

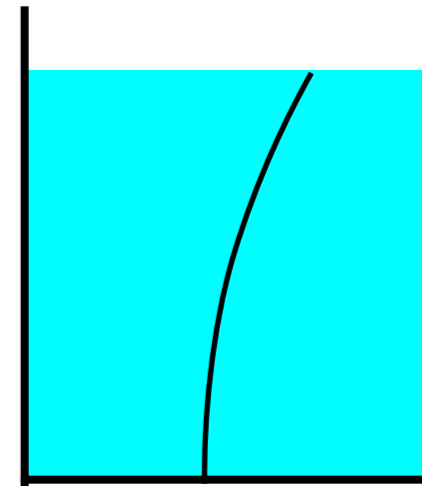
Tiges élastiques : deux exemples

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Elasto-capillarité

José Bico (ESPCI)

Benoît Roman (CNRS & ESPCI)



Noeuds élastiques

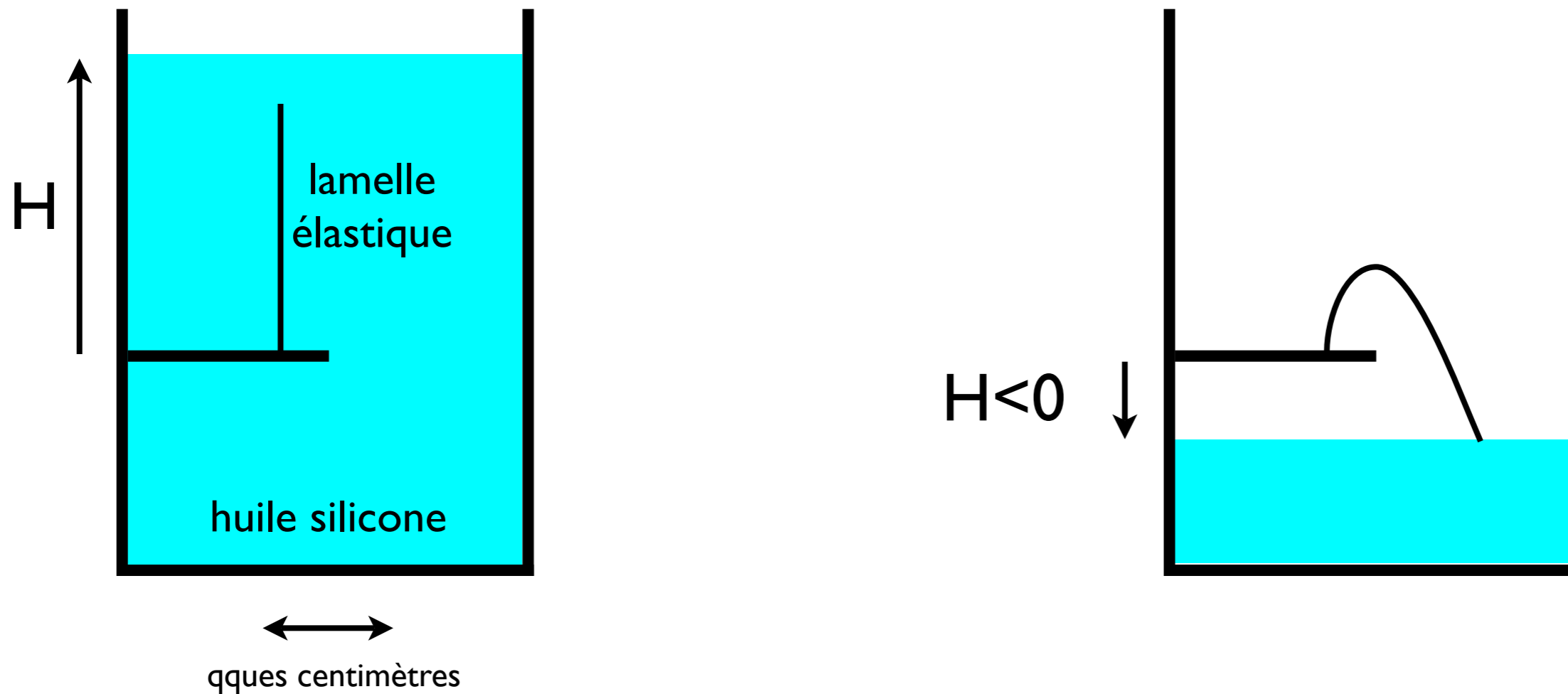
Nicolas Clauvelin (UPMC Univ Paris 6)

Basile Audoly (CNRS & UPMC Univ. Paris 6)



