Control of instabilities in a cavity-driven separated boundary-layer flow

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Boundary layer with cavity

2D flow over a smooth cavity
Inflow: Blasius profile

Long aspect ratio: 20
Reynolds number : 325
Investigation tools

Flow description: **DNS** to compute the base flow:
Chebyshev in wall normal, finite difference in streamwise.
**Stability analysis** by computation of 2D eigenmodes:
Chebyshev/Chebyshev and **Arnoldi**

From eigenmodes: **Optimal growth** by optimization **over initial conditions** :
Singular value decomposition
**Control optimization** by solution of two **Riccati equations**
The eigensolver

2D Navier-Stokes + continuity

\[
\begin{aligned}
-i\omega \hat{u} &= -(U \cdot \nabla) \hat{u} - (\hat{u} \cdot \nabla) U - \frac{\partial \hat{p}}{\partial x} + 1/Re \nabla^2 \hat{u} \\
-i\omega \hat{v} &= -(U \cdot \nabla) \hat{v} - (\hat{u} \cdot \nabla) V - \frac{\partial \hat{p}}{\partial y} + 1/Re \nabla^2 \hat{v} \\
0 &= \nabla \cdot \mathbf{u}
\end{aligned}
\]

Generalized eigenproblem:

\[ B \omega \mathbf{u} = A \mathbf{u} \]

To be rewritten

\[ A^{-1} B \mathbf{u} = \frac{1}{\omega} \mathbf{u} \]

Solved by Arnoldi iterations.

Matrix formulation:

\[
\begin{pmatrix}
-i\omega \hat{u} \\
-i\omega \hat{v} \\
0
\end{pmatrix}
= \begin{pmatrix}
\cdots & \cdots & -\frac{\partial}{\partial x} \\
\cdots & \cdots & -\frac{\partial}{\partial y} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & C
\end{pmatrix}
\begin{pmatrix}
\hat{u} \\
\hat{v} \\
p
\end{pmatrix}
\]

Pressure constraints \( C \)
Eigenmodes

Spectra

\[ \lambda_r \]

\[ \lambda_c \]

(m1) (m2) (m3) (m4) (m5) (m6)

Streamwise velocity, \( U \)
Normal velocity, \( V \)
Pressure
Adjoint streamwise velocity, \( U_a \)
Adjoint normal velocity, \( V_a \)
Optimal growth from initial conditions

System $x(t) = Ax$, $x(0) = x_0$, with solution

$$x(t) = e^{At}x_0$$

Find the initial condition $x_0$ maximizing

$$G(t) = \max_{x_0} \frac{<x(t), x(t)>}{<x_0, x_0>}$$
adjoint: $<Ax_1, x_2> = <x_1, A^+x_2> \forall x_1, x_2$

leads to

$$G(t) = \max \frac{<e^{At}x_0, e^{At}x_0>}{<x_0, x_0>} = \max \frac{<e^{A^+t}e^{At}x_0, x_0>}{<x_0, x_0>}$$

→ Maximum growth at time $t$: eigenvalue of $e^{A^+t}e^{At}$. 
Optimal growth in the cavity

- Global instability
- Potentiality of strong energy growth
- Low frequency cycle

Energy envelopes, number of modes from 1 to 260

Forcing/initial condition
Flow cycle

Generation of **global pressure change** when the wave-packet impacts on the downstream lip

Regeneration of disturbances when the pressure hits the upstream lip
Control

Seek to minimize the energy growth

- One actuator upstream
- One sensor downstream
- Oscillating disturbance in the shear layer
Feedback control

Dynamic model: \[
\begin{align*}
\dot{x} &= Ax + Bu \\
r &= Cx
\end{align*}
\]

Optimize for the feedback \[ u = \mathcal{G}(r) \]

- The model in 2D is too big for optimization \( \rightarrow \text{reduced model} \).
- For reduction: project the dynamics on the least stable eigenmodes.
- Finally, couple the reduced controller and the flow system

**Estimation:** estimate flow state from sensors.

**Control:** Actuate from feedback of estimated flow.
Control and estimation gains

Function used to **extract the actuation signal** from the flow

![Control gain, for U](image)

![Control gain, for V](image)

Function used to **force the estimator** flow

![Estimation gain, for u](image)

![Estimation gain, for v](image)
Compensation performance

Flow, compensated flow

Flow energy, Global cycle + exponential growth

Compensated flow

Good control performance from the second cycle

Reduced order model: 20 states
Flow

Flow, $V(y=4)$

Compensated flow, $V(y=4)$

Flow, pressure ($y=7$)

Compensated flow, pressure ($y=7$)

Actuator signal

Flow signal, $V$ in the vicinity

Signal, $V$, downstream lip

Compensated flow signal

Flow signal
Conclusion

Flow dynamics:
- Incompressible cavity can have global cycle due to pressure.

Global modes:
- Global eigenmodes can be used for analysis and model reduction.
- Convective instability well described by non-normality of global modes

Control:
- Model reduction allows optimal feedback design for large systems.
- Non-parallel effects/global instabilities can be treated.
**Grids & resolution**

The resolution are:

**DNS:**  $n_x=2048$ finite difference, $n_y=97$ Chebyshev, $L_x=409$, $L_y=80$

**EIG:**  $n_x=250$ Chebyshev, $n_y=50$ Chebyshev, $L_x=270$, $L_y=15$.

DNS grid vs eigenmode grid
Control terminology

- **Estimation:** From sensor information, recover the instantaneous flow field.

- **Full information control:** From full knowledge of the flow state, apply control.

- **Compensation:** Close the loop by using the estimated flow state for control.

- **Model reduction:** Project the dynamics on a set of selected basis vectors.

- **Control penalty:** Penalisation of the actuation amplitude.

- **Sensor noise:** Uncertainty in the measured signal.

- **Disturbances:** External forcing exciting the flow.

- **Objective function:** Function of the flow state to be minimized.
Model reduction

Galerkin projection on least stable eigenmodes:

Physical space:
\[
\begin{aligned}
\dot{x} &= Ax + Bu \\
\gamma &= Cx
\end{aligned}
\]

Eigenmode space:
\[
\begin{aligned}
P\dot{x}_k &= PAP^{-1}Px_k + PBu \\
r_k &= CPM^{-1}Px_k
\end{aligned}
\]

Projection on eigenmodes $\rightarrow$ **biorthogonal** set of vectors:

Eigenmodes: $q_i$,

Adjoint operator: $A^+ / <Ax_1, x_2> = <x_1, A^+x_2>$, $\forall x_1, x_2$

Adjoint eigenmodes: $q_i^+$,

Biorthogonality: $\delta_{ij} = <q_i, q_j^+>$, Projection: $k_i = <x, q_i^+>$
Starting the compensator at later times

Compensator cannot affect the disturbance propagation but can affect the disturbance generation.
Dynamic distortion

blue : flow
Red : compensated flow

Spectra with and without compensation