The bending of an elastic beam by a liquid drop

A variational approach

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Arnaud Antkowiak — d'Alembert - CNRS & UPMC - Paris, France
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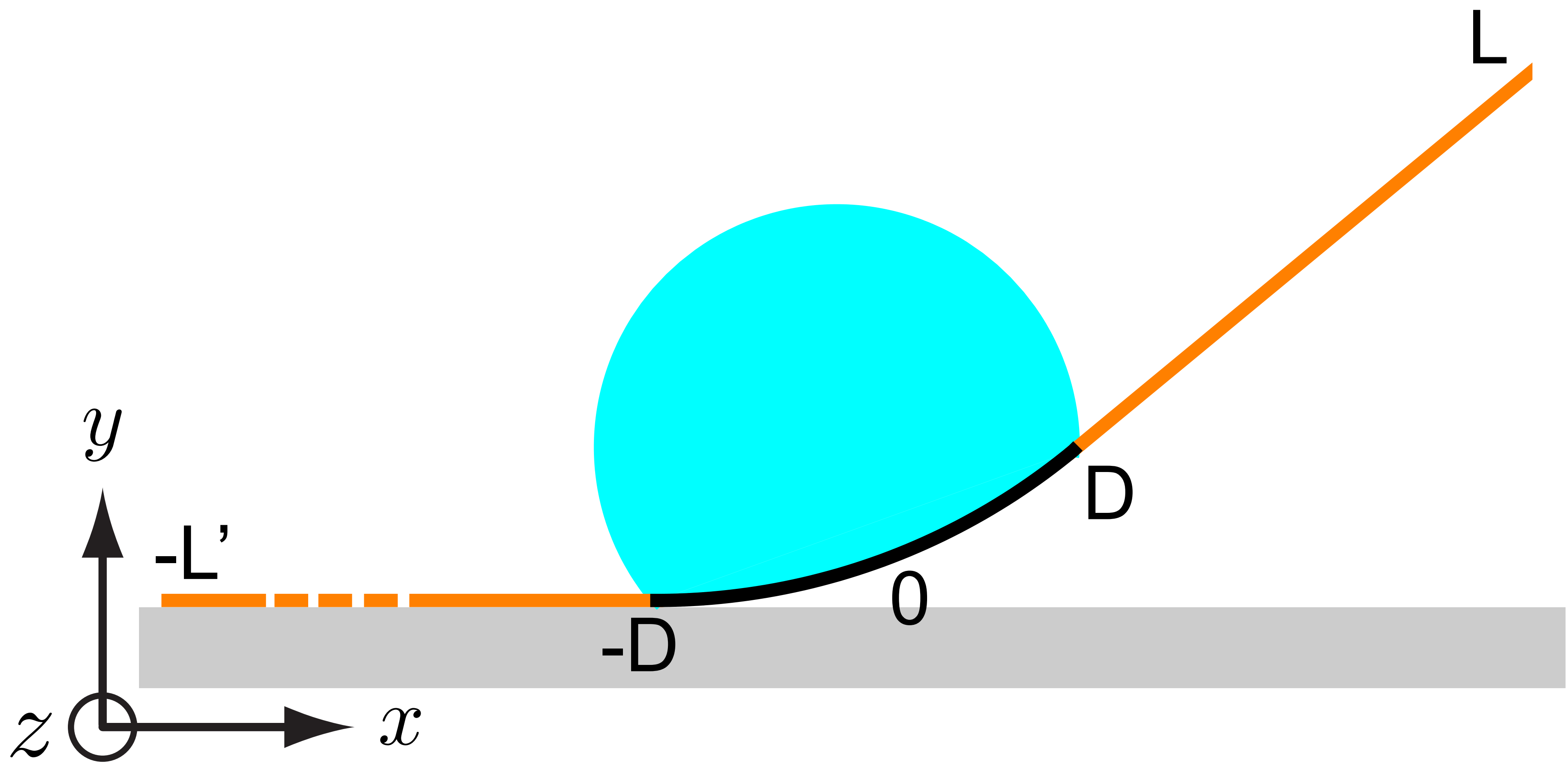
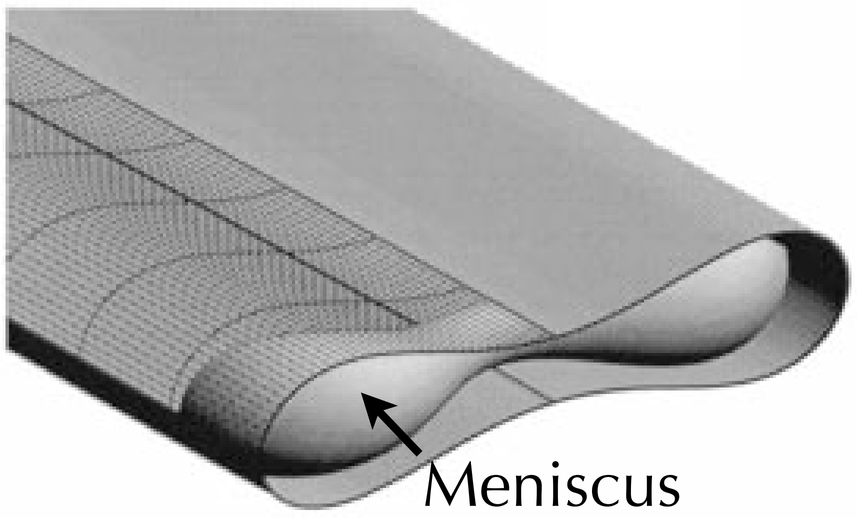


