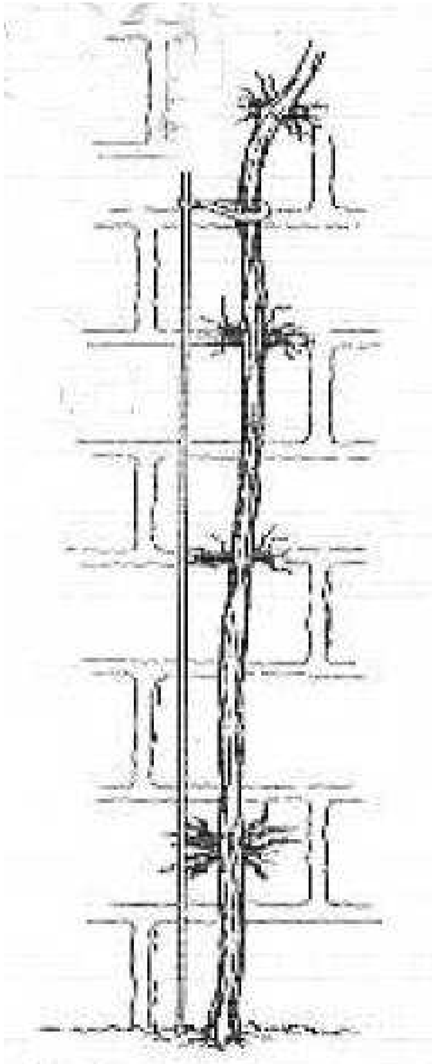
Climbing plants: how thick should their supports be?

Sebastien Neukirch
Universite Paris 6
CNRS – France

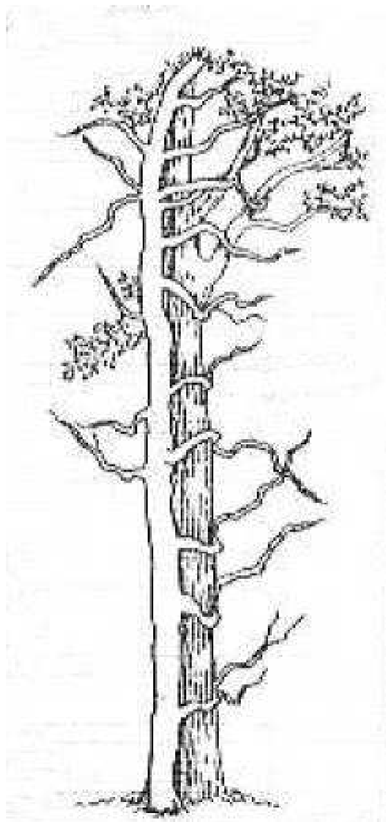
joint work with:
Alain Goriely
University of Arizona
USA

Morning Glory (Ipomoea purpurea) twining up a corn stalk

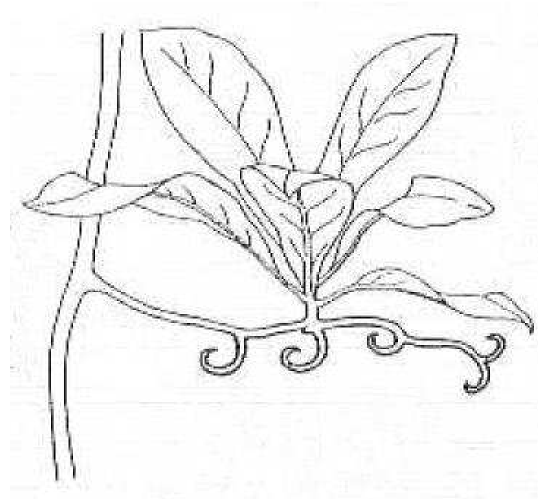
Different kinds of climbing plants



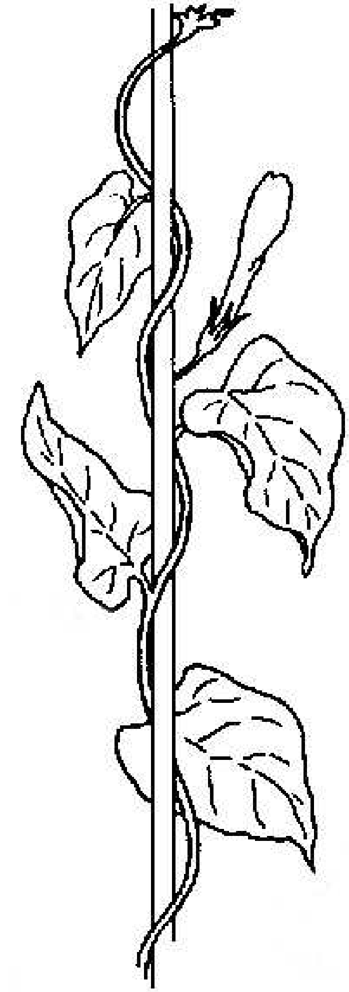
rooter



rooter



hooker



twiner

Twiners: some botanical facts

Goal : reach the canopy (the light).
Use as few structural tissues as possible.
Should be able to twine around different supports
(thick or not, slippery or not)
Evolution from self supporting to supported growth:
smaller stem diameter, more flexible

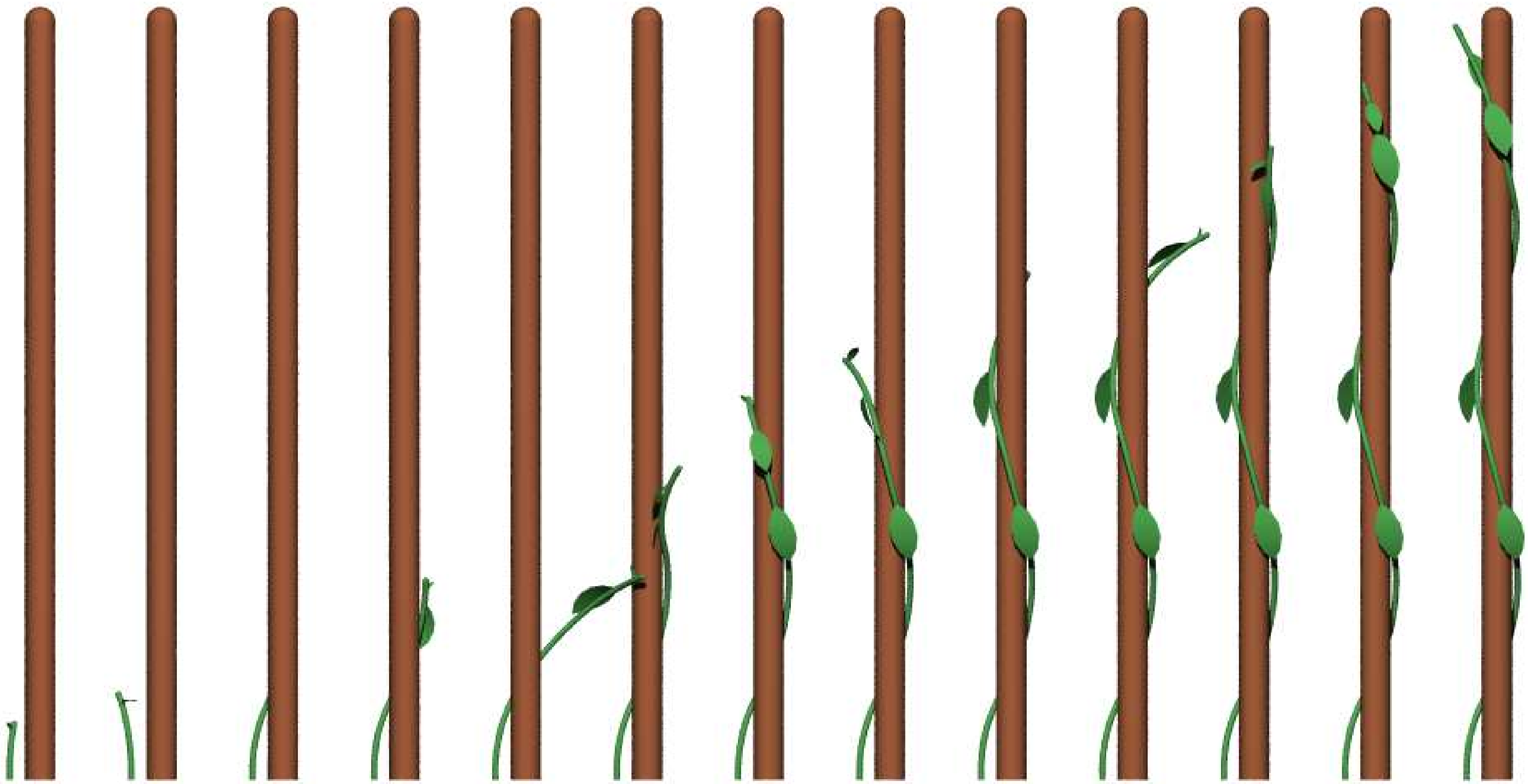
Typical growth speed: 1 cm / hour

Two different zones :

1- apex (search for support, goes around it)
2- lower part of stem (helix)

video

Twining, step by step

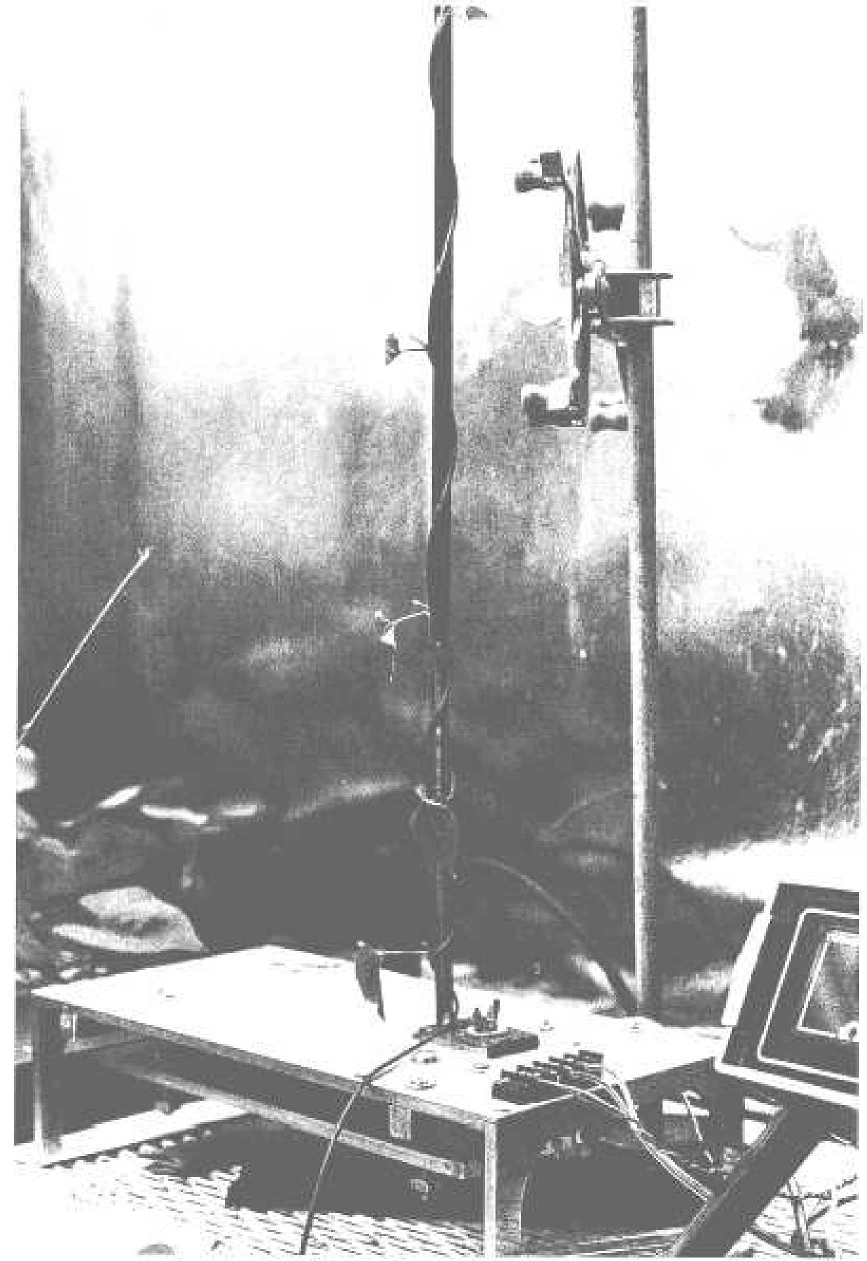


from Knut Arild Erstad www.ii.uib.no/~knute/   (artist view)

