‘Dolphins in phosphorescent sea’. The inspiration for this woodcut, created by M. C. Escher in 1923, was the flow-induced bioluminescence that occurs on dolphins when they swim through waters that contain high levels of bioluminescent plankton.
ABSTRACT. It is common in turbulence research (and in fluid mechanics generally) to put an emphasis on vorticity as a quantity of utmost importance (vorticity obsession). In the context of turbulence this can be put in one sentence: there is no turbulence without vorticity. However, vorticity is only an antisymmetric part of the velocity gradients tensor. Its symmetric part - the rate of strain tensor - is (at least) of equal importance. This is the main theme of this presentation mostly on the basis of experimental evidence (laboratory and numerical).

Formally, the whole (incompressible) flow field is fully determined by the field of vorticity with appropriate boundary conditions. This is true of strain too, e.g. there is a relation between the fields of velocity and strain similar to that of Biot-Savart. Strain controls the flow in the same way as does vorticity in the sense of possible breakdown of smooth solutions for 3D flows. One of the most basic phenomena and distinctive features of three-dimensional turbulence is the predominant vortex stretching. This process occurs via interaction of vorticity and strain. In fact, there exist two nonlocally interconnected and weakly correlated processes: i) predominant vortex stretching (enstrophy production), and (not less important) ii) predominant self-amplification of the rate of strain (production of total strain/dissipation). It is this latter process which is responsible for the enhanced dissipation in turbulent flows.

`Passive' turbulence. Though vorticity has some influence, strain is dominating the evolution and behaviour of passive objects (scalars, vectors) in turbulent and any Lagrangian chaotic flows. The process of self-amplification of vorticity, along with essential differences, has a number of common features with analogous processes in passive vectors: in both the main factor is their interaction with strain. In contrast the production of strain is much more `self'. It should be stressed that the process of self-amplification of strain is a specific feature of the dynamics of genuine turbulence having no counterpart in the behaviour of passive objects.

Evolution of disturbances. The process of evolution and amplification of disturbances (both in genuine and `passive' turbulence) is dominated by the strain field of the basic flow.

Polymer solutions. Though the issue of drag reduction is far from being understood, it is clear that the direct interaction of the dissolved polymers with turbulence is in the small scales. This interaction of polymers and the flow field occurs via the field of strain.
MAIN REFERENCES


WHY STRAIN TOO?

Some reasons. More below

• Most important stresses in fluids (both Newtonian and non-Newtonian) flows are defined by strain.

• Energy dissipation is directly associated with strain and not with vorticity.

• Strain dominated regions appear to be the most active/nonlinear in a number of aspects.

• Though formally all the flow field is determined entirely by the field of vorticity the relation between the strain and vorticity is strongly nonlocal. In many cases, they are only weakly statistically correlated or not correlated at all.

• The energy cascade (whatever this means) and its final result - dissipation, are associated with predominant self-amplification of the rate of strain and vortex compression rather than with vortex stretching. This means that another nonzero odd moment \(- S_{ij} S_{jk} S_{jk}\) responsible for the production of strain, is not less important than the enstrophy production \(\omega_i \omega_j S_{ij}\).

• Vortex stretching is essentially a process of interaction of vorticity and strain. “Vortices” interact via their strain fields.
A POSSIBLE ORIGIN OF `VORTICES`
THE ‘AMOUNT’ OF COMPRESSING IN ‘WORMS’ IS THE SAME AS IN THE WHOLE FIELD! HENCE INADEQUATE REPRESENTATION OF THE FLOW FILED BY A COLLECTION OF PURELY STRETCHED VORTICES (or other ‘simple’ objects), ESPECIALLY THOSE WHICH DO NOT INTERACT WITH STRAIN. THESE LATTER ARE DESCRIBED BY DIFFERENTIAL EQUATIONS. THE REAL ONES ARE DESCRIBED BY INTEGRO–DIFFERENTIAL EQUATIONS.
SOME COMMENTS ON WHAT IS A “VORTEX”?  

I DO NOT KNOW. WHO DOES? THERE ARE SOME DEFINITIONS (MOSTLY AD HOC). A COMMON FEATURE IS THAT (MORE OR LESS) CONCENTRATED VORTICITY IS SURROUNDED BY PLENTY OF STRAIN, E. G. ‘POTENTIAL’ VORTEX: VORTICITY ONLY IS NOT REALLY ‘ONLY’.  

