LINEAR FEEDBACK CONTROL OF TRANSITION IN SHEAR FLOWS

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Abstract This work focuses on the application of linear feedback control to transition to turbulence in shear flows. The controller uses wall-mounted sensor information to estimate the flow disturbances and uses wall actuators to prevent transition to turbulence. The flow disturbances are induced by external sources of perturbations described by means of a stochastic volume forcing. We show that improved performance can be achieved if the proper destabilisation mechanisms are targeted.

Keywords: Feedback control, LQG, state estimation, Kalman filter, stochastic disturbance model.

1. Introduction

In many applications like aeroplane wings, pipes, turbine blades, growth of small disturbances due to external sources of excitation can lead to transition to turbulence and thus increase the friction drag. Control is being increasingly applied to fluid flow as the theories and devices are being developed (see Bewley, 2001, Kim, 2003). A powerful theory for linear feedback control is available and can be applied to flow control, assuming a linear dynamics for the flow (small amplitude disturbances), with a quadratic objective function, and a Gaussian distribution for the external sources of excitation and measurement noise. This method known as LQG (Linear, Quadratic, Gaussian) or L_2 control (see Green and Limebeer, 1995) is used in this work.

2. System and control setup

In this work, the dynamics of small perturbations to a laminar base flow is modelled by the linearized Navier–Stokes equation. Measurements are extracted as the instantaneous value of the two components of the wall skin friction and pressure. Control is applied by means of blowing and suction at the wall. In the LQG control formulation, the systems can be written in state space

$$\begin{cases} \dot{q} = Aq + B_1 f + B_2 u \\ y = Cq + g, \end{cases}, \quad \begin{cases} \dot{\hat{q}} = A\hat{q} + B_2 u - v \\ \hat{y} = C\hat{q} \end{cases}, \quad \begin{array}{l} u = K\hat{q}, \\ v = L(y - \hat{y}). \end{array} \tag{1}$$

The equations above show the four elements of a LQG formulation. The flow system to be controlled, the estimator providing an estimate of the instantaneous flow state, and the estimator and control feedback gains L and K.

The flow state $q = (v, \eta)^T$ is constructed with the wall normal velocity v and wall normal vorticity η. It is affected by external disturbances in the form of a stochastic forcing f. The measurement vector y contains all the available information about the flow state. It is corrupted by the sensor noise g with covariance G.

The estimator is built with analogous form. The estimator state $\hat{q}$ is forced to approach the flow state by a 3D volume forcing v, feedback of the measurements. The flow and estimator states q and $\hat{q}$ are controlled by means of the blowing and suction u. The control actuation is a feedback of the estimated flow state.

For the dynamic operator A, we use the linearised Navier–Stokes equations transformed to Fourier space, i.e. the Orr-Sommerfeld/Squire equations (see e.g. Schmid and Henningson, 2001)

$$A = \begin{pmatrix} \mathcal{L}_{OS} & 0 \\ \mathcal{L}_C & \mathcal{L}_{SQ} \end{pmatrix}, \begin{cases} \mathcal{L}_{OS} = \Delta^{-1}(-ik_x U\Delta + ik_x U'' + \Delta^2/Re), \\ \mathcal{L}_{SQ} = -ik_x U\Delta/Re, \quad \mathcal{L}_C = -ik_z U', \end{cases} \tag{2}$$

where U, U', U'' are the base flow and its wall normal derivatives, Δ denotes the Laplacian operator, k_x and k_z are the streamwise and spanwise wavenumbers, and Re is the Reynolds number.