Mechanical experiments (W. Silk)

<u>Measurements</u>:
- geometrical parameters (on & off pole)
- contact pressure

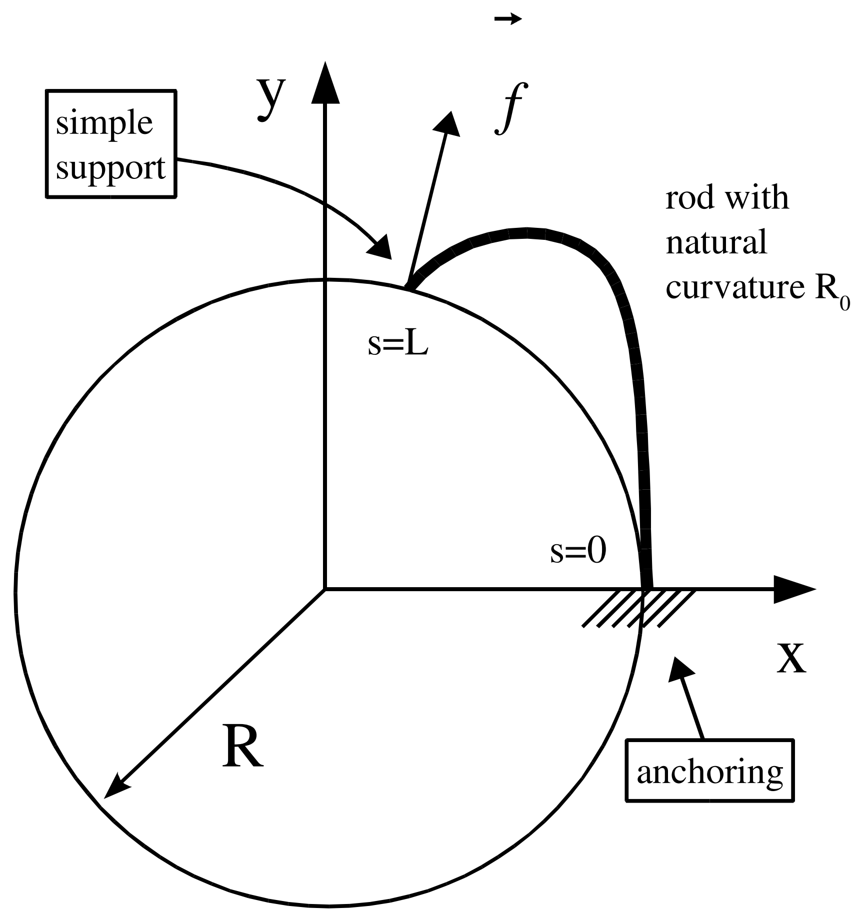
<u>Results</u>:
- stem is in tension
- contact pressure >> weight
- uniform helix
- lower pitch on pole



A model: Equilibrium of an elastic rod (Kirchhoff equations)

$$\begin{cases} \vec{N}' + \vec{p} = \vec{0} \quad : \quad \textit{force balance} \\ \vec{M}' + \vec{r} \times \vec{N} = \vec{0} \quad : \quad \textit{moment balance} \\ \vec{r}' = \vec{t} \quad : \quad \textit{tangent} \\ M_i = B_i(\kappa_i - \kappa_{i0}) \quad : \quad \textit{linear elasticity} \end{cases}$$

$$' \equiv \frac{d}{ds} \quad ; \quad (s : \textit{arclength})$$

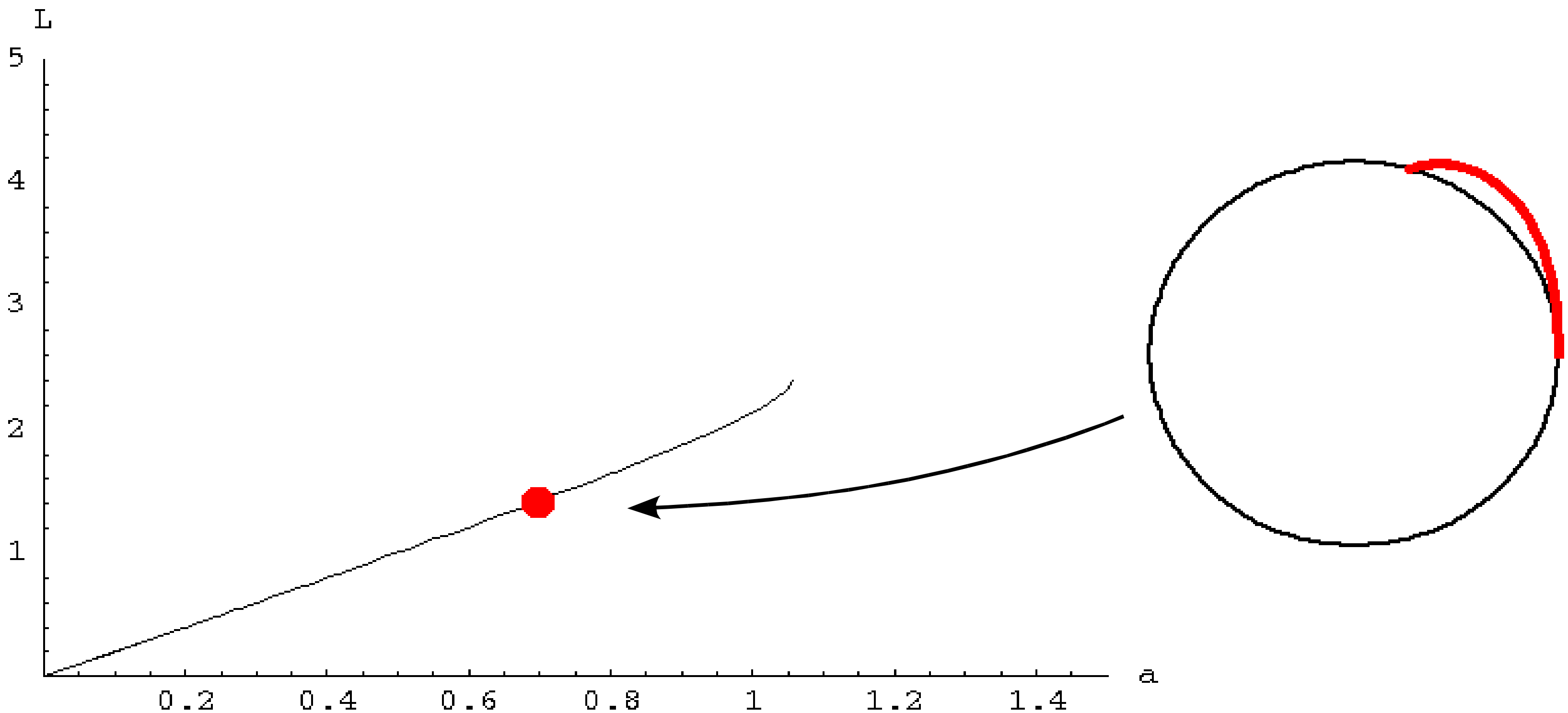


Ordinary differential equations with boundary conditions:

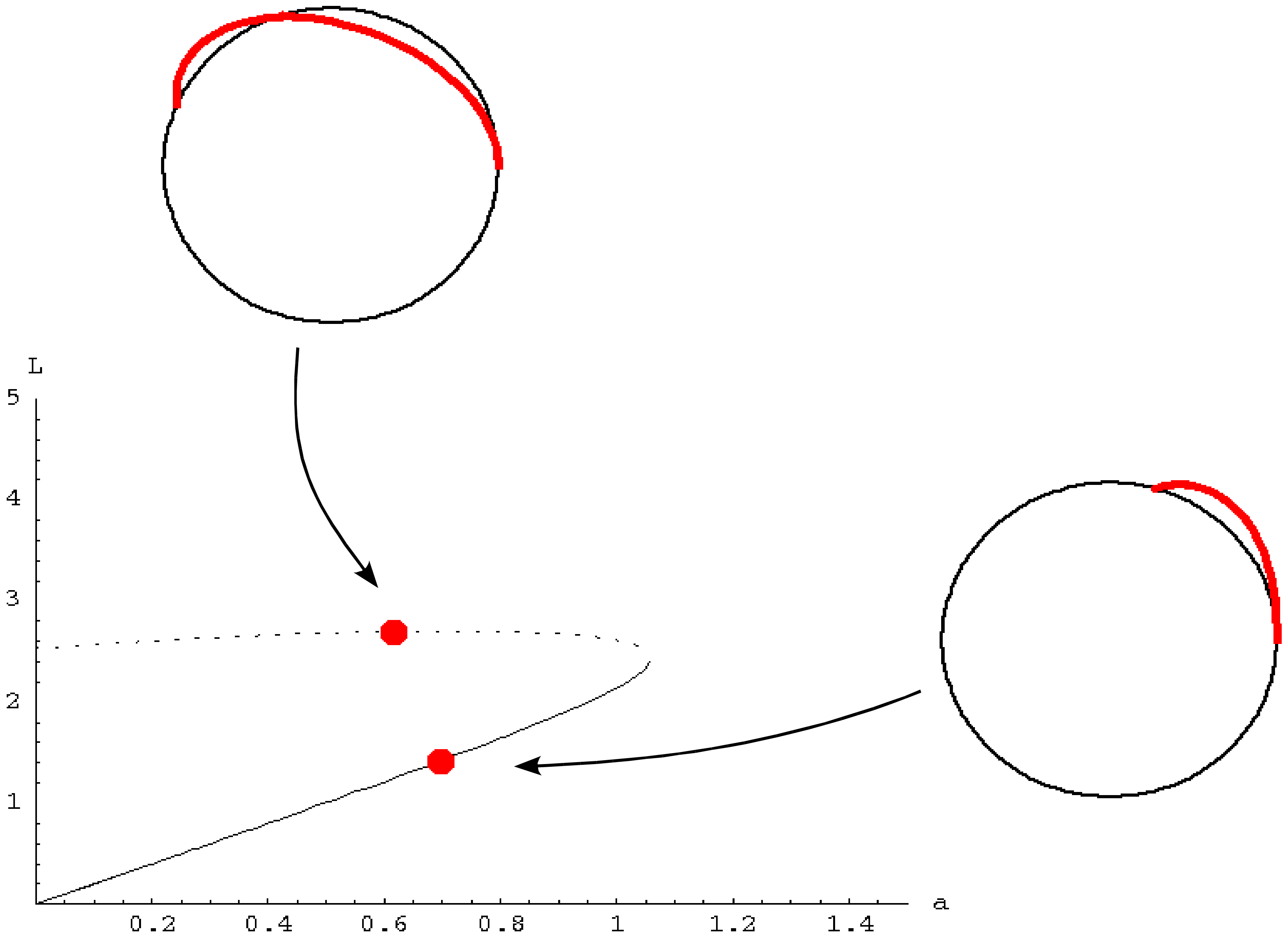
$$s = 0 \quad : \quad \textit{anchoring} \quad : \quad \vec{t}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$s = L \quad : \quad x(L)^2 + y(L)^2 = R^2 \quad \textbf{\textit{with}} \quad \vec{N}(L) = \vec{f} \;\; \| \;\; \begin{pmatrix} x(L) \\ y(L) \end{pmatrix}$$

