MHD Mixed Convection Boundary Layer Flow Toward a Stagnation Point on a Vertical Surface With Induced Magnetic Field

In this paper, the steady magnetohydrodynamic (MHD) mixed convection stagnation point flow of an incompressible, viscous, and electrically conducting fluid over a vertical flat plate is investigated. The effect of induced magnetic field is taken into account. Numerical results are obtained using an implicit finite-difference scheme. Both assisting and opposing flows are considered. The results for skin friction, heat transfer, and induced magnetic field coefficients are obtained and discussed for various parameters. The velocity, temperature, and induced magnetic field profiles are also presented. For the case of the opposing flow, it is found that dual solutions exist for a certain range of the buoyancy parameter. Dual solutions are also obtained for the assisting flow. [DOI: 10.1115/1.4002602]

Keywords: boundary layer, dual solutions, induced magnetic field, MHD, mixed convection

1 Introduction

The case of a steady magnetohydrodynamic (MHD) stagnation point flow of an electrically conducting fluid has many practical applications. Many metallurgical processes, such as drawing, annealing, and tinning of copper wires, involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid. Another important application of hydromagnetics to metallurgy is the purification of molten metals from nonmetallic inclusions by the application of a magnetic field. The study of the MHD stagnation point flow of an electrically conducting fluid in the presence of a uniform magnetic field, which is applied normal to the infinite plane surface, was considered by Ariel [1]. Later, Mahapatra and Gupta [2] studied the MHD stagnation point flow over a stretching surface, and very recently Chen [3] considered the combined effects of Joule heating and viscous dissipation on MHD flow past a permeable stretching surface with free convection and radiative heat transfer. Further, the study of boundary layer flow against a vertical surface problem was considered by Cramer [4], Cobble [5], Raptis et al. [6,7], Soundalgekar et al. [8], Ramachandran et al. [9], Hossain and Ahmed [10], Kumari et al. [11], and Ishak et al. [12–15] in various ways. On the other hand, in micropolar fluid, Lok et al. [16] and Ishak et al. [17] solved the boundary layer flow near a stagnation point on a vertical surface. These are examples where the induced magnetic fields are not considered in their study. The study of the boundary layer flow under the influence of a magnetic field with the induced magnetic field was considered by a few authors. For example, Raptis and Perdikis [18] studied the MHD free convection boundary layer flow past an infinite vertical porous plate. Later, Kumari et al. [19] considered prescribed wall temperature or heat flux, and Takhar et al. [20] studied the time dependence of a free convection flow.

This present paper aims to study the problem of a MHD mixed convection boundary layer flow toward a stagnation point on a vertical flat plate in the presence of a magnetic field where the effect of the induced magnetic field is considered. The governing partial differential equations are reduced to similarity or nonlinear ordinary differential equations that are then solved numerically. The flow depends heavily on the magnetic parameter, the Prandtl number, and the reciprocal of the magnetic Prandtl number.

2 Mathematical Formulation

Consider a steady two-dimensional MHD stagnation point flow of an incompressible, viscous, and electrically conducting fluid past a vertical surface with a velocity proportional to the distance from the fixed origin O of a stationary frame of reference $(x, y)$, as shown in Fig. 1. The assisting flow situation occurs if the upper half of the flat surface is cooled while the lower half of the flat surface is cooled (see Fig. 1(a)). In this case, the flow near the heated flat surface tends to move upward, and the flow near the cooled flat surface tends to move downward; therefore, this behavior acts to assist the flow field. The opposing flow situation occurs if the upper half of the flat surface is cooled while the lower half of the flat surface is heated (see Fig. 1(b)). The frame of reference $(x, y)$ is chosen such that the $x$-axis is along the direction of the surface and the $y$-axis is normal to the surface. It is assumed that the velocity of the external flow $u_e(x)$ and the temperature of the plate $T_w(x)$ are proportional to the distance of $x$ from the stagnation point, where $u_e(x) = ax$ and $T_w(x) = T_w + T_0(x/L)$, where $a$ is a constant, $T_0$ is the reference temperature, and $T_w$ is the uniform ambient temperature. Such assumptions were considered by Ramachandran et al. [9], who studied the steady laminar mixed convection in two-dimensional flows around heated surfaces for both cases of an arbitrary wall temperature and arbitrary surface heat flux. They showed that mixed convection in stagnation flows becomes important when the buoyancy forces due to the temperature difference between the wall and the freestream become high, thereby modifying the flow and thermal fields significantly. It is also assumed that a uniform induced magnetic field of strength $H_0$ is applied in the normal di...
Fig. 1 Physical model and coordinate system for (a) assisting flow and (b) opposing flow

