Asymptotic Study of the Interfacial Crack with Friction

B. Audoly

Laboratoire de Physique Statistique de l'École Normale Supérieure, associé au CNRS, 24 rue Lhomond, 75231 Paris Cedex 05, France.

Abstract

We study a model of interface crack in which contact of the crack faces obeys a Coulomb law of friction. For such cracks, the possibility that the stress has a stronger singularity than $r^{-1/2}$ near the tip has been reported. In this paper, we demonstrate that these strong singularities can in fact be discarded, because they would suppose a backward propagation of the crack. In passing, we prove that near-tip slip is possible in one direction only, which is imposed by the sign of the elastic mismatch. The locking of the stress intensity factor during a non-monotonic cycle of loading is pointed out, as well as the formation of a bubble near the tip under certain loading conditions.

Key words: A. fracture; B. friction; B. layered material; A. stress intensity factor.

1 Introduction

The understanding of the mechanical properties of layered materials is a challenge with many potential applications. These materials are nowadays widely used in the industry, and their structural performances are limited by a variety of mechanisms. Among them is the propagation of interfacial cracks, to which we restrict our attention. As for cracks in homogeneous media, the propagation of interface cracks is believed to depend on the asymptotic expansion of the stress near the crack tip. In this paper, we study the structure of the singularity of the stress at the tip.

We consider a crack propagating quasi-statically at the interface between dissimilar materials. Because of the elastic mismatch of the materials joining

[1] Basile.Audoly@lps.ens.fr

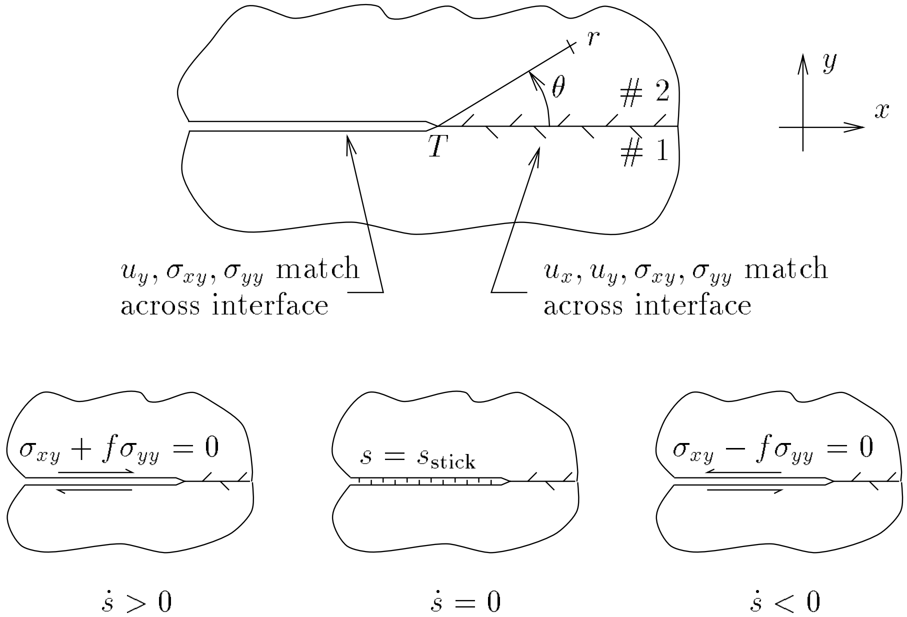

Fig. 1. Asymptotic problem near the tip of a interface crack with friction. Slip can occur in either direction, and stick is also possible.

at the interface, a contact region behind the crack tip must be considered (Comninou and Dundurs, 1980b). This contact region avoids inconsistencies in the model, such as the overlap of the crack faces in the form of microscopic oscillations (Rice, 1988). Moreover, this contact zone is macroscopic under certain loading conditions, and its role in the debonding of the interface has been pointed out by Stringfellow and Freund (1993). The tip is surrounded by the bound interface on one side, and by a contact zone on the other side (see figure 1).

We neglect transverse (mode *III*) forces, and the equations of bidimensional elasticity are used. Let (x, y) be the coordinate in the materials, T the crack tip with coordinates $(x_{\text{tip}}, 0)$, (u_x, u_y) the displacement vector, $\sigma_{\alpha\beta}$ the bidimensional stress tensor. Index 1 and 2 label the materials. We define the relative shift of the materials along the contact region, $s(x) = u_x^2(x, 0) - u_x^1(x, 0)$ for $x < x_{\text{tip}}$. The elastic mismatch between the materials is taken into account, and we introduce the second Dundurs mismatch parameter, β, defined as:

$$\beta = \frac{\mu_2(\kappa_1 - 1) - \mu_1(\kappa_2 - 1)}{\mu_2(\kappa_1 + 1) + \mu_1(\kappa_2 + 1)}, \tag{1}$$

where μ, ν denote respectively the shear modulus and the Poisson ratio of the materials, and $\kappa = (3-4\nu)$ for plane strain, and $(3-\nu)/(1+\nu)$ for plane stress. We assume that the materials have mismatching elastic properties: $\beta \neq 0$.