Elasto-capillarité : poils perçants

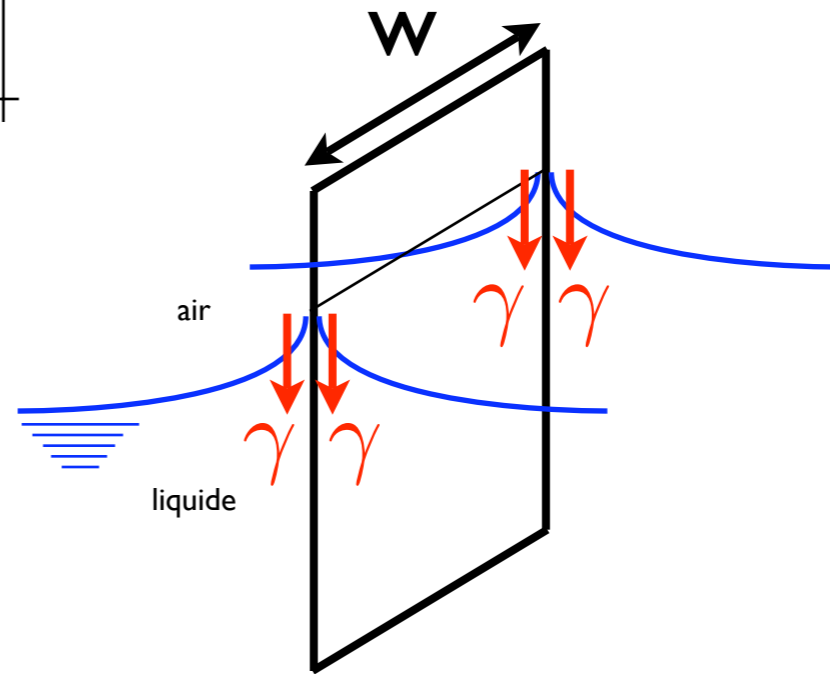
dispositif expérimental



lamelle en polyester :
mouillage parfait avec l'huile

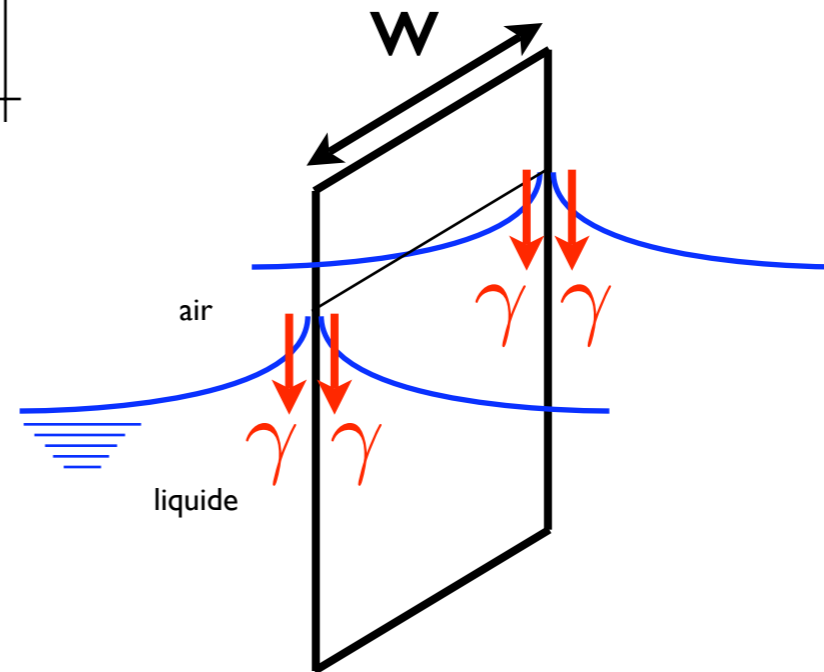
Tension de surface

$$F_{tot} = 2\gamma w$$

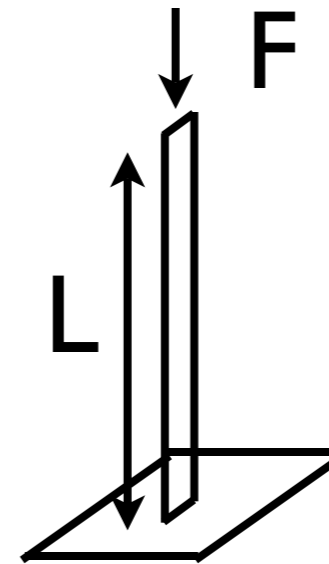


Tension de surface

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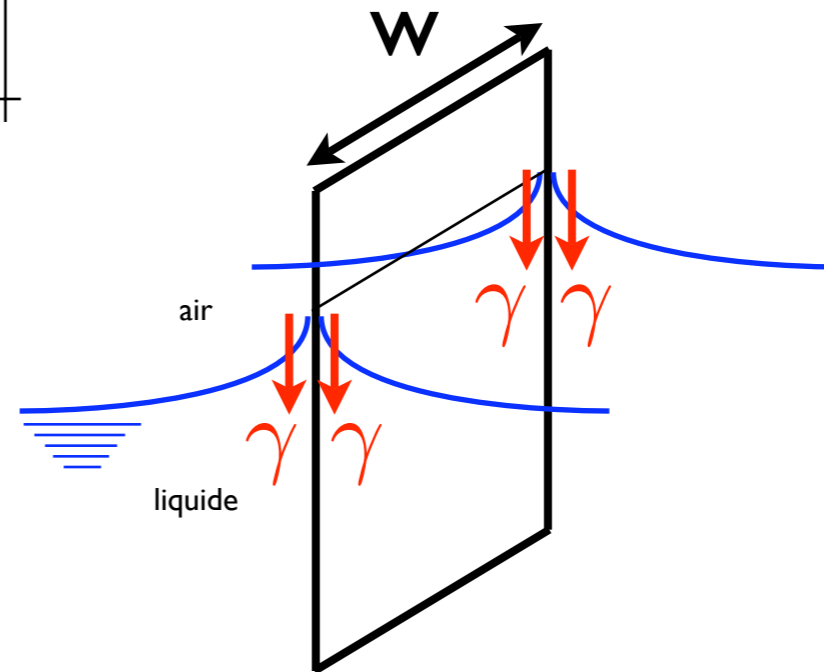


$$F_{euler} = \left(\frac{\pi}{2}\right)^2 \frac{EI}{L^2}$$

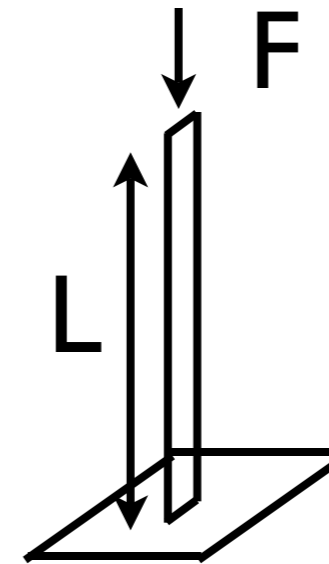


Tension de surface

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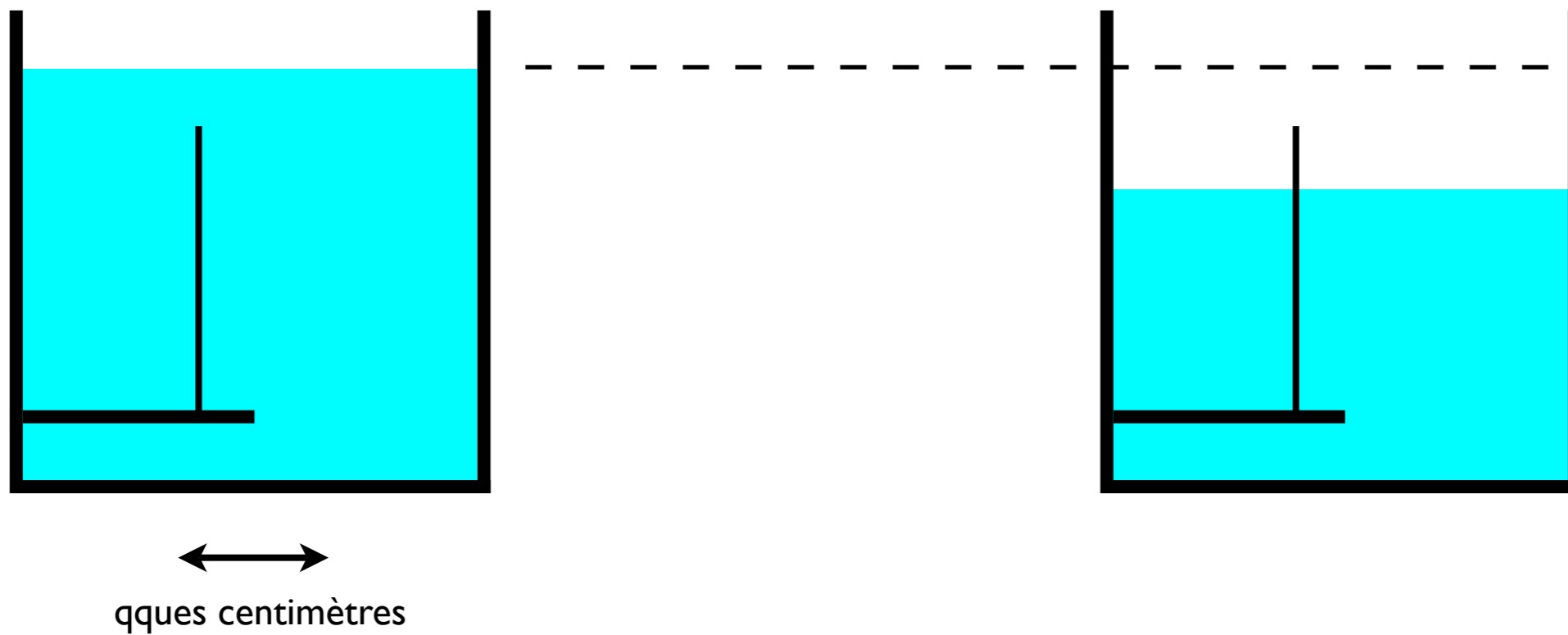