The boundary conditions for Eqs. (1)–(5) are

\[ u = v = 0, \quad \frac{\partial H_1}{\partial y} = H_1 = 0, \quad T = T_w \quad \text{at} \quad y = 0 \]  \hspace{1cm} (6)

where \( H_1(x) = H_0(x/L) \).

Thus, we introduce the following similarity transformations:

\[ \psi = (a'v)^{1/2} x f(\eta), \quad \theta(\eta) = \frac{(T - T_w)}{(T_\infty - T_w)}, \quad \eta = (a'/v)^{1/2} x \]

\[ H_1 = H_0(x/L) h(\eta), \quad H_2 = -\frac{H_0}{L}(v/a)^{1/2} h(\eta) \]  \hspace{1cm} (7)

where \( \psi \) is the stream function, which is defined as \( u = \partial \psi / \partial y \) and \( v = -\partial \psi / \partial x \); hence, Eqs. (1) and (2) are satisfied.

By substituting Eq. (7) into Eqs. (3)–(5), we obtain the following similarity or ordinary nonlinear differential equations:

\[ f'''' + f''(f'')^2 + 1 + M(h''^2 - h'^2) - 1 + \lambda \theta = 0 \]  \hspace{1cm} (8)

\[ a\theta'''' + b\theta'' - h\theta'' = 0 \]  \hspace{1cm} (9)

and the boundary conditions (6) reduce to

\[ f(0) = f'(0) = 0, \quad h(0) = h'(0) = 0, \quad \theta(0) = 1 \]

\[ f'(\infty) = 1, \quad h'(\infty) = 1, \quad \theta(\infty) = 0 \]  \hspace{1cm} (11)

where primes denote differentiation with respect to \( \eta \), \( \alpha = \alpha_1 / v \) is the reciprocal of the magnetic Prandtl number, and \( M = \mu_0 H_0^2 / (\rho L^2 a^2) \) is the magnetic parameter or Hartmann number. Further,

\[ \lambda = \frac{g\beta(T_w - T_\infty)L^2}{Ra} \]  \hspace{1cm} (12)

is the constant buoyancy parameter with \( Gr = g\beta(T_w - T_\infty)L^3 / \nu^2 \) as the Grashof number and \( Re = L^2 a / \nu \) as the Reynolds number, where \( \lambda > 0 \) and \( \lambda < 0 \) correspond to the buoyancy assisting and opposing flows, respectively, and \( \lambda = 0 \) is the pure forced convection flow.

In this study, the physical quantities of interest are the skin friction coefficient \( C_f \), and the local Nusselt number \( Nu \), which are defined as

\[ C_f = \frac{\tau_w}{\rho u_w^2}, \quad Nu = \frac{x q_w}{k(T_w - T_\infty)} \]  \hspace{1cm} (13)

where \( \tau_w \) is the surface shear stress in the direction of \( y \), and \( q_w \) is the surface heat flux, which are given by

\[ \tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0} \]  \hspace{1cm} (14)

with \( \mu \) and \( k \) being the dynamic viscosity and thermal conductivity of the fluid, respectively. Using Eq. (7), we obtain

\[ C_f Re_2^{1/2} = f'(0), \quad Nu Re_2^{1/2} = -\theta'(0) \]  \hspace{1cm} (15)

