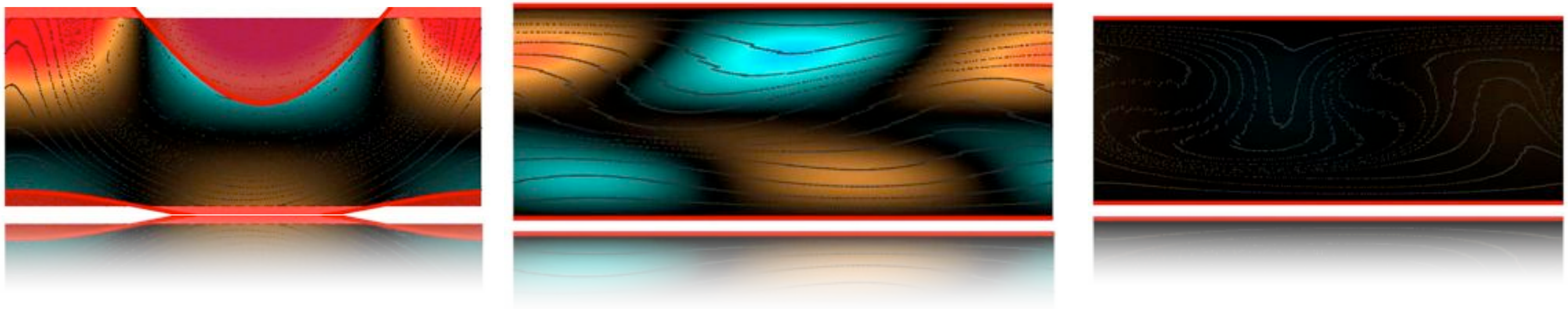
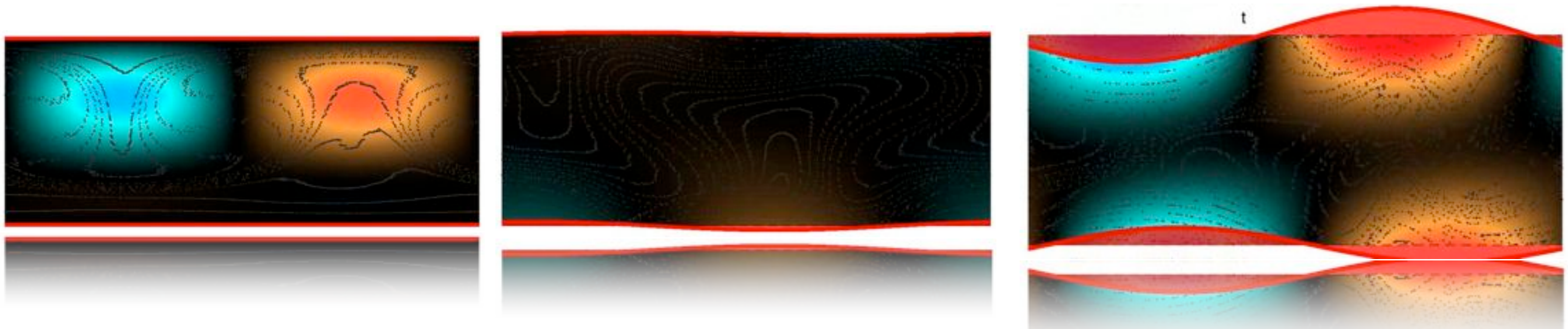


Energy growth in the compliant channel

Jérôme Hœpffner

Julien Favier, Alessandro Bottaro



Compliant surfaces

Ann. Rev. Fluid Mech. 1988. 20 : 393–420

COMPLIANT COATINGS

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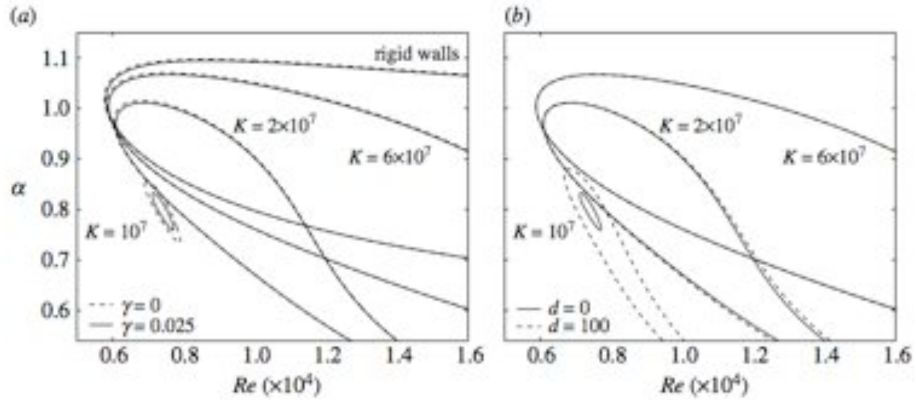
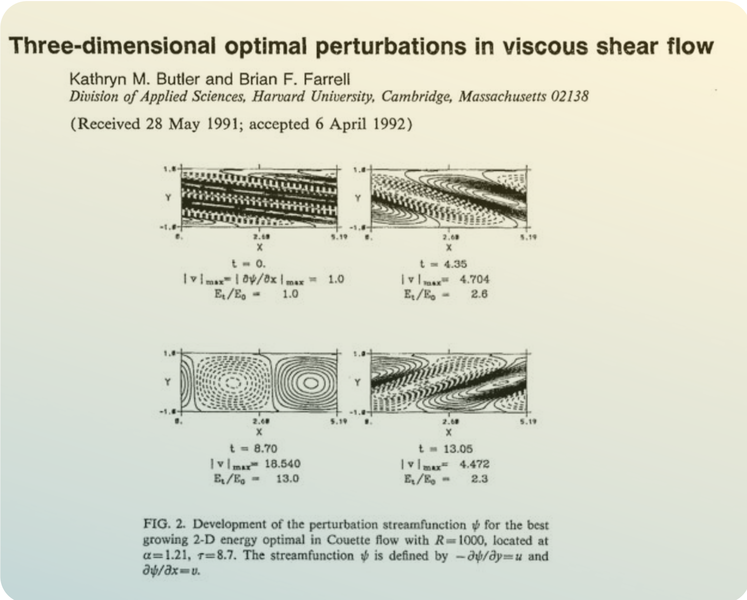


Figure 8. Neutral curves for the Tollmien–Schlichting instability showing the effect of (*a*) wall compliance and wall curvature for $d=0$ and (*b*) wall damping for $\gamma=0.025$. In both cases, we have $B=4K$.

Looking for “special things” in flows using optimization


Three-dimensional optimal perturbations in viscous shear flow

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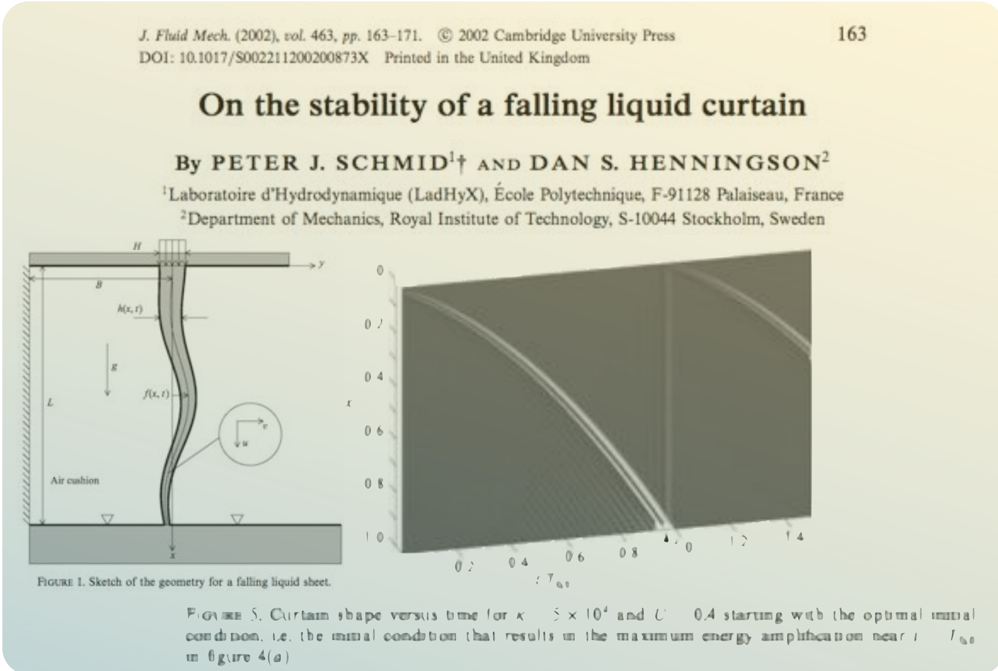
(Received 28 May 1991; accepted 6 April 1992)

FIG. 2. Development of the perturbation streamfunction ψ for the best growing 2-D energy optimal in Couette flow with $R=1000$, located at $\alpha=1.21$, $\tau=8.7$. The streamfunction ψ is defined by $-\partial\psi/\partial y=u$ and $\partial\psi/\partial x=v$.