Following Comninou and Dundurs (1977), we model the contact of the crack

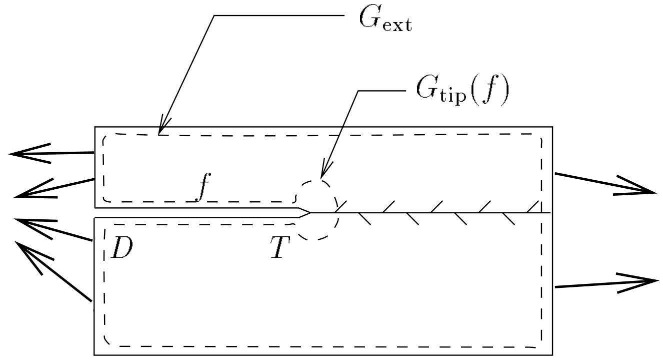

Fig. 2. Path of integration used to calculate the dissipation of the energy release rate, G, for a fixed, arbitrary, external loading (bold arrows).

faces by a Coulomb law of friction. The corresponding boundary conditions are shown in figure 1. In the contact zone, three regimes are possible: the crack faces can slip in either direction, or stick. One of the boundary conditions changes accordingly. Since we make an asymptotic analysis near the crack tip, we shall not be concerned with the possibility that several stick and slip zones coexist along the interface (Comninou and Dundurs, 1979): we only consider the region touching the crack tip.

Before mentioning important results obtained by Comninou and Dundurs for this model of crack, we present simple arguments showing that interfacial friction near the crack tip yields singular effects. Our point is to show that, because of friction, our model cannot be approached by conventional crack analysis.

1.1 Interfacial friction as a singular perturbation

We consider a bimaterial with a partially cracked interface. A fixed, arbitrary, external loading is applied on the sample. For sake of simplicity, we assume that the interface is fully closed, and that slip takes place near the tip (this assumption is not essential). We study the dependence of the energy release rate (Malyshev and Salganik, 1965) at the tip, $G_{\rm tip}$, on the friction coefficient, f, for this fixed external loading (see figure 2). We make a perturbation analysis, and we seek and expansion of $G_{\rm tip}(f)$ in powers of f. At zeroth order, there is no dissipation ($f = 0$); by conservation of the Rice integral (Rice, 1968a–b), the energy release rate at the tip is then given by the external loading: $G_{\rm tip}^{[0]} = G_{\rm ext}$. When friction is turned on, the energy release rate becomes contour dependent. By a straightforward use of the Rice integral on the contour depicted in figure 2, the frictional dissipation of the energy release rate in the contact zone $[DT]$ reads:

$$G_{\text{ext}} - G_{\text{tip}} = \int_{[DT]} (-\sigma_{xy}) \frac{\partial s(x)}{\partial x} \, |dx|, \tag{2}$$

where we remind that s is the relative shift across the interface. By the Coulomb law of friction, the contact shear stress is proportional to the contact pressure: $\sigma_{xy} = \pm f \sigma_{yy}$, with a sign that depends on the direction of slip. The previous equation therefore leads to the first order expansion of $G_{\text{tip}}(f) = G_{\text{tip}}^{[0]} + f \, G_{\text{tip}}^{[1]} + \ldots$ with:

$$G_{\text{tip}}^{[1]} = \pm \int_{[DT]} \sigma_{yy}^{[0]}(x) \frac{\partial s^{[0]}(x)}{\partial x} \, |dx|, \tag{3}$$

where the quantities appearing in the r.h.s. member must be evaluated in the absence of friction.

We now perform a dimensional analysis of the integrand: let $r = |x - x_{\text{tip}}|$ be the distance to the tip along the interface. In the absence of friction, conservation of the J-integral near the tip imposes the scaling law: $\sigma_{yy}^{[0]}(r) \propto r^{-1/2}$, from which $s^{[0]}(r) \propto r^{1/2}$ can be derived. The integrand in the last equation therefore scales like $(x - x_{\text{tip}})^{-1}$ near the tip, and the integral (3) giving $G_{\text{tip}}^{[1]}$ diverges logarithmically near the tip: the expansion of the energy release rate at the tip in powers of f is singular. This result indicates that interfacial friction can have strong effects near the crack tip. Indeed, the conventional crack theory does not apply to our problem: we shall see below that the stress divergence near the crack tip does not satisfy the usual scaling law: $\sigma \propto r^{-1/2}$.

1.2 Expansions obtained by Comninou and Dundurs

We now turn to the direct study of the stress singularity near the tip when interfacial friction is considered, and no longer consider expansions in small f. We present results obtained by Comninou and Dundurs (1979).

