Turbulent Channel Flow Estimation

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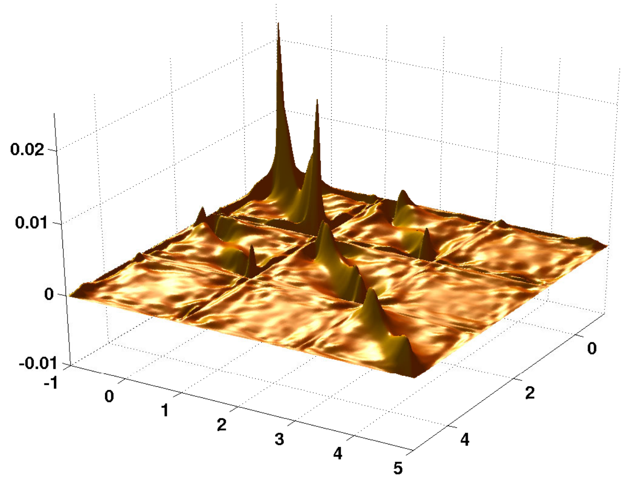

Related talk was given in Session AR.008 by Jérôme Hœpffner

KTH Mechanics

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Aim & Motivation

Estimation – State reconstruction from partial information

Many control strategies (such as LQR) are based on full state information, which may be estimated in the present (nonlinear) problem using an extended Kalman filter

Only partial information about the state is available – generally from wall sensors measuring skin friction and pressure

Though good progress has been made on feedback control of low Re turbulence using full state information (see, e.g., Högberg *et al*, Physics of Fluids 2003), much remains to be done to get better estimator performance

Present Aim

Improve estimation model by using statistical information from DNS

Estimation for a Single Wavenumber Pair

State

$$q = \begin{pmatrix} v \\ \eta \end{pmatrix}$$

Plant

$$\begin{cases} \dot{q} = Aq + Bf, \quad q(0) = q_0, \\ y = Cq + g, \end{cases}$$

Estimator

$$\begin{cases} \dot{\hat{q}} = A\hat{q} - L(y - \hat{y}), \quad \hat{q}(0) = \hat{q}_0, \\ \hat{y} = C\hat{q}, \end{cases}$$

The system is subject to the stochastic quantities

External disturbance f

Sensor noise g

Kalman Filter

The feedback L is optimized to minimize the estimation error, by solving an algebraic Riccati equation

$$0 = AP + PA^* + BRB^* - PC^*G^{-1}CP$$

$$L = -PC^HG^{-1}$$

To be modeled

R : Covariance of the external disturbance

Choice of Stochastic Forcing

Simple choice for the covariance – the identity matrix – doesn't work very well

Main idea: compute statistical quantities of neglected physics in the dynamical model