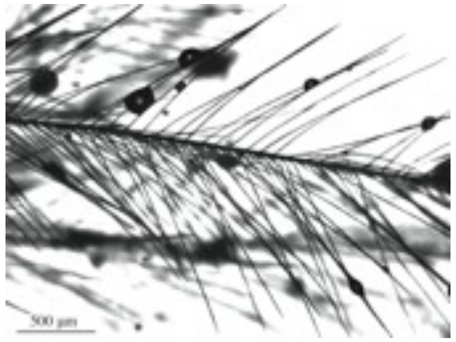
State of the art

Elastocapillarity in Biology



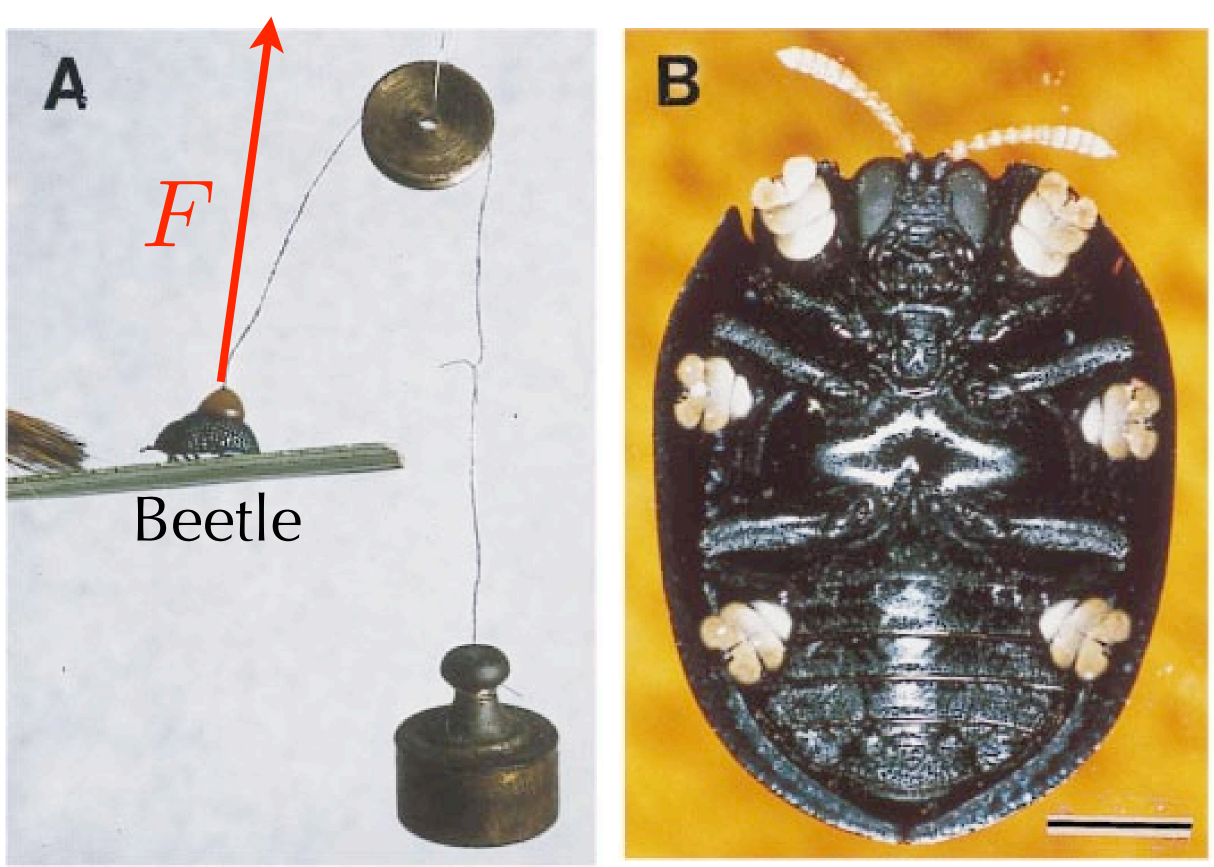
Lung's airway closure

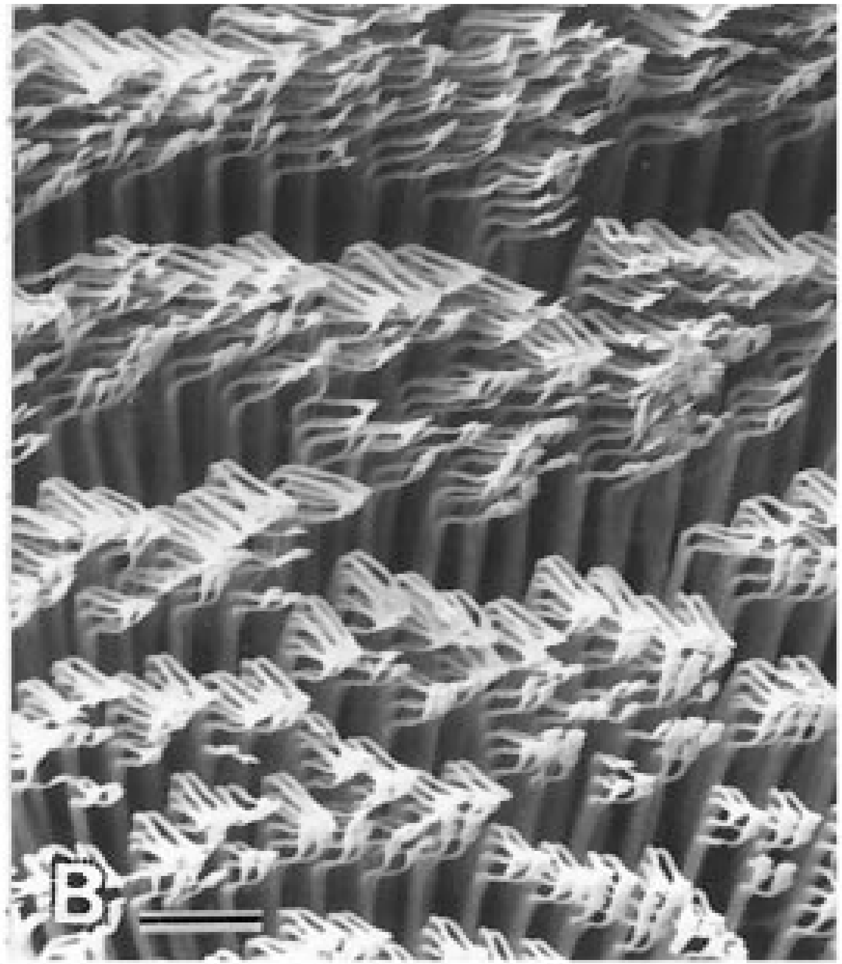
e.g. Heil, J. Fluid Mech 380, 1999



Wet feathers

Duprat, Protière, Beebe and Stone, *Nature* (2012)

