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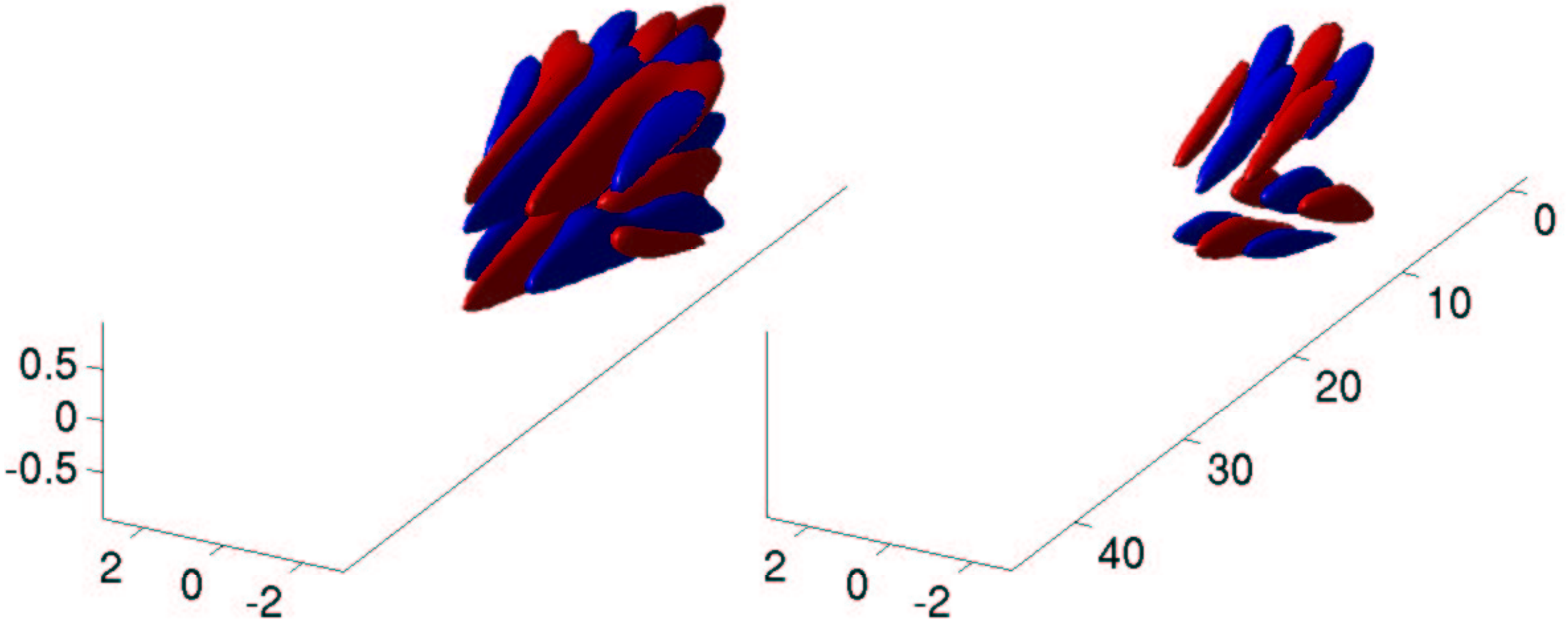
State estimation in wall-bounded flow systems



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In cooperation with

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Aim

Estimate the flow state in the channel from wall mounted sensors

By use of

linear control theory

This requires

modeling of the disturbances affecting the flow

The modeling of the disturbances affecting the flow is as important for the estimation problem as the objective function is for the control problem

Estimation for a single wavenumber pair

state vector $q = \begin{pmatrix} v \\ \eta \end{pmatrix}$

$$\text{Plant} \begin{cases} \dot{q} = Aq + Bf, \quad q(0) = q_0, \\ y = Cq + g, \end{cases}$$

$$\text{Estimator} \begin{cases} \dot{\hat{q}} = A\hat{q} - L(y - \hat{y}), \quad \hat{q}(0) = 0, \\ \hat{y} = C\hat{q}, \end{cases}$$

The system is subject to the stochastic quantities:

Inital condition q_0,

external disturbances f,

and sensor noise g.