Linearize Navier–Stokes equations and add disturbance terms f_1, f_2, and f_3

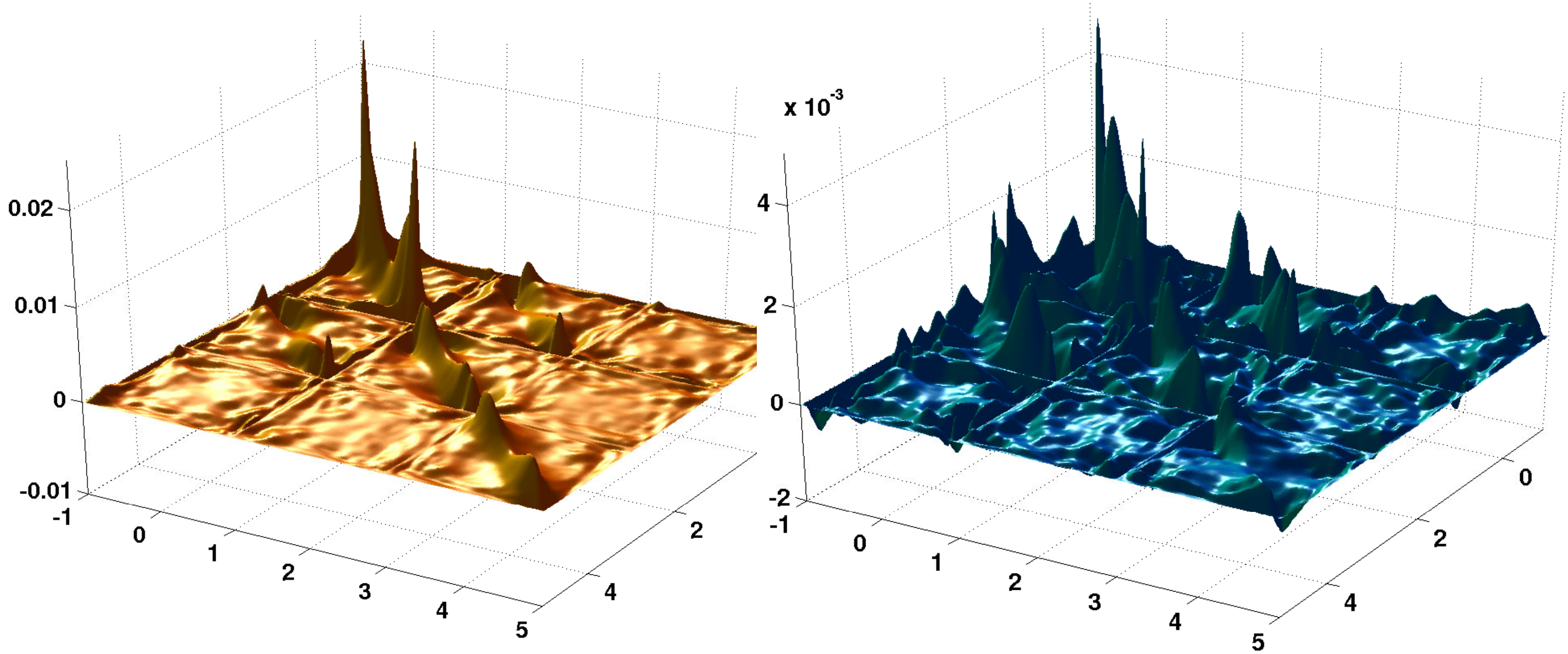
$$
\begin{aligned}
\frac{\partial \check{u}}{\partial t} + U\frac{\partial \check{u}}{\partial x} + \check{v}\frac{\partial U}{\partial y} &= -\frac{\partial \check{p}}{\partial x} + \frac{1}{Re}\Delta \check{u} - \check{u}\frac{\partial \check{u}}{\partial x} - \check{v}\frac{\partial \check{u}}{\partial y} - \check{w}\frac{\partial \check{u}}{\partial z} - \frac{\partial P}{\partial x} + \frac{1}{Re}\frac{\partial^2 U}{\partial y^2} \\
\frac{\partial \check{v}}{\partial t} + U\frac{\partial \check{v}}{\partial x} &= -\frac{\partial \check{p}}{\partial y} + \frac{1}{Re}\Delta \check{v} - \check{u}\frac{\partial \check{v}}{\partial x} - \check{v}\frac{\partial \check{v}}{\partial y} - \check{w}\frac{\partial \check{v}}{\partial z} \\
\frac{\partial \check{w}}{\partial t} + U\frac{\partial \check{w}}{\partial x} &= -\frac{\partial \check{p}}{\partial z} + \frac{1}{Re}\Delta \check{w} - \check{u}\frac{\partial \check{w}}{\partial x} - \check{v}\frac{\partial \check{w}}{\partial y} - \check{w}\frac{\partial \check{w}}{\partial z}
\end{aligned}
$$

Flow variables are divided into mean (U) and fluctuating ($\check{u}$, $\check{v}$, $\check{w}$, and $\check{p}$) part

We will model the relevant statistics (covariance R) of terms in green using DNS database