longueur critique $L_b = \frac{\pi}{2\sqrt{2}} \sqrt{\frac{EI}{\gamma w}}$

longueur
élasto-capillaire

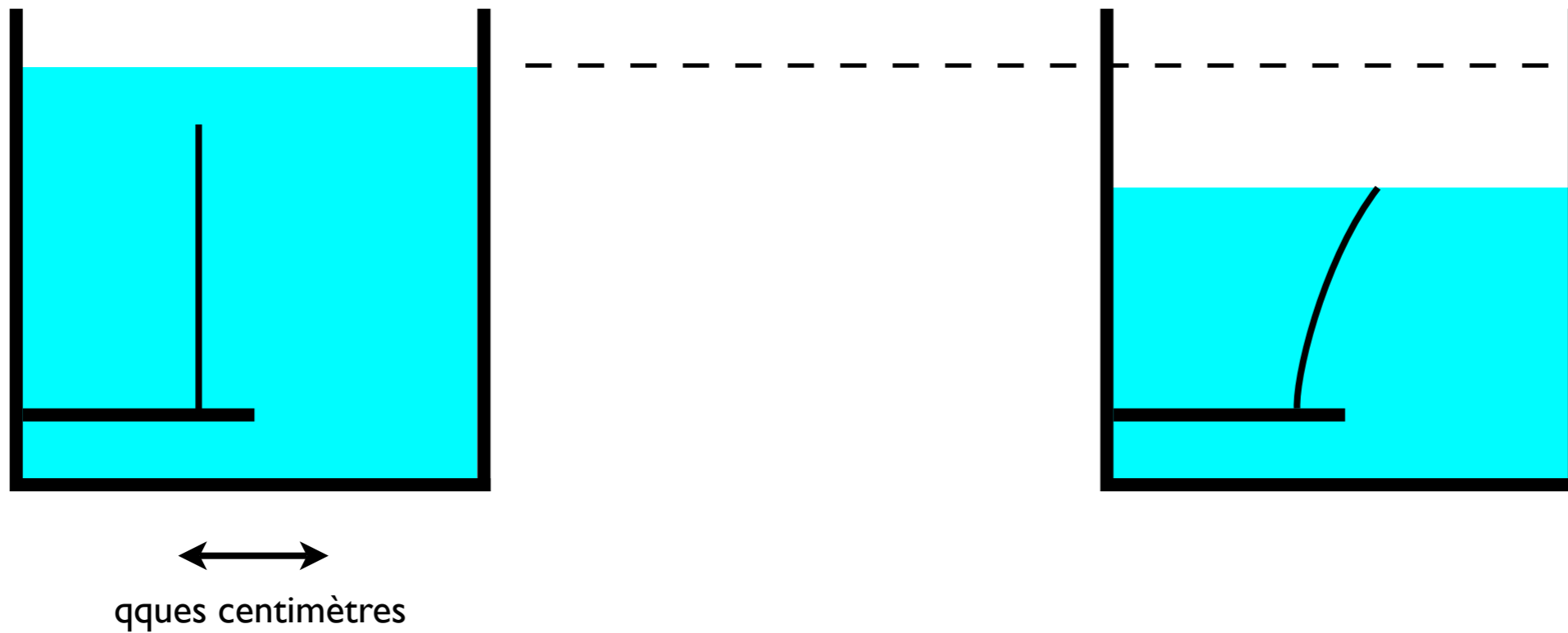
Flambage capillaire

lamelle courte $L < L_b$

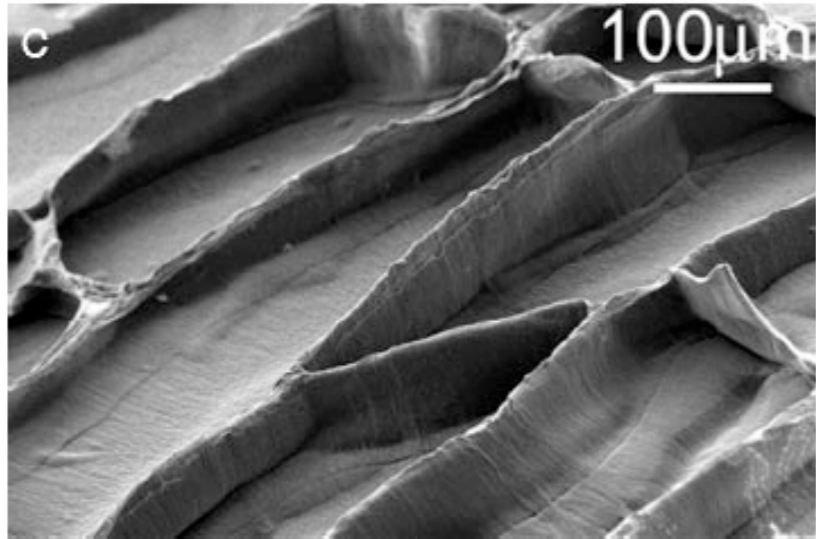


Flambage capillaire

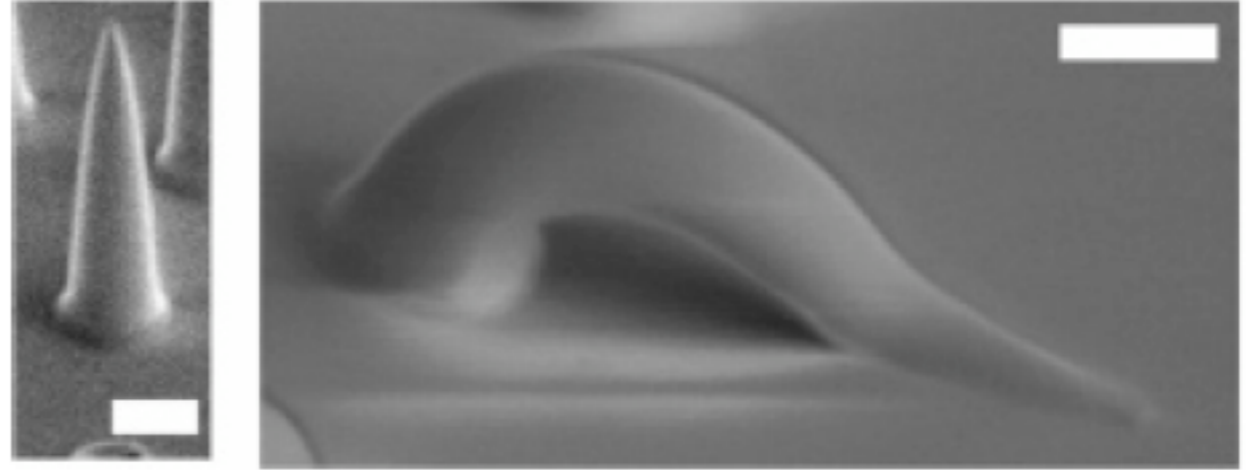
lamelle longue $L > L_b$



Sur le même sujet



nanotubes de carbone après évaporation
N. Chakrapani et al - Nature (2004)



plot de PDMS plié par ethanol
P. Roca-Cusachs et al - Langmuir (2005)