Kalman filter

The feedback $L(t)$ is optimized to minimize the estimation error, by marching in time a differential Riccati equation (DRE) :

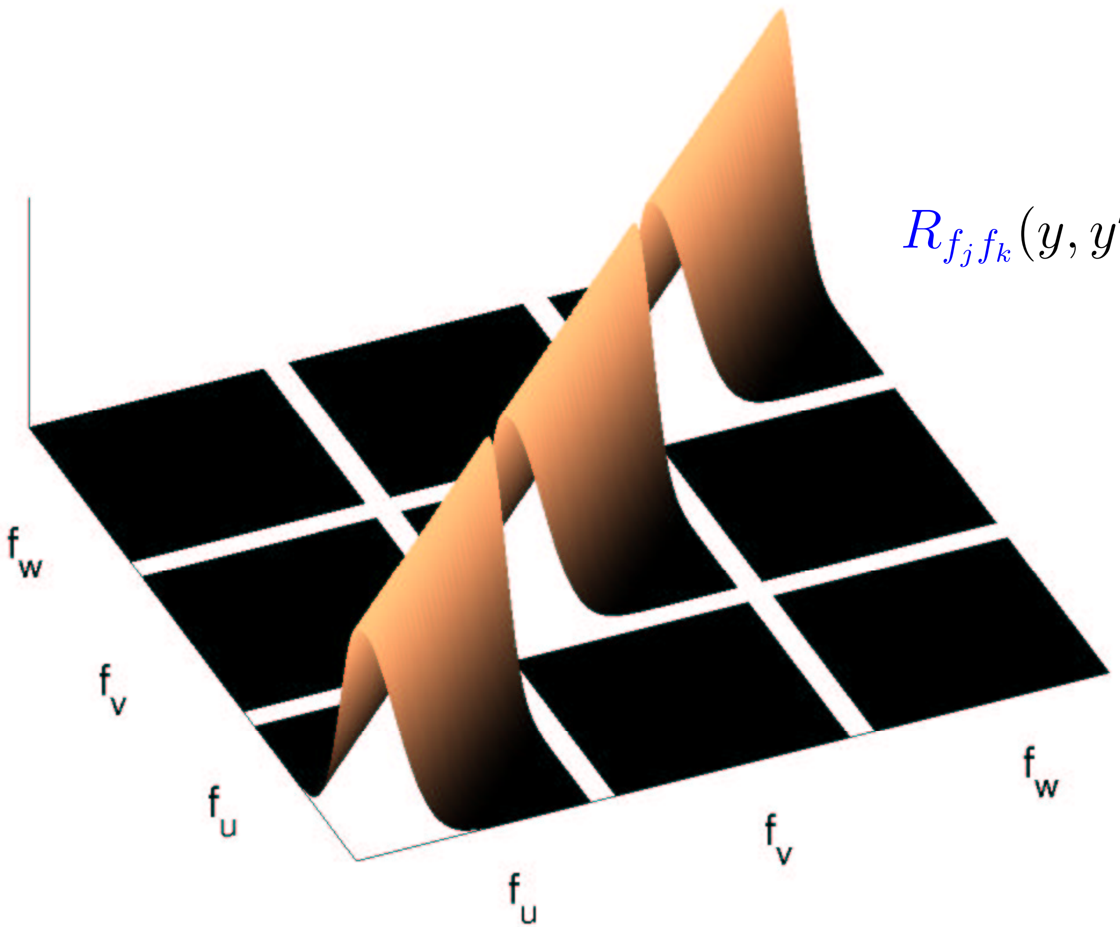
$$\dot{P}(t) = AP(t) + P(t)A^* + BR_{ff}B^* - P(t)C^*G^{-1}CP(t), \quad P(0) = R_{q_0q_0},$$
$$L(t) = -P(t)C^*G^{-1}.$$

To be modeled:

R_{ff} : Covariance of the external disturbance

$R_{q_0q_0}$: Covariance of the initial condition.

1) Model for the covariance R_{ff} of the external disturbances f



$\delta_{jk}\, \mathcal{M}^y(y, y')$

$$R_{f_j f_k}(y, y', r_x, r_z) = v_f\, \delta_{jk}\, \mathcal{M}^x(r_x)\mathcal{M}^z(r_z)\mathcal{M}^y(y, y')$$

$$\begin{cases} \mathcal{M}^x(r_x) = \dfrac{1}{(2\pi s_x)^{1/2}} \mathrm{e}^{-r_x^2/2s_x}, \\ \mathcal{M}^z(r_z) = \dfrac{1}{(2\pi s_z)^{1/2}} \mathrm{e}^{-r_z^2/2s_z}, \\ \mathcal{M}^y(y, y') = \dfrac{1}{(2\pi s_y)^{1/2}} \mathrm{e}^{-(y-y')^2/2s_y}. \end{cases}$$