Covariance Data R

$$R_{ij}(k_x, y, y', k_z) = \langle \hat{\tilde{f}}_i(k_x, y, k_z, t)\, \hat{\tilde{f}}_j^*(k_x, y', k_z, t) \rangle$$

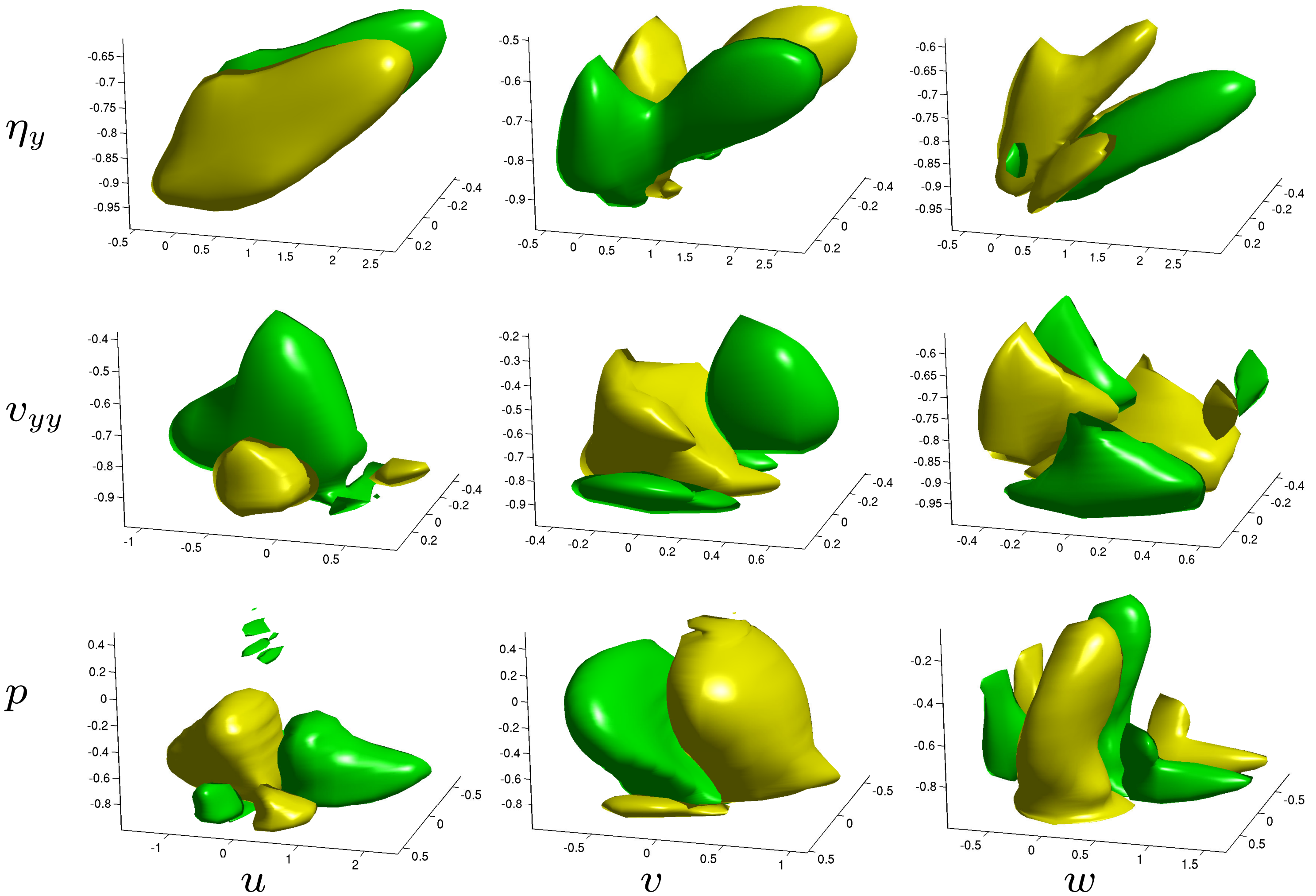


$k_x = 0.500$ and $k_z = 3.008$

Calculated for DNS of $Re_\tau = 100$ turbulent channel flow.