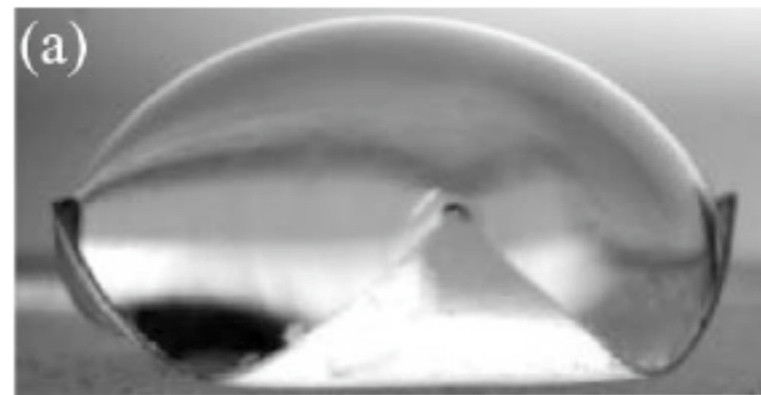


J. Bico et al - Nature
(2004)

agglomération de fibres élastique
et remonté capillaire

C. Py et al - Phys. Rev. Let. (2007)

pliage d'une plaque élastique par capillarité



Modélisation

- pesanteur
- Archimède
- Laplace

$$N'_x = -P_x$$

$$N'_z = -P_z$$

$$M'_y = N_z \sin \theta - N_x \cos \theta$$

$$X' = \sin \theta$$

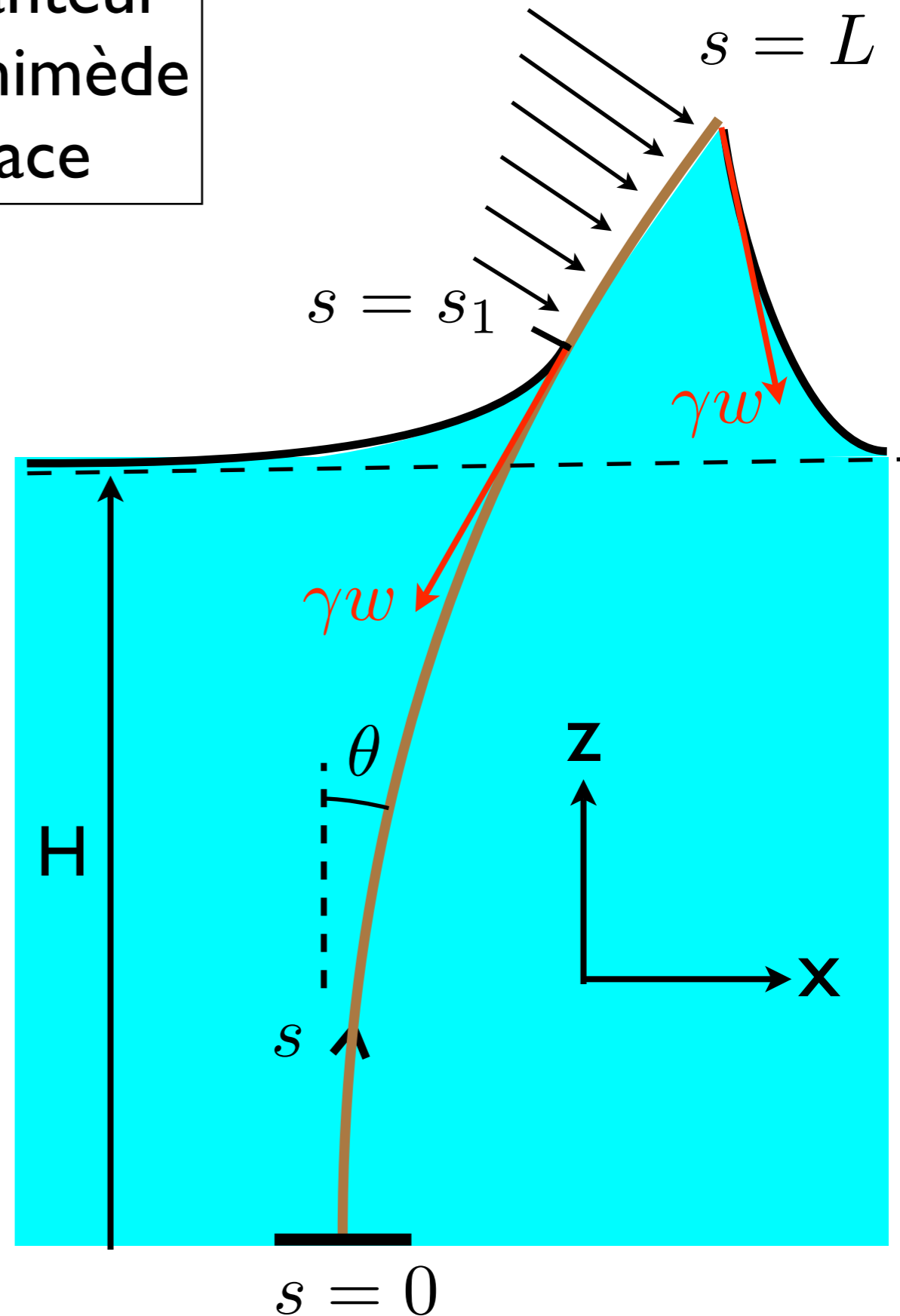
$$Z' = \cos \theta$$

$$\theta' = M_y / (EI)$$