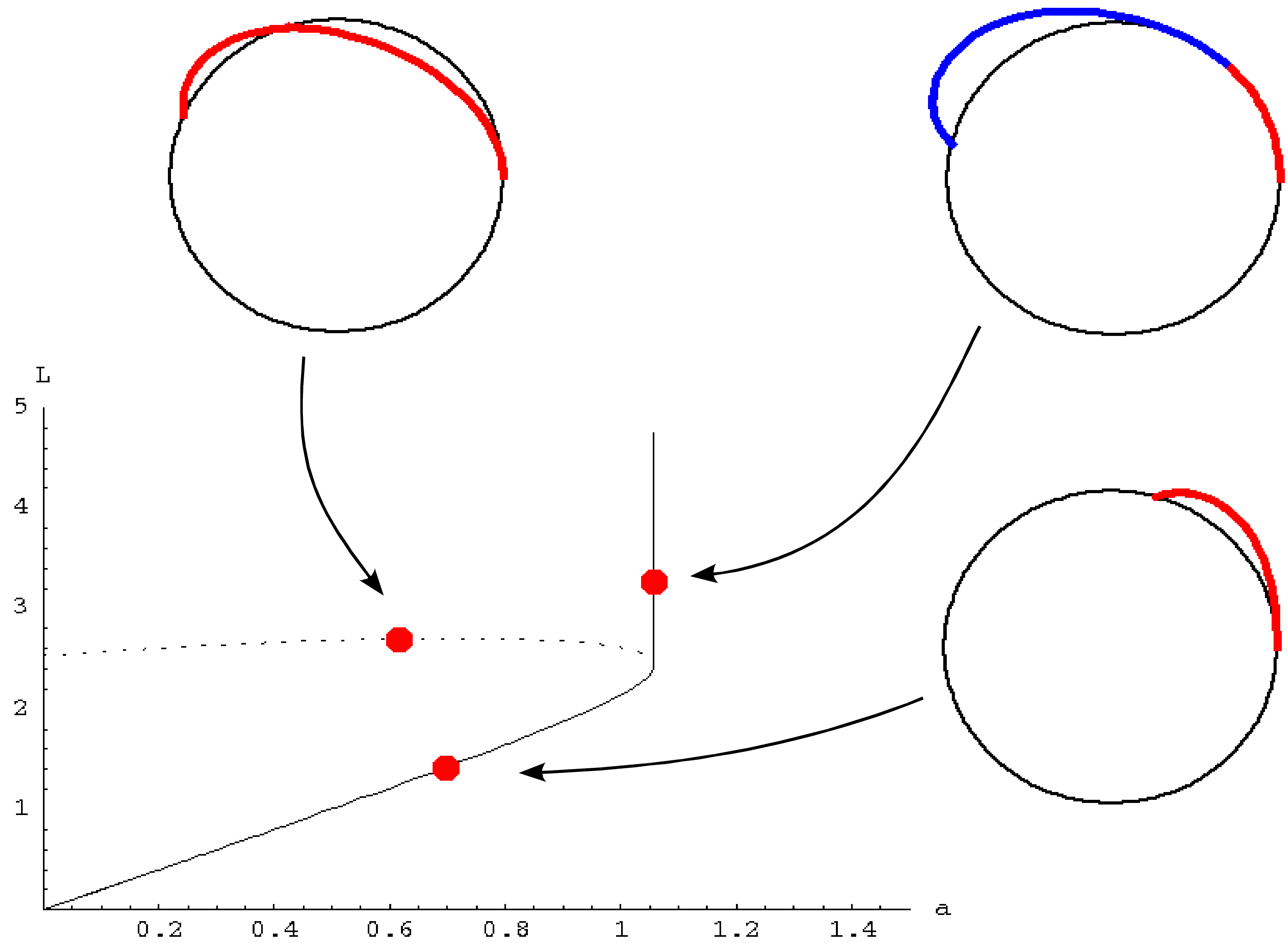
Numerical continuation of solutions : bifurcation diagram K = R / R0 = 3



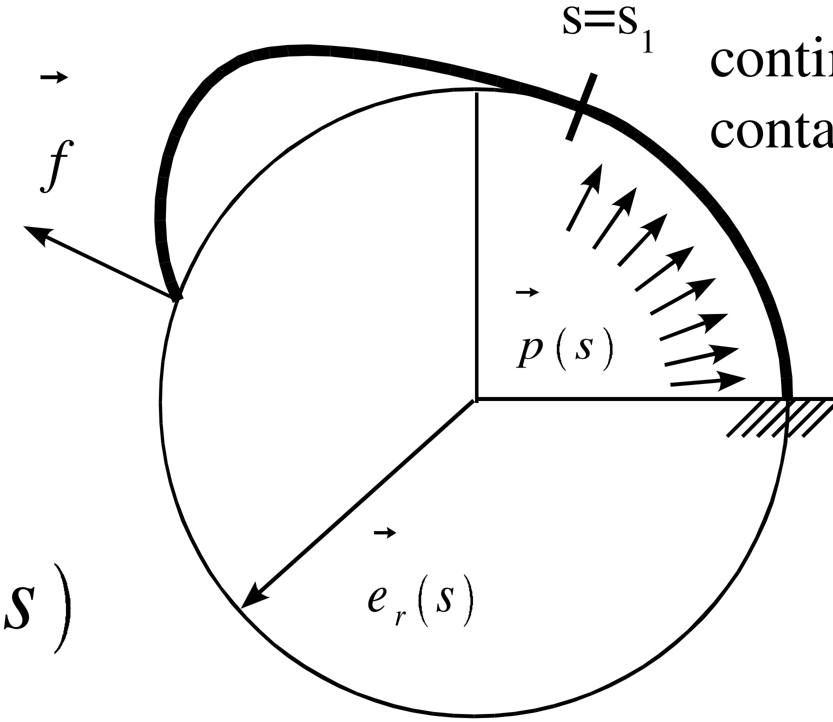
Numerical continuation of solutions : bifurcation diagram K = R / R0 = 3



Numerical continuation of solutions : bifurcation diagram K = R / R0 = 3



Configurations with continuous contact



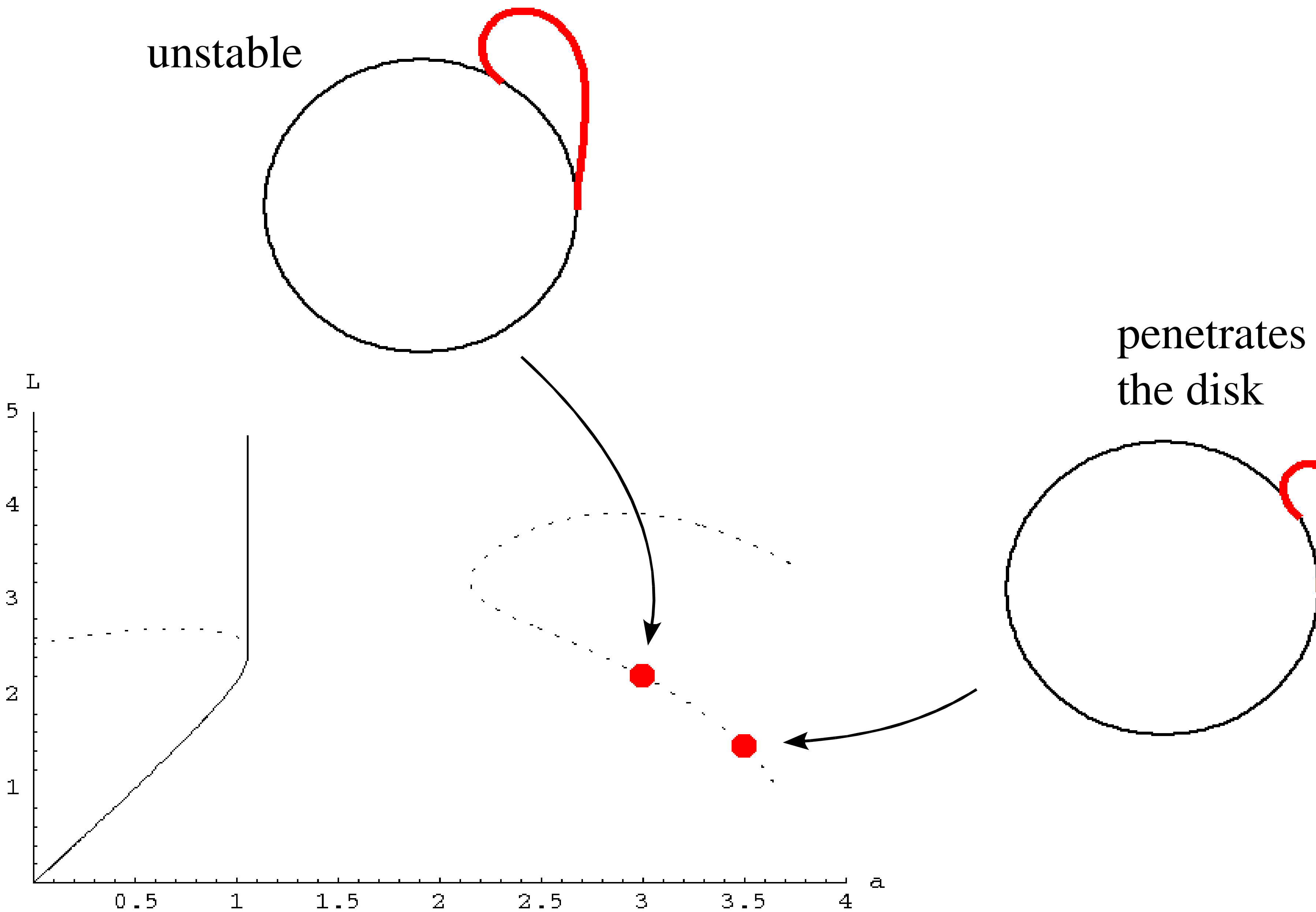
tension: $\vec{N} \cdot \vec{t} = p\,R > 0$

$$\vec{p}(s) = \frac{EI}{R^2}\left(\frac{1}{R_0} - \frac{1}{R}\right)\vec{e}_r(s)$$

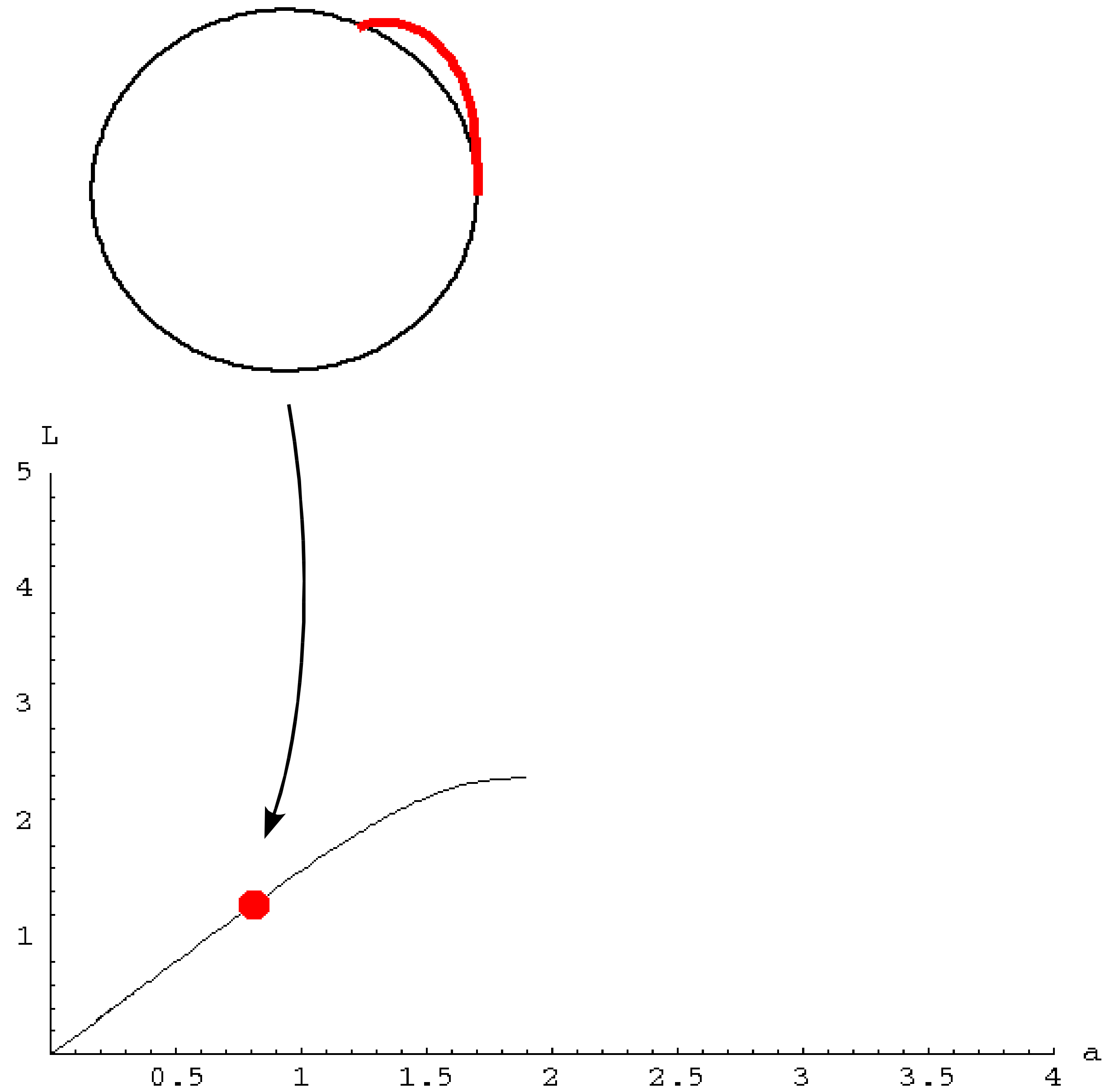
<u>The continuous part can be lengthen arbitrarily</u>