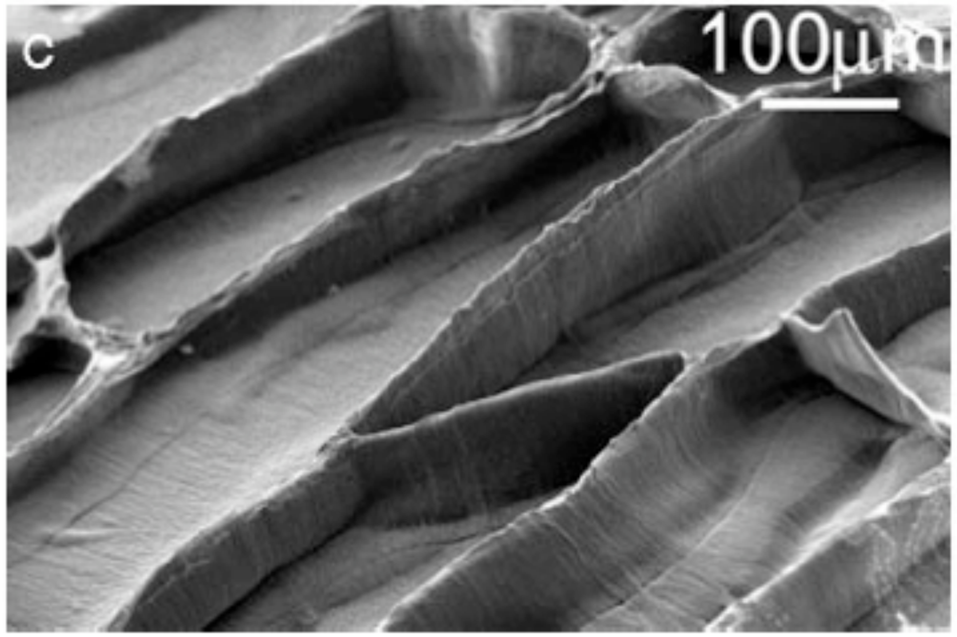




Insect adhesion

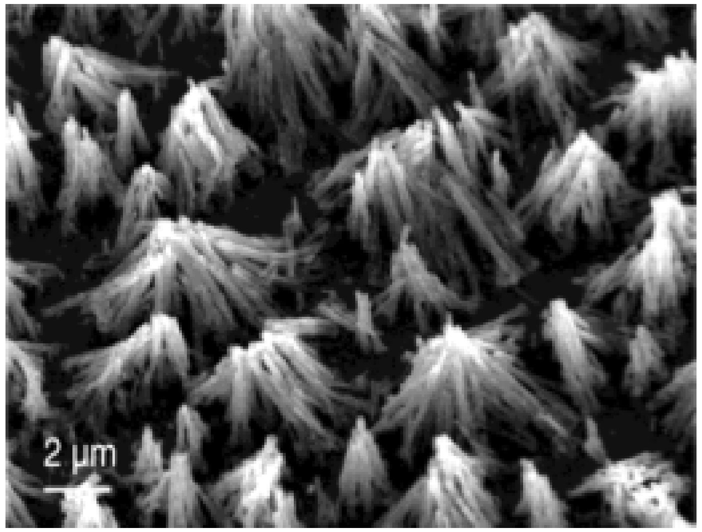
Eisner et al., PNAS, 2000

Elastocapillarity in Industry



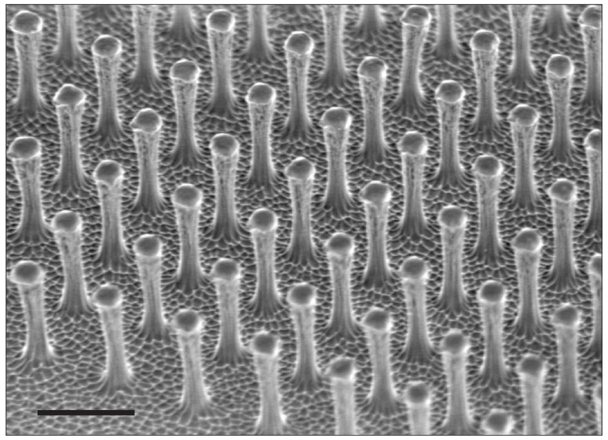
Cellular patterns

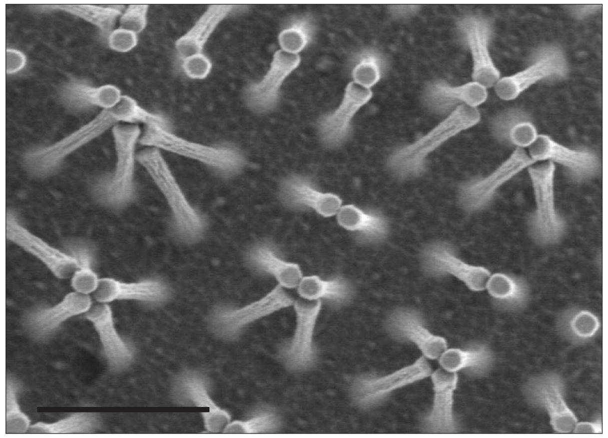
Chakrapani et al., PNAS, 2004



Teepee formation

Lau et al., Nano Lett., 2003



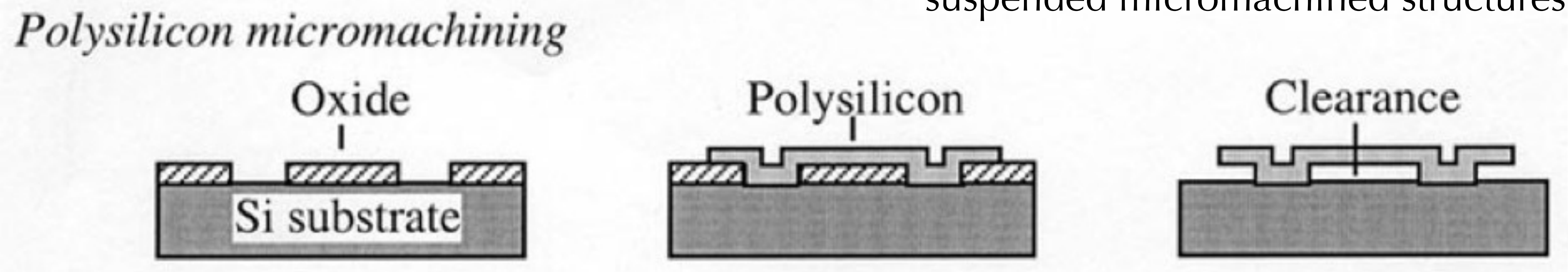


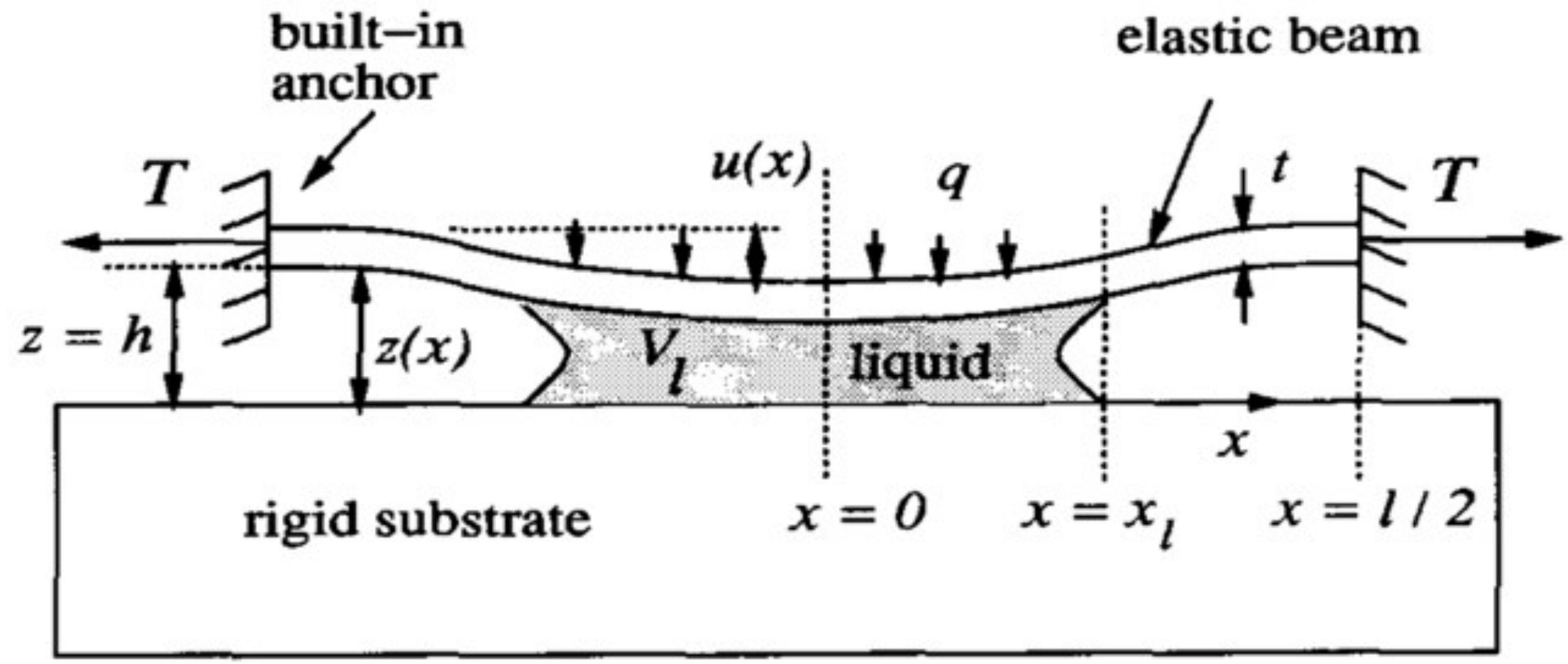
Bio-mimetism

Geim et al., Nature Mat., 2003

Elastocapillarity in Industry

suspended micromachined structures





collapse
during
evaporation

surface tension forces are responsible
for the collapse of microstructures during
removal of sacrificial layers

Mastrangelo, Journal of MEMS (1993)

Elastocapillarity in Industry: Microfabrication

melting of solder

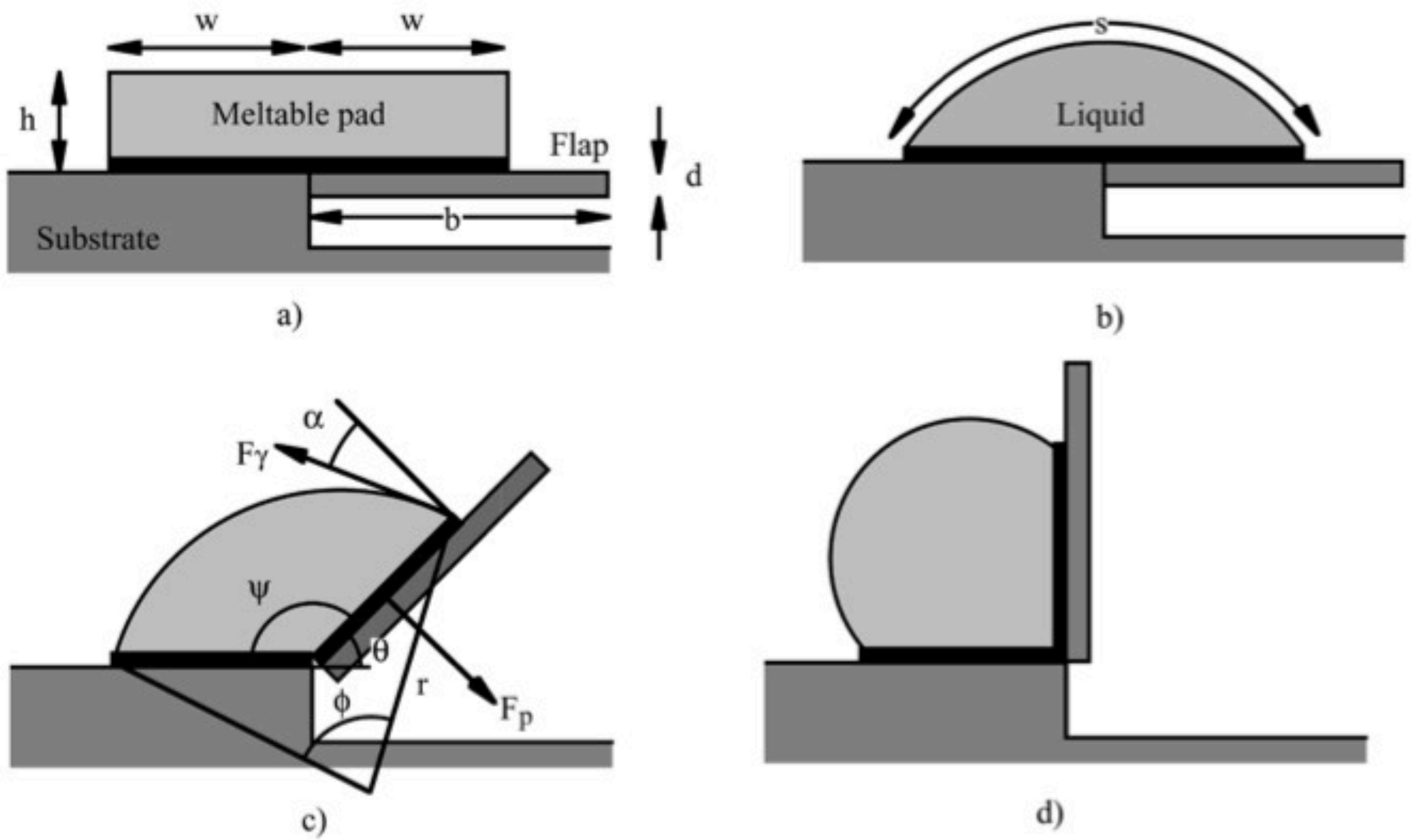


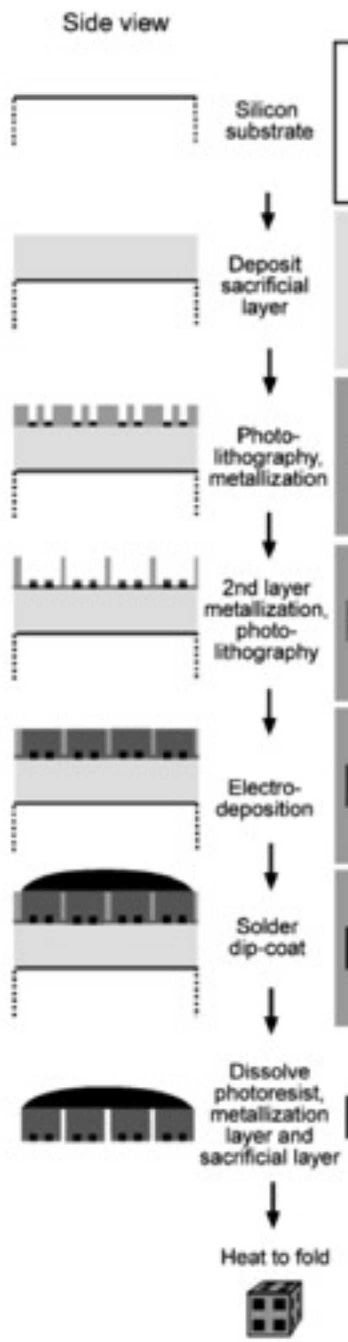
Fig. 3. Two-dimensional geometry for surface tension powered rotation.

