Estimation of wall bounded shear flow

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Outline

• Flow control
• model based control
• linear compensation : control and estimation
• Perturbation model for the estimation
• Results on localized perturbation
Control theory

Mathematically well developed
and central to many engineering applications

- Space ship - satellites trajectory
- Break control (ABS)
- Any automatic pilots
- etc ...

Linear theory pushed to an extremum,
nonlinear theory at its beginning
Flow control

One could like to:

- Postpone transition
- Relaminarise turbulence
- Increase mixing
- Avoid detachment

But also:

- Lock reattachment point
- Lock oscillatory behaviour
- Modify transition scenario
- Modify turbulence statistics
Flow control - continued

To achieve this:

- **passive control**
  - geometry design/optimization
  - roughness element
  - vortex generators

- **active control**
  - constant blowing or suction
  - wall temperature
  - Periodic blowing and suction

- **reactive control (feedback)**
  - sensors and actuators → introduce the estimation
Feedback

The control $u$ is based on measurement $y$ from the system state

$$\begin{aligned}
\dot{q} &= Aq + B_1 u(y) + B_2 f, \quad q(0) = q_0, \\
y &= Cq + g,
\end{aligned}$$

(1)

The system is subject to initial condition $q_0$, volume forcing $f$, and sensor noise $g$. 
Model based control, and optimization

- The feedback law can be based on physical insight.
- But as well on a model → can be optimized.

Note: Even with a model we need physical insight.

1. What is a good model?
2. Which objective function?
3. Which actuation and sensing?

Those three problems are not independent!
Decoupling control–estimation

Plant \[
\begin{align*}
\dot{q} &= Aq + B_1 u + B_2 f, \quad q(0) = q_0, \\
y &= Cq + g,
\end{align*}
\]

Estimator \[
\begin{align*}
\dot{\hat{q}} &= A\hat{q} + B_1 u - v, \quad \hat{q}(0) = \hat{q}_0, \\
\hat{y} &= C\hat{q}, \\
v &= L\hat{y} = L(y - \hat{y}).
\end{align*}
\]

The best feedback controller is composed of the best full information controller and the best estimator. (for linear systems)
Previous achievements

State feedback for streaks

Measurement feedback for oblique wave
Objective function vs noise model

**Control:** act at $T$ to affect the flow later

- We need a policy on how to act:
  objective function & dynamic model

**Estimation:** measure before $T$ to know the flow now.

- We need a policy on how the information is provided:
  perturbation model & dynamic model
Why a stochastic model?

**Deterministic** We know either everything or nothing

**Stochastic** We use the average behaviour to estimate the instantaneous state

- Average over initial condition
- Average over volume forcing

We optimize the performance
averaged over all initial condition
and all volume forcing
Correlation model for the volume forcing

$y$ variation

Fourier space variation: exponential decay
Model for the initial conditions

\( k \) is the realisation number

\[
q_0^{(k)} = \theta_1(k) \left( \begin{array}{c}
\theta_2(k) \frac{q_s}{||q_s||E} + \sum_j \psi_j(k) \frac{r_j}{||r_j||E} \\
\text{Specific} \\
\text{Random}
\end{array} \right),
\]

The corresponding covariance becomes

\[
P_0 = \lambda_1 \left( \frac{E[q_s q_s^*]}{\text{Tr}(E[q_s q_s^*])} + (1 - \lambda_2) \frac{E[r_0 r_0^*]}{\text{Tr}(E[r_0 r_0^*])} \right),
\]

The energy in Fourier space

\[
\lambda_1(k_x, k_z) = v_1 k e^{-s\lambda k^2/2},
\]

\( (2) \)
Results
Three flow cases

- Initial condition + forcing
- Only initial condition
- Only forcing

(0,2)

(1,1)

(1,0)
Measurements
Gain scheduling

Pick a gain from time $t$ and apply it all the time

Steady state gain

Time varying gain
Time varying kernels

$v$

$t=0$

$t=20$

$t=60$

$\eta$
Steady state kernels

\[ \tau_x , \tau_z , p \]

\[ v, \eta \]
Localized perturbation
Estimation performance

Using steady state gain

$E$

Flow energy

$I_2$ variation

Time

$E$

$0$ $50$ $100$ $150$ $200$ $250$ $300$

$10^{-12}$ $10^{-11}$ $10^{-10}$ $10^{-9}$
Flow evolution

flow estimated flow

- $t=0$
- $t=20$
- $t=60$
Conclusion

**Was done**

- A model for perturbations
- Choice of measurements
- Investigation of transient for estimation
- A sub-optimal procedure

**To be done**

- Transient for the control as well