Steady State Kernels

Inverse Fourier transform of feedback L



Direct Numerical Simulations

Application of kernels based on covariance data

Two simulations run in parallel, plant and estimator

Wall measurements fed back as a volume force in estimator

Estimator also nonlinear equations, extended Kalman filter

DNS code

OPUS – incompressible Navier–Stokes equations solver

Constant-mass flux turbulent channel flow at $Re_\tau = 100$

Spectral / finite-difference / spectral discretization $(42 \times 64 \times 42)$

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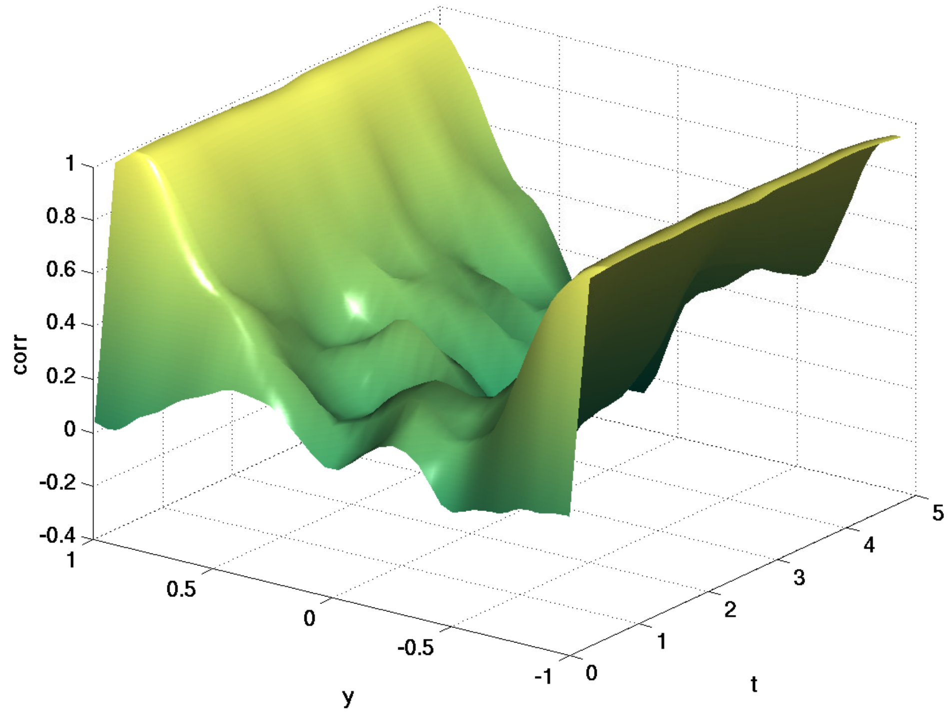
Plant / Estimator Correlation

