagi[j] T[j-1] + diagp[j] T[j] + diags[j] T[j+1] = rhs[j]$$



$$\begin{aligned}
 diags[j] &= 1 \\
 diagp[j] &= -2 - (dx) * (dx) / dt \\
 diagi[j] &= 1 \\
 rhs[j] &= -T_0[j] (dx) * (dx) / dt
 \end{aligned}$$

$$T[j] = a[j] * T[j + 1] + b[j];$$

$$\begin{aligned}
 a[j] &= -diags[j] / (diagp[j] + diagi[j] * a[j - 1]) ; \\
 b[j] &= (rhs[j] - diagi[j] * b[j - 1]) / (diagp[j] + diagi[j] * a[j - 1])
 \end{aligned}$$

Cas des conditions données aux bornes:

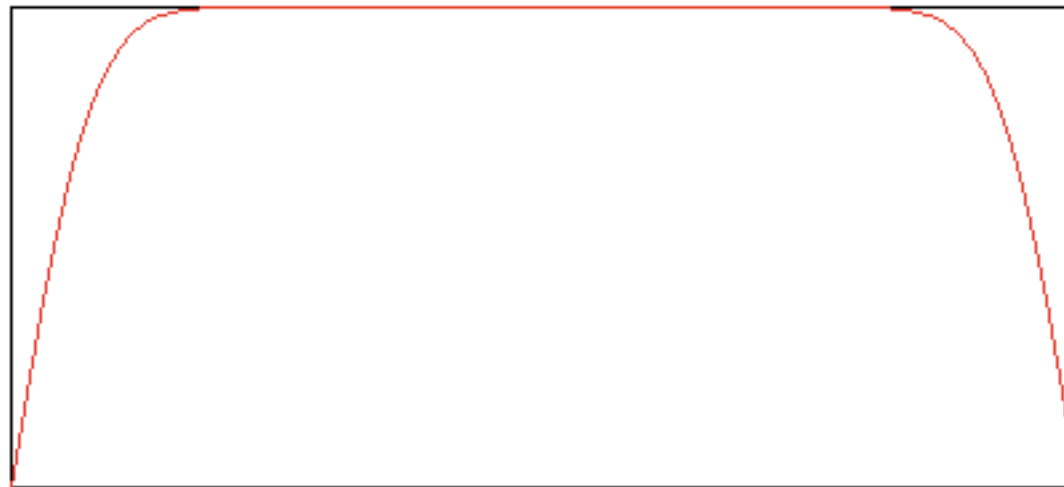
en $j=0$

$$T[0] = T_0 \text{ en } j=n, T[n] = T_1.$$

ce qui nous donne

$$a[0] = 0. \text{ et } b[0] = T_0 \text{ et } T[n] = T_n$$

$T(x, t=0.0010)$



dt=

 T imp....
 Cond. lim.

http://www.lmm.jussieu.fr/~lagree/SOURCES/Appliquette-JavaChal/guiChalomega_v_implicite/index.html