These configurations correspond to climbing cases

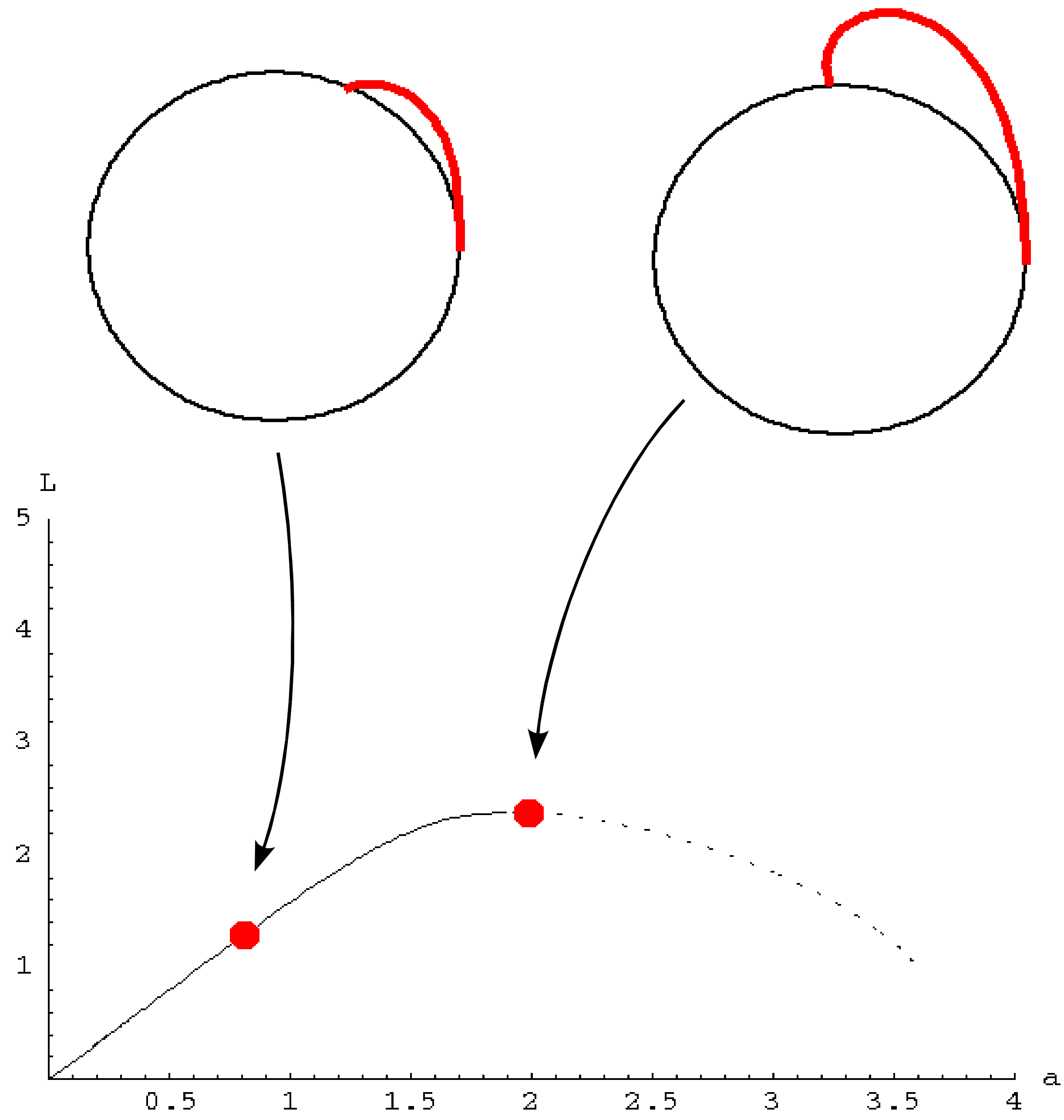
Numerical continuation of solutions : bifurcation diagram K = R / R0 = 3



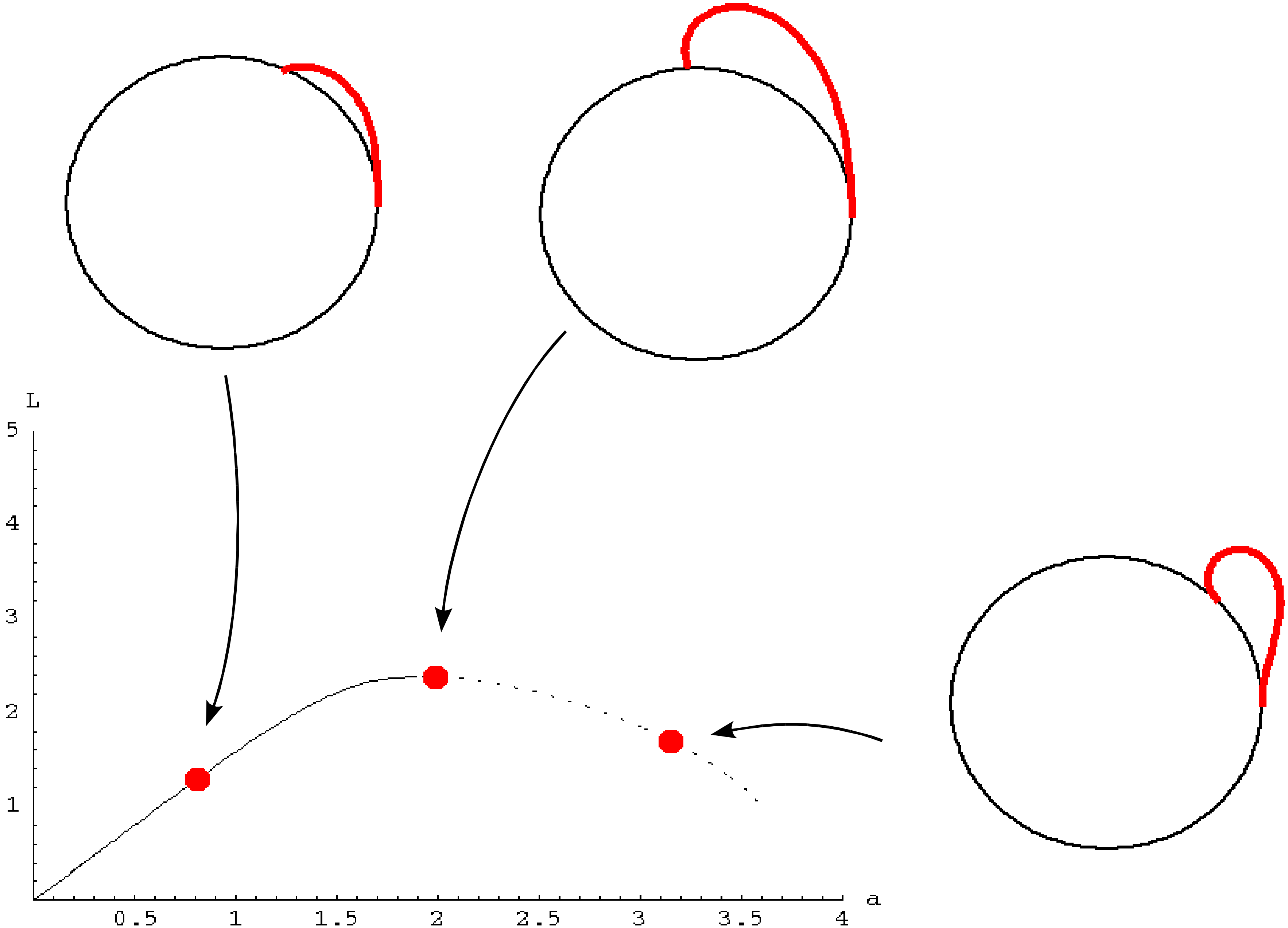
Numerical continuation of solutions : bifurcation diagram K = R / R0 = 3.5



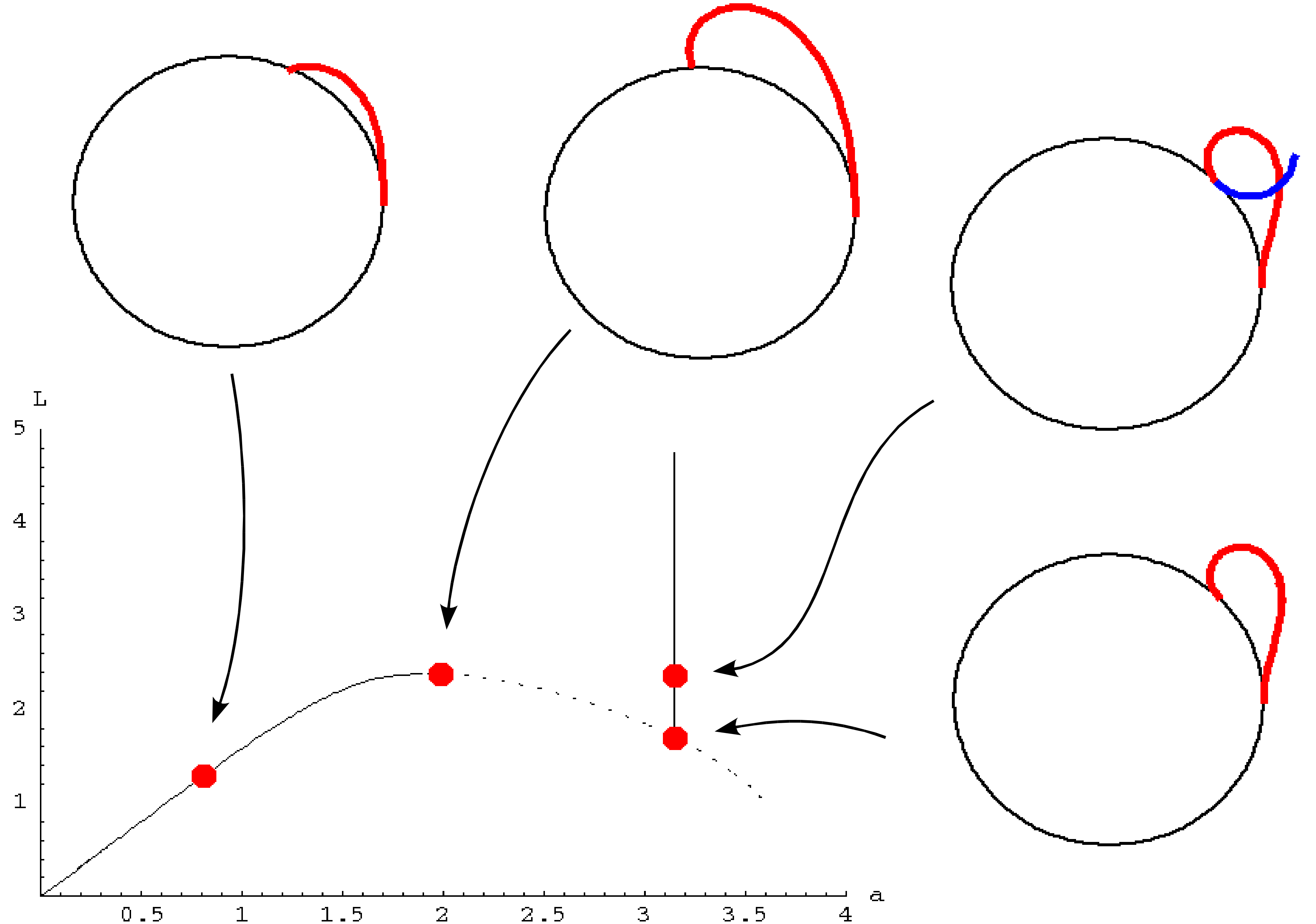
Numerical continuation of solutions : bifurcation diagram K = R / R0 = 3.5