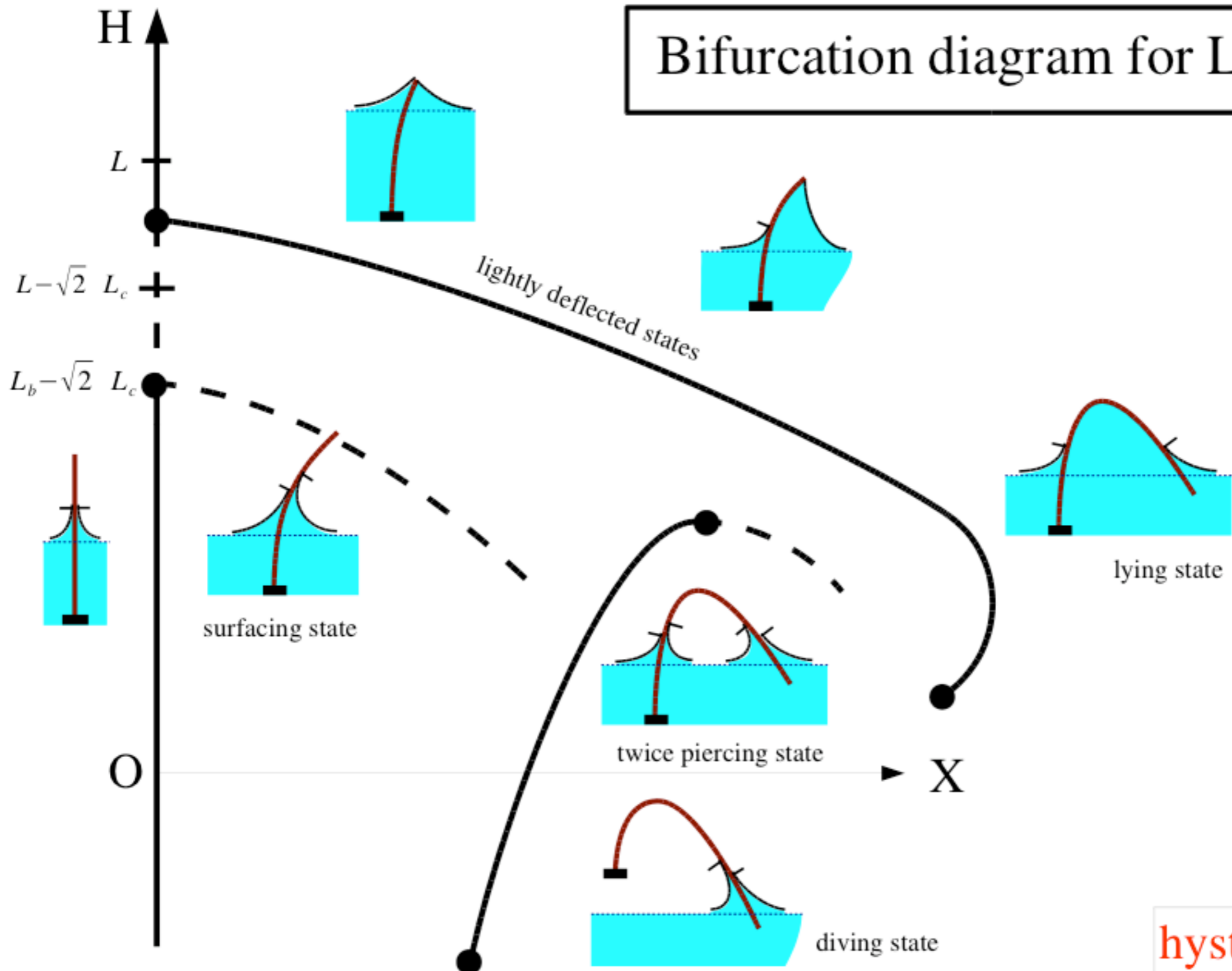
$$' \equiv \frac{d}{ds}$$

- équations de Kirchhoff
- sauts de force aux ménisques
- conditions aux bords

- suivi de courbes : AUTO 94



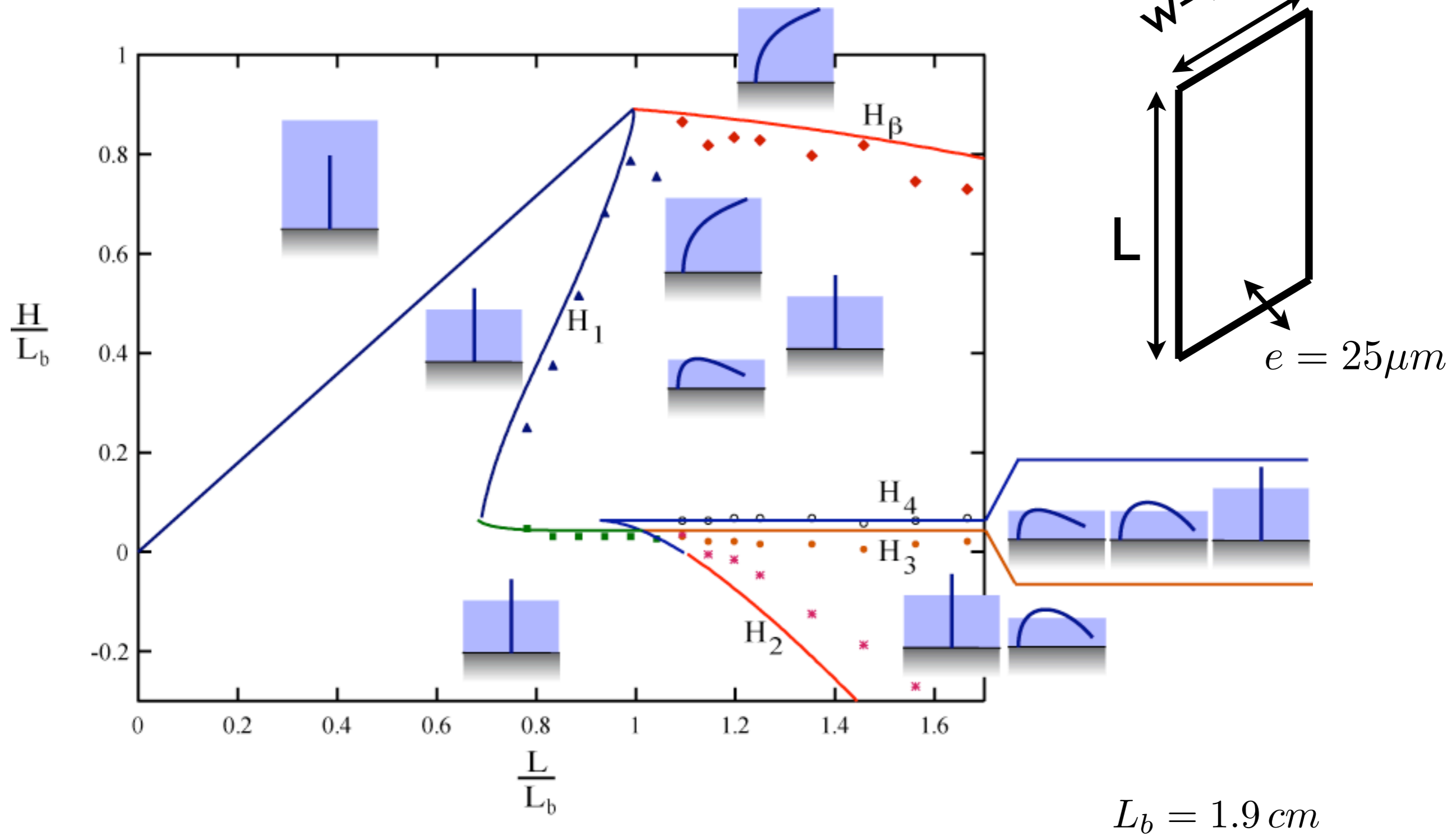
Bifurcation diagram for $L > L_b$



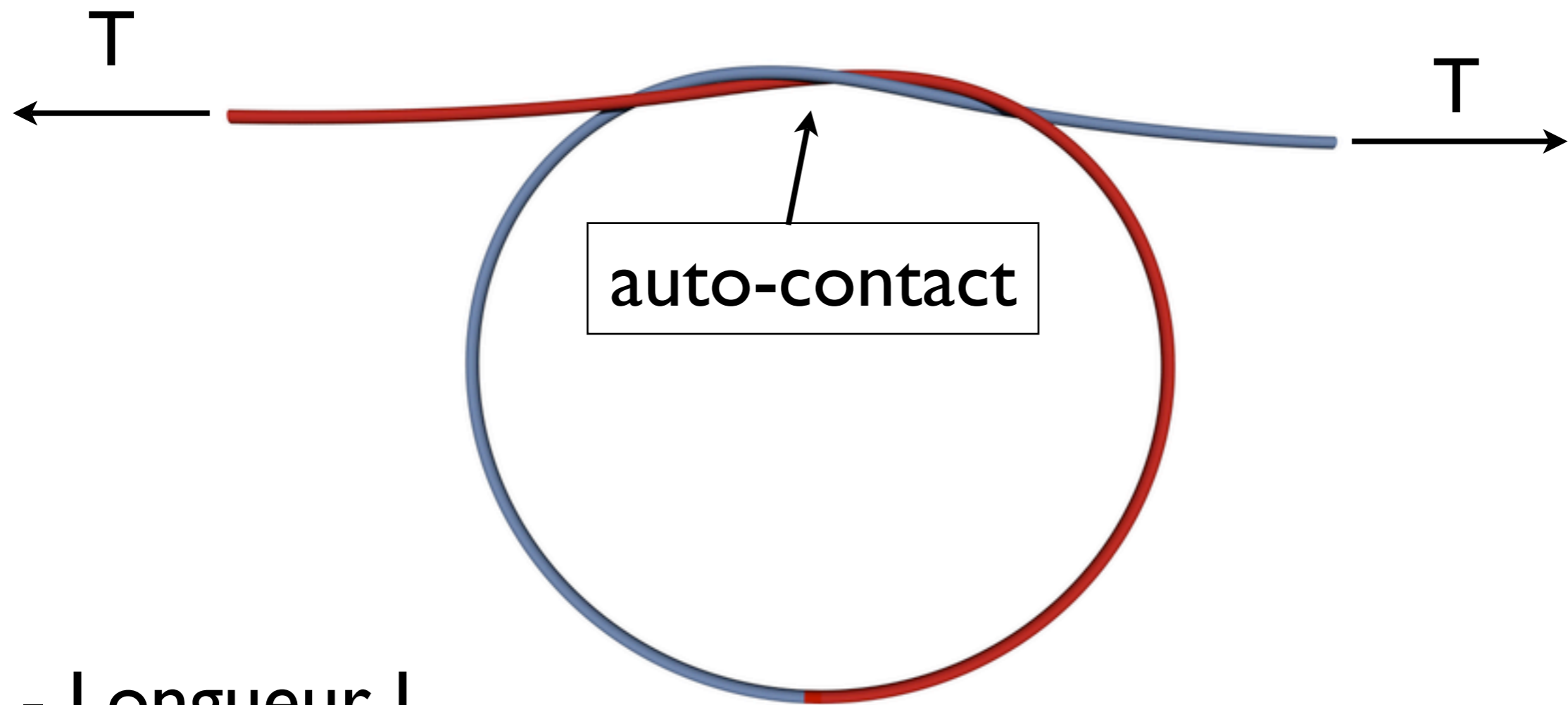
hysteresis

Comparaison théorie/expériences

lamelle polyester



Noeuds élastiques



- Longueur L
- Section circulaire de rayon h
- Rigidité de flexion : $E I$
- Rigidité de torsion : $G J$