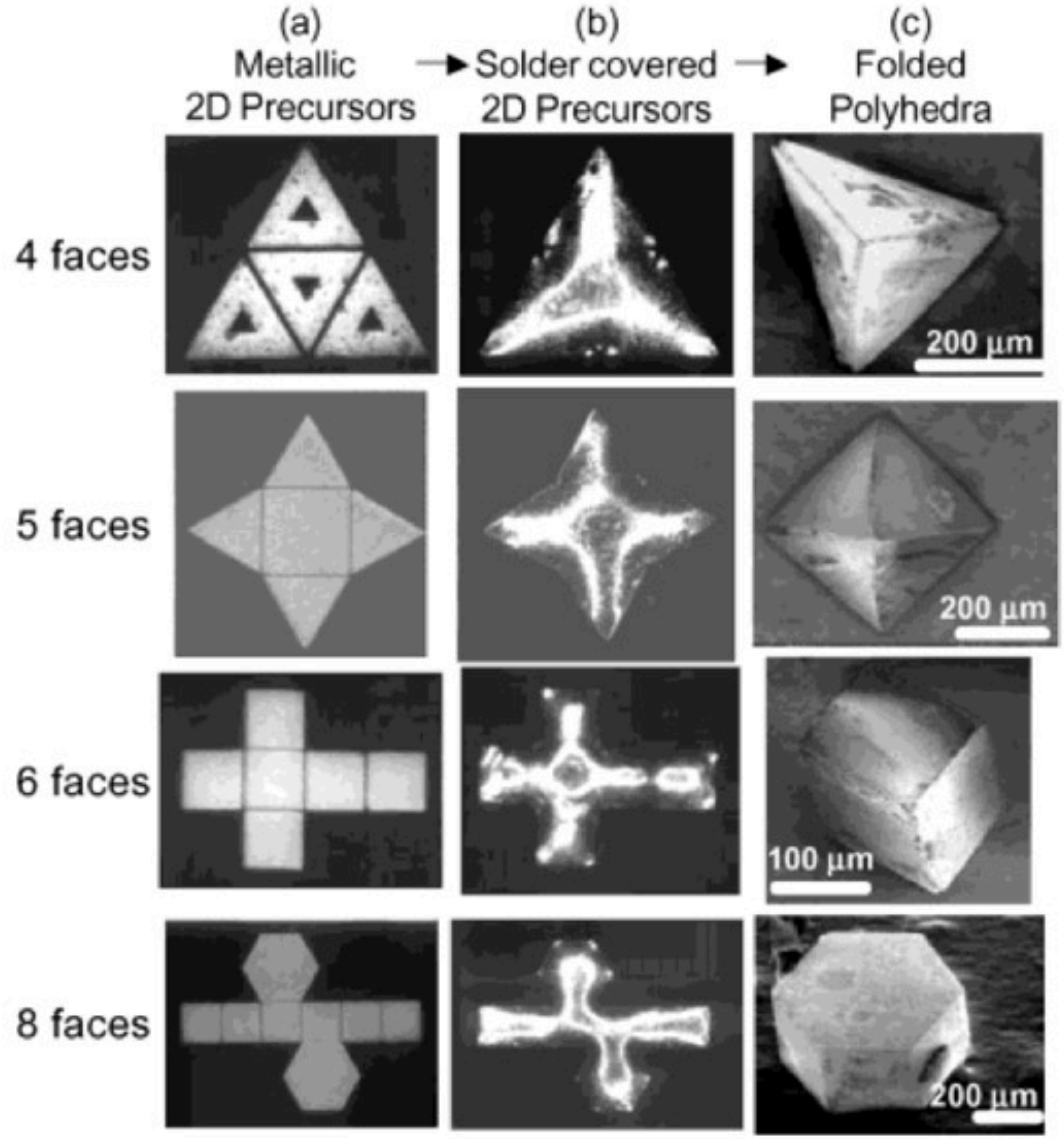
re-solidification

rotate hinged joints for the
self-assembly of 3D microstructures

R. Syms, Journal of MEMS (1995)

Elastocapillarity in Industry: Microfabrication



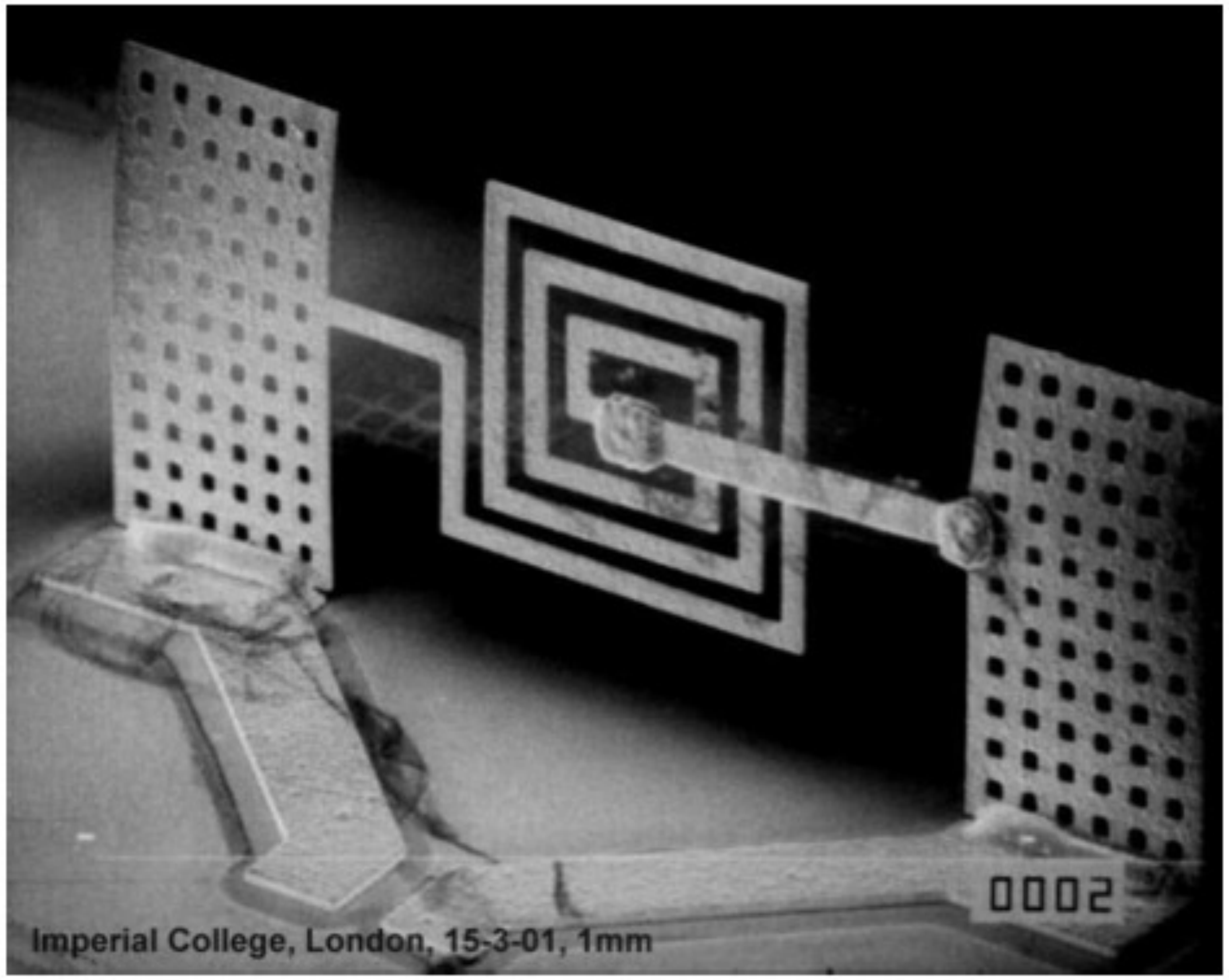


spontaneous folding of 2D structures
under the influence of the
surface tension of liquid solder

Gracias et al, Adv. Mat. (2002)