### Table 1: Variation in the skin friction coefficient for different values of Pr when \( M=0 \) and \( \lambda=1 \)

<table>
<thead>
<tr>
<th>Pr</th>
<th>Ishak et al. [17]</th>
<th>Present results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Upper branch</td>
<td>Lower branch</td>
</tr>
<tr>
<td>0.7</td>
<td>1.7063</td>
<td>1.706376</td>
</tr>
<tr>
<td>1</td>
<td>1.6755</td>
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<tr>
<td>10</td>
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### 3 Results and Discussion

Equations (8)–(10) subject to the boundary conditions (11) have been solved numerically using the Keller-box method as described by Cebeci and Bradshaw [22]. To validate the accuracy of the present method, the numerical result for the local skin friction coefficient when the magnetic parameter is absent in this study is found to be \( f'(0) = 1.2362 \), which is in very good agreement with Wang [23]. Comparison is also made with previously published results for the local skin friction coefficient and the local Nusselt number when \( \lambda = 1 \), as shown in Tables 1 and 2, where the dual solutions are also given, and the comparison is also found to be in very good agreement.

Figures 2–4 respectively display the effects of magnetic parameter on the velocity, temperature, and the induced magnetic field \( H(\eta) \) profiles for both assisting (\( \lambda = 4 \)) and opposing flow (\( \lambda = -0.2 \)) cases when fixed \( \lambda \) and \( Pr = 0.7 \) are applied. Velocity profiles decrease when magnetic parameter \( M \) increases, but the profiles increase with \( M \) after a certain point for the assisting flow. From
Table 2  Variation in the local Nusselt number for different values of Pr when $M=0$ and $\lambda=1$

<table>
<thead>
<tr>
<th>Pr</th>
<th>Ramachandran et al. [9]</th>
<th>Ishak et al. [17]</th>
<th>Present results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Lower branch</td>
<td>Upper branch</td>
</tr>
<tr>
<td>0.7</td>
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<tr>
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<td>–</td>
<td>–</td>
<td>1.9448</td>
</tr>
</tbody>
</table>

Fig. 3, it is seen that the temperature profiles are always increasing when $M$ increases for both assisting and opposing flows, but the increase in $M$ is not very significant for temperature profiles in the assisting flow. In the assisting flow, an increase in $M$ leads to a decrease in $h(\eta)$ profiles, and these profiles increase with the increase in $M$ after a certain point. On the other hand, an increase in $M$ leads to an increase in $h(\eta)$ profiles for the opposing flow. These can be seen from Fig. 4.

Velocity profiles for the assisting flow ($\lambda=1$) and fixed $M=0.2$ decrease as Pr increases. However, the trend reverses for the opposing flow ($\lambda=-0.2$). These profiles are displayed in Fig. 5. Figure 6 shows the temperature profiles for fixed $M=0.2$ and $\lambda=1$, $-0.2$. Both assisting and opposing flows show that the thermal boundary layer thickness decreases as Pr increases. This phenomenon happened because when Pr is increased, the thermal diffusivity decreases; thus, it leads to the decrease of the energy transfer ability that decreases the thermal boundary layer. From Fig. 7, the Prandtl number shows the same effect as $M$ on $h(\eta)$ profiles. In Figs. 8 and 9, the profiles are plotted for various values of the mixed convection parameter $\lambda$ when the magnetic parameter and the Prandtl number are fixed at $M=0.2$ and $Pr=0.7$, respectively. It can be seen that both the velocity profiles and the $h(\eta)$ profiles reduce with the increase in $\lambda$. Figure 10 shows the opposite trend for the temperature profiles.
Figures 11 and 12 display the dual solutions for the quantities of physical interest, which are the skin friction coefficient $f'$ and the local Nusselt number $-\theta'(0)$. It can be seen that dual solutions of Eqs. (8–10) can be obtained for assisting and opposing flows for both $M=0.0$ and 0.3. For $\lambda>0$ (assisting flow), dual solutions exist for all $\lambda$, and the skin friction coefficient increases with $\lambda$ as the pressure gradient due to the buoyancy forces accelerates the flow. For $\lambda<0$ (opposing flow), solutions do not exist beyond certain critical values of $\lambda_c$, dual solutions exist at $\lambda>\lambda_c$, and a unique solution is obtained when $\lambda=\lambda_c$. Hence, at $\lambda=\lambda_c$, the boundary layer separation occurs. In this study, for $M=0$, $\lambda_c=-2.2$, while for $M=0.3$, $\lambda_c$ reduces to $-1.6$. The critical value $|\lambda_c|$ shown in Fig. 11 decreases as the magnetic parameter increases; therefore, the magnetic field induces earlier boundary layer separation or the boundary layer separation becomes faster when the magnetic field is applied. Numerical values of these results are presented in Table 3 for both assisting and opposing flows. Figure 12 shows that $-\theta'(0)$ becomes unbounded...
when \( \lambda \rightarrow 0^+ \) and \( \lambda \rightarrow 0^- \) for the second solution, and for a certain value of the magnetic field \( M \), the heat transfer coefficient for the first solution increases with \( \lambda \). It is worth mentioning that as in similar physical situations, the upper branch (first) solutions are physically stable and occur in practice, while the lower branch (second) solutions are not physically obtained. This can be verified by performing a stability analysis, but this is beyond the scope of the present paper. Such an analysis was done by Merkin [24], Weidman et al. [25], and Harris et al. [26].