Numerical continuation of solutions : bifurcation diagram K = R / R0 = 3.5



Numerical continuation of solutions : bifurcation diagram K = R / R0 = 3.5



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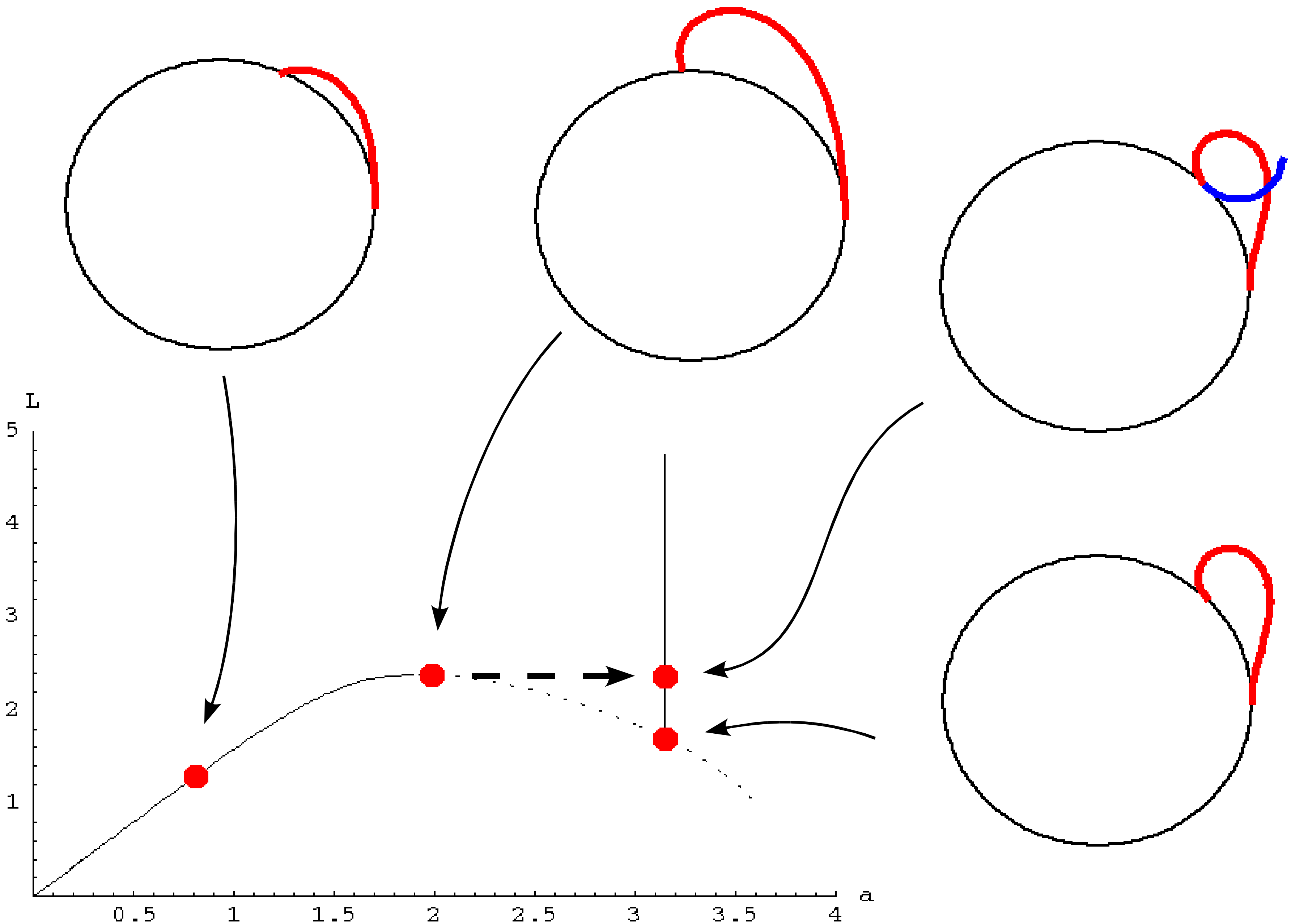
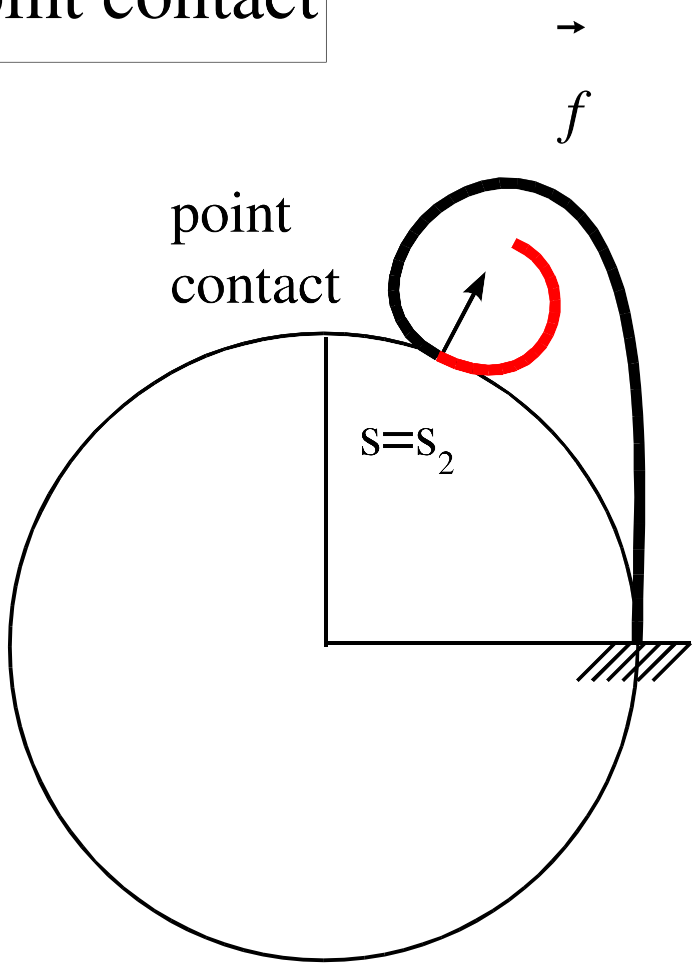


Configurations with point contact

$$at\ s = s_2: \quad \vec{N}^+ - \vec{N}^- + \vec{f} = 0$$

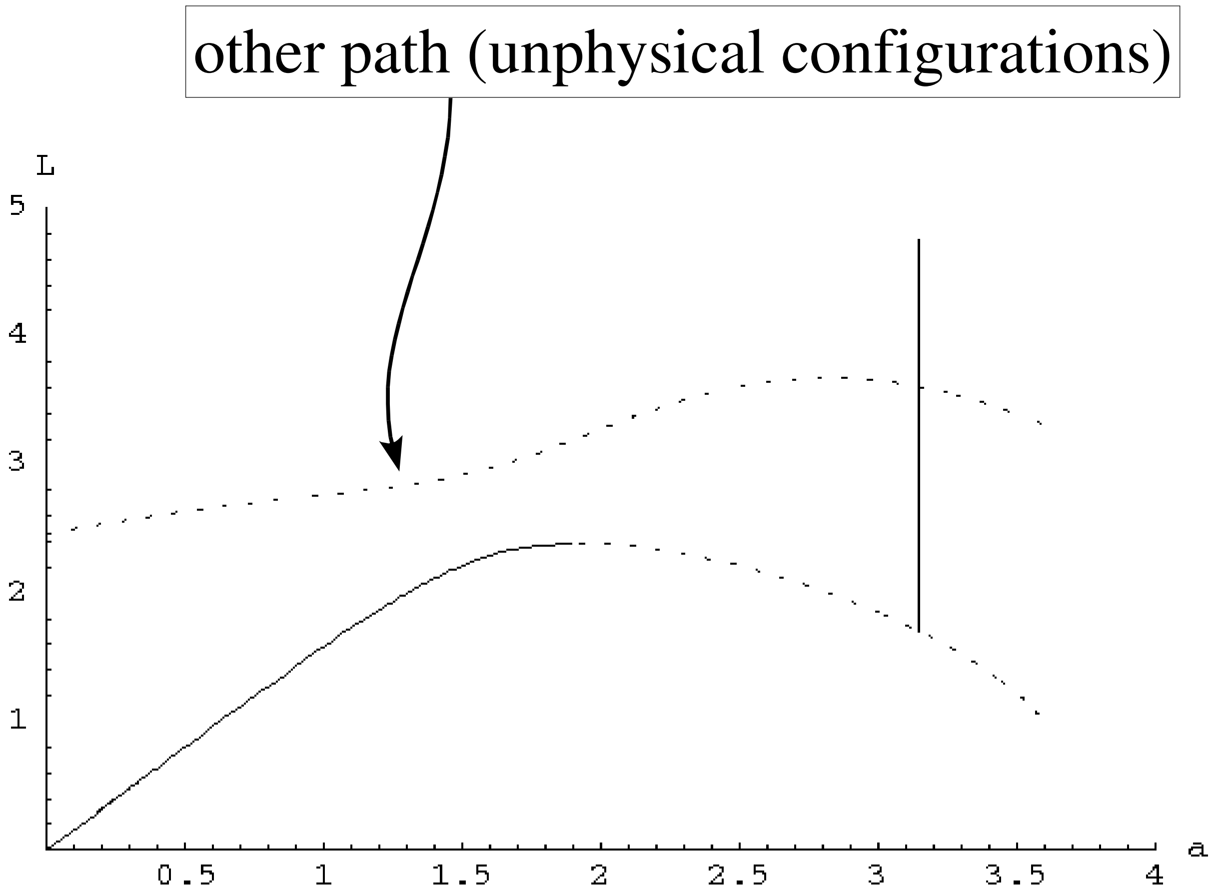
for $s > s_2$, unstressed shape



<u>The free part can be lengthen arbitrarily</u>

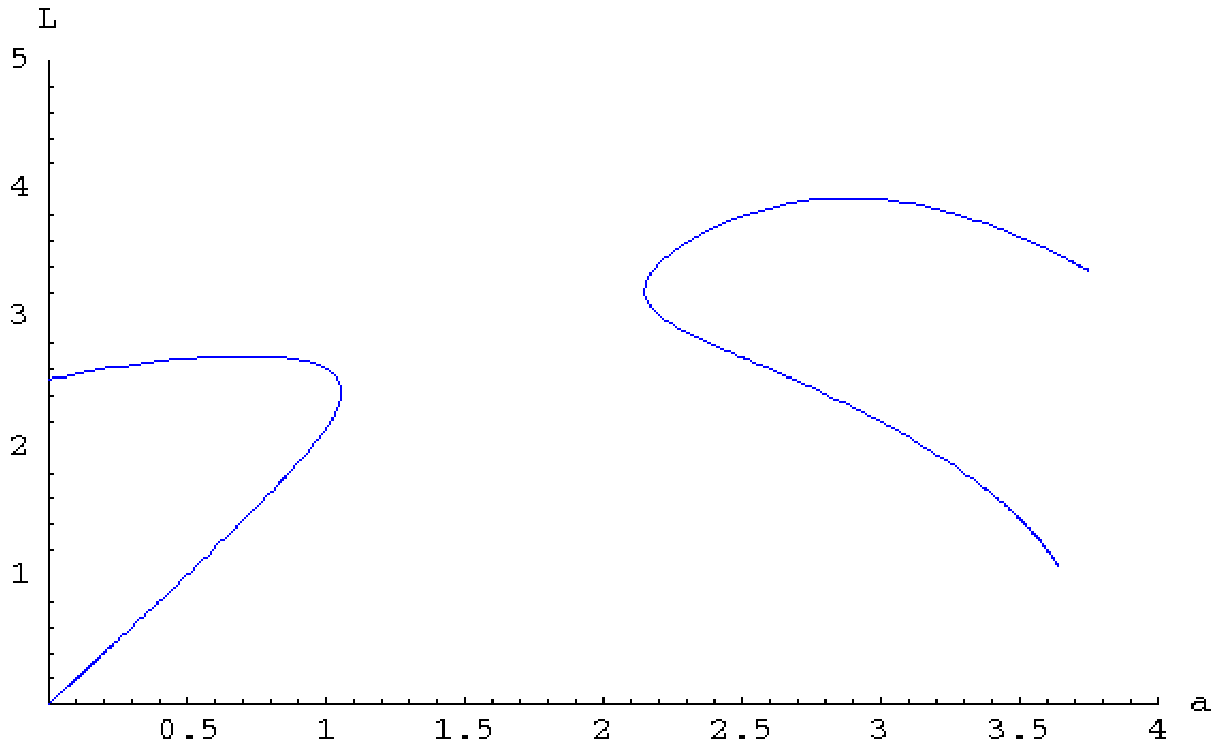
These configurations correspond to *non-climbing* cases