Elastocapillarity in Industry: Microfabrication

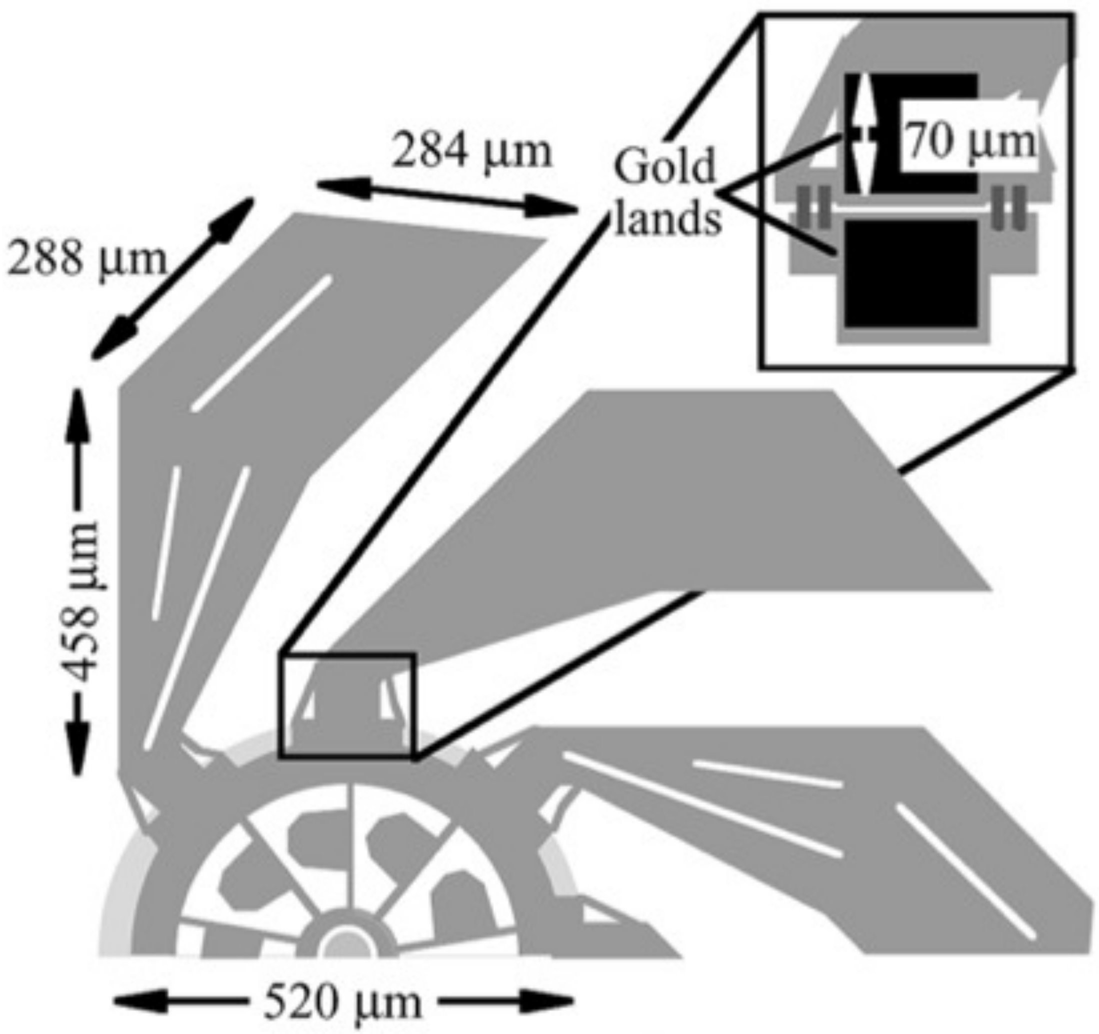
inductor has to be
away from (metallic) substrate to
prevent magnetic field loss

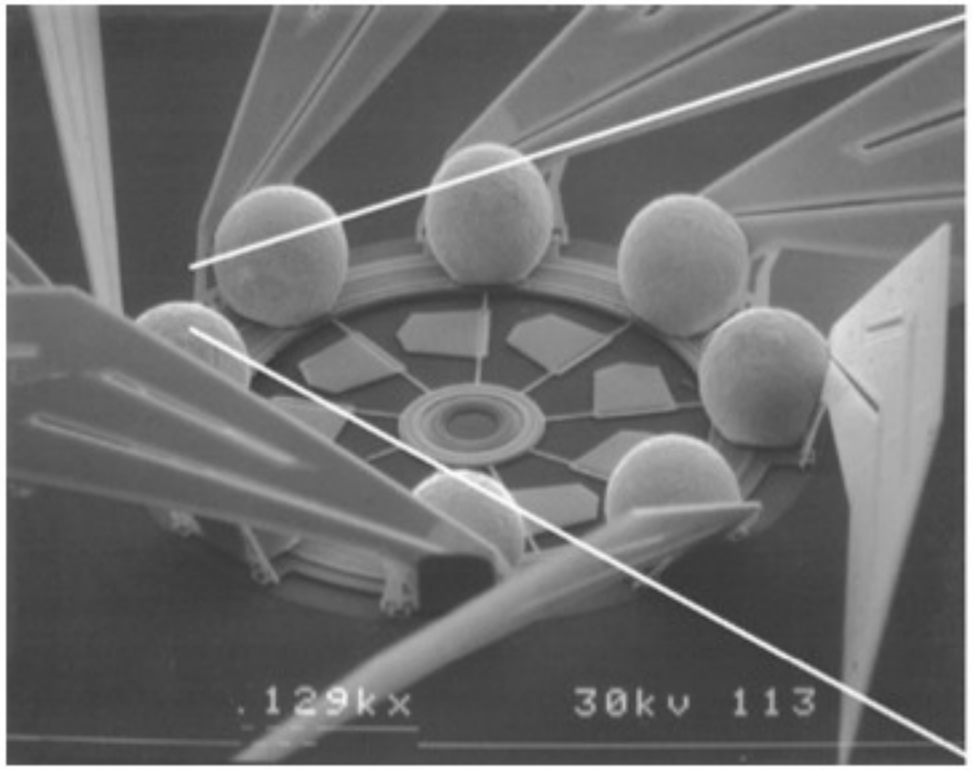


3D electrical components (here an inductor)
assembled by surface tension

Dahlmann, Electron Lett. (2000)

Elastocapillarity in Industry: Microfabrication





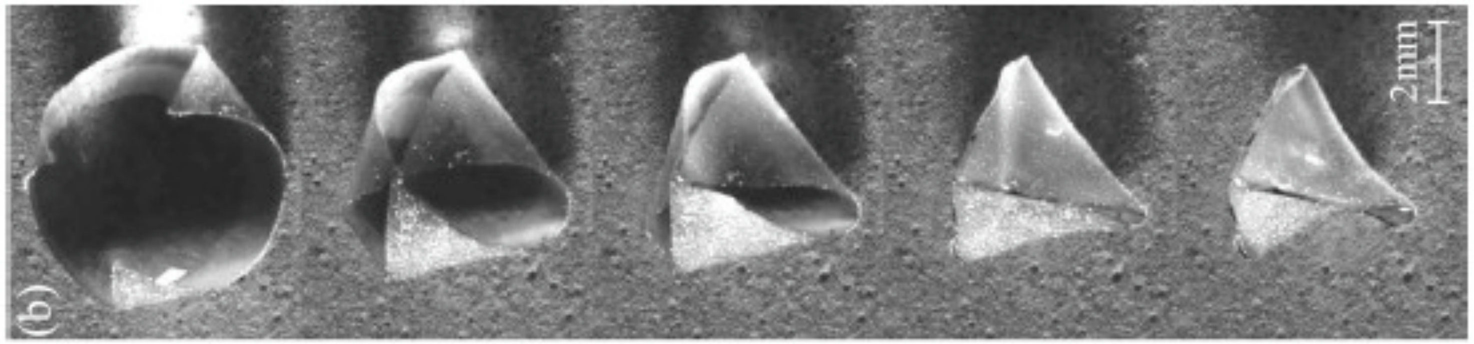
folding by surface tension of Pb:Sn solder spheres

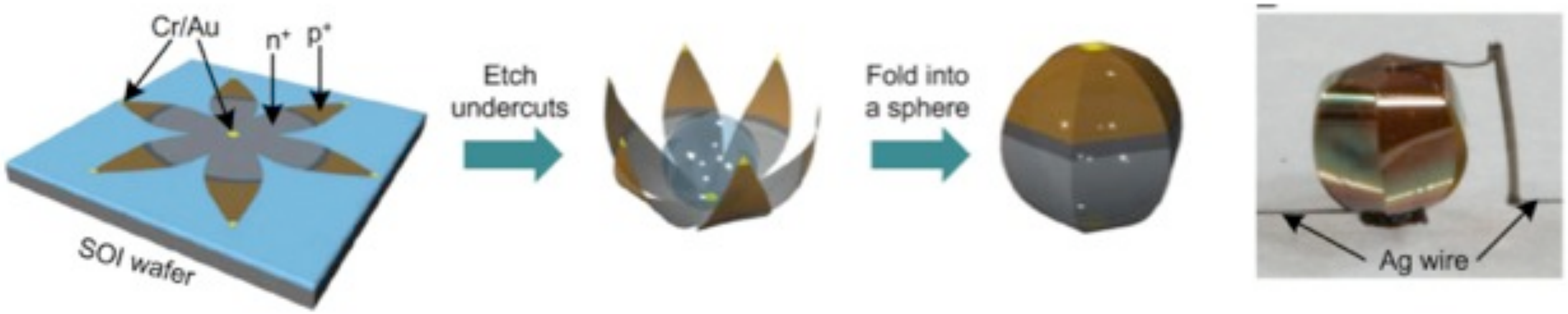
microfan with polysilicon
180 rpm
micro-fluidic systems

Linderman et al, Sens. Actuators (2002)

Elastocapillarity in Industry: Microfabrication

Py et al
Capillary origami
Phys. Rev. Lett. 2007



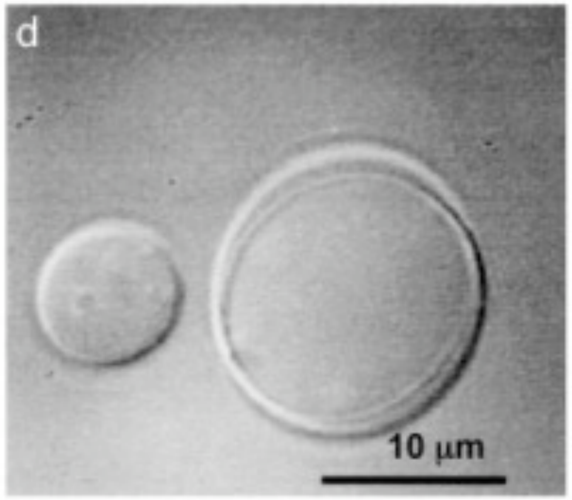


Guo et al., PNAS, 2009

Applications: non-spherical lenses, 3D electronic circuits, curved micro solar panels, wrapping of active substances for targeted drug delivery...

