State estimation in wall-bounded flow systems

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Aim
Estimate the flow state in the channel from wall mounted sensors

By use of
linear control theory

This requires
modeling of the disturbances affecting the flow

The modeling of the disturbances affecting the flow is as important for the estimation problem as the objective function is for the control problem
Estimation for a single wavenumber pair

State vector \( q = \begin{pmatrix} v \\ \eta \end{pmatrix} \)

Plant
\[
\begin{align*}
\dot{q} &= Aq + Bf, \\
q(0) &= q_0, \\
y &= Cq + g,
\end{align*}
\]

Estimator
\[
\begin{align*}
\dot{\hat{q}} &= A\hat{q} - L(y - \hat{y}), \\
\hat{q}(0) &= 0, \\
\hat{y} &= C\hat{q},
\end{align*}
\]

The system is subject to the stochastic quantities:

Initial condition \( q_0 \),
external disturbances \( f \),
and sensor noise \( g \).
Kalman filter

The feedback $L(t)$ is optimized to minimize the estimation error, by marching in time a differential Riccati equation (DRE):

$$
\dot{P}(t) = AP(t) + P(t)A^* + BR_{ff}B^* - P(t)C^*G^{-1}CP(t), \quad P(0) = R_{q_0q_0},
$$

$$
L(t) = -P(t)C^*G^{-1}.
$$

To be modeled:

- $R_{ff}$ : Covariance of the external disturbance
- $R_{q_0q_0}$ : Covariance of the initial condition.
1) Model for the covariance $R_{ff}$ of the external disturbances $f$

$$R_{fjf_k}(y, y', r_x, r_z) = v_f \delta_{jk} M^x(r_x) M^z(r_z) M^y(y, y')$$

$$\begin{cases} 
M^x(r_x) = \frac{1}{(2\pi s_x)^{1/2}} e^{-r^2_x/2s_x}, \\
M^z(r_z) = \frac{1}{(2\pi s_z)^{1/2}} e^{-r^2_z/2s_z}, \\
M^y(y, y') = \frac{1}{(2\pi s_y)^{1/2}} e^{-(y-y')^2/2s_y}.
\end{cases}$$

Model parameters $v_f, s_x, s_z, s_y$
2) Model for covariance $R_{q_0q_0}$ of the initial conditions $q_0$

Assume two main components:

$$R_{q_0q_0} = \lambda_1 \left( \lambda_2 \underbrace{R_{ss}}_{\text{specific}} + (1 - \lambda_2) \sum_{j=1}^{p} R_{\xi_j\xi_j} \right)$$

$s$: TS waves, streamwise vortices ...

$\xi_j$: eigenmodes of OSS

Model parameters $\lambda_1(k_x, k_z), \lambda_2$

$\lambda_2 \in [0, 1]$ : how much we know about the statistics of the initial condition
Steady state kernels

Inverse Fourier transform of the feedback $L(t = \infty)$
Time varying kernels for $\tau_x$

Inverse Fourier transform of the feedback $L(t)$
Red: estimation error using time varying gains.

This demonstrates how an accurate estimate of the assumed statistics of the initial conditions ($\lambda_2$) can greatly improve the estimator behavior.
Flow evolution

flow     estimated flow

\[ t = 0 \]

\[ t = 20 \]

\[ t = 60 \]
Conclusion

**External disturbances**

A physically relevant stochastic model allows well-behaved kernels for all independent measurements at the wall: $\tau_x$, $\tau_z$, and $p$.

**Initial conditions**

Stochastic description of the initial conditions leads to a time-varying estimation law. It improves the estimation of transitional flows.

A careful modeling of the stochastic disturbances affecting the flow is key to a better estimation performance.
**Related paper:** State estimation in wall bounded flow systems, J. Høepffner, M. Chevalier, T. R. Bewley and D. S. Henningson, Submitted to JFM

**Related talk:** Mattias chevalier, Turbulent flow estimation
Monday afternoon, session JA, 15:20