Comninou and Dundurs have solved the static linear 2D elasticity equations near the tip of a closed interfacial crack; they consider slip of the crack faces against each other. They obtain a full expansion for the near-tip stress in the materials, the leading term of which reads (Comninou and Dundurs, 1977, 1980b):

$$\sigma_{ij}(r, \theta) = C \, \Sigma_{ij}(\theta) \, r^{-\lambda}, \tag{4}$$

where (r, θ) are polar coordinates with origin T, $\Sigma_{ij}(\theta)$ are universal functions of θ given by COMNINOU and DUNDURS (1980b). C is a stress intensity factor which depends on the external loading. As shown in figure 1, the coefficient of friction, $f > 0$, comes in the equations with a sign that depends on the direction of slip, $(\operatorname{sgn} \dot{s})$, and they indeed propose that the exponent λ should be determined by:

$$\cot \lambda\pi = \operatorname{sgn}(\dot{s})\beta f \quad \text{with } 0 < \lambda < 1. \tag{5}$$

For $\lambda \geq 1$, the elastic energy stored near the crack tip would be infinite: the proposed range for λ, $0 < \lambda < 1$, therefore corresponds to the most diverging term in the expansion compatible with a finite energy. The fact that λ is generically different from $\frac{1}{2}$ can be understood in the light of §1.1.

The relationship between the quantities s and λ is complex. On one hand, λ obviously depends on the direction of slip, $(\operatorname{sgn} \dot{s})$, through equation (5). On the other hand, from equation (4) or [(3.18), DUNDURS and COMNINOU, 1979], the near-tip expansion of s has a leading term $s \propto (x_{\text{tip}} - x)^{1-\lambda}$; therefore, s and $(\operatorname{sgn} \dot{s})$ in turn depend on λ. The quantities $(\operatorname{sgn} \dot{s})$ and λ are thus self-referencing. As a result, it is possible that $(\operatorname{sgn} \dot{s})$ and λ depend not only on the current loading, but also on the crack history: they should be determined by successive attempts to make the equations self-consistent (COMNINOU and DUNDURS, 1980b; DENG, 1994). For certain loading histories of the interfacial crack, it has been observed (DUNDURS and COMNINOU, 1979) that the singularity of the stress is smoother than $r^{-1/2}$ ($\lambda \leq \frac{1}{2}$). On the other hand, the possibility that $\frac{1}{2} < \lambda < 1$ in equation (5) is problematic (p. 79, DUNDURS and COMNINOU, 1979); this eventuality, although not exemplified so far, has not been shown inconsistent either.

2 Singularity of the stress near the tip

In this section, we present new results about the singularity of the stress in the presence of friction. First, we prove that λ in fact cannot be larger than $\frac{1}{2}$. This is consistent with the intuition that friction tends to make the stress *less* divergent near the crack tip (as noted above, $\lambda = \frac{1}{2}$ in the absence of friction); moreover, infinite energy flow towards the crack tip, which would occur for $\lambda > \frac{1}{2}$, are removed from the theory. Secondly, we show that λ is actually *independent* of the loading history, and that equation (5) can be replaced by a simpler one:

$$\cot \lambda\pi = |\beta| f \quad \text{with } 0 < \lambda < \frac{1}{2}. \tag{6}$$

We shall note λ_0 the unique solution of this equation, which is now history independent: λ_0 is a function of the materials constants only. Thanks to this result, the asymptotic study of a closed crack will be greatly simplified because equations (4) and (6) are no longer self-referencing: λ, as determined from equation (6), should simply be put into equation (4). The location of stick and slip zones along the interface, and the stress intensity factor C, however, remain history dependent.

2.1 The stress is less singular than $r^{-\frac{1}{2}}$

If the lips are slipping ($\dot{s}(x_{\text{tip}}^-) \neq 0$), the expansion of the stress near the tip of a closed crack reads (Dundurs and Comninou, 1979):

$$\sigma_{ij}(r,\theta) = C_{1a}\Sigma_{ij}^{1a}(\theta)r^{-1+\lambda_0} + C_{1b}\Sigma_{ij}^{1b}(\theta)r^{-\lambda_0} + C_{1c}\Sigma_{ij}^{1c}(\theta)r^0 + \\ C_{2a}\Sigma_{ij}^{2a}(\theta)r^{+\lambda_0} + C_{2b}\Sigma_{ij}^{2b}(\theta)r^{1-\lambda_0} + C_{2c}\Sigma_{ij}^{2c}(\theta)r^1 + \ldots \tag{7}$$

This is the same expansion as in equation (4) with terms added beyond the dominant order. We also have used a trick to suppress artificially the time-dependence of the exponent in equation (4): from equation (5), the dominant exponent for the stress is $\lambda = \lambda_0$ or $1 - \lambda_0$, depending on $\operatorname{sgn}(\beta\,\dot{s})$. Both expansions have been written in the above equation, with the convention that one of these vanishes:

$$\text{if } \beta\,\dot{s} > 0 \text{ at } x = (x_{\text{tip}})^-, \quad C_{1a} = C_{2a} = \ldots = 0 \tag{8-a}$$

$$\text{if } \beta\,\dot{s} < 0 \text{ at } x = (x_{\text{tip}})^-, \quad C_{1b} = C_{2b} = \ldots = 0. \tag{8-b}$$

This convention expresses equation (5).