The Elastocapillary lengthscale

A tubulin rod growing inside a lipid vesicle



Cohen & Mahadevan, PNAS (2003)

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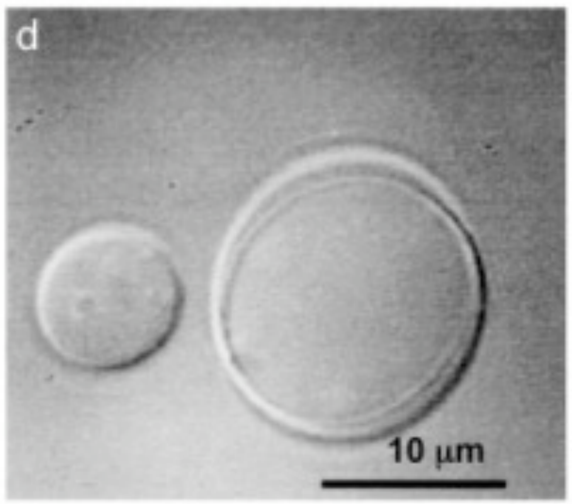
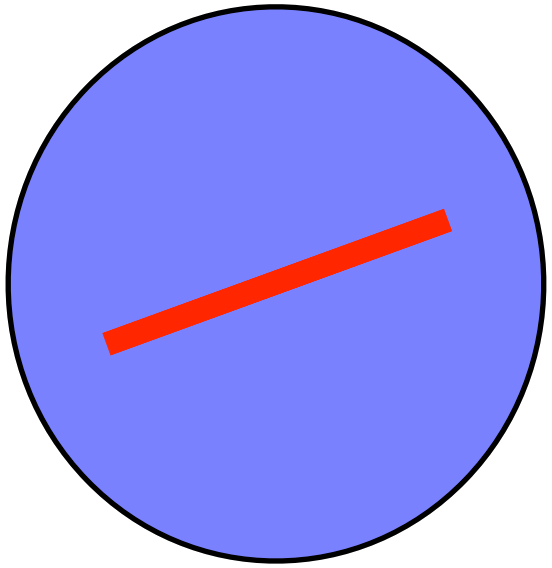


Cohen & Mahadevan, PNAS (2003)



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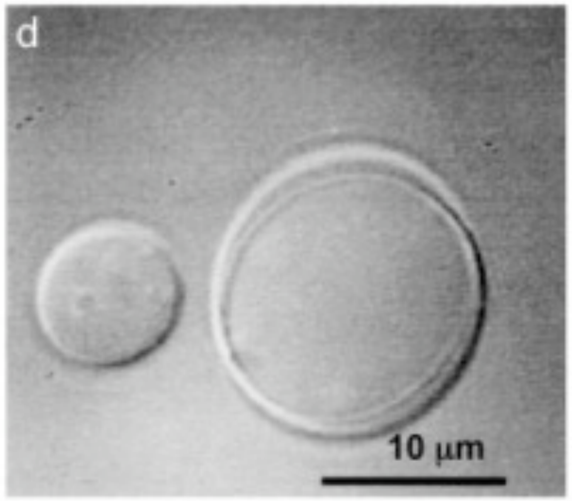
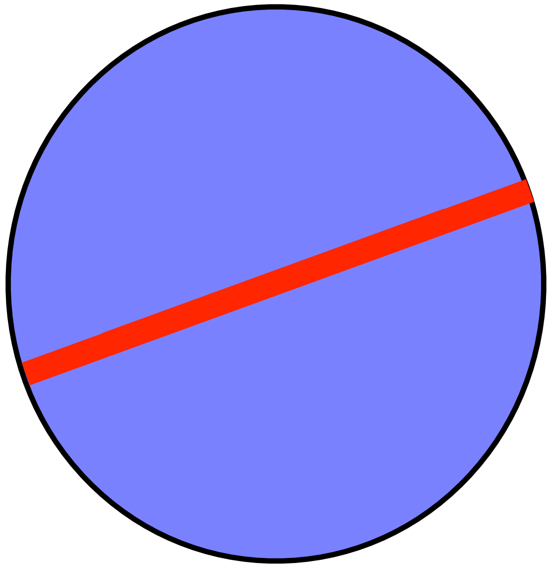


Cohen & Mahadevan, PNAS (2003)



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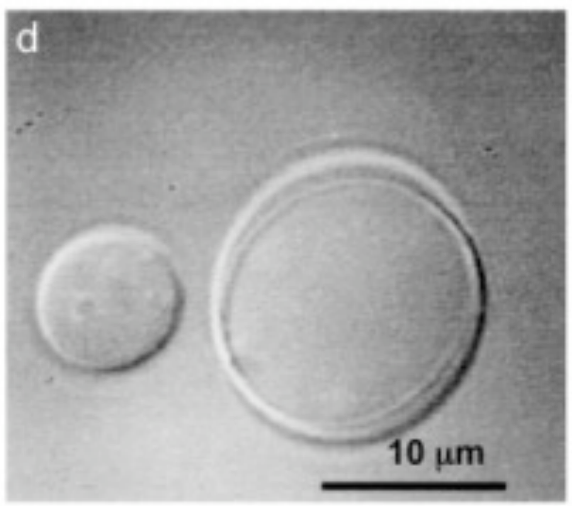
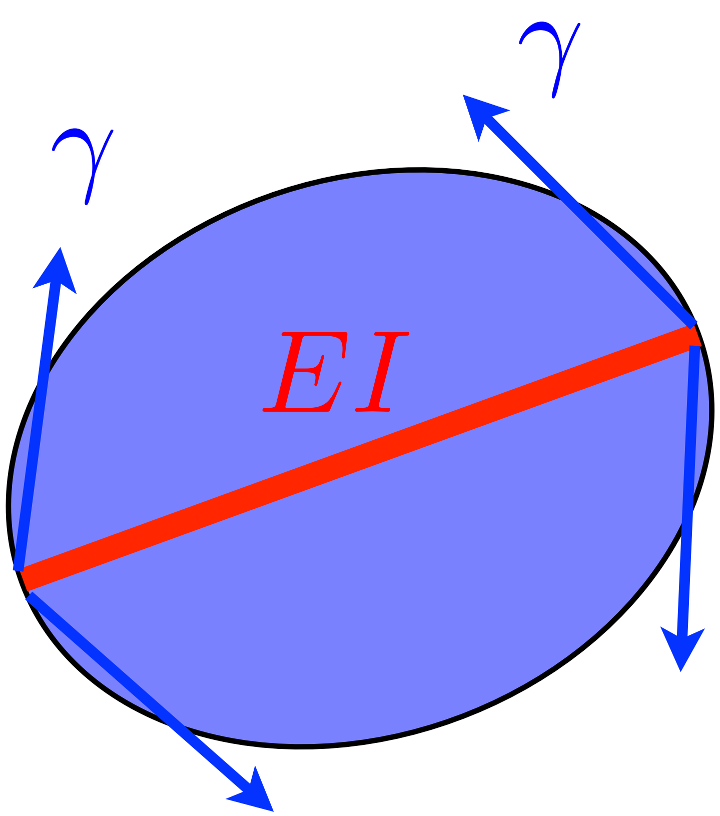


Cohen & Mahadevan, PNAS (2003)