Numerical continuation of solutions : bifurcation diagram K = R / R0 = 3.5

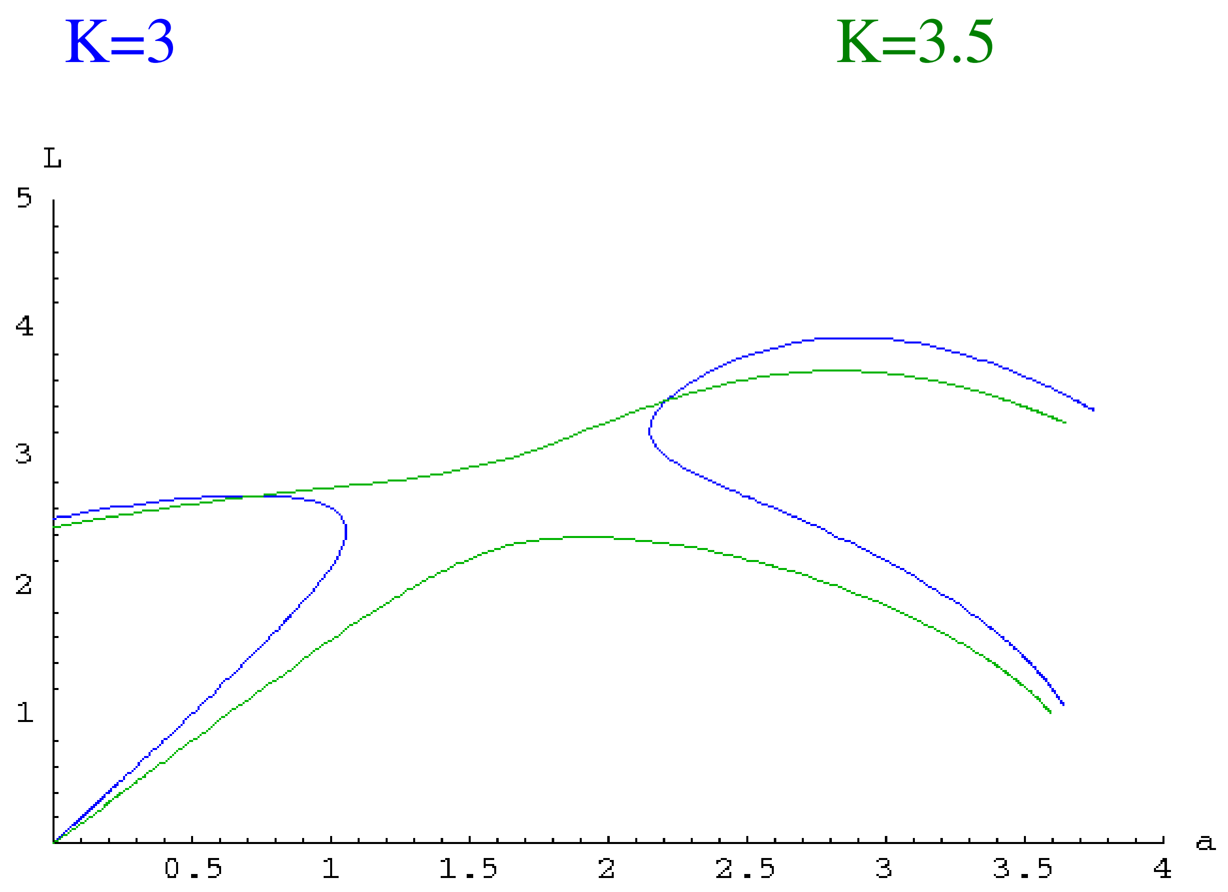


Numerical continuation of solutions : bifurcation diagrams when K varies

K=3

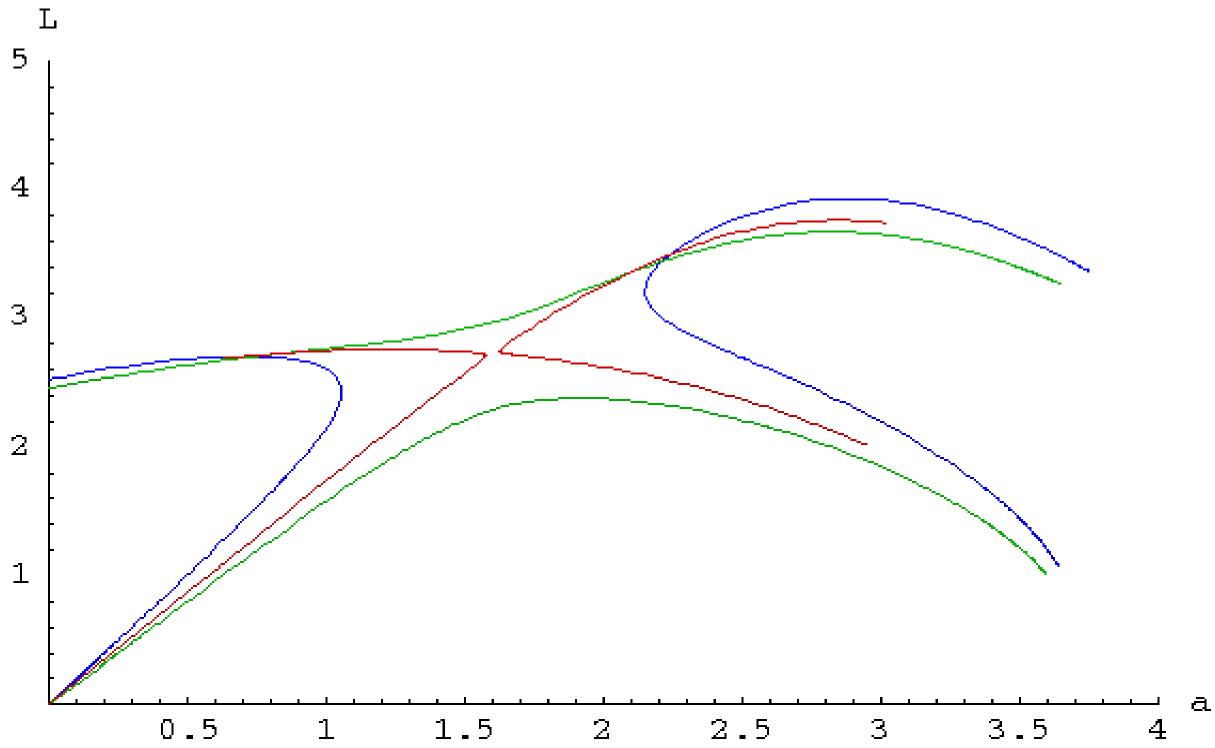


Numerical continuation of solutions : bifurcation diagrams when K varies



Numerical continuation of solutions : bifurcation diagrams when K varies

K=3 K=3.3085 K=3.5



Numerical continuation of solutions : bifurcation diagrams when K varies

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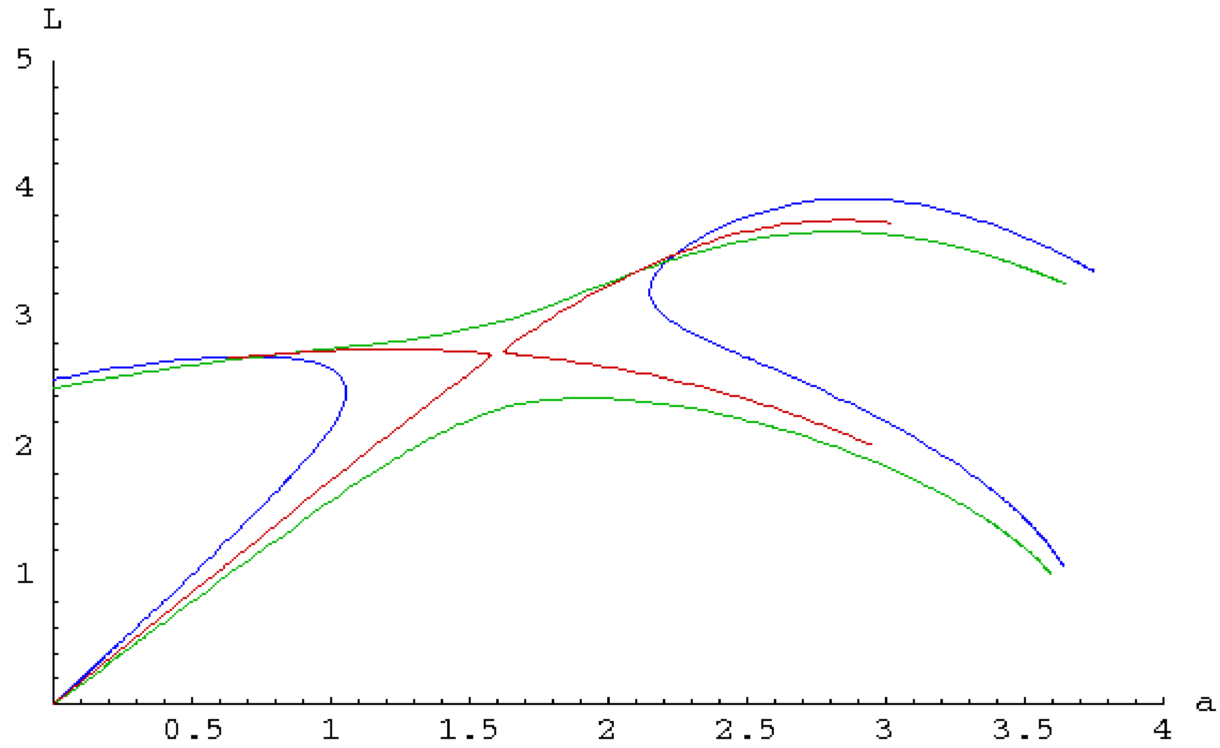
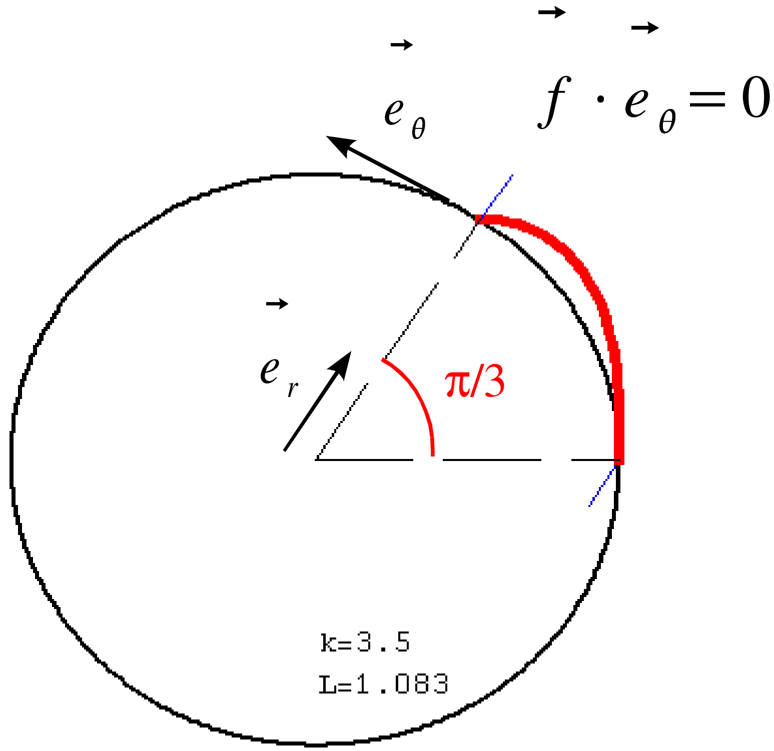