In the following, it will be convenient to introduce the index d, which we define as b if $\beta\,\dot{s} > 0$ or a if $\beta\,\dot{s} < 0$: among the a and b terms in equation (7), only the d terms are non vanishing. We also define $\lambda_{1a} = 1 - \lambda_0$ and $\lambda_{1b} = \lambda_0$ for obvious reasons. $\lambda_d = \lambda_{1a}$ or λ_{1b} is in the range $0 < \lambda_d < 1$.

As explained above, Comninou and Dundurs have found solutions to the equations of linear elasticity near the crack tip, T. In particular, they have given the asymptotic expression for the relative shift, s, and for the normal stress at the interface, N, in the slip zone [(3.18–19), Dundurs and Comninou, 1979]:

$$s(x,t) = -\frac{1}{\mu^*}\,C_{1d}\,(x_{\text{tip}} - x)^{1-\lambda_d}\,\sin\lambda_0\pi + \ldots \tag{9-a}$$

$$N(x,t) = \beta\,C_{1d}\,(x_{\text{tip}} - x)^{-\lambda_d}\,\sin\lambda_0\pi + \ldots, \tag{9-b}$$

where $\mu^*(\mu_2, \mu_1, \nu_2, \nu_1, \lambda)$ is a combination of the elastic modulus of the materials and β is the mismatch parameter defined in (1).

Consistency requires that the normal stress along the slipping part of the interface, $N(x) = \sigma_{yy}(x, y = 0)$, be compressive ($N(x) \leq 0$), so that the lips are pressed one against the other. This yields [(3.40), Dundurs and Comninou, 1979]:

$$\beta\, C_{1d} \leq 0. \tag{10}$$

We shall consider that this condition is automatically satisfied. Indeed, a tensile contact pressure $N > 0$ would indicate that the crack is open near the tip, which, again, is not possible when $\beta \neq 0$. $N \leq 0$ is in fact guaranteed by the tuning of the size of the contact region with the applied loading (Audoly, 1999).

We consider the possibility that the crack advances quasi-statically: the crack tip is at $x_{\text{tip}}(t)$ at time t, and, we shall call $v_{\text{tip}}(t) = \dot{x}_{\text{tip}}(t)$ the instantaneous crack tip velocity. The sign of $(\beta\, \dot{s})$ needed in equations (8-a) and (8-b) can be obtained as follows; derivation of equation (9-a) with respect to time yields:

$$\beta\, \dot{s} = -\frac{\beta\, C_{1d}}{\mu^*} \sin \lambda_0 \pi \, \left(\frac{\dot{C}_{1d}}{C_{1d}} + (1 - \lambda_d)\, \frac{v_{\text{tip}}}{x_{\text{tip}} - x} \right) \, (x_{\text{tip}} - x)^{1-\lambda_d}. \tag{11}$$

μ^* is positive [(3.21), Dundurs and Comninou, 1979] and βC_{1d} is negative from equation (10); the sign of $(\beta\, \dot{s})$ near the tip in the last equation is thus given by the bracketed term:

$$\begin{aligned} \operatorname{sgn}(\beta\, \dot{s})|_{x=x_{\text{tip}}^-} &= \operatorname{sgn} \left(\frac{d}{dt} \ln |C_{1d}| + (1 - \lambda_d) \frac{v_{\text{tip}}}{x_{\text{tip}} - x} \right) \\ &= \begin{cases} + & \text{if } v_{\text{tip}} > 0, \\ \operatorname{sgn}\left(\frac{d}{dt} \ln |C_{1d}| \right) & \text{if } v_{\text{tip}} = 0, \end{cases} \end{aligned} \tag{12}$$

where we have used $\lambda_d < 1$. Equation (12) holds under the conditions that $C_{1d} \neq 0$ and that the lips are slipping in the vicinity of the crack tip ($\beta\, \dot{s} \neq 0$). At this point, we recover the fact that singularities stronger than $r^{-1/2}$ are absent when the crack propagates (Deng, 1994): then, $\beta\, \dot{s} > 0$ from the equation above, and equation (5) shows that $\lambda \leq \frac{1}{2}$.

We shall now review all the possible motions of the lips near the crack tip (slip in either direction, stick), and study the corresponding dynamics of C_{1a}. Let us first assume $C_{1a} \neq 0$ and that the lips are slipping in such a direction that $\beta\,\dot{s} < 0$; then $d = a$ and equation (12) shows that:

$$\text{if } \beta\,\dot{s}(x_{\text{tip}}{}^{-}) < 0 \text{ and } C_{1a} \neq 0, \quad \text{then } v_{\text{tip}} = 0 \text{ and } \frac{d}{dt}\ln|C_{1a}| < 0. \quad \text{(13-a)}$$

If the lips are slipping in the other direction, it follows from equation (8-a) that:

$$\text{if } \beta\,\dot{s}(x_{\text{tip}}^{-}) > 0, \quad \text{then } C_{1a} = 0. \qquad \text{(13-b)}$$