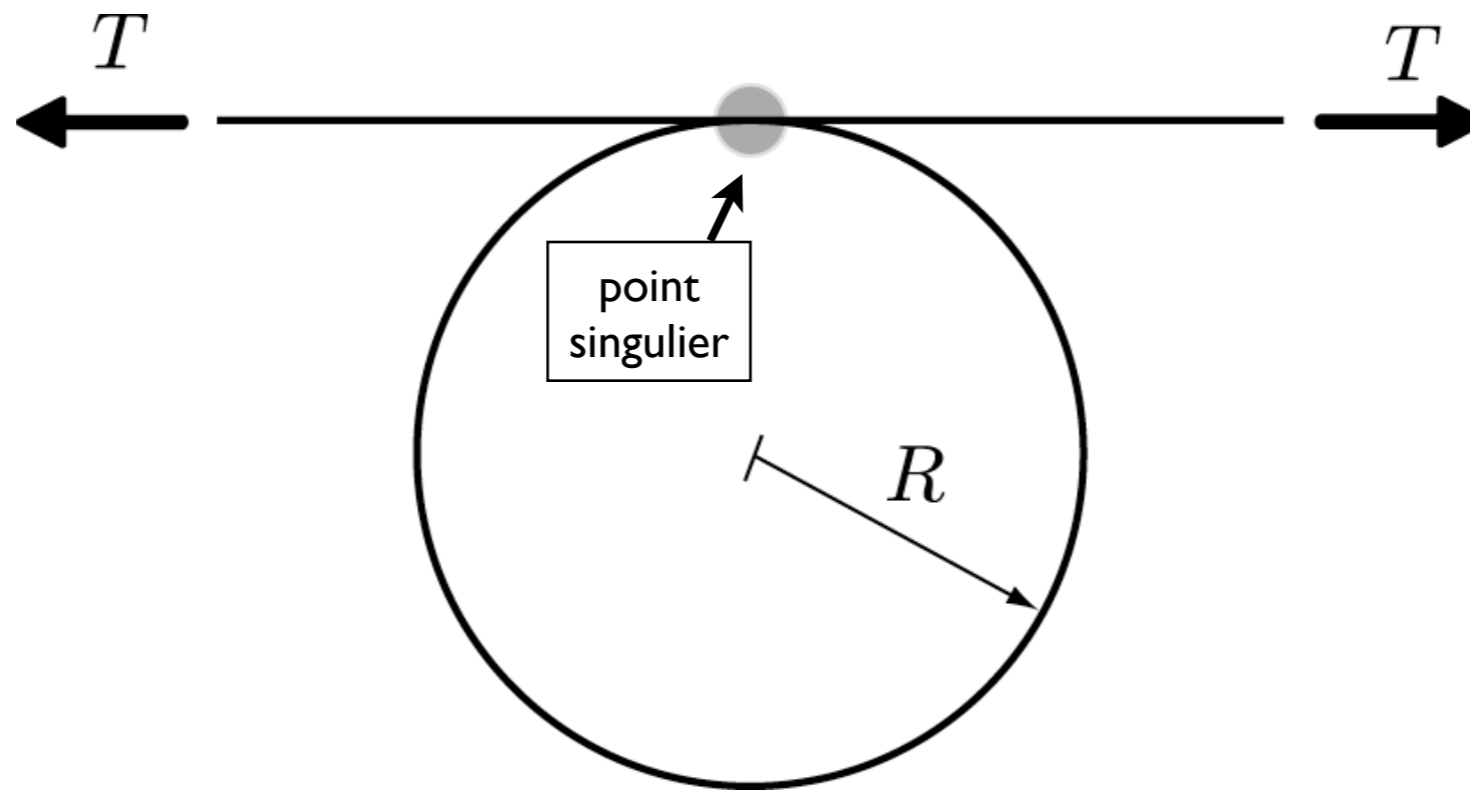
$$I = \frac{\pi h^4}{4}$$

$$J = \frac{\pi h^4}{2}$$

E : module d'Young

G : module de cisaillement

Limite d'épaisseur nulle

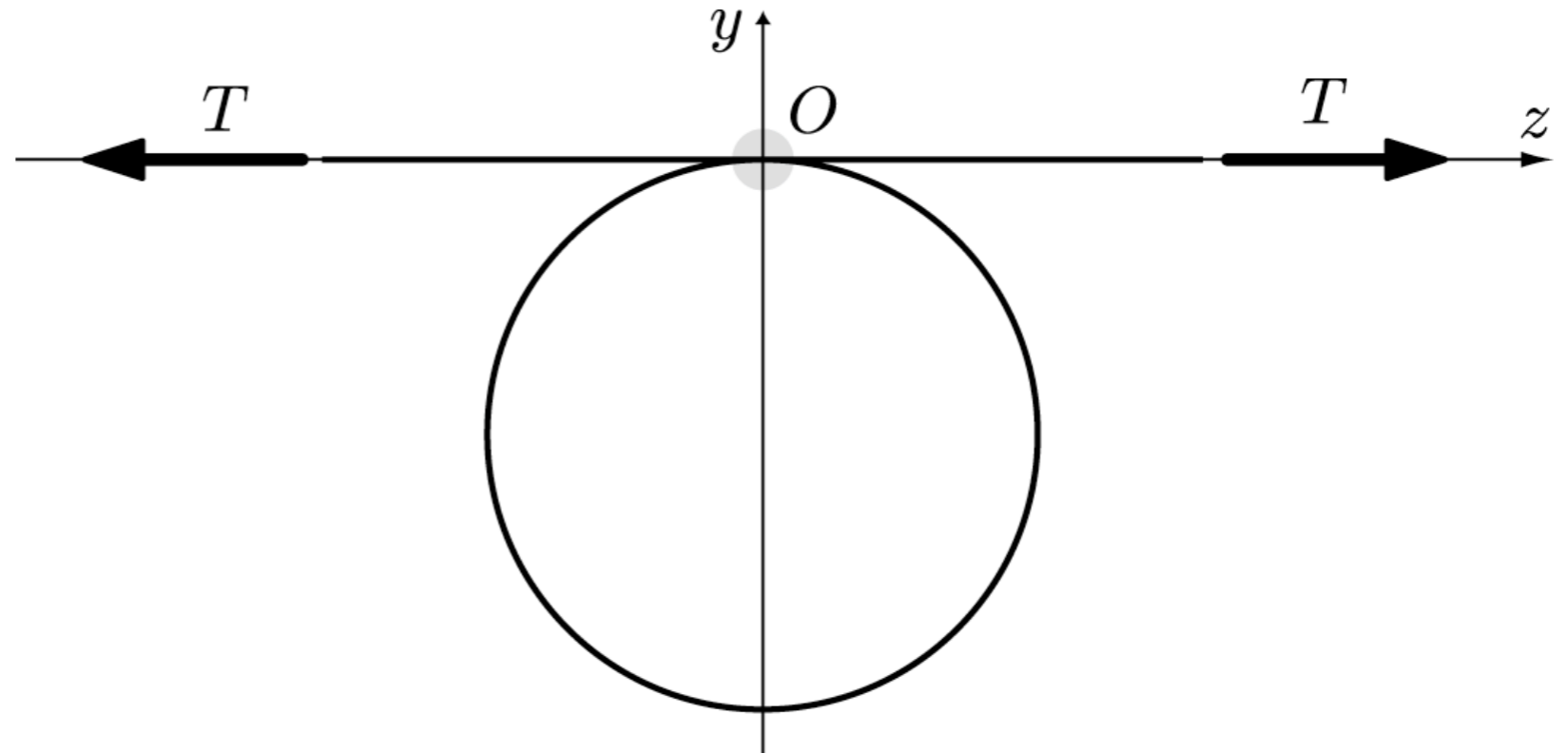


équilibre : $T = \frac{EI}{2R^2}$

tension T point singulier ➔ courbure $\frac{1}{R}$

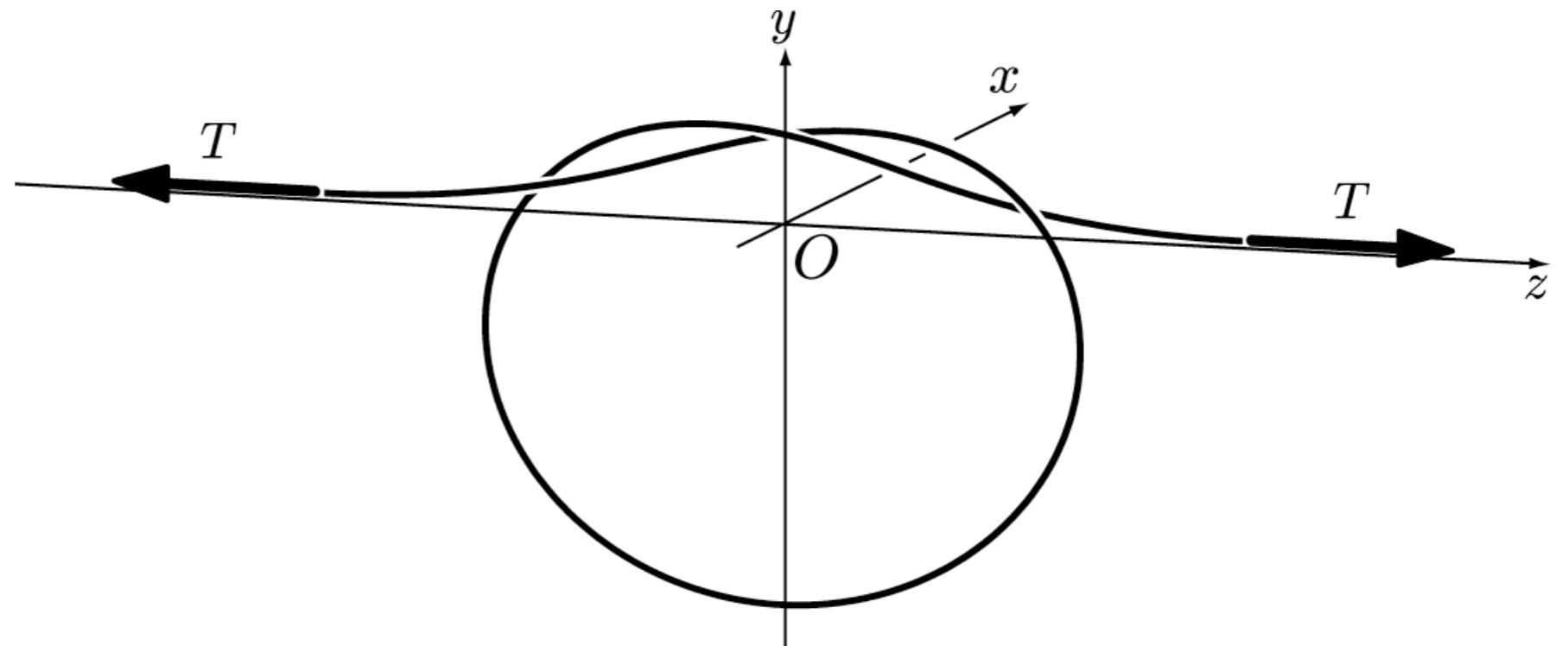
Faible épaisseur

$$h = 0$$

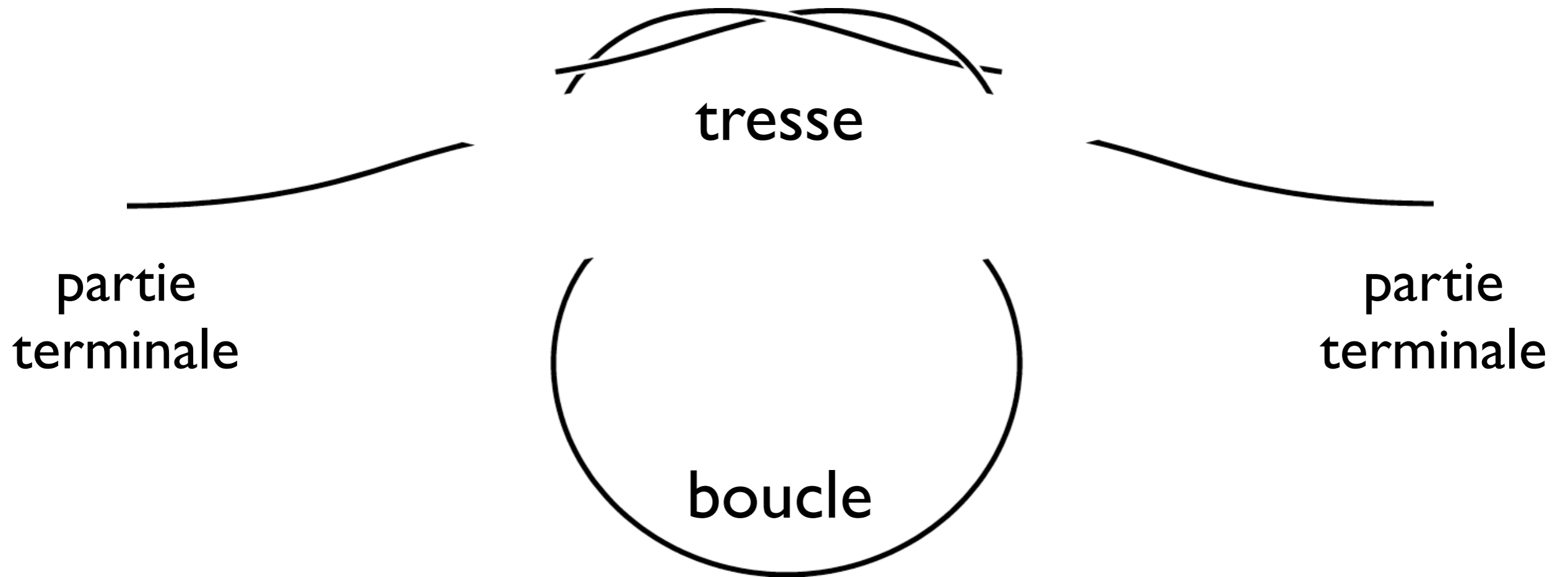


$$h \ll L$$

$$h \ll \sqrt{\frac{EI}{T}}$$



Dévelop. asymptotiques raccordés