J. Fluid Mech. (2002), *vol.* 463, *pp.* 163–171. © 2002 Cambridge University Press 163
DOI: 10.1017/S002211200200873X Printed in the United Kingdom

On the stability of a falling liquid curtain

By PETER J. SCHMID[1]† AND DAN S. HENNINGSON[2]
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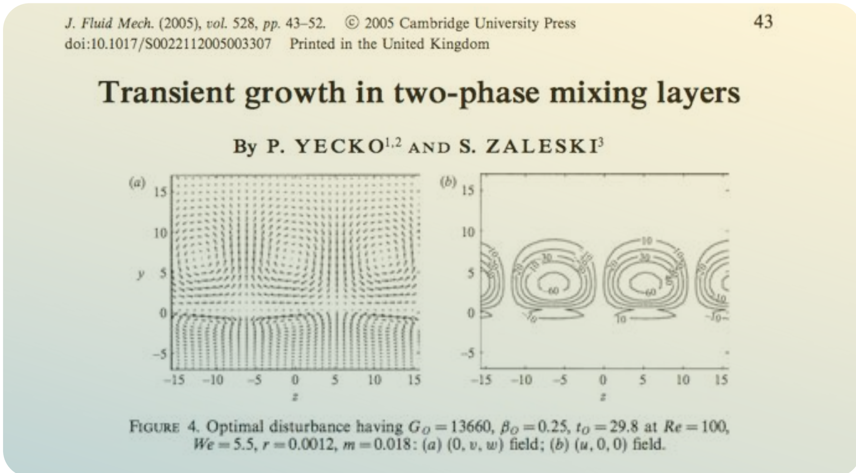
FIGURE 1. Sketch of the geometry for a falling liquid sheet.

FIGURE 5. Curtain shape versus time for $\kappa = 5\times10^4$ and $U = 0.4$ starting with the optimal initial condition, i.e. the initial condition that results in the maximum energy amplification near $t = T_{opt}$ in figure 4(*a*).


J. Fluid Mech. (2005), *vol.* 528, *pp.* 43–52. © 2005 Cambridge University Press 43
doi:10.1017/S0022112005003307 Printed in the United Kingdom

Transient growth in two-phase mixing layers

By P. YECKO[1,2] AND S. ZALESKI[3]

FIGURE 4. Optimal disturbance having $G_O=13660$, $\beta_O=0.25$, $t_O=29.8$ at $Re=100$, $We=5.5$, $r=0.0012$, $m=0.018$: (*a*) $(0, v, w)$ field; (*b*) $(u, 0, 0)$ field.

Flow and wall dynamics

η: wall displacement:

$$m\eta_{tt} + \frac{d}{Re}\eta_t + \frac{B\Delta_{2D}^2 + T\Delta_{2D} + K}{Re^2}\eta = \binom{+}{-} p|_{\text{wall}}$$

mass, acceleration

Damping

Tension T, rigidity B, spring stiffness K

Forcing by the pressure

Navier-Stokes, linearized about Poiseuille U:

$$\begin{aligned} u_t + Uu_x + U_y v &= -p_x + \Delta u/Re, \\ v_t + Uv_x &= -p_y + \Delta v/Re, \\ u_t + Uw_x &= -p_z + \Delta w/Re, \\ u_x + v_y + w_z &= 0 \end{aligned}$$

Boundary conditions:

$$u(1) = 2\eta_{\text{top}}, \quad u(-1) = -2\eta_{\text{bot}}, \quad v(\pm 1) = \eta_t, \quad w(\pm 1) = 0.$$

Energy

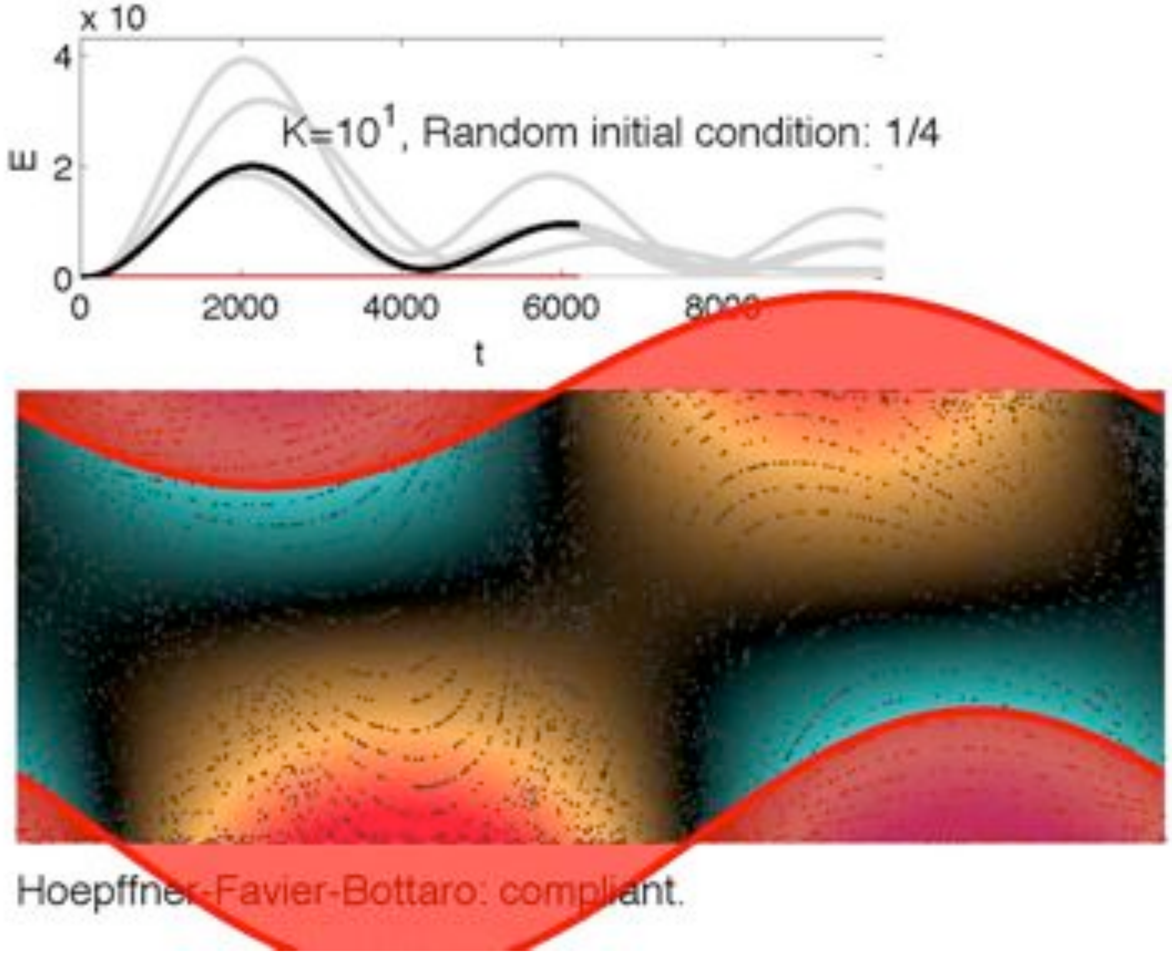
Flow energy+wall kinetic and potential energy:

$$E \triangleq \underbrace{\frac{1}{2}\int_y \overline{u^2+v^2+w^2}\mathrm{d}y}_{\text{Flow}} + \underbrace{\sum_{\text{bot}}^{\text{top}} \frac{1}{2}\left(m\overline{\eta_t^2} + \frac{B\Delta_{2D}^2 + T\Delta_{2D} + K}{Re^2}\overline{\eta^2}\right)}_{\text{Walls}}$$