Finally, if the lips are sticking near the crack tip, the expansion (7) is no longer valid. However, in this case, the relative slip displacement of the lips, s, is frozen to $s = s_{\text{stick}}$. Furthermore, if s_{stick} of the form (9-a) with $d = a$ and $C_{1a} \neq 0$ is put in the stick problem in figure 1 as a boundary condition, it is clear that the leading order of the stress remains given by equation (7). For this reason, the stress intensity factor C_{1a} is also frozen by the stick zone:

$$\text{if } \beta\,\dot{s}(x_{\text{tip}}^{-}) = 0 \text{ and } C_{1a} \neq 0, \quad \text{then } C_{1a} \text{ is constant in time.} \qquad \text{(13-c)}$$

Since we consider only quasi-static propagation of the cracks, all displacements are continuous functions of time, and $C_{1a}(t)$ is continuous. Collecting equations (13-a)–(13-c), one shows that $|C_{1a}(t)|$ decreases at all times. The only assumption that we now make on the crack history is very weak: we assume that the crack was not loaded sometime in the past. This is true, for example, if the sample was initially free of loads, as happens in most practical situations (note that we do not require that the loading has been turned on monotonically, like in (Comninou and Dundurs, 1980a); this would be a much stronger assumption). In the case of delamination of thin films obtained by vapor deposition, for example, the mismatch strain which loads the interface crack appears progressively when the sample cools down after high temperature deposition (Hutchinson and Suo, 1991), hence $C_{1a}(t = -\infty) = 0$.

Under the assumption stated above, C_{1a} vanishes at all times, and divergences of the stress stronger than the usual inverse square root law are removed from the theory. This result is based on the irreversibility of the crack opening: in the preceding proof, a key argument was that the crack can only propagate forward—see equation (12).

2.2 The dominant exponent λ is not history dependent

Let us now review the possible stick or slip states of the interface near the tip (at $x = x_{\text{tip}}^{-}$). We shall write that there is near-tip stick when $\dot{s}(x_{\text{tip}}^{-}) = 0$, and that there is *positive* near-tip slip when $\beta\,\dot{s}(x_{\rm tip}^-) > 0$, and *negative* near-tip slip when $\beta\,\dot{s}(x_{\rm tip}^-) < 0$. The near-tip slip direction determines the near-tip stress divergence: for positive near-tip slip, the stress intensity factor C_{1b} is generically non-vanishing (we remind that C_{1a} vanishes from §2.1); for negative near-tip slip, however, the near-tip stress divergence shall be suppressed because $C_{1b} = 0$ from equation (8-b). We note that, by the same arguments as those used to derive equation (13-c), $C_{1b}(t)$, which is now the leading stress intensity factor, is a continuous function of time and is conserved during near-tip stick. Below, we establish that near-tip negative slip is not possible; this result will permit a definitive answer to the question of the history dependence of the exponent λ.

Assume, first, that positive near-tip slip takes place sometime in the crack history, and lasts until time $t = t_0$. At time t_0, the crack lips can *a priori* either start to stick near the tip, or to slip in the negative direction ($\beta\dot{s} < 0$). According to Comninou and Dundurs, this change can be traced in the equations as follow: if the assumption $\beta\,\dot{s}(x_{\rm tip}^-) > 0$ is used at times later than t_0, either equation (10) or (12) becomes inconsistent (Dundurs and Comninou, 1979). We have mentioned that equation (10) is automatically satisfied; therefore, equation (12) must become inconsistent with $\beta\,\dot{s}(x_{\rm tip}^-) > 0$ at time t_0. As a result, $v_{\rm tip}(t_0) = 0$ and the stress intensity factor $|C_{1b}^{\rm err}|$, calculated under the (erroneous) assumption $\beta\dot{s}(x_{\rm tip}^-) > 0$, is strictly decreasing at times just after t_0 (see figure 3). We shall label by "err" all quantities calculated under this erroneous assumption at times later than t_0.

That $|C_{1b}^{\rm err}|$ strictly decreases just after t_0 obviously implies $|C_{1b}^{\rm err}(t_0^+)| > 0$. Since $C_{1b}(t)$ is a continuous function of time, the actual stress intensity factor C_{1b} satisfies: $|C_{1b}(t_0^+)| = |C_{1b}(t_0^-)| = |C_{1b}^{\rm err}(t_0^+)| > 0$ and, by continuity, C_{1b} remains nonzero during a time interval $\Delta t > 0$ after time t_0. Using equation (8-b), $\beta\,\dot{s}(x_{\rm tip}^-) \geq 0$ on this finite interval $t \in [t_0, t_0 + \Delta t]$. By assumption, positive slip ends at time t_0, hence stick starting at time t_0. Therefore, we have proved that when the crack faces stop slipping in the direction $\beta\,\dot{s}(x_{\rm tip}^-) > 0$ near the tip, a stick zone develops from the tip; moreover, this stick zone will continue to exist near the tip for a finite time interval before negative slip can eventually be reached ($\Delta t > 0$).

We can now examine what happens when this stick zone disappears, say, at time t_0'. We remind that C_{1b} has been conserved during near-tip stick (see figure 3). Moreover, C_{1b} was non-vanishing when the stick zone appeared (see previous paragraph). Therefore, $C_{1b}(t)$, which, again, is continuous in time, is non-vanishing at times just after t_0'. From equation (8-b), negative near-tip slip cannot take place just after time t_0', and the interface necessarily goes back to positive near-tip slip.