Model parameters v_f, s_x, s_z, s_y

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2) Model for covariance $R_{q_0q_0}$ of the initial conditions q_0

Assume two main components:

$$R_{q_0q_0} = \lambda_1 \left(\lambda_2 \underbrace{R_{ss}}_{\text{specific}} + (1 - \lambda_2) \underbrace{\sum_{j=1}^{p} R_{\xi^j\xi^j}}_{\text{Random}} \right)$$

s : TS waves, streamwise vortices ...

ξ^j : eigenmodes of OSS

Model parameters $\lambda_1(k_x, k_z)$, λ_2

$\lambda_2 \in [0, 1]$: how much we know about the statistics of the initial condition

Steady state kernels

Inverse Fourier transform of the feedback $L(t = \infty)$

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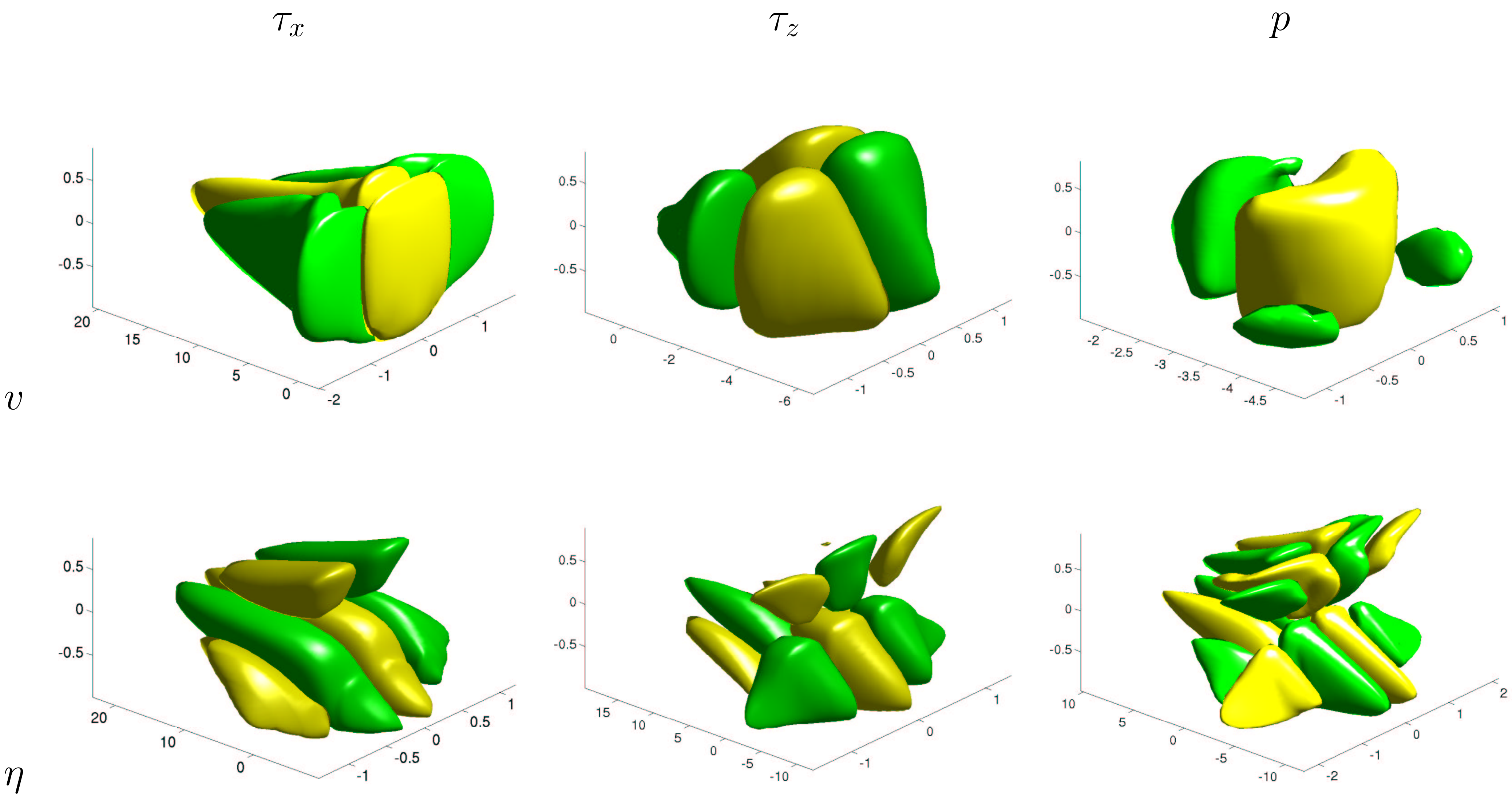
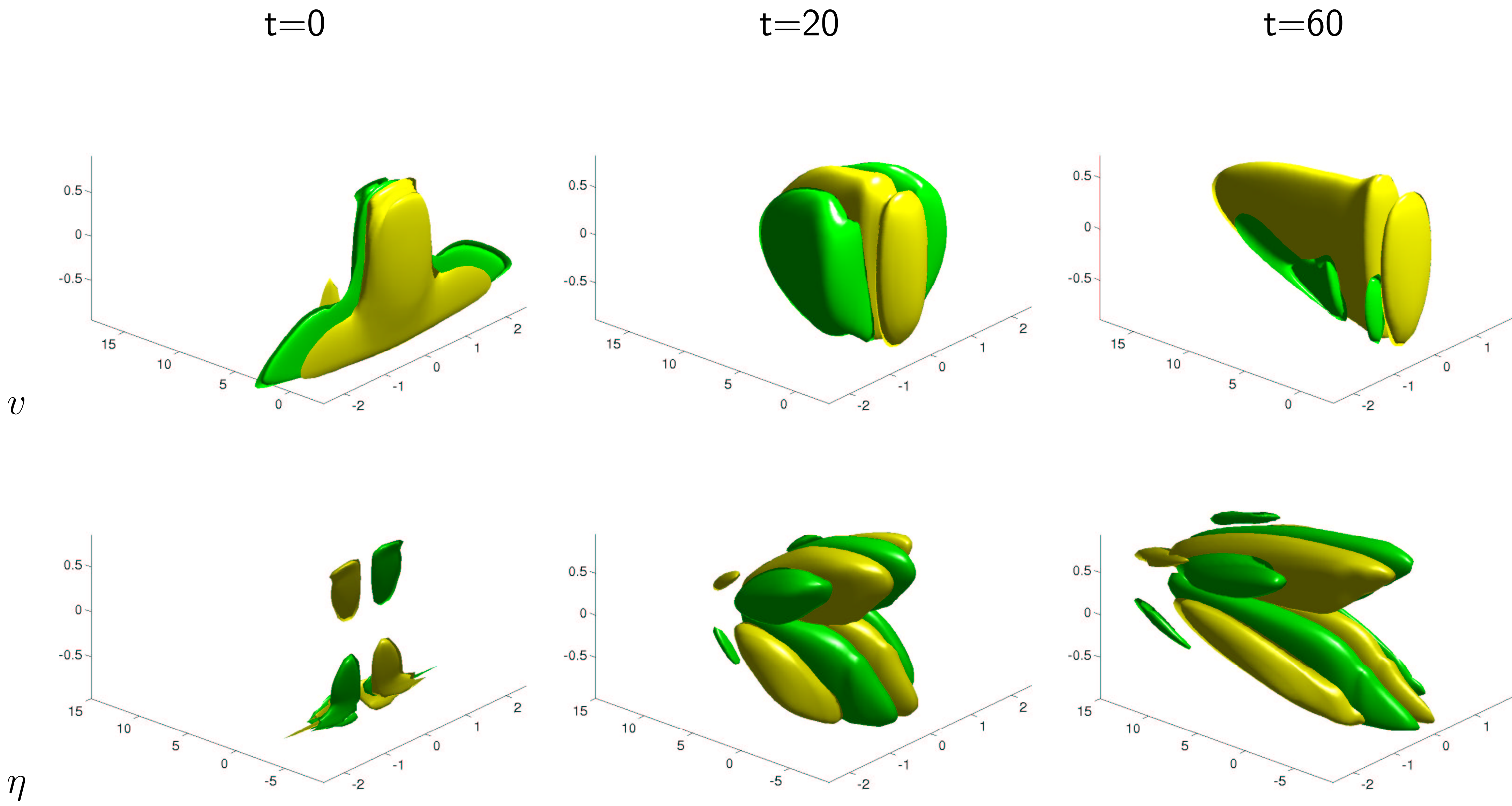