Energy exchange:

$$E_t = \underbrace{-\int_y U_y\overline{uv}\mathrm{d}y + \frac{1}{Re}\left[\overline{(u^2+v^2+w^2)_y}\right]_{\text{bot}}^{\text{top}}}_{\text{Energy exchange with base flow}} \underbrace{-\frac{1}{Re}\int_y \overline{\omega.\omega}\,\mathrm{d}y - \sum_{\text{bot}}^{\text{top}} \frac{d}{Re}\overline{\eta_t^2}}_{\text{Viscous damping}}.$$

Flow response to random inital conditions

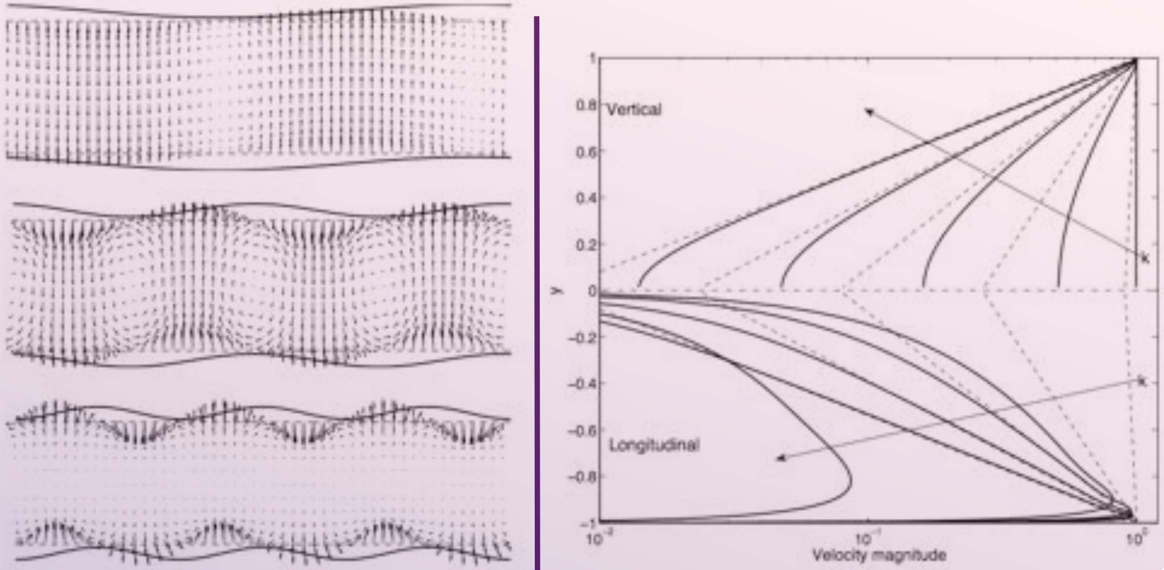


Hoepffner-Favier-Bottaro: compliant.

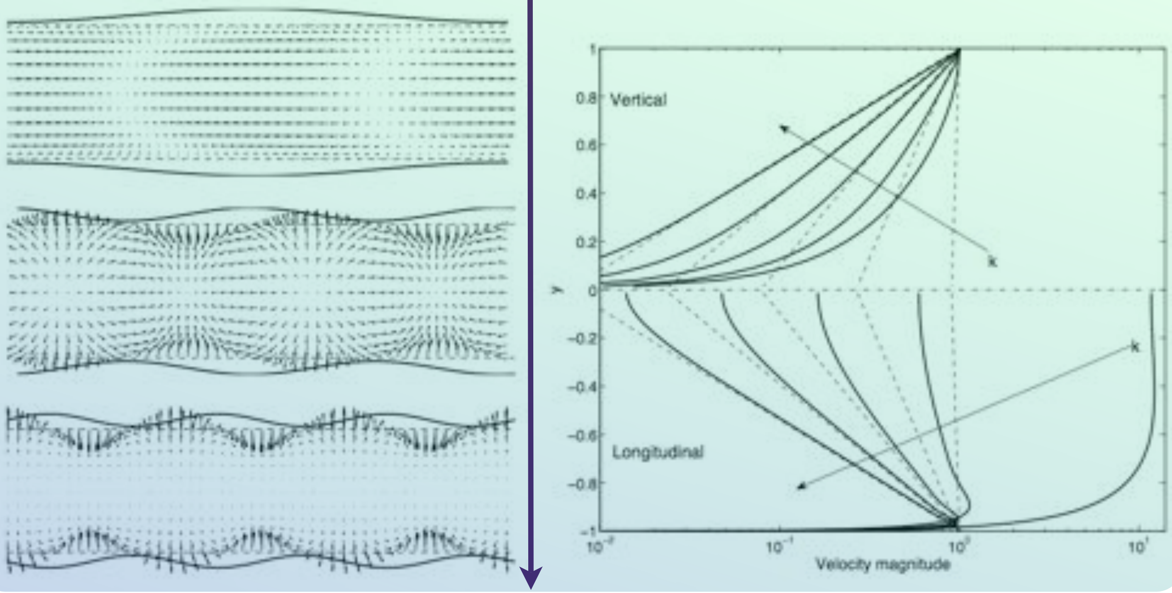
Wall waves

Decrease wavelength

Sinuous:



Varicose:

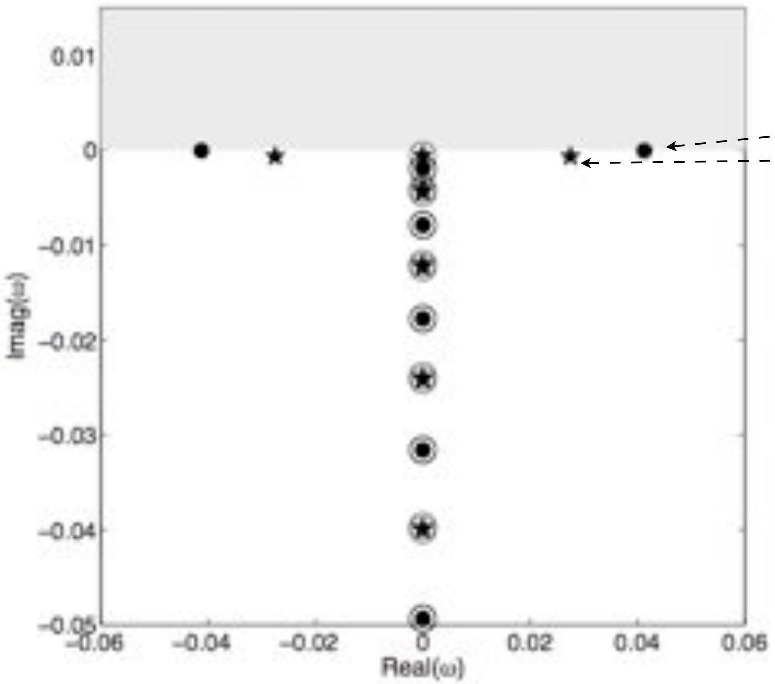


Pulsation of the free wall:

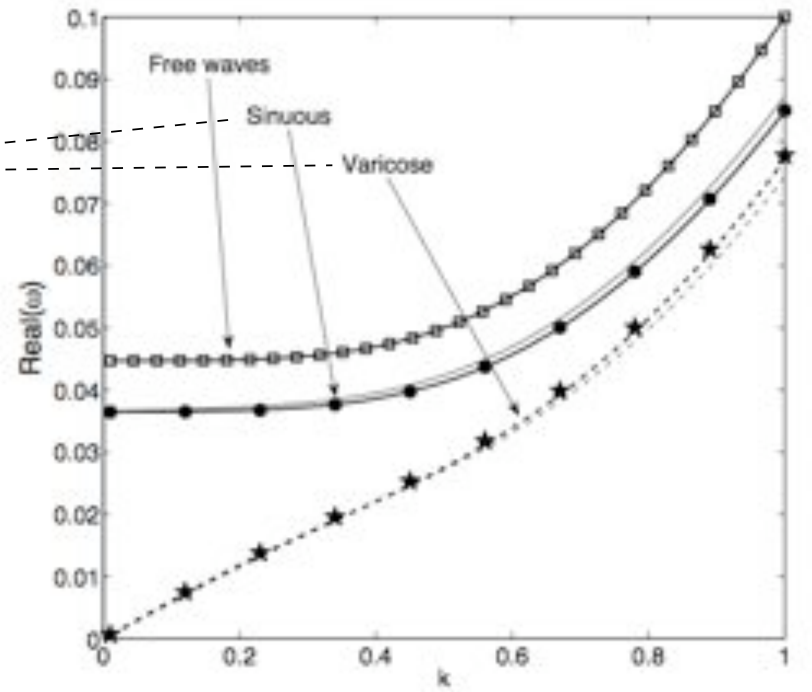
$$m\omega^2 = \frac{Bk^4 + Tk^2 + K}{Re^2}$$

Fluid effect: added mass

sinuous: $m_a^s = (1 - e^{-k})/k$
varicose: $m_a^v = (1 - e^{-k})/k + 1/k^2$



Spectra



Pulsation:
free wall, computed eigenmodes,
added mass model, inviscid

Optimization of the initial conditions

1) Projection on eigenmodes:

$$\dot{\kappa} = \Lambda\kappa, \quad \mathcal{Q} = U^H Q U = F^H F$$

2) Optimality:

$$G(t) = \max_{\kappa_0} \frac{\|\kappa(t)\|_{\mathcal{Q}}}{\|\kappa_0\|_{\mathcal{Q}}} = \max_{\kappa_0} \frac{\|e^{\Lambda t}\kappa_0\|_{\mathcal{Q}}}{\|\kappa_0\|_{\mathcal{Q}}} = \|e^{\Lambda t}\|_{\mathcal{Q}} = \|\underbrace{F^{-1}e^{\Lambda t}F}_{\mathcal{H}}\|_2$$

α=0, stable

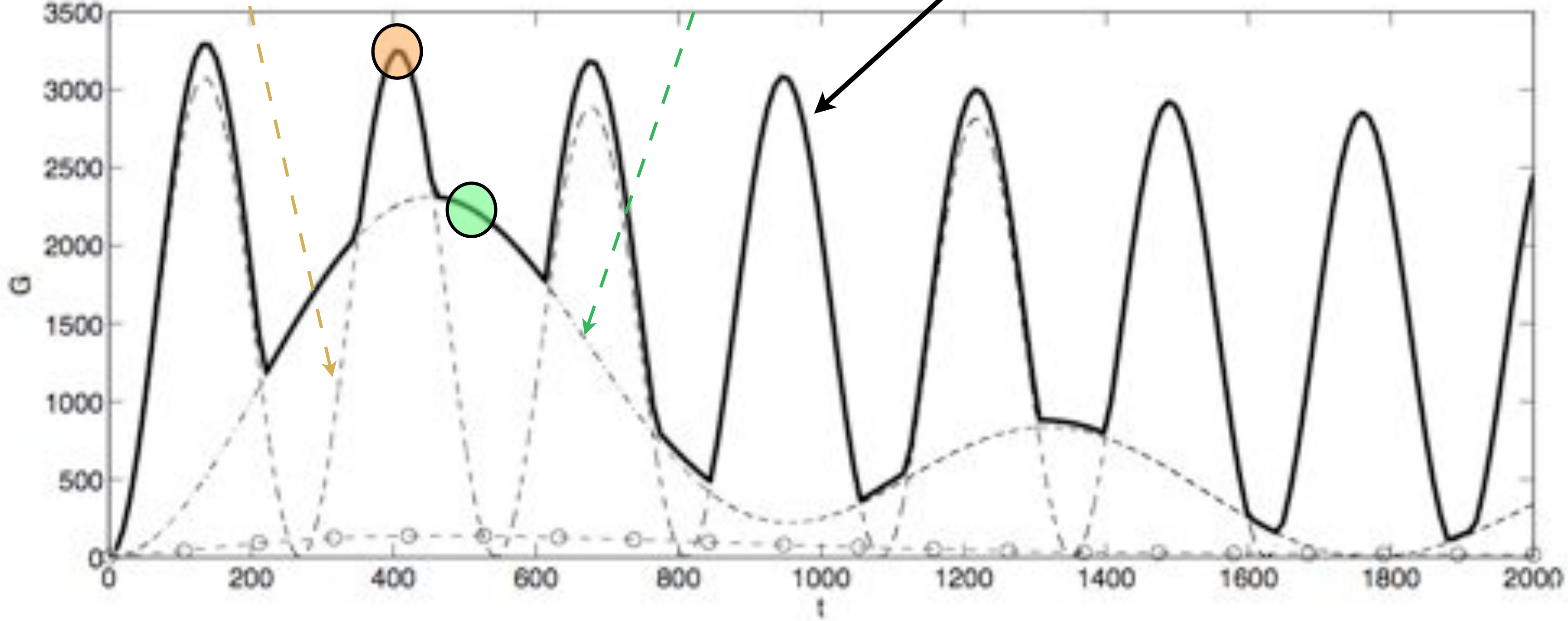
Optimization results

Energy evolution of the initial condition optimal for t=500

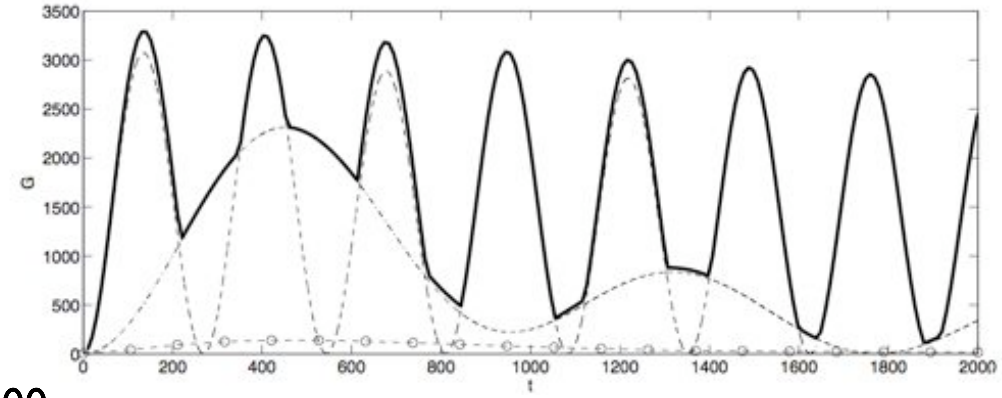
There are clearly two mechanisms, with different growth and different time scales