A popular method to look for structure(s) (and `vortices') is to use a criterion based on one parameter only, e.g. enstrophy $\omega^2$. Though such an approach is useful and `easy', it is inherently limited and reflects the simplest aspects of the problem. For example, even for characterization of some aspects of the local (i.e. in a sense `point'-wise) structure of the flow field in the frame following a fluid particle requires at least two parameters: the second and the third invariants of the velocity gradient tensor $\partial u_i/\partial x_k$: $Q = 1/4(\omega^2 - 2 s_{ij}s_{ij})$ and $R = - 1/3(s_{ij}s_{jk}s_{ki} + 3/4\omega_i\omega_j\omega_{ij})$. Therefore attempts to adequately identify/characterize finite size structure(s) – and `vortices’ seem to be of this kind - by one parameter only are unlikely to be successful, and one needs something like pattern recognition based on some conditional sampling scheme involving definitely more (perhaps much more) than two parameters. But then all the `simplicity' (and attraction) will be gone.  

So it is safer to use well defined quantities – VORTICITY AND STRAIN. VORTICITY ALONE IS NOT ‘ENOUGH’. Concentrated vorticity (very popular) is not that important and other regions (e.g. regions dominated by strain) play essential role in the evolution and dynamics of turbulent flows (Tsinober, 1998, 2001).
Summary of three-dimensional, incompressible flow patterns/
local structure of the flow field in the frame following a fluid particle

\[ Q = \frac{1}{4}(\omega^2 - 2s_{ij}s_{ij}), \]
\[ R = - \frac{1}{3}(s_{ij}s_{jk}s_{ki} + \frac{3}{4}\omega_i\omega_j s_{ij}). \]
VORTICITY AND STRAIN
LIVE TOGETHER
Shiyi CHEN, 2000

DNS of forced NSE in a periodic box

Re_\lambda = 220,
\omega = 3 \omega_{\text{mean}},
\sigma = 3 \sigma_{\text{mean}}
‘STRUCTURES’ OF INTENSE VORTICITY AND STRAIN, Moisy & Jimenez, 2004

(a) Big structure with $|\omega| > 3\omega$;  
(b) Small structure with $|\omega| > 3\omega$;  
(c) Big structure with $|\omega| > 6\omega$

SIMILAR TO THOSE IN  
SHE et al., 1991 and  
BORATAV AND PELZ, 1997  
CHEN, 2000

(a) $|s| > 2.8s$; (b) $|s| > 4.2s$
GENUINE TURBULENCE

Formally, the whole (incompressible) flow field is fully determined by the field of vorticity with appropriate boundary conditions. This is true of strain too, e.g. there is a relation between the fields of velocity and strain similar to that of Biot-Savart

\[ u_i(x,t) = \int \mu_j(r) s_{ij}(y,t) dy, \]
\[ \mu_j(r) = - (2\pi)^{-1} r_j/r^3, \quad r_i = x_i - y_i, \]

One of the most basic phenomena and distinctive features of three-dimensional turbulence is the predominant vortex stretching. This process occurs via *interaction* of *vorticity* and *strain*.

$$(\frac{1}{2})D\omega^2/Dt = \omega_i \omega_j S_{ij} + \nu \omega_i \Delta \omega_i + \varepsilon_{ijk} \omega_i \partial F_k / \partial x_j$$

In fact, there exist *two* nonlocally interconnected and weakly correlated processes: *i)* predominant vortex stretching/enstrophy production, and *ii)* predominant self-amplification of strain. It is this latter process which is responsible for the enhanced dissipation in turbulent flows. Note that the production of strain is much more "self".

The process of self-amplification of strain is a specific feature of the dynamics of genuine turbulence having no counterpart in the behavior of passive objects. Enstrophy production $\omega_i \omega_j S_{ij}$ has an additional role in exchanging "energy" between enstrophy and strain.

$$(\frac{1}{2})D s^2 / D t = - S_{ij} S_{jk} S_{ki} - (1/4) \omega_i \omega_j S_{ij} - S_{ij} \partial^2 p / \partial x_i \partial x_j + \nu s_{ij} \Delta s_{ij} + s_{ij} F_{ij}$$
Rate of enstrophy production and its viscous reduction conditioned on strain and vorticity

NOTE THE LARGE RATE OF ENSTROPHY PRODUCTION IN STRAIN DOMINATED REGIONS (RED CURVE) AS COMPARED TO REGIONS OF LARGE ENSTROPHY (BLUE CURVE)

ALL NONLINEAR TERMS BEHAVE THIS WAY!
Enstrophy production (left) and its rate (right) conditioned on strain and vorticity.
JOINT PDFs OF PRODUCTION OF ENSTROPY (top) AND STRAIN (bottom) WITH ENSTROPHY (left) AND STRAIN (right)

