Oscillatory fracture paths in thin elastic sheets

Fissures oscillantes dans les feuilles élastiques minces

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Abstract

We report a novel mode of quasi-static oscillatory crack propagation when a cutting tip of moderately large width is driven through a thin brittle polymer film. Experiments show that the amplitude and wavelength of the oscillatory crack paths scale linearly with the width of the cutting tip over a wide range of length scales but are independent of the width of the sheet and of the cutting speed. We propose a mechanism for this instability, based on the coupling between crack propagation and out-of-plane deformations of the film. *To cite this article: B. Roman, P.M. Reis, B. Audoly, S. De Villiers, V. Viguié, D. Vallet, C. R. Mecanique 331 (2003).*

Résumé

Nous présentons une nouvelle instabilité oscillante de fissure qui se produit lorsqu'un indenteur assez large est forcé à travers une feuille mince fragile préalablement entaillée. Nos expériences révèlent que l'amplitude et la longueur d'onde de la fissure dépendent linéairement de la largeur de l'indenteur sur une grande gamme de largeurs, et ne dépendent ni de le la largeur du film, ni de la vitesse de découpe. Nous proposons un mécanisme pour cette instabilité, fondé sur un couplage entre l'avancée de la fissure et les déformations transverses du film. *Pour citer cet article : B. Roman, P.M. Reis,B. Audoly,, S. De Villiers, V. Viguié, D. Vallet, C. R. Mecanique 331 (2003).*

Thin elastic plates ; fracture ; oscillatory crack path

Mots-clés : Plaques élastiques minces ; fissure ; fissure oscillante

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L'étude de la rupture est un sujet ancien mais qui reste toujours actif, notamment en ce qui concerne la compréhension du chemin de propagation et de son contrôle. Nous présentons ici une expérience simple où une fissure se propage en oscillant dans une feuille elastique mince fragile. L'étude de la rupture d'une bande de verre soumise à un gradient thermique [1-5] a permis les premières études de fissures oscillantes dans des conditions expérimentales bien contrôlées. Le phénomène que nous décrivons ici est à notre connaissance inédit et repose sur un mécanisme tout à fait différent : on coupe une feuille mince au moyen d'un indenteur cylindrique ou rectangulaire de largeur w orienté perpendiculairement à la feuille (figure 1). Sur une grande gamme de largeurs w, on observe des oscillations de la découpe, dont nous montrons qu'elles résultent du couplage de la fissure avec les déformations hors du plan de la feuille. L'étude de cette instabilité et la possibilité de l'inhiber présentent des intérêts pratiques pour la découpe de feuilles minces.

L'expérience consiste à fixer une bande mince (polypropylène mince ou acétate de cellulose ; épaisseur de 25 à 130 μm) sur un cadre que l'on déplace le long d'un rail à l'aide d'une vis-sans-fin actionnée par un moteur. La feuille est ainsi déplacée à vitesse constante v vers l'indenteur fixe dont la largeur w varie de 0.05 à 60 mm. Les fissures observées sont fragiles (pas ou peu de déformation plastique). Au dessus du seuil d'oscillation ($w > w_c$), la fissure oscille périodiquement, créant ainsi un motif de découpe de la feuille très régulier et reproductible. Pour des objets minces ($w < w_c$), il ne se produit pas d'instabilité et la découpe reste droite.

Nous décrivons dans cette Note le régime d'oscillation bien au dessus du seuil ($w \gg w_c$). L'amplitude crête à crête A et la longueur d'onde λ du motif varient linéairement avec la largeur w (figure 2), sur près de trois décades, indépendament de la nature de matériau et de son épaisseur. D'autre part, le motif ne dépend pas de la vitesse de déplacement, ni de la largeur D de la feuille, ce qui est a priori surprenant.