Energy evolution of the initial condition optimal for t=400

Growth enveloppe



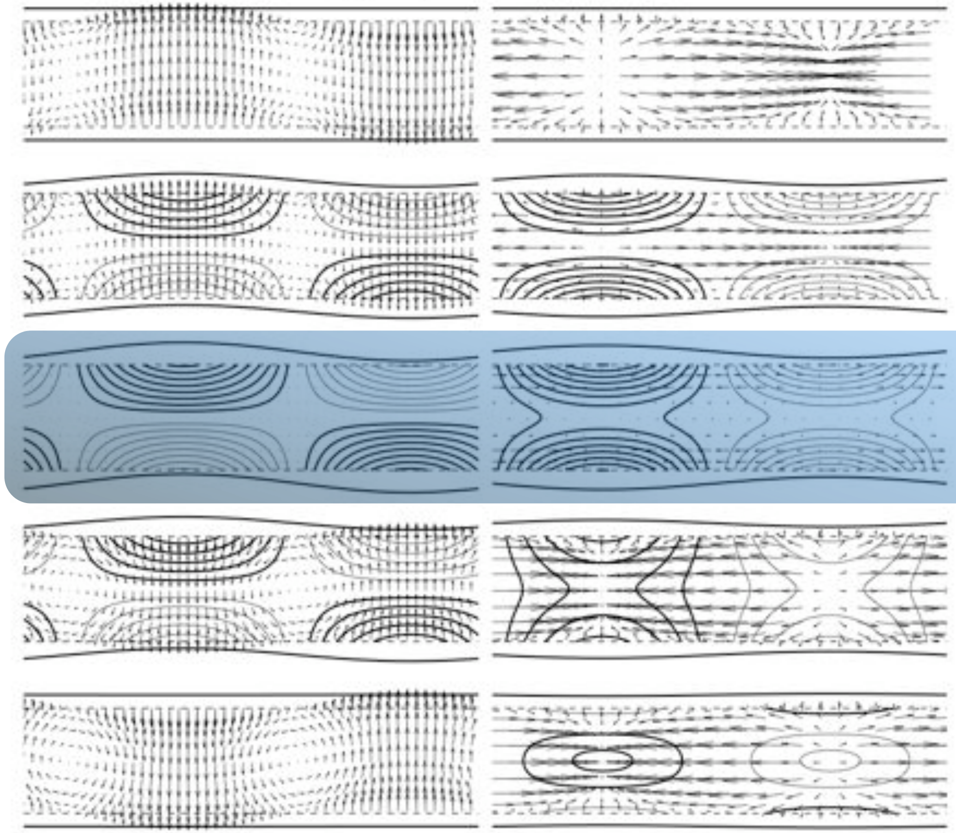
Optimization results



Growth enveloppe

Optimal at t=400

Optimal at t=500

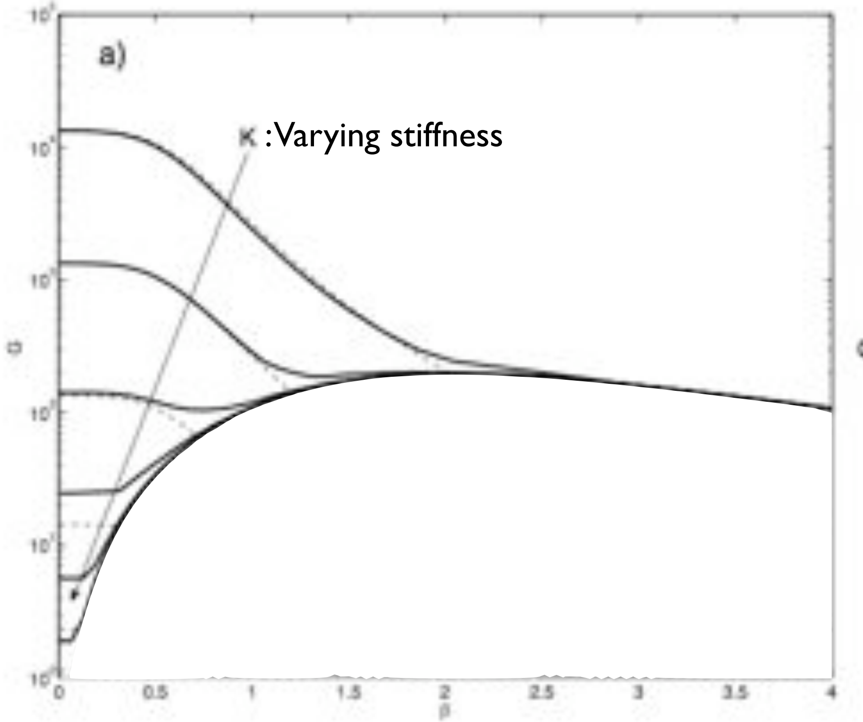


Time of largest energy

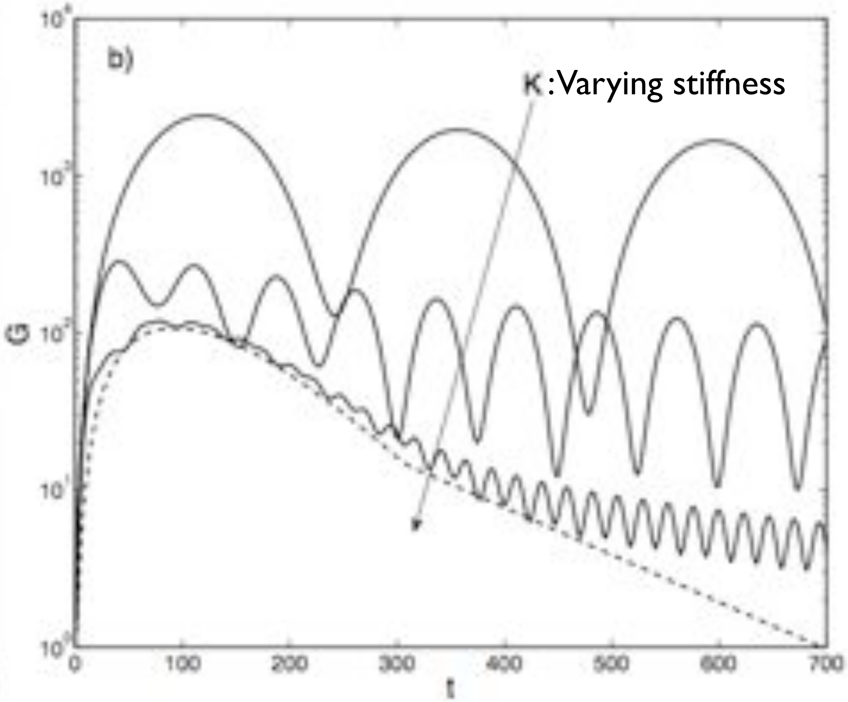
Time

Two mechanisms:
sinuous and varicose

Optimization results

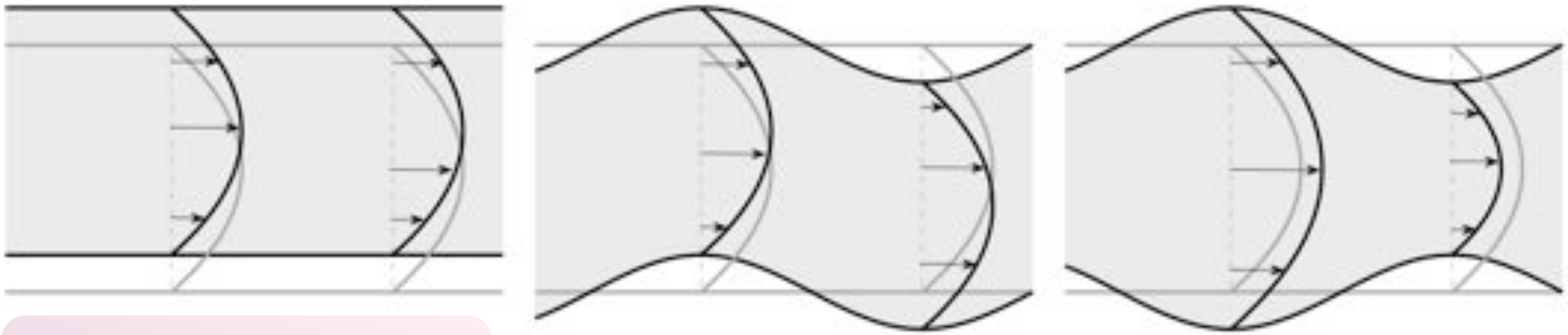


Maximum growth, varying spanwise wavelength, compare to rigid growth



Enveloppes in time, varying stiffness

Candidate mechanisms



Channel moving up&down

$$u'(y) = 1 - y^2 + 2\eta y$$

Up&down sinuous

$$u'(y) = 1 - y^2 + 2\eta y$$