Table 4 shows the effect of the reciprocal of the magnetic Prandtl number \( \alpha \) on the skin friction, the induced magnetic field, and the heat transfer coefficients. All three coefficients increase with \( \alpha \) for both assisting and opposing flows. It can be seen that the induced magnetic field coefficients are affected the most by \( \alpha \) as it appears in the induced magnetic field equation.

### 4 Conclusions

A numerical study is performed for the problem of the steady laminar mixed convection boundary layer flow on a vertical surface in the presence of a magnetic field. The induced magnetic field is also taken into account. The velocity, temperature, and the induced magnetic field profiles are affected by the magnetic parameter, the Prandtl number, and the buoyancy parameter for both assisting and opposing flows. In this study, it is also found that the induced magnetic field are affected the most by the reciprocal of the magnetic Prandtl number \( \alpha \) compared with the skin friction and heat transfer coefficients. Dual solutions are also obtained for opposing and assisting flows in this study. It is also found that the magnetic field induces earlier boundary layer separation; i.e., the boundary layer separation becomes faster when the magnetic field is applied.

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### Nomenclature

- \( \alpha \) = constant
- \( C_{f \alpha} \) = skin friction coefficient
- \( f(\eta) \) = dimensionless stream function
- \( g \) = acceleration due to gravity
- \( Gr \) = Grashof number
- \( h(\eta) \) = dimensionless induced magnetic field
- \( H_0 \) = applied magnetic field
- \( H_1, H_2 \) = induced magnetic field components along the \( x \) and \( y \) directions, respectively
- \( k \) = thermal conductivity
- \( L \) = characteristic length
- \( M \) = magnetic parameter
- \( Nu \) = Nusselt number
- \( Pr \) = Prandtl number
- \( q_0 \) = surface heat flux
- \( Re \) = Reynolds number
- \( T \) = fluid temperature
- \( T_e(x) \) = temperature of the surface
- \( T_a \) = ambient temperature
- \( u, v \) = velocity components along the \( x \) and \( y \) directions, respectively
- \( u_e(x) \) = velocity of the external flow
- \( x, y \) = Cartesian coordinates along the surface and normal to it, respectively

### Greek Symbols

- \( \alpha \) = reciprocal of the magnetic Prandtl number
- \( \alpha_1 \) = magnetic diffusivity
- \( \beta \) = thermal expansion coefficient
- \( \eta \) = similarity variable
- \( \theta \) = dimensionless temperature
- \( \lambda \) = buoyancy or mixed convection parameter
- \( \mu \) = dynamic viscosity
- \( \mu_0 \) = magnetic permeability
- \( \nu \) = kinematic viscosity
- \( \rho \) = fluid density
- \( \psi \) = stream function
- \( \tau_w \) = surface shear stress

### References