La séquence de la figure 3, prise depuis une caméra placée au dessus du montage, illustre le mécanisme d'oscillation dans le repère de l'indenteur. Celui-ci repose sur une caractéristiques propres des feuilles minces : l'énergie de courbure est négligeable devant l'énergie d'extension et on suppose qu'elle ne contribue pas à la propagation de la fissure. Pendant $t_1 < t < t_2$ la propagation de la fissure (au point T) se fait vers la gauche en raison de la présence d'un pli (ligne en pointillé et double flèche) que l'indenteur repousse au point de contact N, le mettant sous tension (flèches grises). La propagation se fait ainsi dans la direction perpendiculaire au pli (flèche noire). Il apparaît ensuite un nouveau point N' de contact sur l'autre lèvre de la fissure (entre t_1 et t_2) supportant des contraintes croissantes, qui finissent par devenir plus grandes que celles dues au premier pli N. Par conséquent, au temps t_3, la direction principale des contraintes près de la pointe de la fissure change ; il s'ensuit une phase de propagation dynamique. Après t_4, la propagation se fait à nouveau de façon quasistatique, dans une configuration symétrique de celle en t_1 : sous l'action de l'indenteur sur la lèvre de droite, la fissure se propage vers la droite. Un nouveau point de contact N'' est créé, conduisant finalement à un changement de direction suivi d'une phase dynamique entre t_5 et t_6, qui ramène à un état identique à t_1 après une période complète. Un modèle basé sur ces arguments qualitatifs fera l'objet d'une prochaine publication (voir [7]).

Au voisinage de la transition entre fissure droite et oscillante, la feuille ne peut plus être considérée comme beaucoup plus fine que l'indenteur et les énergies d'extension et de courbures deviennent du même ordre de grandeur ; le mécanisme proposé ici n'est plus valable, et on observe un mode de propagation rectiligne des fissures. Nous observons une zone de bistabilité pour des largeur proches de w_c.

Nous avons donc présenté une nouvelle instabilité oscillante de fissures dans une feuille mince, lorsque la taille de l'indenteur est assez grande $w > w_c$. L'amplitude et la longueur d'onde dépendent linéairement de la taille w de l'indenteur, et le motif est indépendant le la largeur de la feuille et de la vitesse de découpe.

1. Introduction

Despite a long history of research into the field of fracture, many puzzling issues remain unsolved. An interesting problem concerns the direction of propagation of the crack tip: when a glass breaks, can the shape of the resulting pieces be predicted? Recent well controlled experiments have yielded a variety of interesting behaviours that are a challenge to existing theoretical formulations. An oscillatory instability during dynamic crack propagation was recently observed in a pre-tensioned thin rubber sheet [6] these mechanism behind which is still unresolved. Another example is the controllable quasi-static propagation of oscillatory cracks in a thin strip of glass submitted to a thermal field [1,2,5] which, despite its apparent simplicity, has been stimulating much theoretical study [3,4].

Here we report results on oscillatory fracture paths in a new experimental context: an object, which we denote by *cutting tip*, is perpendicularly driven through a thin polymer sheet held along its lateral boundaries, that progressively cuts the material as it advances. For large enough cutting tip widths, the crack follows a well defined and highly reproducible oscillatory path that spans a wide range of length scales, as shown in the two examples presented in Fig. 1d) and e). In fact, even doing the experiment by hand yields surprisingly regular patterns. The experimental observations of oscillatory motion in this new geometry present a challenge, from a fundamental point of view, to our understanding of crack propagation in thin sheets. Moreover, these ideas should have practical applications since the precise cutting of brittle thin sheets is common in the manufacturing industry.