The main issues in designing such a controller, is the description of the external disturbances f by their covariance R_{ff} (Hœpffner et al., 2003), and the control objective $\mathcal{J}$ by the quadratic norm Q,

$$R_{ff} = E[ff^*], \quad \mathcal{J} = \frac{1}{2}\int_0^\infty (q^*Qq + \ell^2 u^* u)\,\mathrm{d}t, \tag{3}$$

where $E[\cdot]$ denotes the expectation operator, and ℓ plays the role of a penalty on the control effort. The optimal feedback gains L and K can be computed independently for each wave number pair by solving two

Riccati equations (see e.g. Glover et al., 1989)

$$\left\{\begin{array}{ll} A^*X + XA - \frac{1}{l^2}XB_2B_2^*X + Q = 0, & K = -\frac{1}{l^2}B^*X, \\ AP + PA^* + B_1R_{ff}B_1^* - PC^*G^{-1}CP = 0, & L = -PC^*G^{-1}. \end{array}\right. \tag{4}$$

The performance of the estimator is monitored by the estimation error energy, i.e. the kinetic energy of $(q - \hat{q})$. The flow state is well estimated when this energy is low compared to the flow energy. The performance of the controller is seen by the controlled flow energy. The flow is well controlled when its energy is low compared to the uncontrolled flow. We will see in the following sections how these performance can be fine-tuned.

3. Results

3.1 Control and estimation

The evolution of a localized initial condition in channel flow, and its evolution when controlled are depicted in figure 1 . The objective function is designed to minimize the kinetic energy of the disturbance to the laminar flow profile. The model for the disturbances assumes finite length correlation for the external disturbances (see Hœpffner et al., 2003). The covariance model for the disturbances assumes the form

$$\left\{\begin{array}{l} R_{f_j f_k}(y, y', k_x, k_z) = d_1\, \delta_{jk}\mathcal{M}(y, y'), \\ \mathcal{M}(y, y') = e^{-(y-y')^2/2d_y}, \\ d_1(k_x, k_z) = d_a\, k_d^2\, \mathrm{e}^{-k_d^2+1} \quad \text{with} \quad k_d^2 = (k_x/d_x)^2 + (k_z/d_z)^2, \end{array}\right. \tag{5}$$

where $\mathcal{M}$ describes the covariance of f in the wall normal direction for a single wavenumber pair, and d_1 accounts for the variation in wavenumber pair space of the strength of this forcing. The model parameters d_y, d_a, d_x, and d_z can be freely chosen to fit the flow type at hand.

The initial condition presented is the localized perturbation that may originate from a jet normal from the bottom channel wall. This perturbation initially grows in energy and is finally damped by viscous effects. We show two cases of controller. The first one is turned on at time 0 and the second one at time 20. In both cases the feedback uses estimated flow state information. The performance for both cases for full information controller and estimation error is depicted with dashed dotted and dotted thin lines. When the controller is started at time 0, there is an initial growth of the controlled flow energy and estimation error energy due to a strong component of growing disturbance. But when controller and estimator are started up later, monotonous decay is observed.

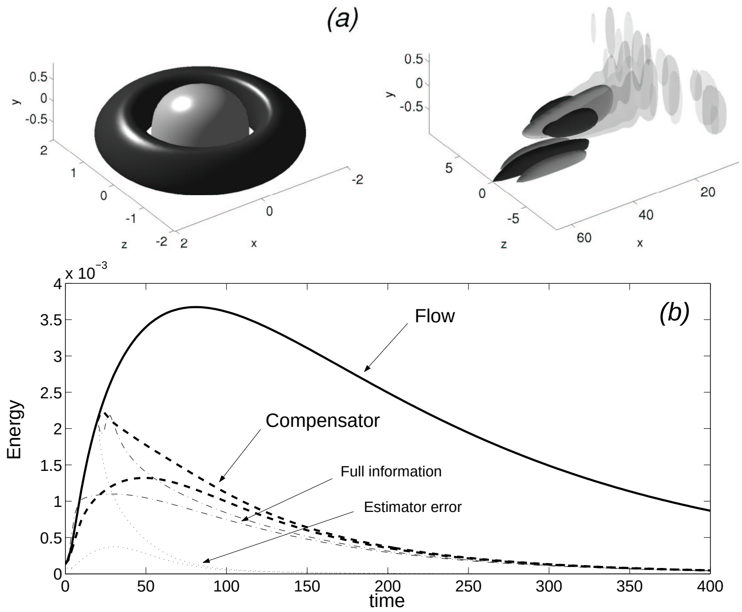

Figure 1. *(a)* Isosurfaces of the wall normal velocity, for time 0 and 90 of the evolution of the localised initial condition in a channel. Flow without control is transparent and with control is opaque. *(b)* Energy evolution in time for the flow (bold solid) and the compensated flow (bold dashed). Two cases are represented: when the compensator is turned on at initial time and turned on at time 20. The flow energy evolution with full information control is represented with a dash-dotted line and the estimation error energy with dotted lines.