Finally, we use again the assumption that the crack was not loaded sometime in the past: C_{1b} was then vanishing, and $|C_{1b}|$ has necessarily started to increase as the loading has been turned on. Equation (12) shows that the system has then entered either state: $\dot{s}(x_{\rm tip}^-) = 0$ (stick near the tip), or $\beta\dot{s}(x_{\rm tip}^-) > 0$ (slip in the positive direction near the tip). At subsequent times, the system eventually goes from one state to the other, but slip in the forbidden direction, $\beta\dot{s}(x_{\rm tip}^-) < 0$, can never be reached: at all times,

$$\beta\ \dot{s}\big|_{x=x_{\rm tip}^-} \geq 0. \tag{14}$$

As a result, the dependence of λ on $(\operatorname{sgn}\dot{s})$ in equation (5) is made-up, and it is much easier to determine the exponent λ using equation (6). We hereby have proved the important result: the exponent of the divergent term in the stress expansion near the tip is $-\lambda_0 \geq -\frac{1}{2}$, and it does not depend on the loading history. It would be history-dependent only if the crack could propagate backwards.

3 Prevention of slip in the forbidden direction

In the previous section, we have established a striking property of the interface crack with friction: in the vicinity of the tip, slip can take place in one direction only, which depends on the sign of the elastic mismatch. Below, we point out two mechanisms that prevent slip in the opposite direction.

From equation (14), the applied loading tends to induce slip in the forbidden direction when the stress intensity factor obtained from the conventional crack analysis satisfies: $\operatorname{sgn}\frac{d}{dt}K_{II} = \operatorname{sgn}(-\beta)$. This can happen in two situations: first, when applied shear stress is compatible with the authorized slip direction ($\operatorname{sgn} K_{II} = \operatorname{sgn}\beta$), but is decreasing in magnitude ($d|K_{II}|/dt < 0$). This situation is typically encountered during cyclic loading sequences; it is studied in §3.1. The second situation is when the applied shear stress is opposite the intrinsic slip direction ($\operatorname{sgn} K_{II} = -\operatorname{sgn}\beta$), and increasing ($d|K_{II}|/dt > 0$): in §3.2, we study the effect of an applied shear conflicting with the intrinsic slip direction.

3.1 Locking of the stress intensity factor

We consider a bimaterial with a partially cracked interface, submitted to a non monotonic sequence of loading, as in figure 3. For sake of definiteness, we consider the case $\beta > 0$, so that the slip direction near the crack tip imposed by equation (14) is $\dot{s} > 0$. The loading is assumed to be compatible with the

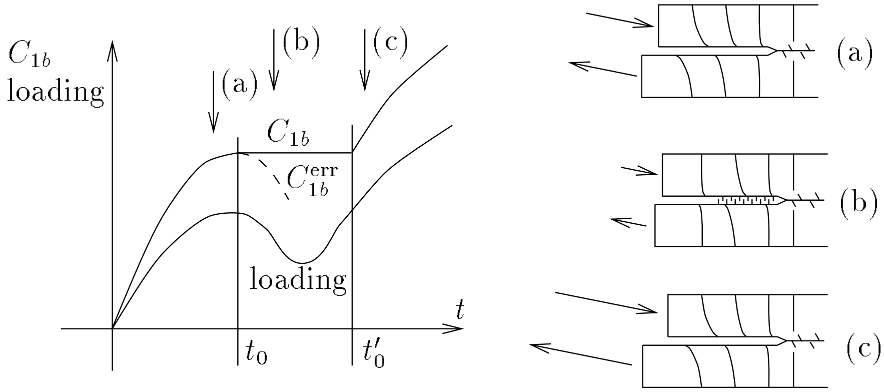

Fig. 3. Locking of the stress intensity factor C_{1b} during a non-monotonic variation of the loading, when imposed loading complies with the intrinsic slip direction. Crack is assumed not to propagate ($v_{\rm tip} = 0$). C_{1b} is proportional to loading as long as the loading is monotonic (a); at time t_0, the loading starts decreasing, and C_{1b} is locked by a stick zone near the tip (b). As the loading is increased again, the size of the stick zone decreases, and it disappears at time t'_0 when the loading reaches again the same value as at time t_0 (c).

intrinsic slip direction: it induces a stress intensity factor $K_{II} > 0$. The crack is assumed not to propagate ($v_{\rm tip} = 0$).

From the results of §2.2, the stress intensity factor C_{1b} increases proportionally to the loading, as long as the loads are increased monotonically; at time t_0, when the loading starts decreasing, $C_{1b}^{\rm err}$, which is proportional to loading, also decreases; equations (12) and (14) then becomes inconsistent. This indicates that a stick zone develops near the crack tip. As the loading is being decreased more and more, it is reasonable to expect that the stick zone spreads outwards from the crack tip. Similarly, it can be expected that this stick zone will shrink when the loading is increased again. We remind that C_{1b} is frozen as long as a stick zone exists near the tip. At time t'_0, when the loading comes back to the same level as at time t_0, the system is in the same state as at time t_0, and the near-tip stick zone disappears. C_{1b} then again follows the applied loading.