Conclusion : in the 2D case, $K_{max} \simeq 3.31$

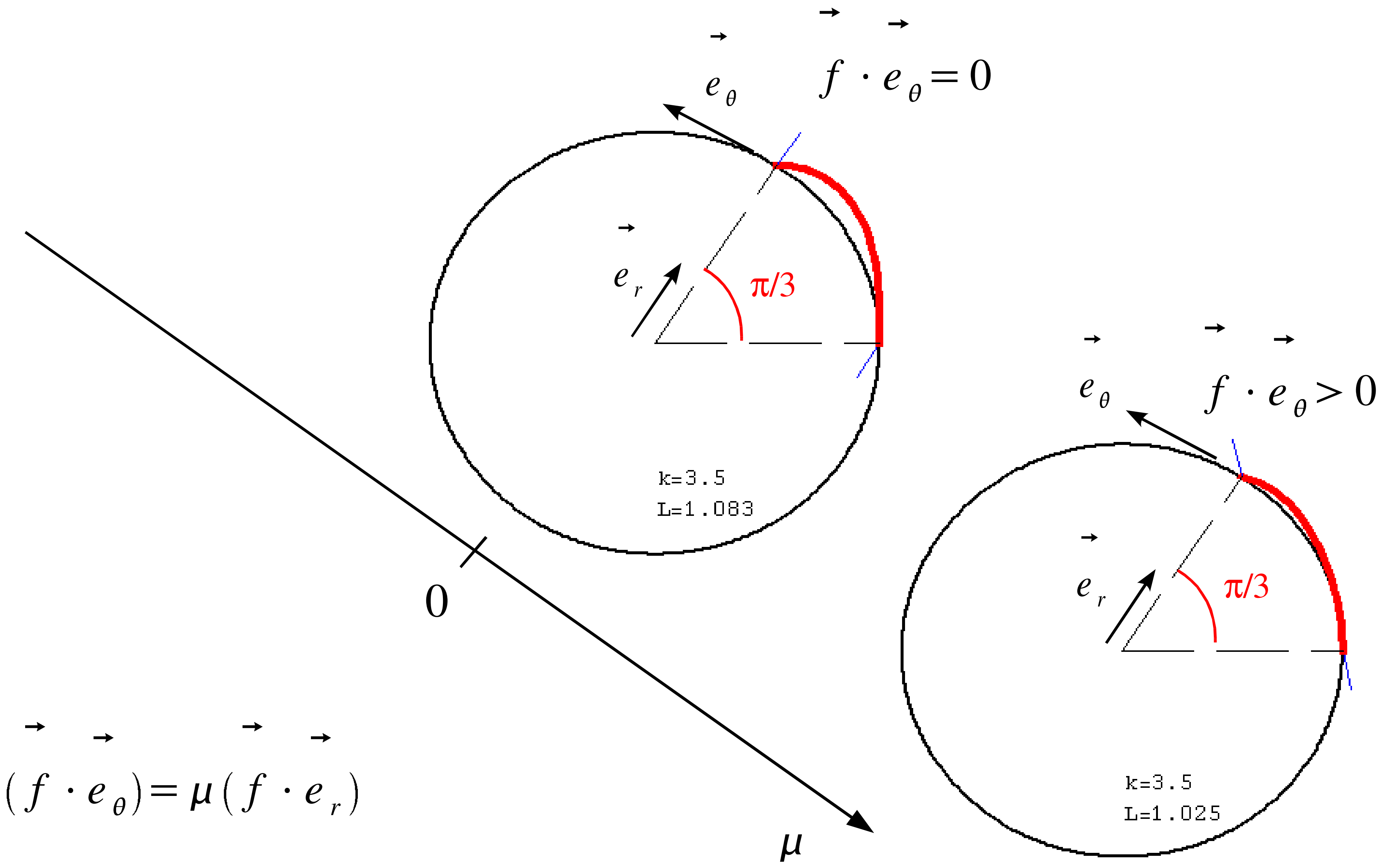
Friction



$$\mu = 0$$

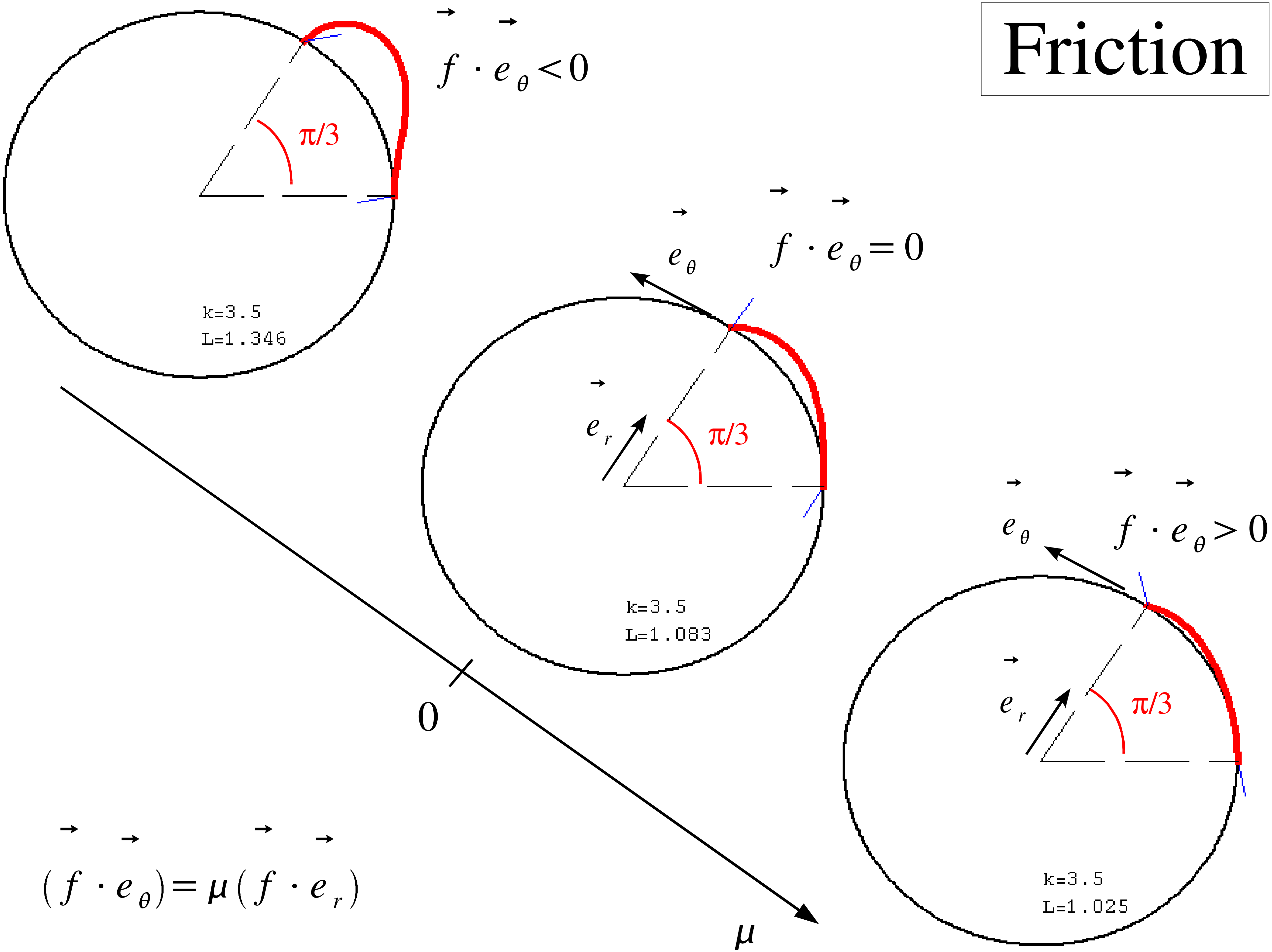
$$(\vec{f} \cdot \vec{e}_\theta) = \mu (\vec{f} \cdot \vec{e}_r)$$

Friction



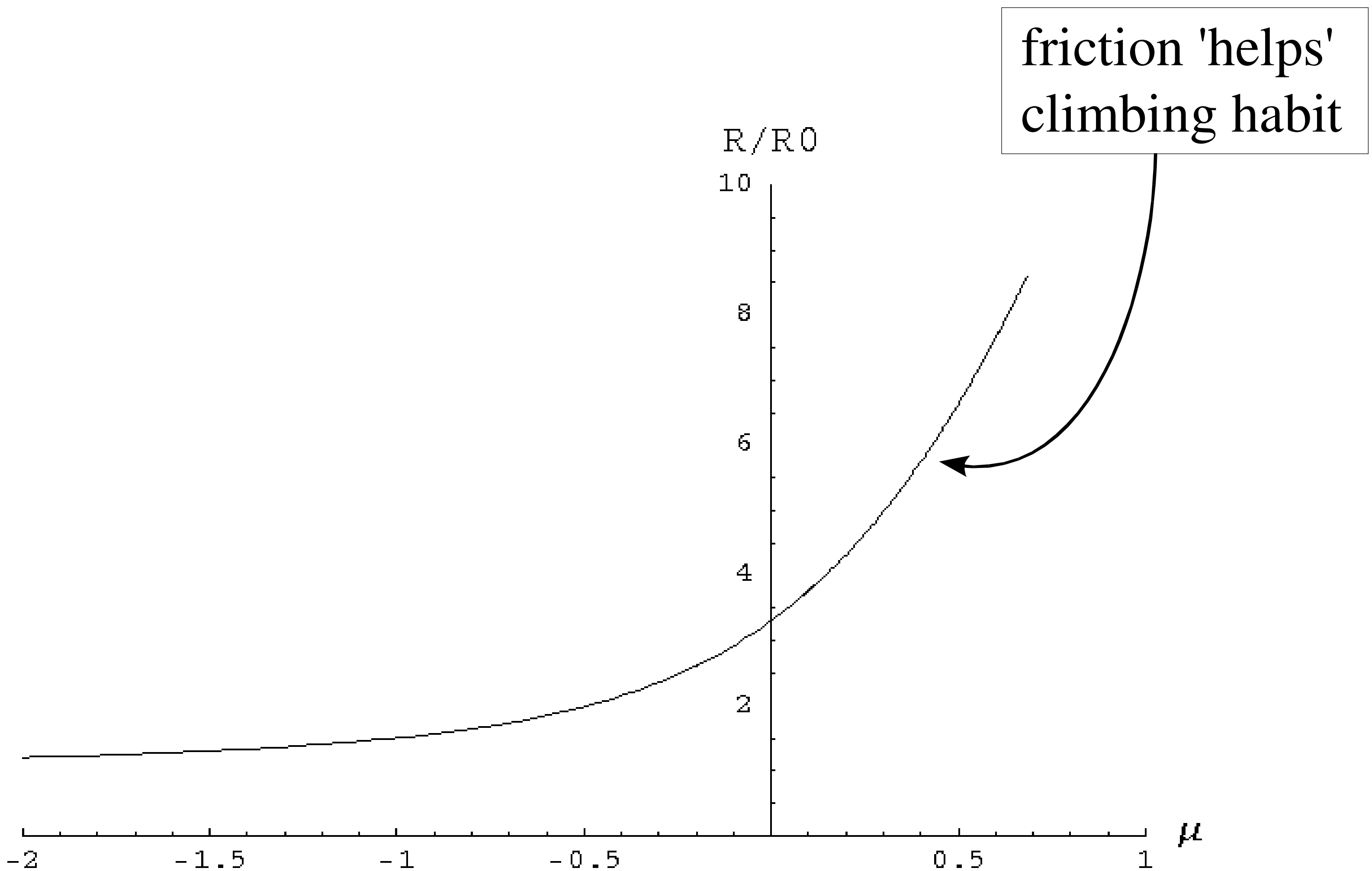
$\vec{e}_\theta$
$\vec{f} \cdot \vec{e}_\theta = 0$
$\vec{e}_r$
π/3
k=3.5
L=1.083
0
$\vec{e}_\theta$
$\vec{f} \cdot \vec{e}_\theta > 0$
$\vec{e}_r$
π/3
k=3.5
L=1.025
$(\vec{f} \cdot \vec{e}_\theta) = \mu (\vec{f} \cdot \vec{e}_r)$
μ