Up&down varicose

$$u'(y) = (1+\eta) - y^2/(1+\eta)$$

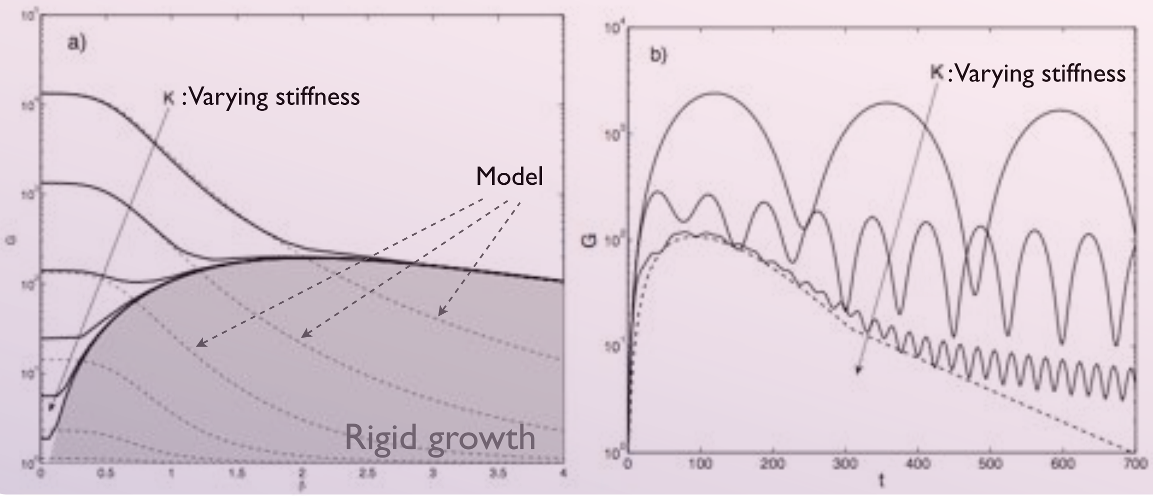
Energy evolution (perturbation to Poiseuille):

$$2E/\epsilon^2 = 2\left(\frac{4}{3} + \frac{K}{Re^2 m}\right)\cos(\omega t)^2 + 2\left(\omega^2(m+1)\right)\sin(\omega t)^2$$

Oscillatory energy:

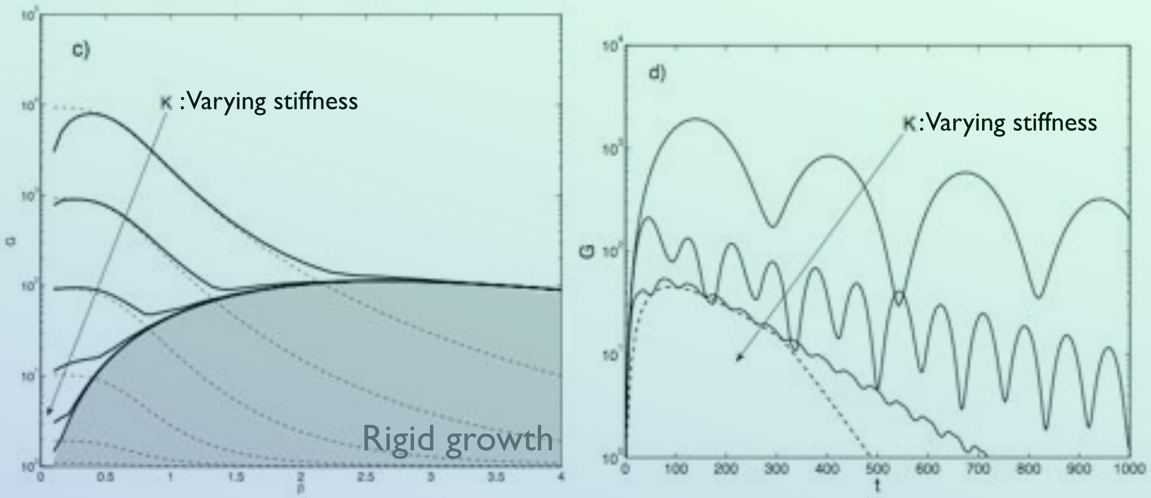
$$G^s_{k=0} = 1 + \frac{4}{3}\frac{Re^2}{K}, \quad T^s_{k=0} = \frac{\pi Re}{2}\sqrt{\frac{m+1}{K}}$$

Model/computations



Sinuous u&d:

$$G^s = 1 + \frac{4}{3}\frac{Re^2}{Bk^4 + Tk^2 + K}$$



Varicose u&d:

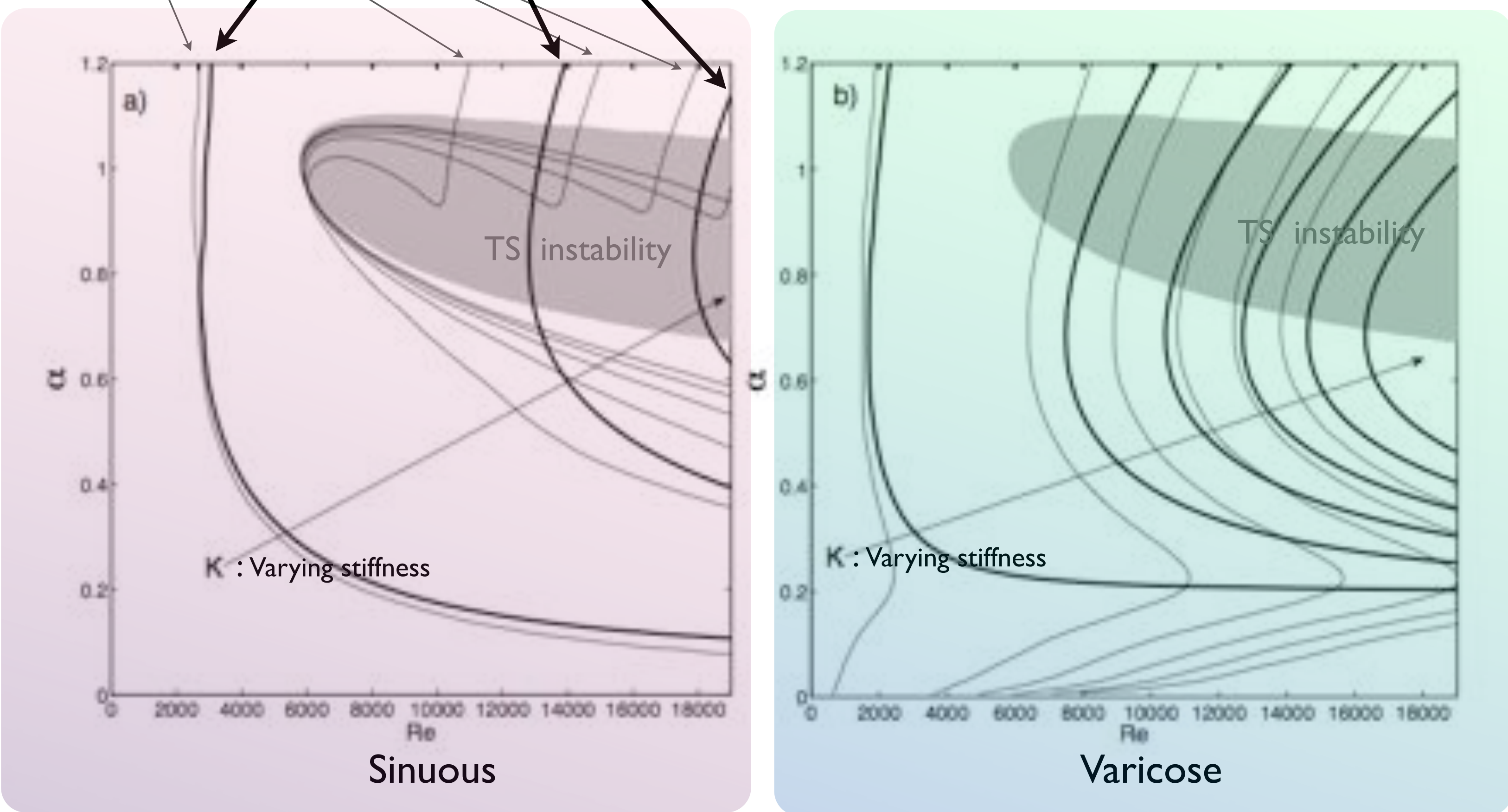
$$G^v = 1 + \frac{14}{15}\frac{Re^2}{Bk^4 + Tk^2 + K}$$