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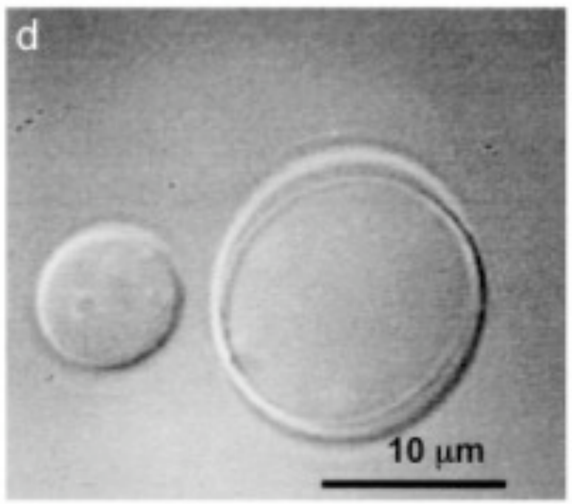
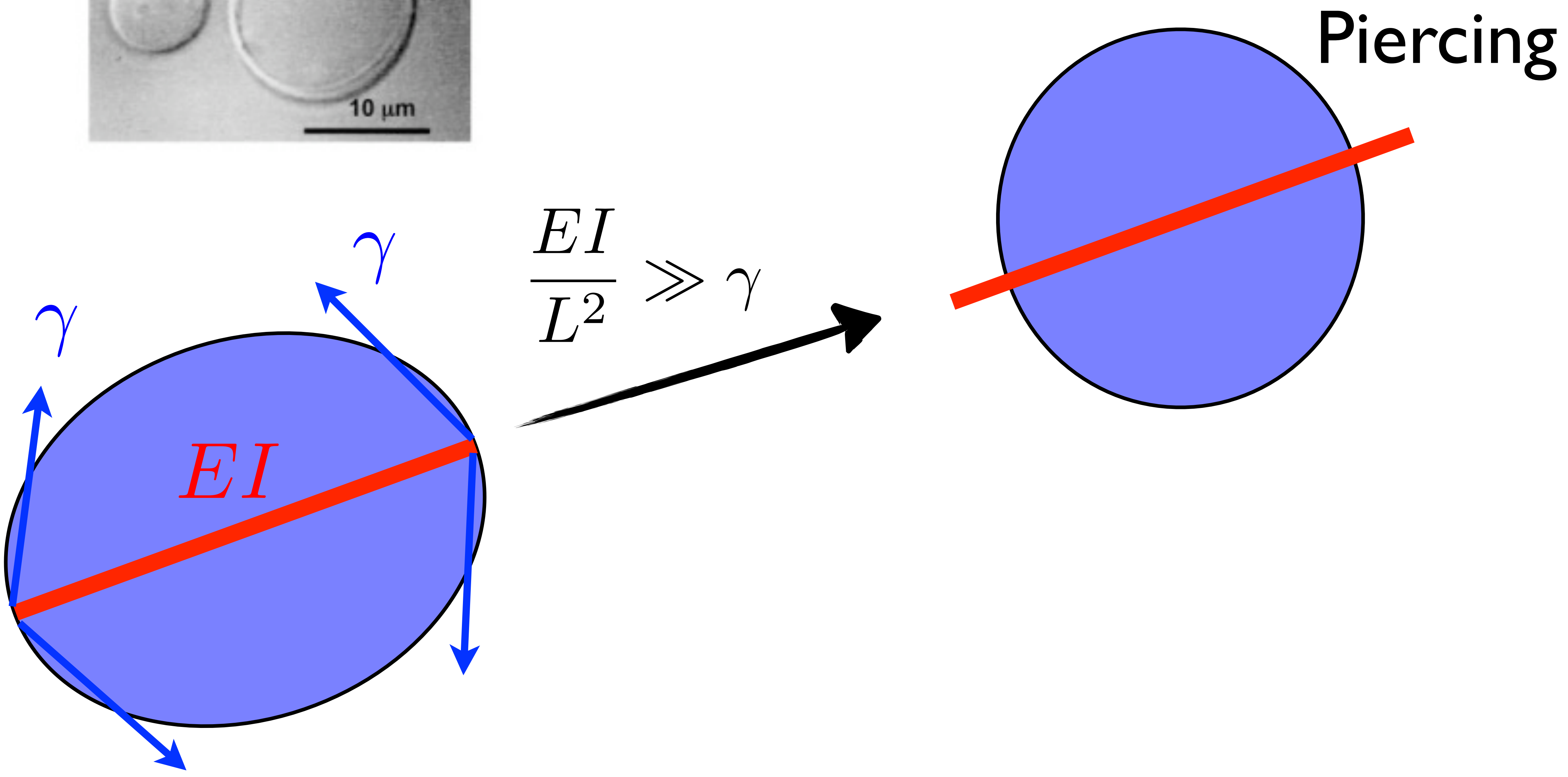


Cohen & Mahadevan, PNAS (2003)



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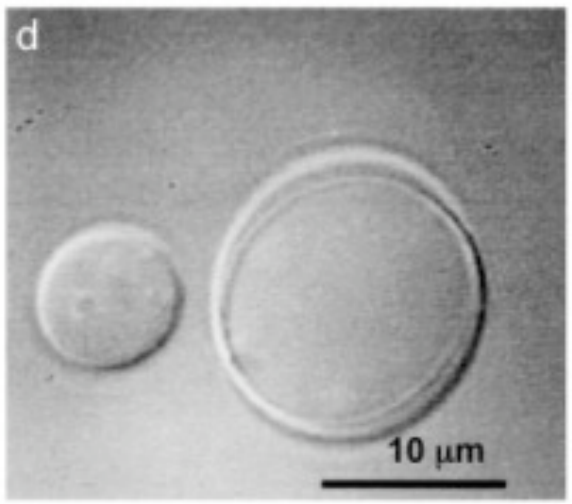
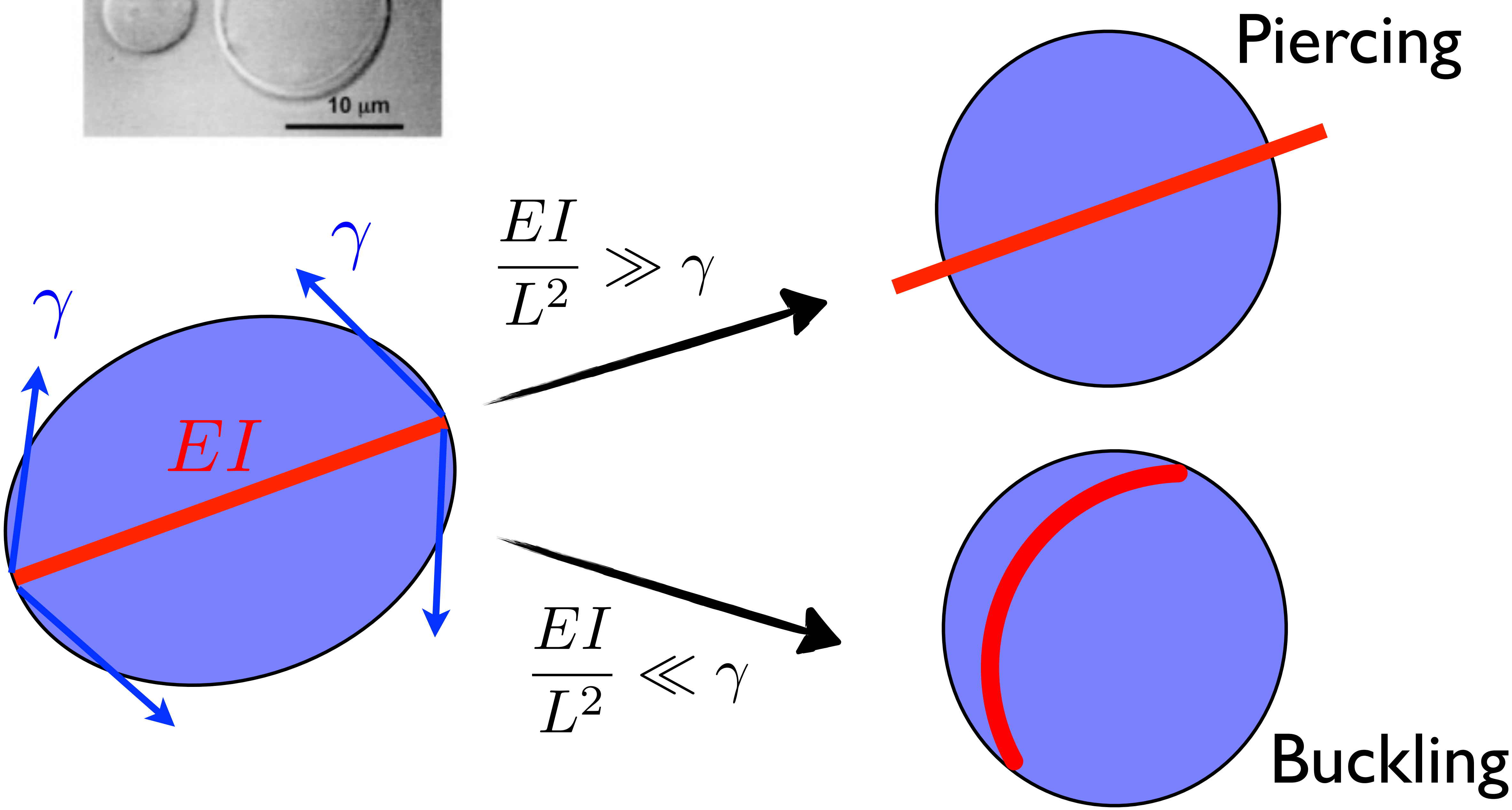
A tubulin rod growing inside a lipid vesicle



Cohen & Mahadevan, PNAS (2003)