In figure 3, the stress intensity factor C_{1b} is seen to increase irreversibly. Using equations (12) and (14), it can indeed be seen that $|C_{1b}|$ increases as long as the crack does not advance. During decrease of the loads, a decrease of the stress intensity factor C_{1b} is prevented by the formation of a stick zone at the crack tip (see figure 3). This irreversible increase of the stress intensity factor is an example of memory effects in cracks. It may be relevant in the study of fatigue.

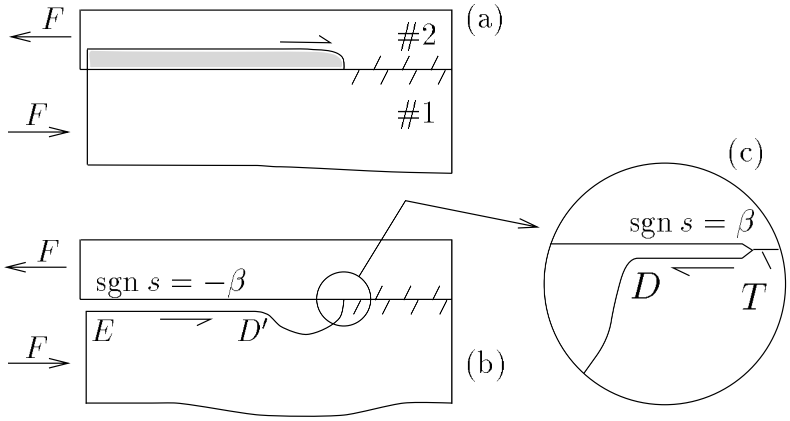

Fig. 4. Formation of a bubble near the crack tip when the loading is opposite the intrinsic slip direction of the interface. Macroscopic interpenetration is obtained in the conventional crack analysis (a). Interpenetration is forbidden in our model, and a bubble is formed near the tip (b). Very close to the tip, the crack closes again, and the intrinsic slip direction of the interface is recovered (c).

3.2 Formation of a bubble near the crack tip

We shall now discuss the state of the system when the imposed loading is opposite the intrinsic slip direction ($\operatorname{sgn} K_{II} = -\operatorname{sgn}\beta$). Again, we consider a partially cracked bimaterial. Integration of equation (14) with respect to time is possible, because the crack remains closed near the tip at all times; assuming moreover, as in §2.1, that the crack was initially free of load, one obtains: $\beta\, s(x,t) \geq 0$ at all times in the left vicinity of the crack tip, hence equation (15-a) below. We impose a loading such that $\operatorname{sgn} K_{II} = -\operatorname{sgn}\beta$; then, the induced slip direction at the interface can be expected to be: $\operatorname{sgn} s = \operatorname{sgn} K_{II}$, hence equation (15-b).

$$\operatorname{sgn} s = \operatorname{sgn}(+\beta) \quad \text{imposed by near-tip analysis} \tag{15-a}$$
$$\operatorname{sgn} s = \operatorname{sgn}(-\beta) \quad \text{imposed by external loading.} \tag{15-b}$$

In these conditions, the direction of slip imposed by the material constant on one hand, and by the external loading on the other hand, are indeed in conflict.

Noticing that equation (15-a) applies at microscopic scales near the tip, while equation (15-b) has a macroscopic origin (the loading), one can solve the contradiction as follow: the materials contact along two different regions separated by a bubble (see figure 4). One region, $[ED']$, is macroscopic; the other one, $[DT]$, touches the crack tip T and has microscopic extent. In the macroscopic contact zone, $[ED']$, the direction of slip is imposed by the loading, and satisfies

$\operatorname{sgn} s = \operatorname{sgn} K_{II} = \operatorname{sgn}(-\beta)$. In the microscopic contact region near the crack tip, the slip direction is imposed by the materials constants: $\operatorname{sgn} s = \operatorname{sgn} \beta$. The bubble can therefore be seen as a transition between a region where the slip direction is imposed by the loading, as in equation (15-b), to a region where the slip direction is imposed by the properties of the interface, as in equation (15-a), when these directions are incompatible.

In figure 4, a bubble has been represented. For sake of simplicity, we take the material #2, above, infinitely rigid ($\beta > 0$). Then, the authorized slip direction near the crack tip is $\beta \dot{s} > 0$, and the loading is assumed to impose $K_{II} < 0$, hence the direction of the applied force on the picture. We note that in this geometry, macroscopic interpenetration of the materials is obtained in the conventional crack analysis: $K_I < 0$ (Suo and Hutchinson, 1990). Consideration of the interfacial contact is therefore essential. For the geometry in figure 4 ($K_{II} < 0$), a bubble has indeed been observed numerically by Stringfellow and Freund (1993) when $\beta > 0$. Our analysis offers a simple interpretation to the apparition of this bubble.