$\alpha \neq 0$, instabilities

Modal instability, Re/α

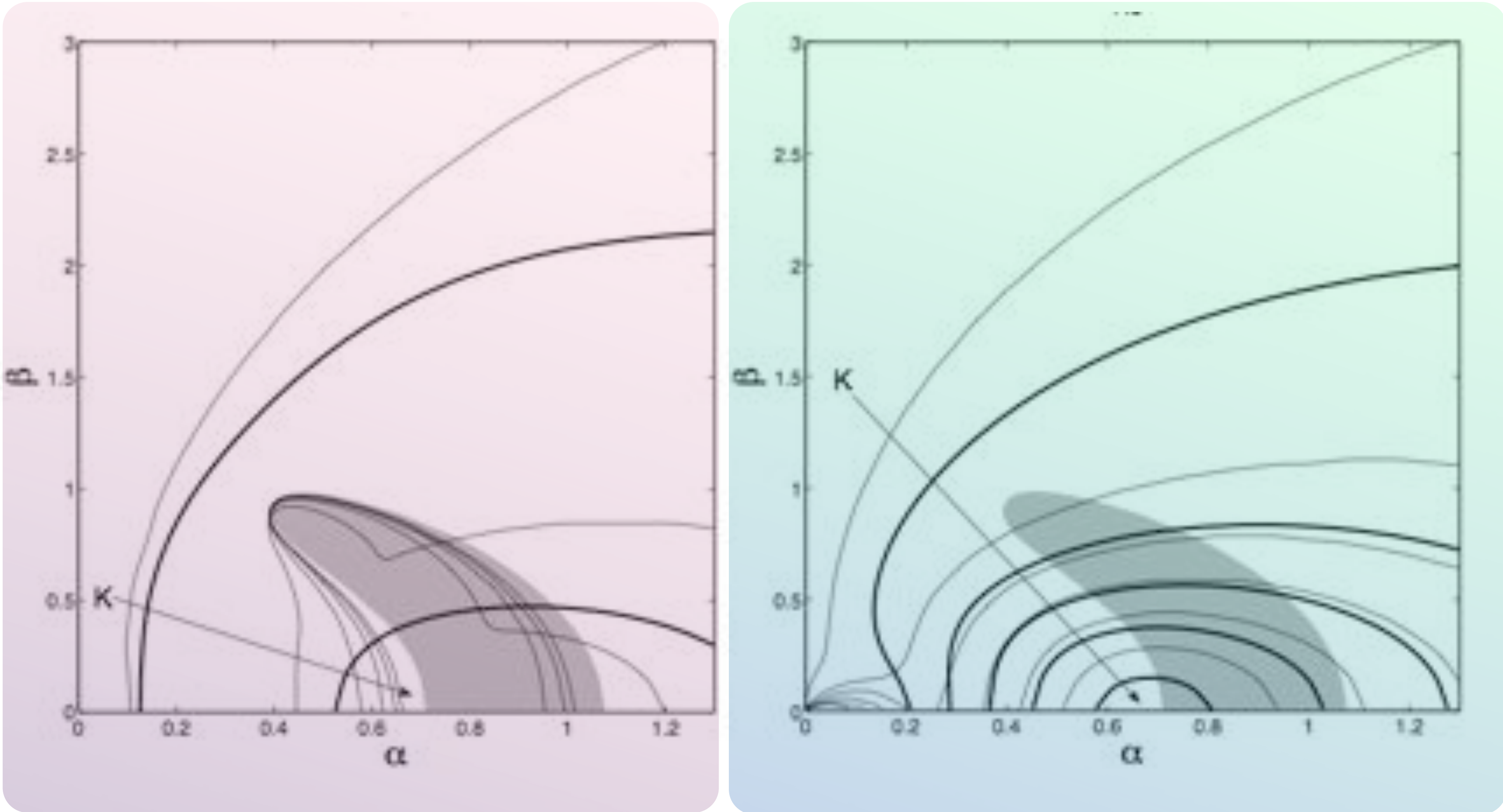
Exponential growth=100 at t=100

Neutral curves

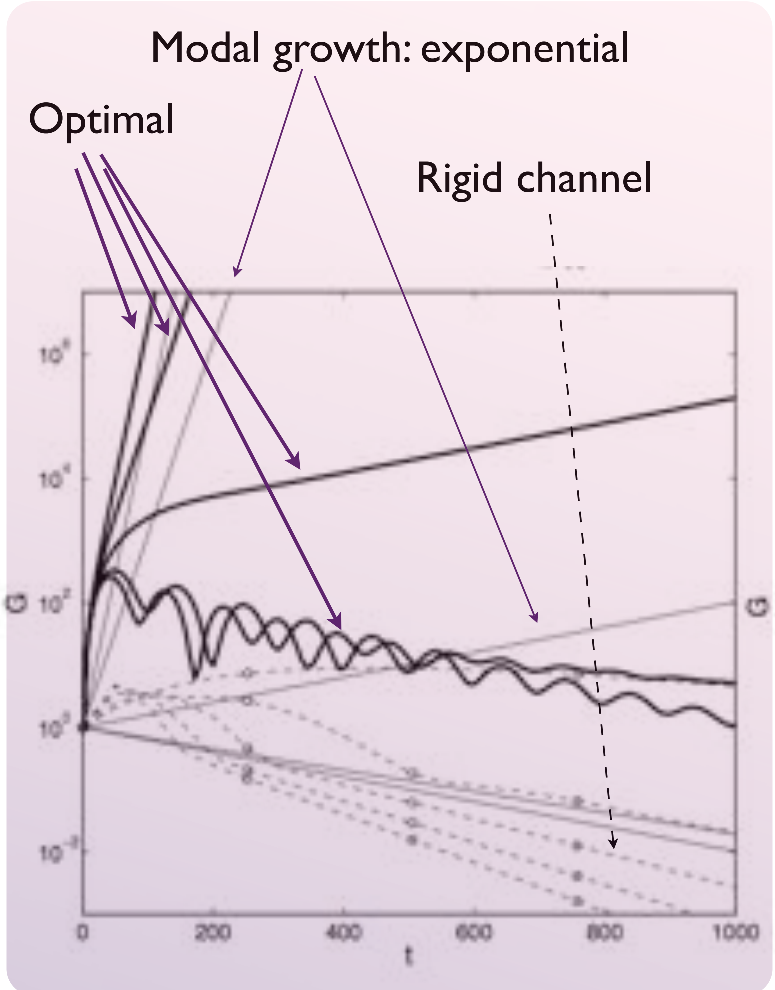


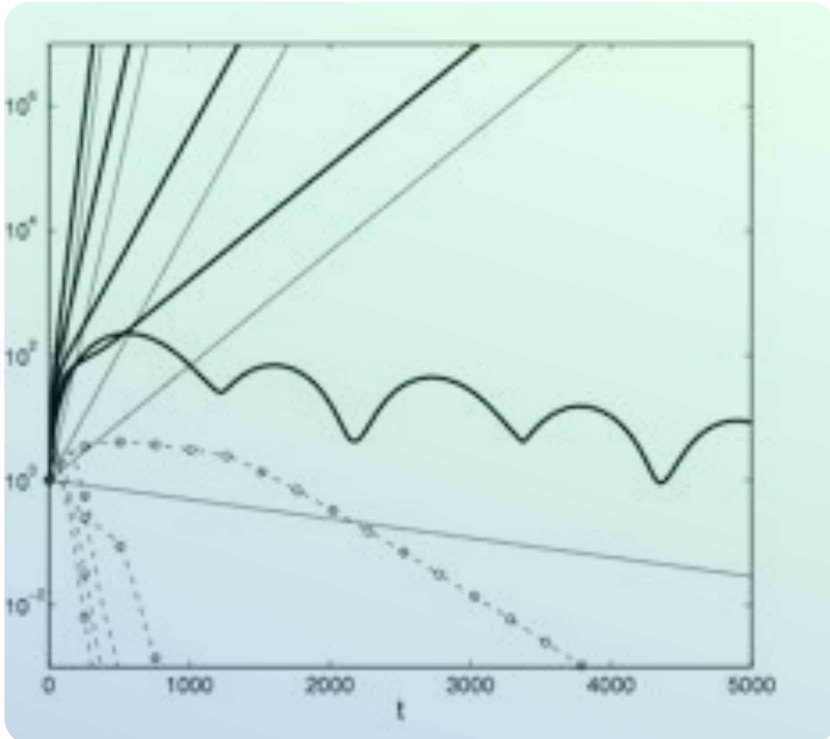
Modal instability, α/β

With Re=15000



Growth + Modal instability

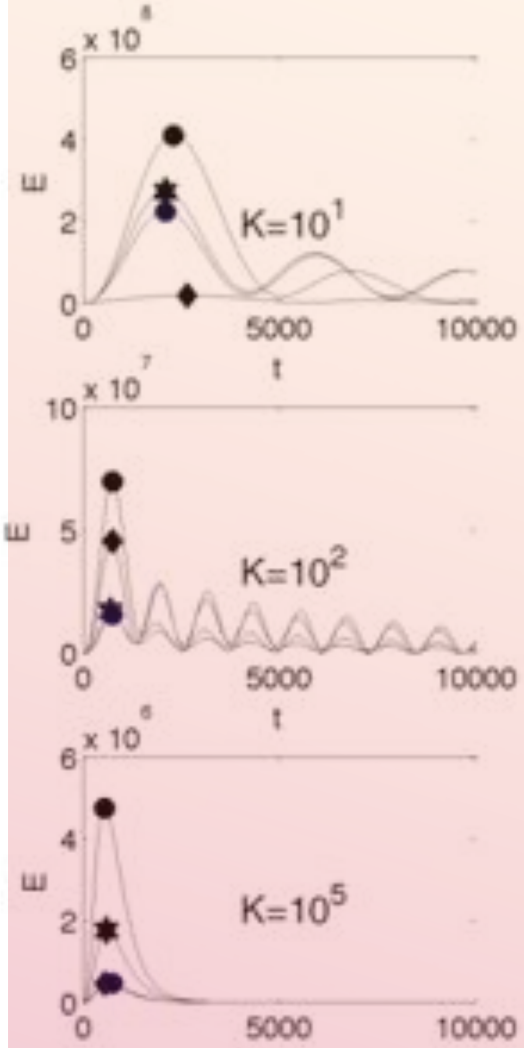


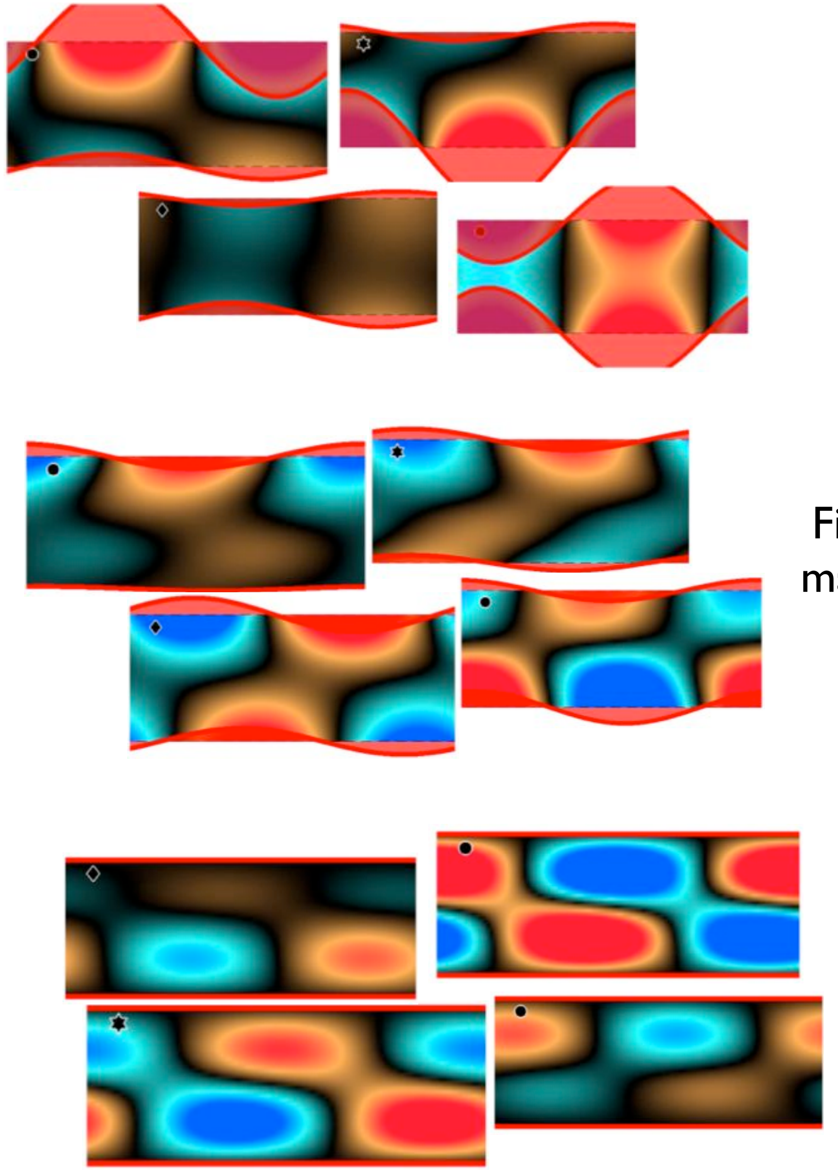


Even when the flow is unstable, mechanism is active to initially propel the modal instability.

From the random initial conditions

Energy evolution:





Fields at time of maximum energy

Extra slides

Growth derivation

where we have accounted for the kinetic and potential energy of both walls. The contribution of u' is

$$\int_y u'^2 \mathrm{d}y = \int_y \left(1 - (y-\eta)^2\right)^2 dy = \frac{16}{15} + \mathcal{O}(\eta^4)$$

The kinetic energy in u is thus constant in time up to order 4 in the wall displacement. Using $v = \eta_t = \epsilon\omega \sin(\omega t)$ we have