FROM OUR FIELD EXPERIMENT AT Re$_\lambda$ = 10$^4$

*Physics of Fluids*, 13, 311 (2001)
All the prongs are made of manganine with resistance temperature coefficient 400 smaller than that of tungsten. Five cold-wires added for temperature and its gradient measurements.
NONLOCALITY OF VORTICITY/STRAIN RELATION
**HOW THIS LOOKS ON THE R – Q PLANE**

\[ Q = \frac{1}{4} \{ \omega^2 - 2s^2 \}; \quad R = - \frac{1}{3} \{ s_{ij} s_{jk} s_{ki} + \frac{3}{4} \omega_i \omega_j s_{ij} \} \]

CONDITIONAL AVERAGES ON THE R-Q PLANE OF **REYNOLDS STRESS** (LEFT) AND **TKE ‘generating events’** (RIGHT), **CHASIN AND CANTWELL**, 2001

This quadrant is dominated by strain and strain production

- This quadrant is dominated by strain and strain production
- Sweeps (+u-v)
- Ejections (-u+v)

\[ D=0 \]

\[ uv/u_t^2 \]

\[ u_t/u_t^2 \]
HOW THIS LOOKS ON THE $R - Q$ PLANE

$Q = (1/4)\{\omega^2 - 2s^2\}; \quad R = - (1/3)\{s_{ij}s_{jk}s_{ki} + (3/4)\omega_i\omega_j\epsilon_{ij}\}$

CONDITIONAL AVERAGES ON THE R-Q PLANE OF REYNOLDS STRESS (LEFT) AND DISSIPATION (RIGHT), CHASIN AND CANTWELL, 2001
SELF-AMPLIFICATION OF VORTICITY AND STRAIN

SELF-RANDOMIZATION: NO SOURCE OF RANDOMNESS IS NEEDED

AT THE LEVEL OF VELOCITY
DERIVATIVES: VORTICITY
AND STRAIN (DISSIPATION)

THE EXTERNAL FORCING IS
IRRELEVANT

Three cases:
1. DNS in a periodic box, $Re_\lambda=10^2$
2. DNS in a channel flow, $Re=5600$
3. Atmospheric SL, $Re_\lambda=10^4$; $Re=10^8$
The process of evolution and amplification of disturbances - both in genuine and `passive' turbulence - is dominated by the strain field of the basic flow. In all their energy production has the form
\[ \Delta u_i \Delta u_j S_{ij} \]
*Tsinober & Galanti, 2003*
`PASSIVE' TURBULENCE

Though vorticity has some influence, strain is dominating the evolution and behaviour of passive objects (scalars, vectors) in turbulent and any Lagrangian chaotic flows. The process of self-amplification of vorticity, along with essential differences, has a number of common features with analogous processes in passive vectors: in both the main factor is their interaction with strain. In contrast the production of strain is much more `self'. Moreover, it should be stressed that the process of self-amplification of strain is a specific feature of the dynamics of genuine turbulence having no counterpart in the behavior of passive objects.
MATERIAL LINES AND ENSTROPHY PRODUCTION CONDITIONED ON STRAIN (red) AND VORTICITY (green), (re)plotted from DNS by Huang (1996)
MATERIAL LINES – PRODUCTION, $|i|jS_{ij}$ (left) AND ITS RATE, $|i|jS_{ij}$ (right) CONDITIONED ON STRAIN (red) AND VORTICITY (blue), From PTV experiments by LÜTHI et al, 2003
\[
\frac{DG_i}{Dt} = -G_k \frac{\partial u_k}{\partial x_i} + D \nabla^2 G_i; \quad G_i \equiv \partial T / \partial x_i
\]

\[
- G_k \frac{\partial u_k}{\partial x_i} \equiv -G_j s_{ij} - \frac{1}{2} \varepsilon_{ijk} \omega_j G_k
\]

\[
\frac{1}{2} \frac{D}{Dt} G^2 = -G_i G_k s_{ik} + D G_i \nabla^2 G_i
\]

\[
- \left\langle G_i G_k s_{ik} \right\rangle > 0
\]
Conditional averages on vorticity and strain

Production of $G_i = \partial T/\partial x_i$

Note the essential difference in the behavior of the production of temperature gradients in regions dominated by vorticity and strain: the production of temperature gradients is much more intensive in the regions dominated by strain. Another feature is that the production of temperature gradients is practically independent of the magnitude of vorticity.
The results shown seem to be of universal nature, at least qualitatively, as can be seen from comparison of the plots \(c, d\) with the plots \(a, b\).
\[ \frac{DB}{Dt} = (B \cdot \nabla)u + \eta \Delta B; \quad \frac{1}{2}\frac{DB^2}{Dt} = B_i B_j s_{ij} + \eta B_i \Delta B_i \]

Conditional averages of production \( B_i B_j s_{ij} \) and its rate \( B_i B_j s_{ij}/B^2 \) on vorticity and strain.
EIGENCONTRIBUTIONS TO THE RATE \( \frac{B_i B_j s_{ij}}{B^2} = \Lambda_k \cos^2(\omega, \lambda_k) \)