petit paramètre : $\epsilon = \left(\frac{2h^2 T}{EI} \right)^{1/4} \ll 1$

Equations de Kirchhoff

$\vec{p}(s)$ pression ext.

$\vec{N}(s)$ force interne

$\vec{M}(s)$ moment interne

$\vec{R}(s)$ position

$\vec{t}(s)$ tangente

$$\vec{N}' = -\vec{p}$$

$$\vec{M}' = \vec{N} \times \vec{t}$$

$$\vec{R}' = \vec{t}$$

$$\vec{t}' = \frac{1}{EI} \vec{M} \times \vec{t}$$

équil. forces

équil. moments

déf. tangente

déformation

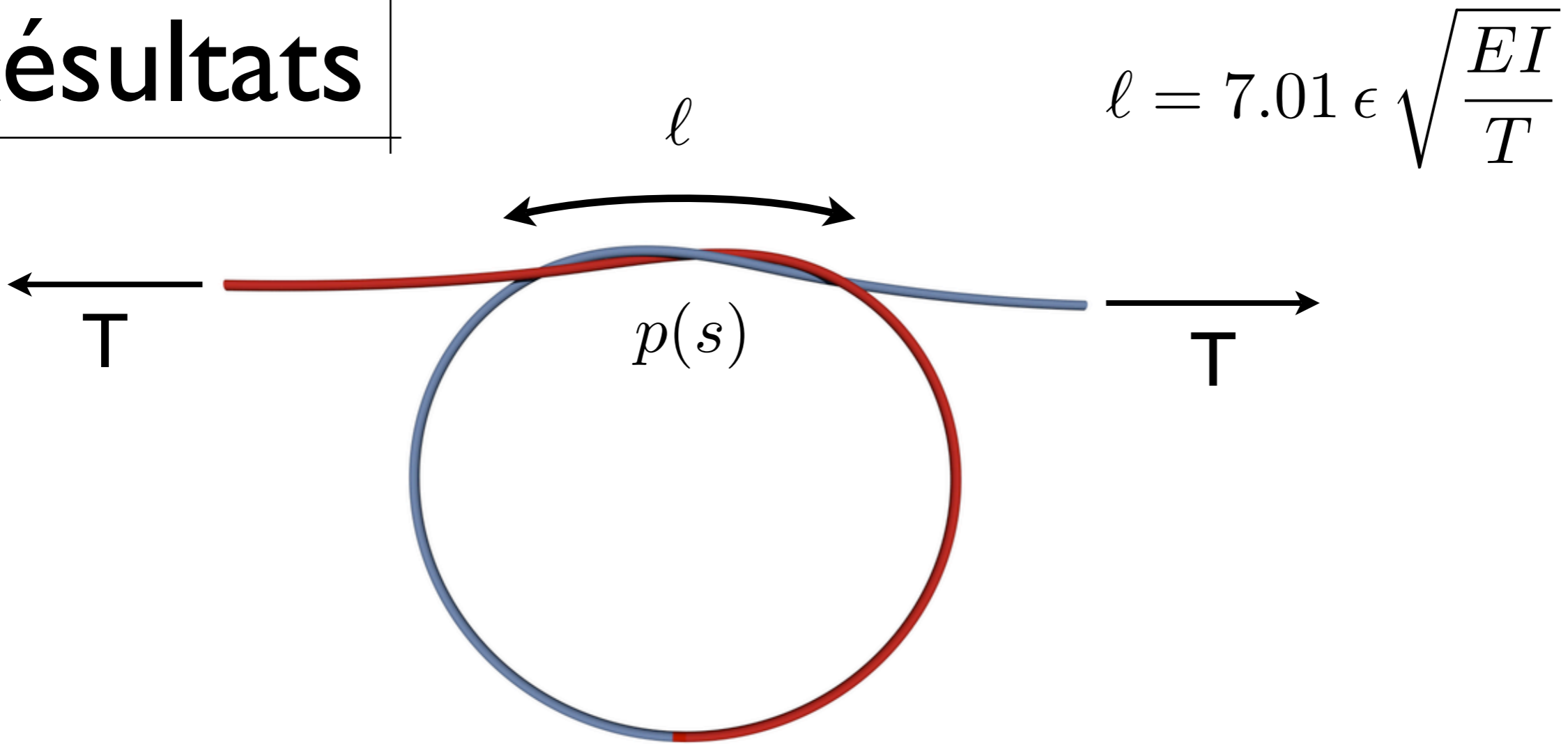
loi de comportement :