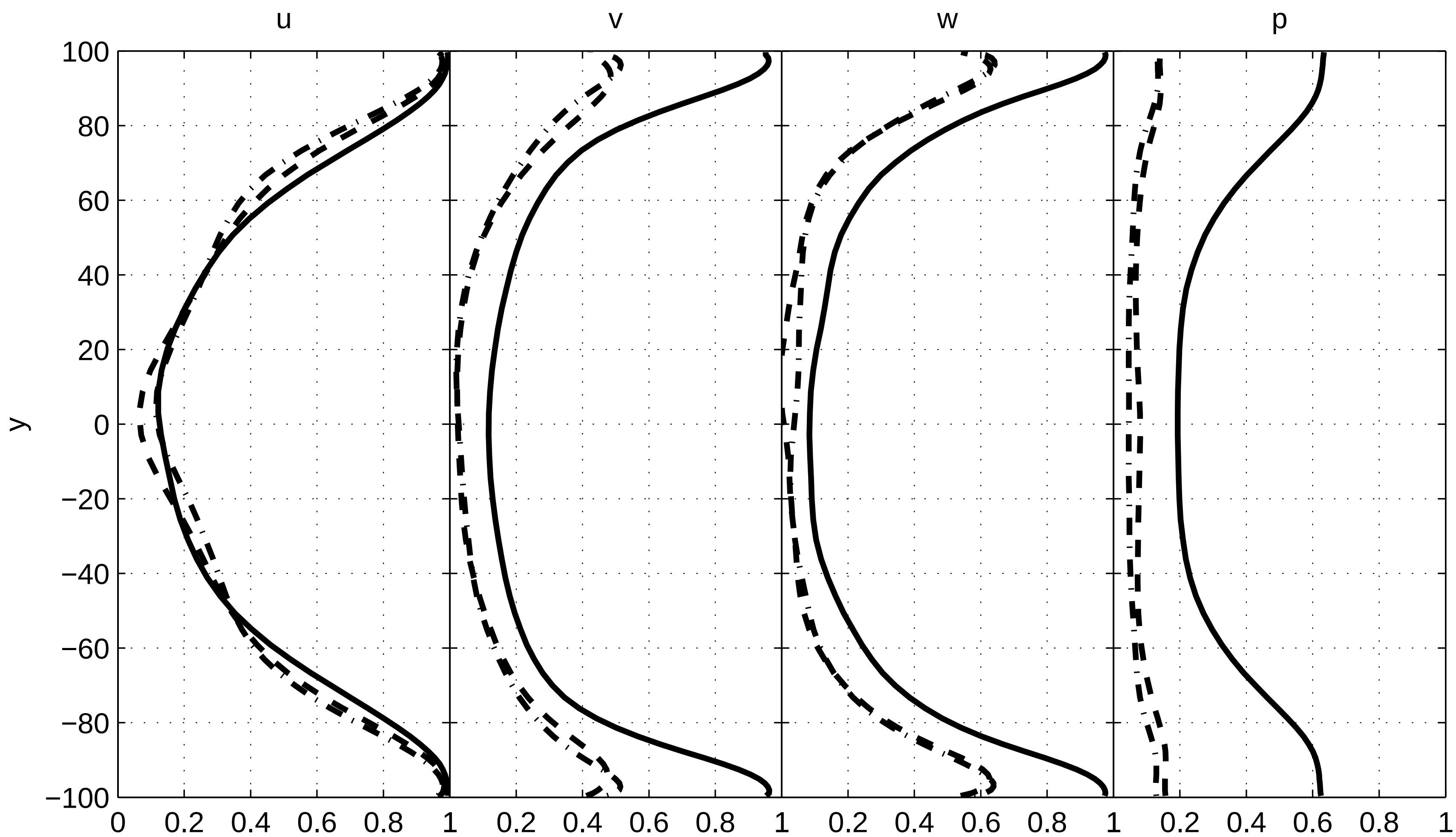
Correlation between state (u) and estimator ($\check{u}$) for the streamwise velocity

$$\mathrm{corr}_y(u,\check{u}) = \frac{\int_0^{Lx}\int_0^{Lz} u\check{u}\,\mathrm{d}x\mathrm{d}z}{\sqrt{\int_0^{Lx}\int_0^{Lz} u^2\,\mathrm{d}x\mathrm{d}z}\sqrt{\int_0^{Lx}\int_0^{Lz} \check{u}^2\,\mathrm{d}x\mathrm{d}z}}$$



Correlation for streamwise velocity component.

Plant / Estimator Correlation



Plant / estimator correlation of velocity and pressure in wall-normal direction.

Conclusions

Appropriate covariance data has been computed from a DNS database in order to improve reconstruction of a turbulent flow system

Estimation gains based on the covariance data has been computed

Well-behaved estimation kernels are obtained for three measurements

Extended Kalman filter for new gains gives improved estimator performance