An equivalent picture to the formation of the bubble can be given. In the conventional analysis of interface cracks, where contact of the crack lips is not considered, the stress intensity factors K_I and K_{II} are scale dependent in the presence of elastic mismatch (Rice, 1988):

$$(K_I + \imath K_{II})|_{l^*} = (K_I + \imath K_{II})|_l \left(\frac{l^*}{l}\right)^{\imath\varepsilon}, \tag{16}$$

where $\varepsilon = \ln((1-\beta)/(1+\beta))/2\pi$ is a mismatch parameter, and l and l^* are two different lengthscales at which the stress intensity factor are evaluated. The parameter ε is numerically small for a variety of interfaces, and this scale dependence can often be neglected (Rice, 1988); this is why we have introduced stress intensity factors without mention to any lengthscale elsewhere in this paper. Assume that, for some given external loading, the crack closes at a macroscopic scale l: $K_I|_l = 0$. Using the equation above, and noticing that $\operatorname{sgn} \varepsilon = \operatorname{sgn}(-\beta)$, it is easily seen that $K_I|_{l*}$ becomes positive at lengthscales l^* smaller than l if $\operatorname{sgn} K_{II}|_l = \operatorname{sgn}(-\beta)$. That K_I becomes positive at small scales means that the crack reopens, hence the formation of a bubble for $\operatorname{sgn} K_{II} = -\operatorname{sgn} \beta$. Contrarily, $K_I|_{l*}$ becomes negative at small scales l^* when $\operatorname{sgn} K_{II}|_l = \operatorname{sgn} \beta$, which indicates that the crack is fully closed.

The formation of a bubble near the tip has important consequences for the interfacial crack: because a bubble is present only for a definite sign of K_{II}, $\operatorname{sgn} K_{II} = \operatorname{sgn}(-\beta)$, the geometry of the contact regions at the interface strongly depends on this sign. As a result, the apparent toughness of the interface $\Gamma(\psi)$ resulting from frictional screening of the external loads (Stringfellow and Freund, 1993) should be asymmetric (Audoly, 1999).

4 Summary and conclusion

We have studied the concentration of stress near the tip of an interface crack with friction. We have shown that the dominant term in the expansion of the stress is given by equation (4), where the exponent λ is smaller than $\frac{1}{2}$: divergences stronger than in the conventional crack analysis ($\sigma \propto r^{-1/2}$) have therefore been removed from the theory. In contrast to what had been postulated (Comninou and Dundurs, 1980b; Deng, 1994), this exponent is history independent. Moreover, we have shown that the near-tip slip can occur in one direction only; this direction in imposed by the properties of the materials—see equation (14). All these results derive from the fact that the crack can only propagate forward. Two mechanisms prevent slip in the forbidden direction. First, a stick zone spreads outwards from the crack tip when a non monotonic cycle of loading is applied. Secondly, a bubble is nucleated near the tip when the imposed loading would induce slip in the forbidden direction.

A memory effect at the crack tip has been discussed: the crack tip retains the highest value of the stress intensity factor ever reached since the crack is arrested. It has also been pointed out that the formation of a bubble at the crack tip should make the effective toughness of the interface asymmetric. Finally, our analysis allows an approach of the interfacial crack with friction using the concept of energy release rate: infinite flows of energy towards the crack tip have been ruled out from the theory. In a subsequent paper (Audoly, 1999), the interface toughness induced by friction is investigated on the basis of the present analysis.

Acknowledgements

I wish to thank M. Adda-Bedia, J. W. Hutchinson, Y. Pomeau and J. R. Rice for helpful discussions, comments and encouragements, as well as R. Magis for technical support. The Laboratoire de Physique Statistique is associated with the CNRS, the École Normale Supérieure, and the Universités Paris 6 and Paris 7.

References

AUDOLY, B.	1999	Submitted to *J. Mech. Phys. Solids.*
COMNINOU, M.	1977	*J. Appl. Mech.* **44**, 780.
COMNINOU, M. and DUNDURS, J.	1979	*Eng. Frac. Mech.* **12**, 191.
COMNINOU, M. and DUNDURS, J.	1980a	*J. Elasticity* **10**, 203.
COMNINOU, M. and DUNDURS, J.	1980b	*Res Mechanica* **1**, 249.
DENG, X.	1994	*Int. J. Sol. Struct.* **31**, 2407.
DUNDURS, J. and COMNINOU, M.	1979	*J. Elasticity* **9**, 71.
HUTCHINSON, J. W. and SUO, Z.	1991	*Adv. Appl. Mech.* **29**, 63.
MALYSHEV, B. M. and SALGANIK, R. L.	1965	*Int. J. Frac. Mech.* **1**, 114.
RICE, J. R.	1968a	In *Fracture: an advanced treatise* (edited by H. LIEBOWITZ), Vol. 2, p. 191. Academic press, New York.
RICE, J. R.	1968b	*J. Appl. Mech.* **35**, 379.
RICE, J. R.	1988	*J. Appl. Mech.*, **110**, 98.
STRINGFELLOW, R. G. and FREUND, L. B.	1993	*Int. J. Sol. Struct.* **30**, 1379.
SUO, Z. and HUTCHINSON, J. W.	1990	*Int. J. Frac.* **43**, 1.