Though vorticity has some influence, strain is dominating the evolution and behaviour of passive objects (scalars, vectors) in turbulent and any Lagrangian chaotic flows.
PASSIVE VECTORS VERSUS VORTICITY

COMPARISON OF ENSTROPHY PRODUCTION WITH SIMILAR TERMS FOR PASSIVE VECTORS FOR GENUINE TURBULENCE

RANDOM GAUSSIAN FIELD

Note symmetric PDF of $\omega_i\omega_j^*$ (green curve)
GENUINE VERSUS “PASSIVE” TURBULENCE:

A REMINDING

The process of self-amplification of vorticity, along with essential differences, has a number of common features with analogous processes in passive vectors: in both the main factor is their interaction with strain. In contrast the production of strain is much more `self'. Moreover, the process of self-amplification of strain is a specific feature of the dynamics of genuine turbulence having no counterpart in the behaviour of passive objects.
Though the issue of drag reduction (more generally the interaction of polymers with turbulence) is far from being understood, it is clear that the direct (inter)action of the dissolved polymers with the fluid flow is in the small scales. This interaction of polymers and the flow field occurs via the field of strain.

The additional term appearing in the NSE is \( \frac{\partial \tau_{ik}}{\partial x_k} \) expressed in a variety of models via the so-called conformation tensor \( R_{ik} \)

\[
\tau_{ik} = \frac{(v_p)}{(\tau_p)} \{f(x,t)\rho_0^{-2}R_{ik} - \delta_{ik}\}, \text{f(x,t)=} \{((\rho_m^2 - \rho_0^2)/(\rho_m^2 - R_{kk}(x,t)) \}
\]

and \( R_{ik} \) is governed by the following equation

\[
\frac{D R_{ij}}{D t} = \frac{\partial u_i}{\partial x_k} R_{kj} + \frac{\partial u_k}{\partial x_j} R_{ik} \equiv \frac{1}{\tau_p} \{f(x)R_{ij} - \rho_0^2\delta_{ij}\}
\]

Since

\[
\frac{\partial u_i}{\partial x_k} R_{kj} + \frac{\partial u_k}{\partial x_j} R_{ik} \equiv s_{ik} R_{kj} + R_{ik} s_{kj}
\]

it is seen that the stress tensor \( \tau_{ij} \) is indeed a (Lagrangian and pretty complicated nonlinear) functional of strain only as should be.
Inhibition of vortex stretching first suggested by Pfenninger in 1967. 

Potential vortex is an $\omega$-singularity surrounded by plenty of strain.

Vortex cavitation of finite span hydrofoil is inhibited in a similar way by drag-reducing polymer-solutions, Fruman and Aflalo (1989).
TURBULENCE IN DILUTE POLYMER SOLUTIONS
A PARTICLE TRACKING EXPERIMENT IN QUASI-HOMOGENEOUS TRUBULENT FLOW \(\text{(Liberzon et al., 2004)}\). 
SUPPRESSION OF 
(entrainment and enstrophy production are suppressed too but less)

\[
\text{STRAIN, } S^2 = S_{ij} S_{ij} \\
\text{PRODUCTION OF STRAIN, } - S_{ij} S_{jk} S_{ki}
\]
ADDITIONAL ASPECTS

PRODUCTION OF TURBULENT ENERGY IN SHEAR FLOWS IS THE RESULT OF INTERACTION OF THE FIELD OF FLUCTUATIONS WITH MEAN STRAIN AND IS DUE TO PREDOMINANT COMPRESSION OF FLUID ELEMENTS: 

\[ (u_i u_k s_{ik}) \]

IN TWO-DIMENSIONAL ‘TURBULENCE’ VORTICITY IS QUASI - ‘PASSIVE’ WHEREAS STRAIN IS ‘ACTIVE’: IT ACTS IN PRODUCTION OF GRADIENTS OF PASSIVE SCALAR \(- G_i G_k s_{ij}\); PASSIVE VECTORS: material lines \(- I_i I_k s_{ij}\), magnetic field \(- B_i B_k s_{ij}\), AND ALSO THE GRADIENTS OF VORTICITY \(- \xi_i \xi_k s_{ij}\).
TO CONCLUDE: STATEMENTS LIKE

“In the presence of a velocity field, the diffusion is altered by the stirring of the substrate and is usually enhanced, as for example in a straining field or in a vortex”.

CONTAIN A MISLEADING ELEMENT, SINCE VORTEX IS NOT JUST VORTICITY: IT IS ALSO PLENTY OF STRAIN — THERE IS HARDLY ANY MIXING WITHOUT STRAIN