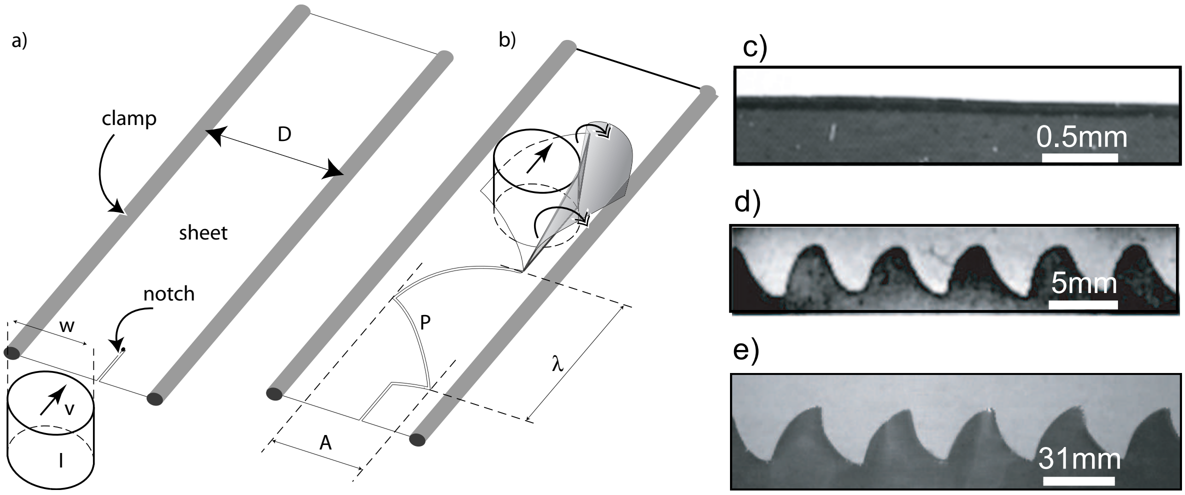


Figure 1. a) and b) Schematic diagram of the experimental set up. A cylindrical cutting tip is forced into a clamped thin polymer sheet with a notch, leading to an oscillatory crack path P. a) Initial configuration of the experiment. b) Typical configuration during fracture. The advance of the cutting tip through the sheet leads to out of plane deformations (double arrows and region in grey) along with mode III propagation at the crack tip. c–e) Edge of the sheet as seen from above (polypropylene 27μm thick) for three cutting tip widths: c) $w = 0.15$mm (straight path), d) $w = 5$mm (oscillatory path), e) $w = 31$mm (oscillatory path). White rectangles are scale bars.

2. The experiment

We have performed reproducible experiments with thin sheets of different polymeric materials in a range of thicknesses and have investigated the dependence of the resulting fracture paths on the width of the cutting tip. A schematic diagram of the apparatus is presented in Fig. 1a) and b). It consists of a thin

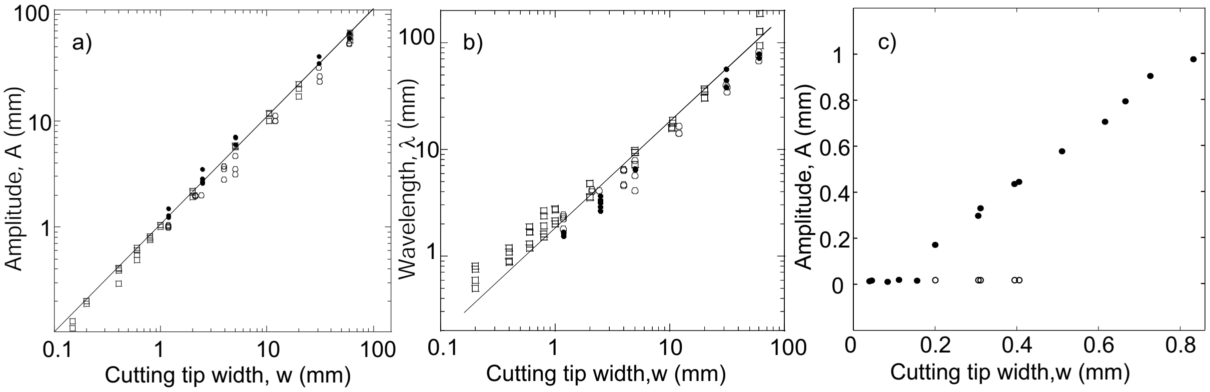


Figure 2. Log-log plots of the (a) peak-peak amplitude, A, and (b) wavelength, λ, of the oscillations as a function of tip width w for various materials and thicknesses (polypropylene 25 to 53 μm: open symbols, cellulose acetate 100 and 130 μm : closed symbols), and for both cylinders (○●) and rectangular blades (□). Amplitude and wavelength fall on slope 1 lines, over a range of almost 3 decades. The straight lines represent the best linear fits with slope 1. c) Peak-to-peak amplitude of paths for various cutting tip widths (with rectangular blades), showing the transition between the straight and oscillatory states. Open symbols (○) correspond to regions of bi-stability where both oscillatory and straight paths can be observed.

flat sheet (dimensions ranging from 6 × 60 mm to 120 × 500 mm) clamped along its lateral boundaries and mounted on on a linear translation stage. This stage was driven at constant speed, v, towards a fixed object, the *cutting tip*, which could have either rectangular or cylindrical profile, with a variety of widths (0.05 mm $\leq w \leq$ 60 mm). A camera was mounted directly above the apparatus such that the propagating crack was imaged in the cutting tip's frame of reference.

The sheet was initially prepared with a notch on one of its side boundaries to position and initiate the crack. Both bi-oriented polypropylene and cellulose acetate thin sheets were investigated, with thicknesses ranging between 25 and 130 μm. The sheet's Young's modulus and fracture energy were measured to lie within the ranges $E = 1$–2 GPa and $\gamma = 2$–5 kJ/m^2, respectively. Although polymeric, these materials are brittle since they have been severely stretched when processed into thin sheets. As a result, they undergo minimal plastic deformation during fracture propagation but, being thin, can sustain large bending without crack initiation. This explains why they are widely used in the packaging industry (resistant but easy to tear once a notch is started). The oscillatory paths discussed below were not observed in ductile materials.