$$2E'(t) = \frac{16}{15} + 2\epsilon^2 \left[\omega^2 (m+1)\sin(\omega t)^2 + \frac{K}{Re^2}\cos(\omega t)^2\right] + \mathcal{O}(\epsilon^4)$$

This total energy should be conserved in time, thus the coefficients of the sinus and the cosinus should be equal. This leads to

$$\omega^2 = \frac{K}{Re^2(m+1)}$$

thus the added mass at infinite wavelength is 1 a discussed in §3. Turning now to the energy in the perturbation to the static Poiseuille profile $U = 1 - y^2$, we have

$$\int_y u \mathrm{d}y = \epsilon^2 \cos(\omega t) \int_y (2y)^2 \mathrm{d}y = \frac{8}{3}\epsilon^2 \cos(\omega t)$$

The energy evolution of the perturbation is thus

$$2E/\epsilon^2 = 2\left(\frac{4}{3} + \frac{K}{Re^2 m}\right)\cos(\omega t)^2 + 2\left(\omega^2(m+1)\right)\sin(\omega t)^2$$

which is the radius of an ellipse at an angle ω from its principal axis. Expressing ω as in (3.1), we obtain the energy growth along one fourth of the rotation period

$$G^s_{k=0} = 1 + \frac{4}{3}\frac{Re^2}{K}, \quad T^s_{k=0} = \frac{\pi Re}{2}\sqrt{\frac{m+1}{K}}$$