$$M_{\kappa} = EI \kappa \quad \text{flexion } \kappa$$

$$M_{\tau} = GJ \tau \quad \text{torsion } \tau$$

$$' \equiv \frac{d}{ds}$$

Résultats

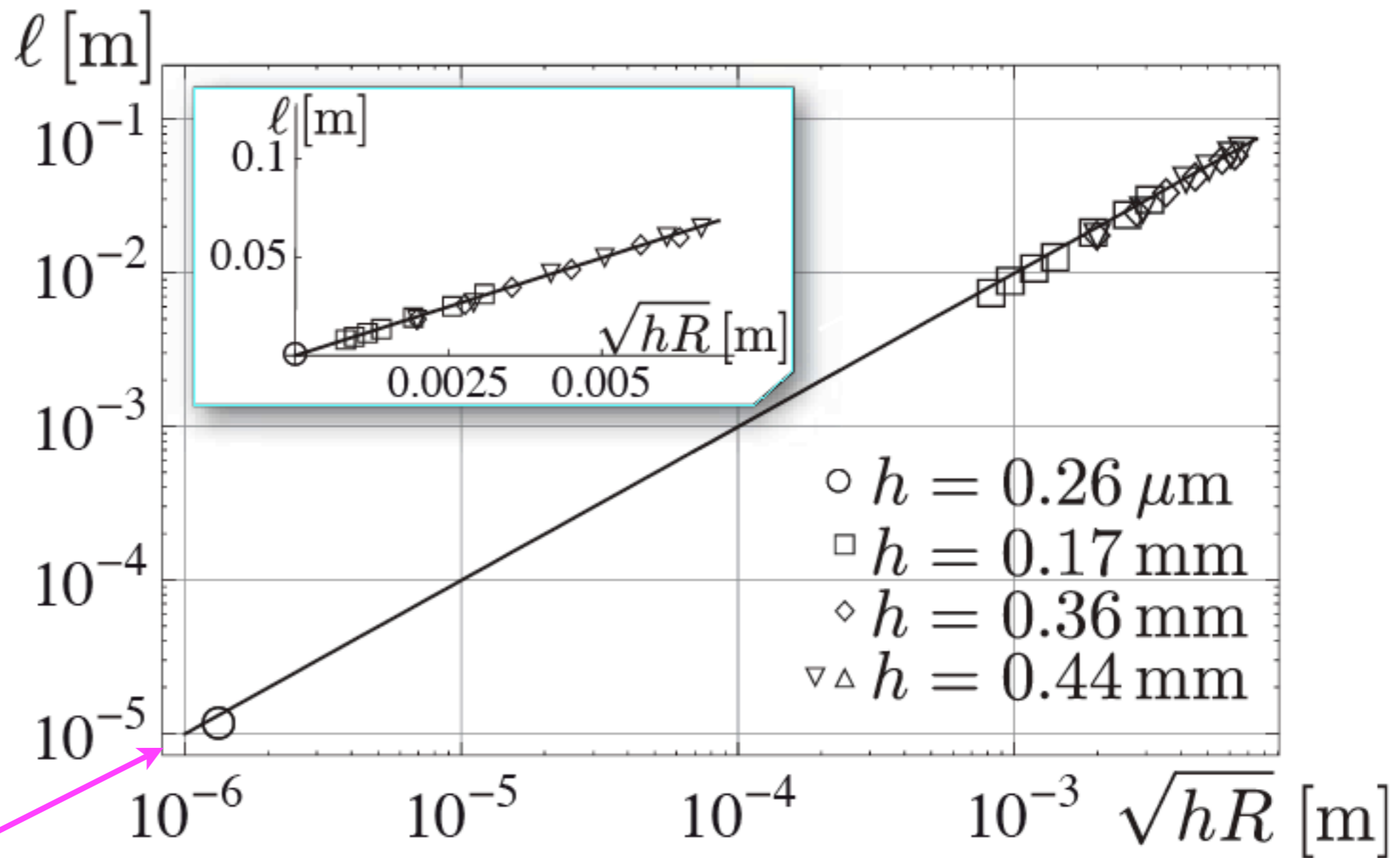
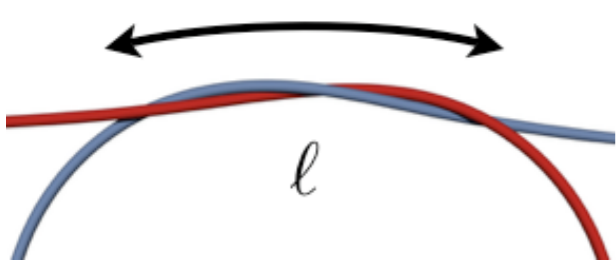


$$\ell = 7.01 \epsilon \sqrt{\frac{EI}{T}}$$

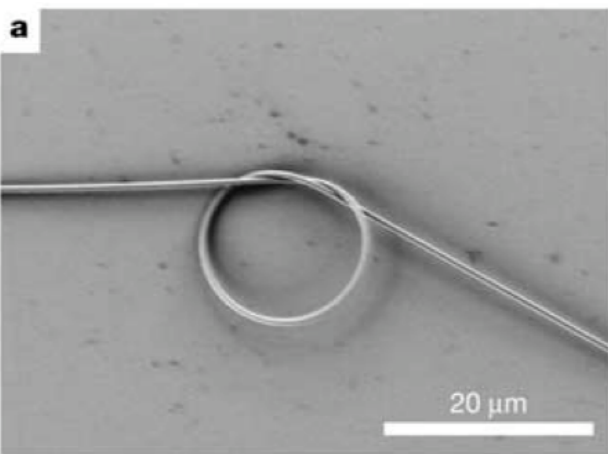
Pression de contact : $p(s)$

Force totale
$$P = \int_0^{\ell} p(s) ds = 0.98 \frac{T}{\epsilon}$$

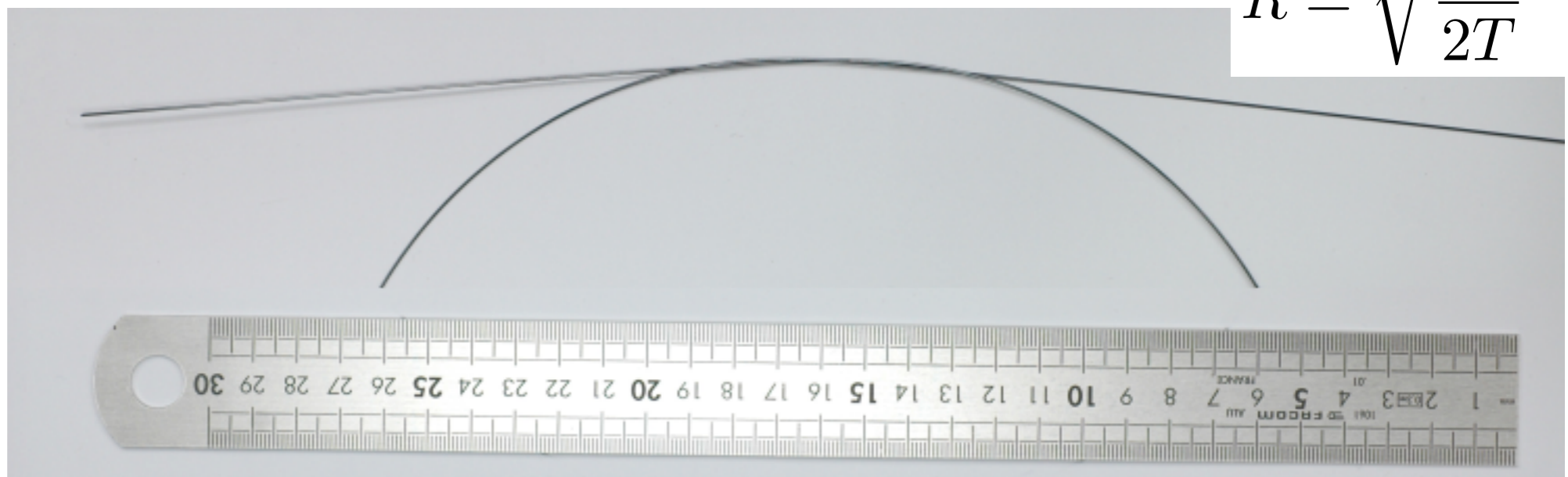
Expériences



Tong et al., Nature 2003



fil en silice
 $h = 1/2$ micron



$$R = \sqrt{\frac{EI}{2T}}$$

Expériences