Time varying kernels for τ_x

Inverse Fourier transform of the feedback $L(t)$



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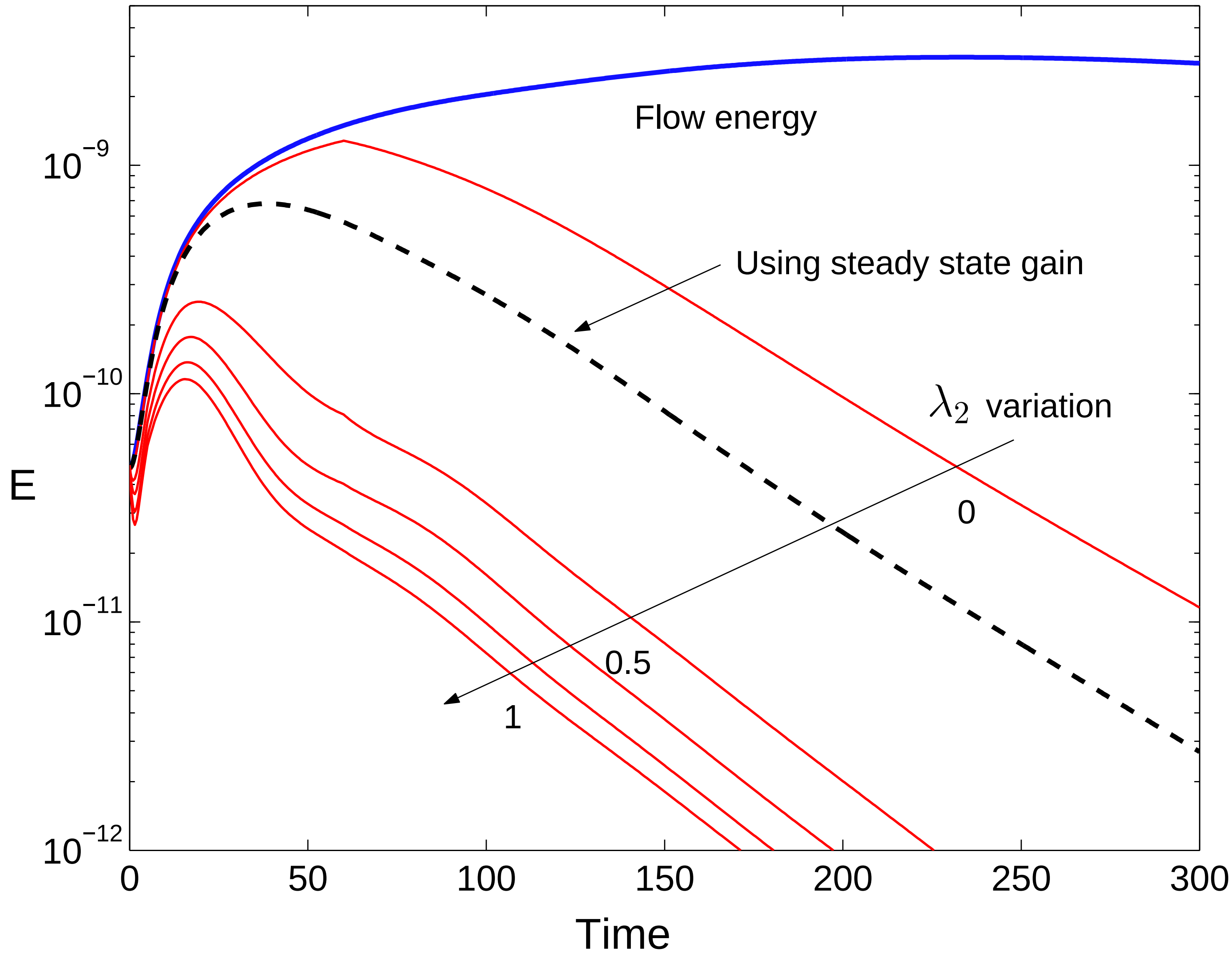
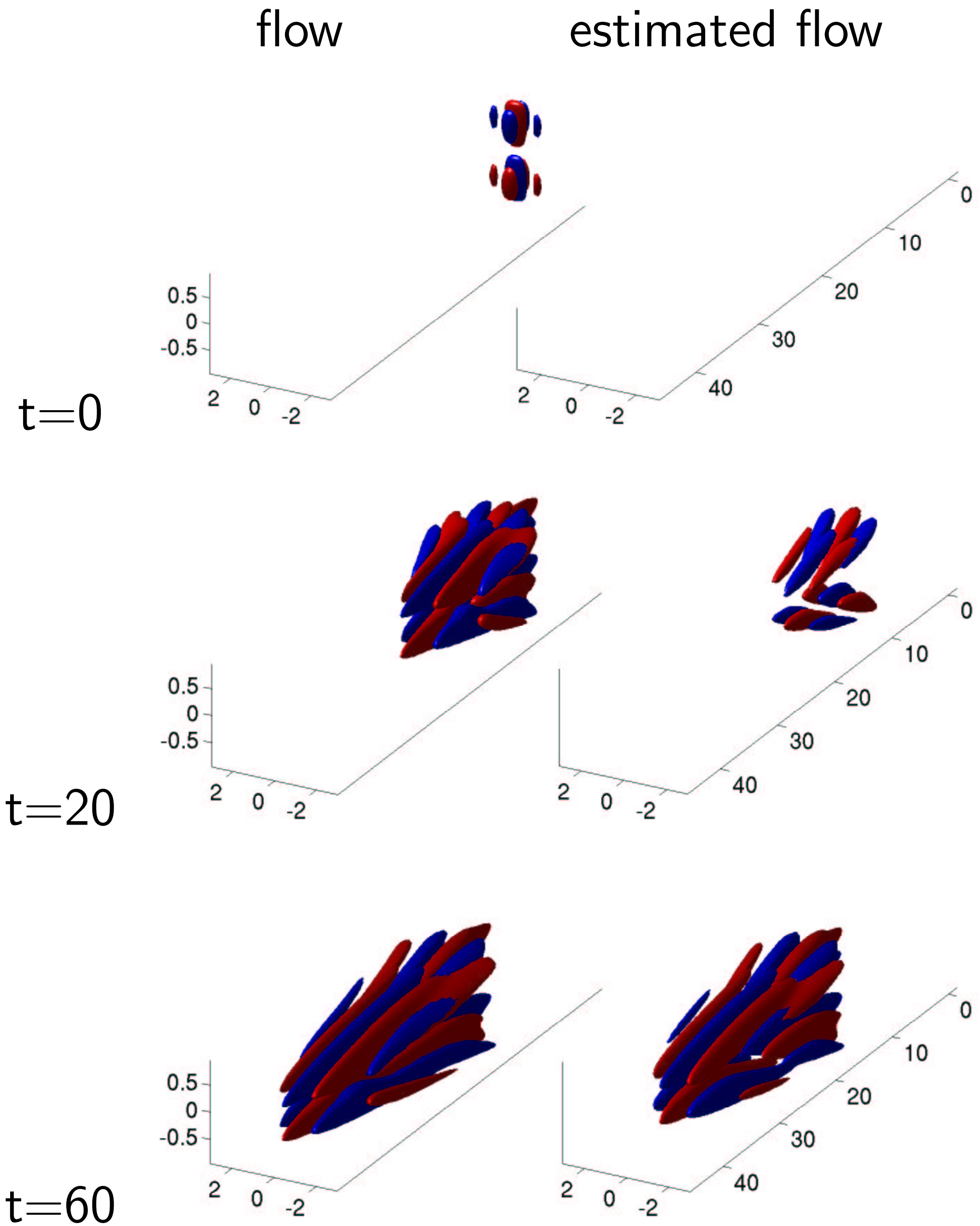
Estimation performance

Red : estimation error using time varying gains.

This demonstrates how an accurate estimate of the assumed statistics of the initial conditions (λ_2) can greatly improve the estimator behavior



Flow evolution



Conclusion

External disturbances

A physically relevant stochastic model allow well behaved kernels for all independent measurements at the wall: τ_x, τ_z, and p.

Initial conditions

Stochastic description of the initial conditions leads to a time varying estimation law. It improves the estimation of transitional flows.

A careful modeling of the stochastic disturbances affecting the flow is key to a better estimation performance.

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Related paper: State estimation in wall bounded flow systems, J. Hœpffner, M. Chevalier, T. R. Bewley and D. S. Henningson, Submitted to JFM

Related talk: Mattias chevalier, Turbulent flow estimation Monday afternoon, session JA,15:20