Friction



$$(\vec{f} \cdot \vec{e}_\theta) = \mu (\vec{f} \cdot \vec{e}_r)$$

Friction



The 3D case

R : cylindrical support radius

R_0 : natural (intrinsic) radius of curvature

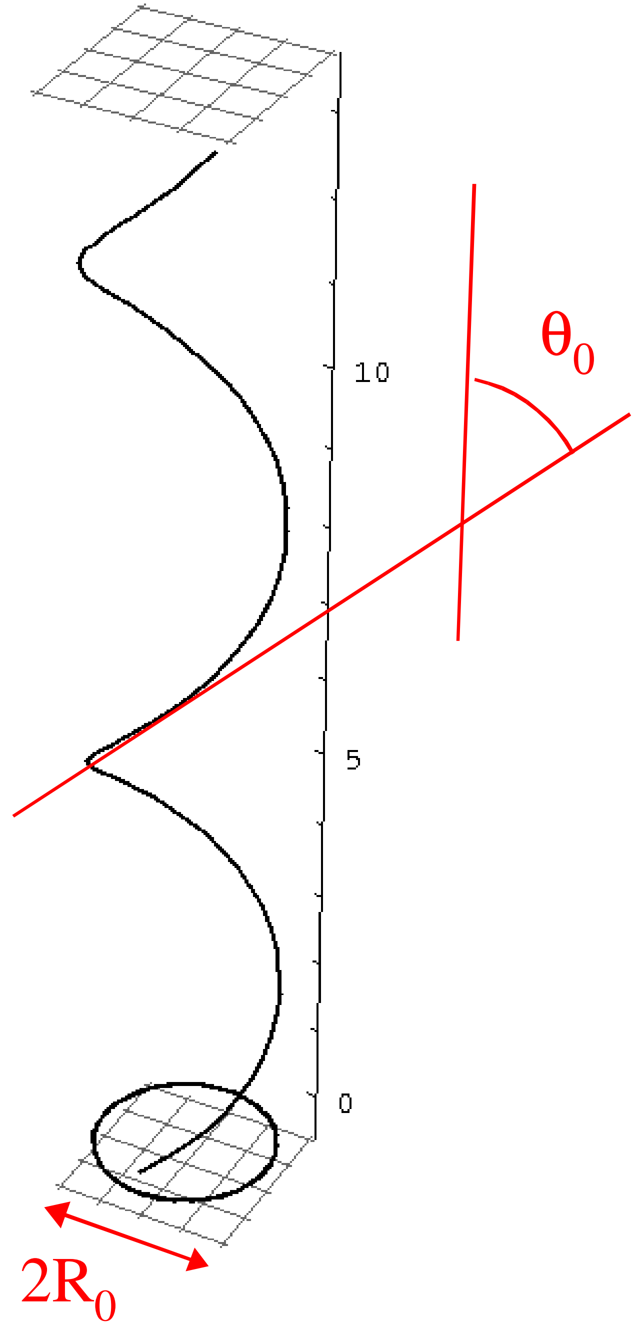
θ_0 : natural (intrinsic) helical angle

nearly helical solutions : climbing angle θ

-climbing angle $\theta = \theta(R_0, \theta_0, R)$

-contact pressure $P = P(R_0, \theta_0, R)$

-limit $K_{max} = \frac{R}{R_0} = K_{max}(\theta_0)$

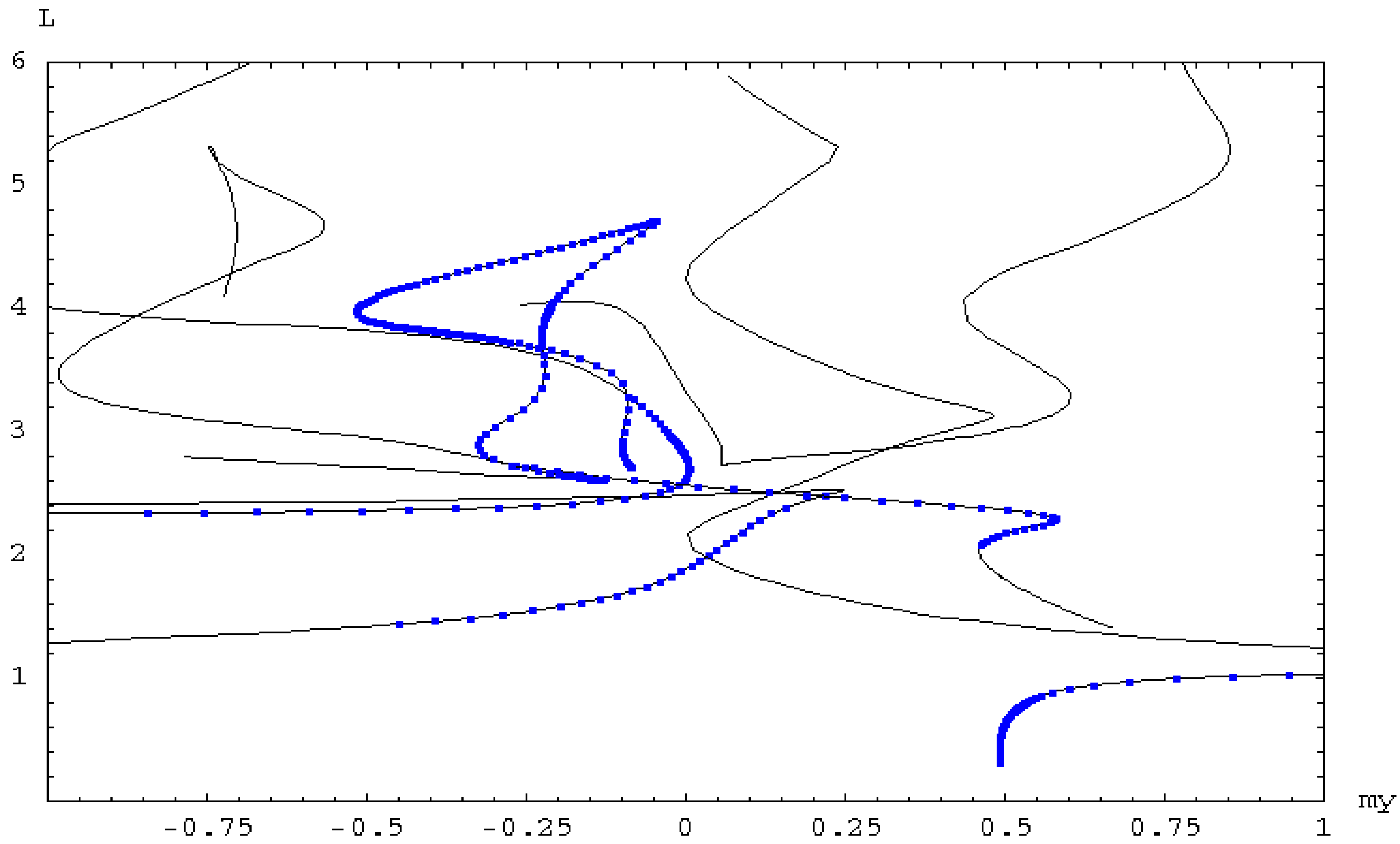


The 3D case : a bifurcation diagram

$$\theta_0 = 1.4 < \frac{\pi}{2}$$

$$\frac{R}{R_0} = 3$$

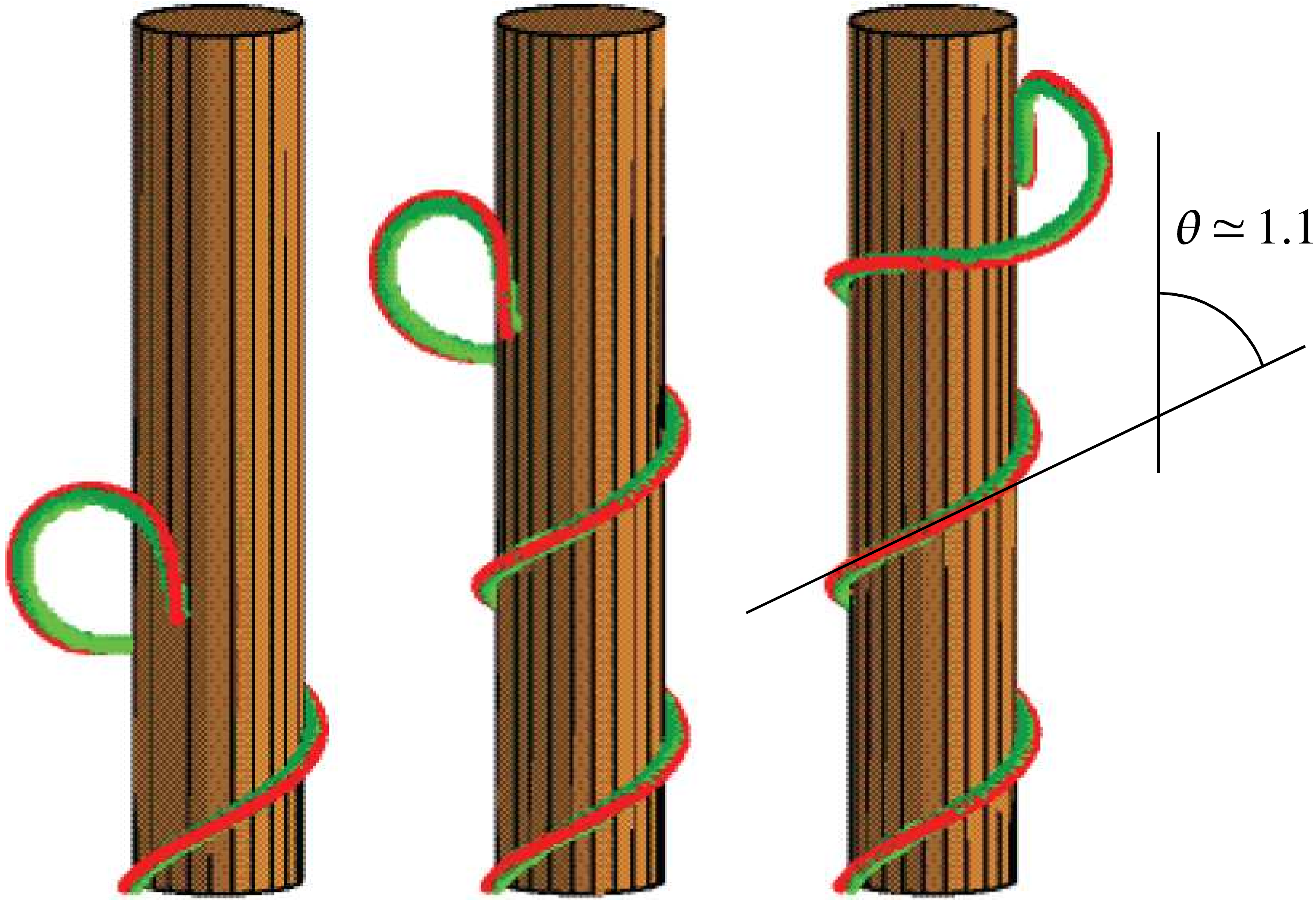
no "climbing" configurations
=> K_{max} decreases in 3D



$\theta_0 = 0.8$

$\frac{R}{R_0} = 2$

Shapes in 3D



liana in Cairns (Queensland), Australia [www.botgard.ucla.edu]