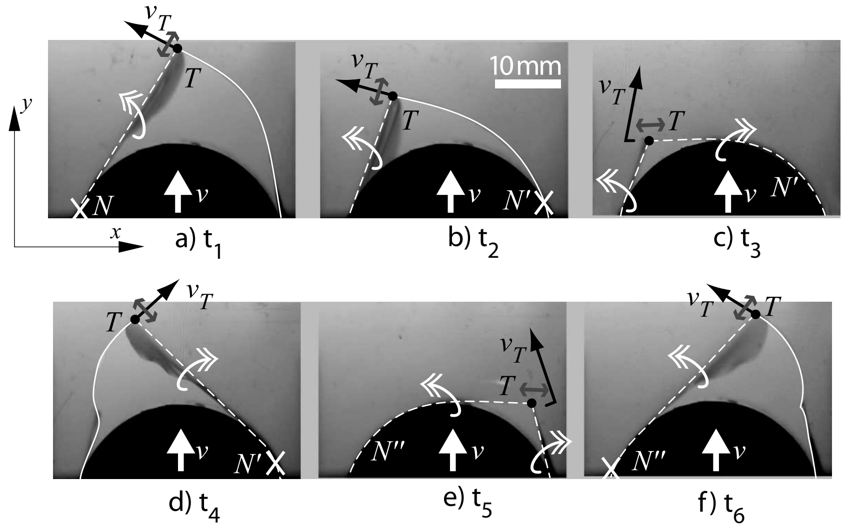
3. Oscillatory fracture paths

As the thin sheet is forced through the fixed tip, the material is cut, leaving behind a well defined and highly reproducible path. For large enough cutting tips, the resulting path is oscillatory, two examples of which, for significantly different sizes of the cutting tip, are shown in Fig 1d) and e). In this oscillatory regime, the non-sinusoidal oscillatory paths resembles a series of shark fins; the fracture path is made up of smooth curves connected by sharp *kinks*. However, and as one would expect, for thin enough objects, the path left behind the cutting tip is straight; Fig 1c). Our results point to a new instability in the fracture of thin polymer films from straight to oscillatory patterns, as the size of the cutting tip is increased.

In this Note, we mainly focus on the regime well above threshold. As shown in Fig. 2, we find that both the amplitude A and wavelength λ of the oscillatory paths depend linearly on the width of the cutting tip in this regime, over almost three decades. This linear dependence is valid for a variety of materials and thicknesses.

Furthermore, we have performed experiments for speeds in the range $0.06\ \text{mm/s} < v < 64\ \text{mm/s}$ and found no influence of the cutting speed on the patterns. We have also checked, for a fixed tip size $w = 2$ mm, that the pattern is independent of the sheet's width, D, within $3.3w < D < 21w$. This feature is *a priori* surprising since one could have thought of D as one of the natural length scales in the problem.

The sequential process of crack propagation over a single period of oscillation, as seen from above in the frame of reference of the cutting tip, is presented in Fig. 3. Let T be the position of the crack tip which propagates with a velocity v_T. Throughout the frames in Fig. 3a)-f) the cutting tip is moving at a steady velocity v, relative to the thin film, as indicated by the bold white arrows. In between t_1 and t_2, T propagates lefwards. During this slow propagation, the y-component of v_T is slower than v and eventually a new contact N' is established between the cutting tip and the right planar edge of the film (solid white line). In this period, $t_1 < t < t_2$, a fold (white dashed line and double white arrow) induces increasingly stronger out-of-plane deformations in the left edge of the film by bending. Because the point of contact N is pushed leftwards by the cutting tool, the fold is under tension. This yields stresses near the crack tip as indicated by grey arrows and the direction of crack propagation is perpendicular to this fold (black arrow v_T). Between t_2 and t_3, both rims of the crack are pushed onto the cutting tool and are, therefore, stretched. Stretching of the right-hand-side rim eventually dominates at t_3 and the main direction of stresses near the crack tip changes accordingly; a period of dynamic propagation follows. This releases most of the stretching energy stored in the right rim. After this dynamic stage the crack tip comes to a rest, ahead of the cutting tip. Between $t_4 < t < t_5$, a new point of contact, N'', is now established between the cutting tip and the left edge of the film. Frame t_5 corresponds to the half-cycle point, the mirror image of t_2, and the stresses at T are again such that dynamic propagation occurs leftwards. The second half-cycle is similar and t_6 brings us back to the starting point t_1 after one full period of the oscillation.



