Comment on "Crack street: The cycloidal wake of a cylinder tearing through a thin sheet"

A new mode of oscillatory fracture in brittle thin films has recently been reported in two independent experiments [1, 2]. When an object is driven through a thin film clamped along its lateral boundaries, the film tears following a striking sawtooth pattern, as the crack tip propagates in an oscillatory manner.

In their paper, Ghatak and Mahadevan [1] make an analogy with the classical construction of a cycloid and claim that the crack path consists of series of arches of prolate cycloids (APC). This approach is based on the assumptions that: (i) the crack tip moves along an arc of circle in the cutting tool's frame of reference and (ii) it does so with an angular velocity that is constant with an alternating sign. Whereas these assumptions roughly mimic the oscillatory behaviour, they bear some important limitations. First, they lack a physical justification, making their construction an *ad hoc* description of the experiments. For instance, this construction is justified by the fact that the crack tip moves '*on the circumference of the tool*' but a heuristic fitting parameter, $\alpha > 1$, is later introduced in the model, implying that the crack is *ahead* of the tool. Secondly, these assumptions are inconsistent with some of the experimental observations. In particular, (i) in our own experiments [2], we have measured fluctuations of the distance of the crack tip to the center of the cutting tool by as much as 40% of the tool radius and (ii) the velocity of the crack tip presented in their Fig. 4b is far from constant, even within the quasi-static regime (stick phase).

In the absence of a solid physical basis, the claim that the morphology of fracture paths is cycloidal is supported solely by a superposition of an experimental path with an APC curve, in their Fig. 2c. Although visually satisfactory, the similarity of the curves mostly reflects the ingredients that have been included in the construction by hand, namely the periodicity, existence of angular points and invariance under axial symmetry combined with translation by a half-period.

With the aim of testing the APC construction on a more quantitative ground, we have performed an error analysis of this model: we compared its predictions to those of two reference models which were purposely chosen to be unphysical. To avoid any bias, the comparison was based on the experimental crack path presented by the authors in Fig. 2c, and the reference models were chosen with the same number of adjustable parameters, *two*, as in the APC construction. The first reference model is a plain sinusoid $y(x) = \alpha \sin(\beta x)$, with adjustable amplitude, α, and wavenumber, β. Note that experimental patterns are far from sinusoidal [1]; hence, as intended, this model should provide a poor fit and establish an upper bound on the error estimation. The other reference model is based on *replicated arches of parabolas* (RAP): every half period follows the equation $y(x) = \alpha x^2$ for $0 \leq x \leq \beta$, where α and β are adjustable, and is replicated as in the APC construction.

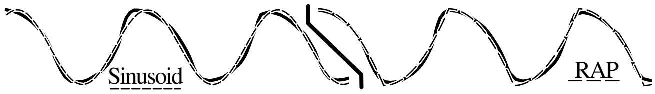

FIG. 1: The experimental crack path used in Ref. [1] (solid curve) is much better fitted by our two reference models than by the cycloidal construction (compare to their Fig. 2c).

The error of the fit provided by all three models was measured using the r.m.s. of the transverse deviation: $E = (2/d)\,\langle[y_{\mathrm{model}}(x) - y_{\mathrm{exp}}(x)]^2\rangle^{1/2}$, where x and y are the axial and transverse coordinates, the brackets denote average with respect to x, and the prefactor (inverse radius of the cylinder) makes the error dimensionless. The reference models fit the experimental crack path of Fig. 2c with an error $E_{\mathrm{sinusoid}} = 7.8\%$ and $E_{\mathrm{RAP}} = 6.4\%$. In comparison, the fit intended to support the APC construction in their Fig. 2c yields a much larger error, $E_{\mathrm{APC}} = 18.7\%$. We conclude that the two reference models, even though just as unphysical, provide a more effective description of the experiments than the APC construction. In this sense, the statement that the crack morphology is cycloidal appears arbitrary and the construction proposed in [1] seems to be an unnatural starting point in understanding oscillatory fracture paths in the tearing of thin sheets.

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[1] A. Ghatak and L. Mahadevan, Phys. Rev. Lett. **91**(21), 215507 (2003)

[2] B. Roman, P.M. Reis, B. Audoly, S. De Villiers, V. Viguié, D. Vallet, C. R. Mécanique **331**(12) 811-816 (2003)