See on figure 1(b) an isosurface plot of the wall normal velocity for the initial disturbance and its evolved state at time 90 for no control (transparent) and estimation-based control (opaque). When controlled, the wave packet is prevented from spreading in the channel. The actuation is visible in this figure as the non-zero value of the wall normal velocity at the lower wall. It can be seen that the wall blowing and suction is of the same order of magnitude as the wall normal velocity of the flow disturbance to be controlled. The control effort is though of relatively low amplitude since most of the energy of the disturbance is carried by the streamwise velocity component.

3.2 Flexibility in the objective function

If one seeks to minimize the disturbance kinetic energy everywhere and all the time, the objective function (3) is well suited. We will now demonstrate that different goals can be reached by simple modifications of the quadratic norm defined by Q.

Two examples of the flexibility of the quadratic objective function for the case of a single wavenumber pair $(k_x, k_z) = (0, 0.5)$ of a Blasius boundary layer are depicted in figure 2 .

On figure 2(a) the control is turned off at time 100 in the time evolution of the initial condition that lead to the greatest reachable energy growth. If a strong (though not energetic) component of the potentially growing initial condition is still present at time 100, the growth resumes when the actuation is stopped as can be seen for case 1. To avoid this the controller in case 2 targets the growth mechanism by an extra penalty Re on the wall normal velocity responsible for the lift up effect (see e.g. Schmid and Henningson, 2001). This way, the growth is reduced after the actuation interval. Note that the control performance is only marginally affected by the extra penalization if the control is not stopped (dashed lines).

In figure 2(b) the same flow system is constantly excited by stochastic disturbances and has reached statistical steady state. For the three cases presented, the controller targets respectively the total kinetic energy, the kinetic energy integrated in the wall normal direction up to 2 and up to 1 (in displacement thickness units). Such a controller seeks to eject the disturbances away from the wall instead of killing disturbance energy everywhere. When only targeting the disturbance energy up to 1, the goal is met, and very little disturbances are in contact with the wall surface.

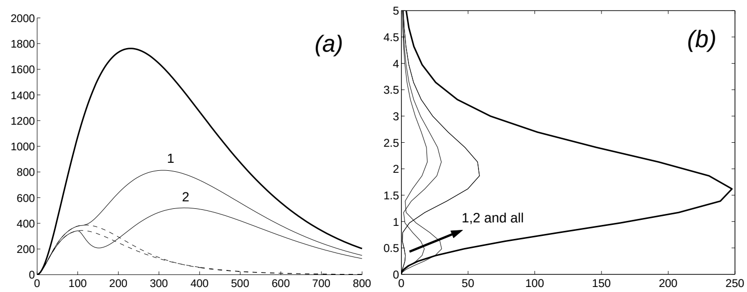

Figure 2. Illustration of the flexibility in the choice of the objective function. *(a)* time evolution of the flow energy with full information control all the time (thin dashed) and when the controller is turned off at time 100 (thin solid). In case 1, the controller minimizes the kinetic energy, and in case 2 there is an additional Reynolds number penalisation on the wall normal velocity component. *(b)* Steady state distribution of perturbation kinetic energy in the wall normal direction for a boundary layer constantly excited by external forcing. The bold line is the flow kinetic energy and the thin lines are the full information controlled flow when only the kinetic energy up to 1, 2, y_{max} is minimised in the objective function, as shown by the arrow.

4. Conclusion

We show in this work that improved control performance can be achieved for control of transition to turbulence in shear flows, if the proper destabilisation mechanisms are targeted. This means that a thorough physical understanding of the phenomenon to be controlled is necessary. We use a stochastic model for external disturbances, that may account for a wide range of flow disturbances. We design the quadratic objective function to target the main destabilisation mechanism, as for example streamwise elongated vortices in bypass transition.

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