a) t_1 b) t_2 c) t_3

d) t_4 e) t_5 f) t_6

Figure 3. Sequence of experimental frames of the crack propagation, over a single period of oscillation $\Delta t = 32$ s, as seen from above in the frame of reference of the cutting tip. The crack tip is positioned at T and propagates at a velocity v_T. White lines have been been superposed on the thin film to aid visualization: solid white lines represents a fractured edge, while a fold is indicated by a white dashed line. Near these folds, the cutting forces the film to deform out-of-plane plane (darker region), as indicated by the double arrows. The arrows in grey represent the main direction of stress felt at T. A representative movie is available online [7].

The linear dependence of A and λ on w discussed above breaks down in the vicinity of the transition region between straight and oscillatory paths. In these regions the sheet can no longer be considered thin relative to the cutting tip and the plate bending and stretching energies become of the same order of magnitude. For the transition experiments we have used fine rectangular blades (precision feeler gauges) in the range $0.0375 < w < 0.8$mm. At this lengthscales the imaging was done with a microscope with a 2× objective. As shown in the plot of Fig. 2, a clear transition is observed, from a straight crack to an oscillatory regime, as w was incrementally increased. Above a critical cutting tip width, w_c, the amplitude of the oscillatory paths scales linearly with w, as discussed above. Above the transition point, in the vicinity of w_c, there is a region of bi-stability where both straight and oscillatory paths could be observed.

4. Discussion and conclusion

The instability scenario presented above suggests that the origin of the oscillations lies in the mechanical properties of thin elastic sheets and in their connection with the geometry of surfaces. Under external constraints, thin elastic sheets bend in order to avoid stretching, since bending energy is small compared to stretching energy [8]. In order to conserve their in-plane lengths and remain unstretched, thin sheets tend to adopt developable shapes, that are the reunion of straight lines, *generatrices*. To aid further understanding the cutting process, it is useful to divide the thin sheet, as it is cut, into two regions. *Soft-regions* are defined as those in which the sheet can deform by pure bending, therefore easily accommodating the presence of the advancing cutting tip via out-of-plane deformations. On the other hand, in the *hard-regions*, the sheet can no longer bend and the presence of the cutting tool induces large in-plane stresses, eventually leading to crack propagation. These arguments are the basis of a quantitative model of this fracture instability, which will be presented in future work. An preview of this model and its predictions are available in [7].

To conclude, we have reported a novel mode of oscillatory crack propagation as a cutting tip is driven through a thin polymer sheet with an associated instability as the cutting tip size is varied. Although these oscillatory patterns are reminiscent of those observed in thermal quenching experiments [1,2,5], an important difference is that the oscillation mechanism here arises from a coupling between fracture and the out-of-plane deformations of the sheet. The crack path amplitude and wavelength were found independent of the driving speed, of the thickness and lateral width of the sheet, and scale linearly with the size of the object. For cutting tip widths below a critical value w_c no oscillatory cracks are observed and the crack path is straight.

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References

[1] Yuse A. and Sano M., Transition between crack patterns in quenched glass plates, Nature **362** (1993) 329–331.

[2] Ronsin O., Heslot F. and Perrin B., Experimental study of quasistatic brittle crack propagation, Phys. Rev. Let. **75** (1995).

[3] Adda-Bedia M. and Pomeau Y., Crack instabilities of a heated glass strip, Phys. Rev. E **52** (1995) 4105–4113.

[4] Yang B. and Ravi-Chandar K., Crack path instability in a quenched glass plate, J. Mech. Phys. Solids **49** (2001) 91–130.

[5] Deegan R., Chheda S., Patel L., Marder M. and Swinney H., Wavy and rough crack in Silicon, 2002 preprint.

[6] Deegan R., Petersan P., Marder M. and Swinney H., Oscillating fracture paths in rubber, Phys. Rev. Let. **88** (2002).

[7] `http://www.lmm.jussieu.fr/platefracture/`

[8] Pogorelov A., Bending of surfaces and stability of shells, Translation of mathematical monographs, AMS, 1988.

[9] Ghatak A. and Mahadevan L., *preprint.*