The Elastocapillary lengthscale

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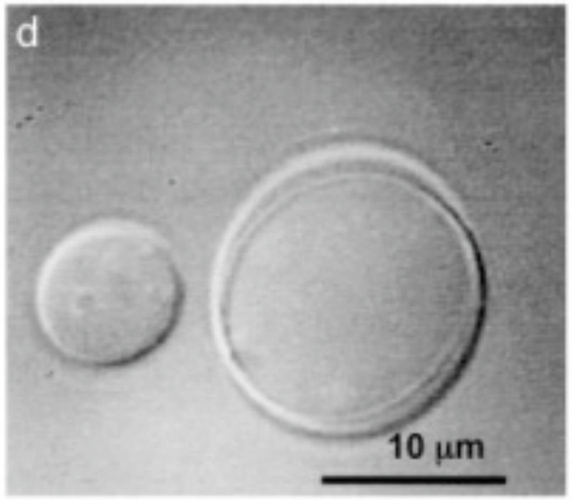
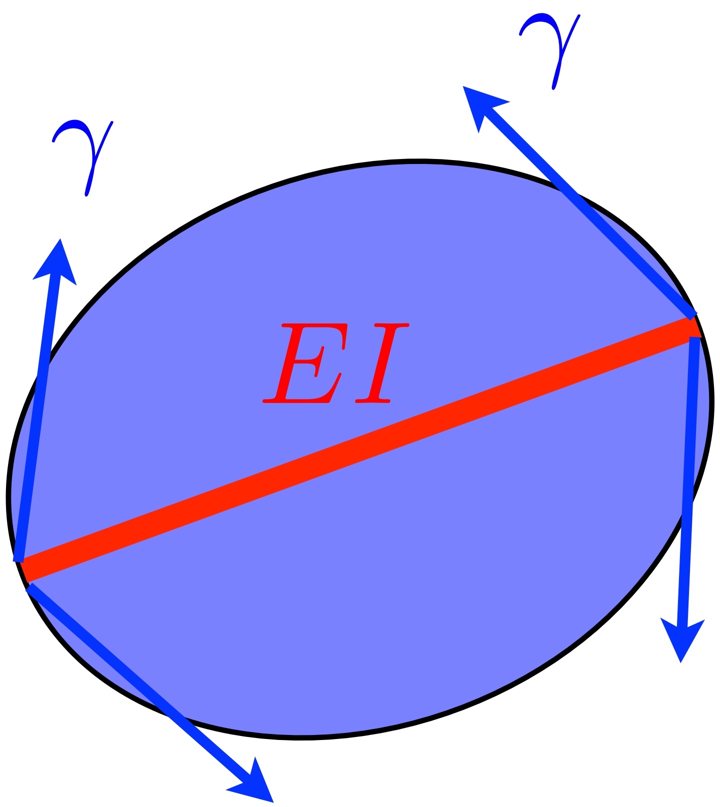


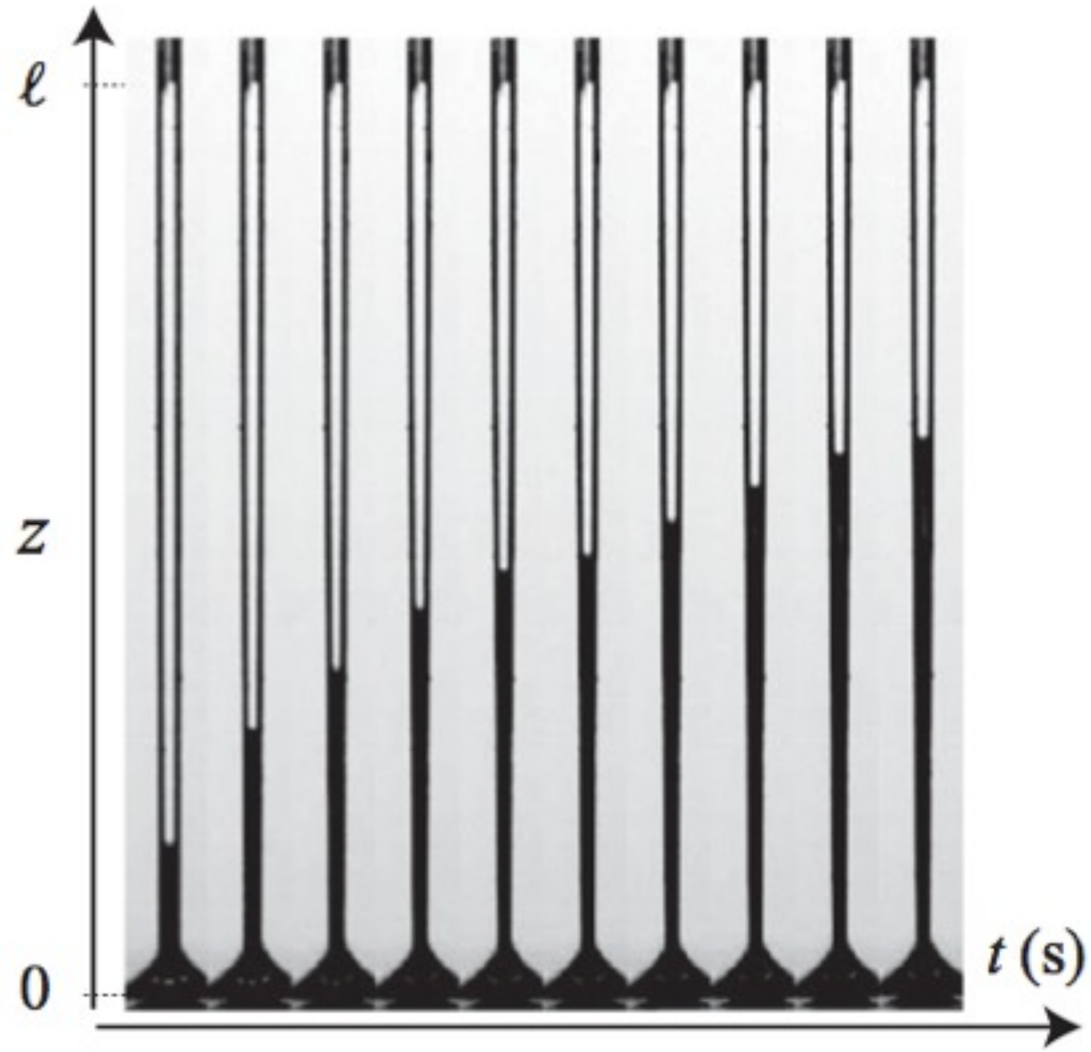
Cohen & Mahadevan, PNAS (2003)



$$\frac{EI}{L^2} \sim \gamma \longrightarrow L_{\mathrm{EC}} = \sqrt{\frac{EI}{\gamma}}$$

Mechanisms: Aggregation

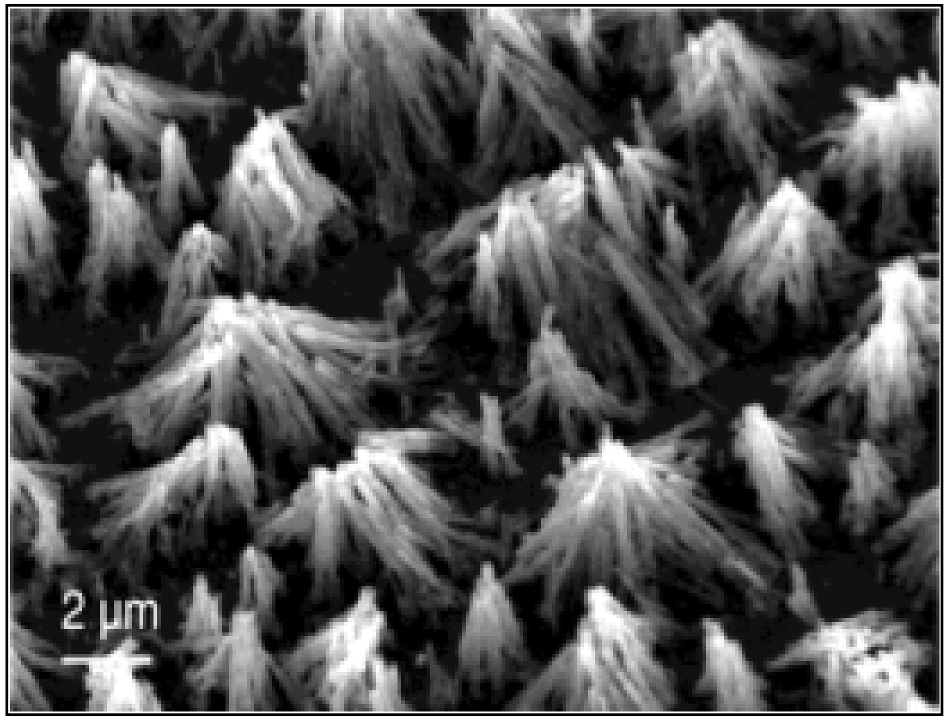
Wet hairs: elastic Jurin's law and aggregation

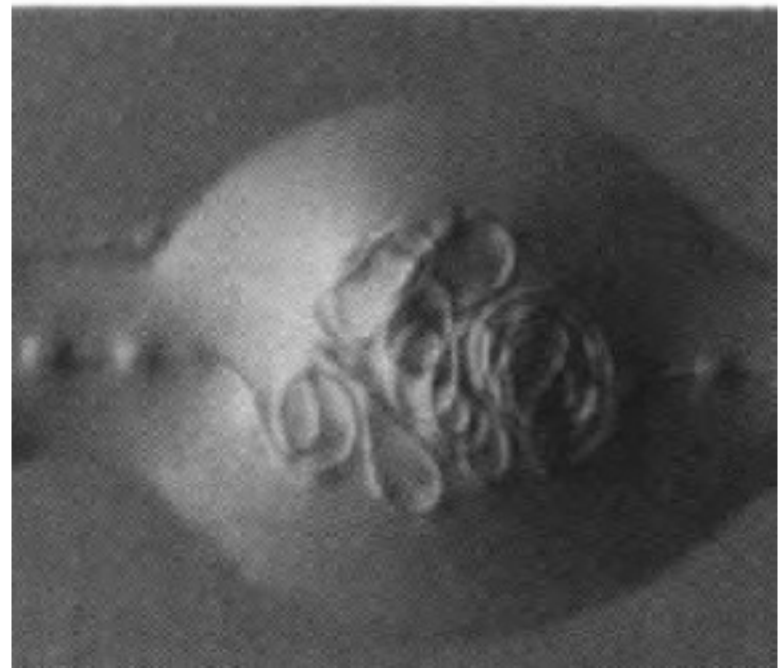


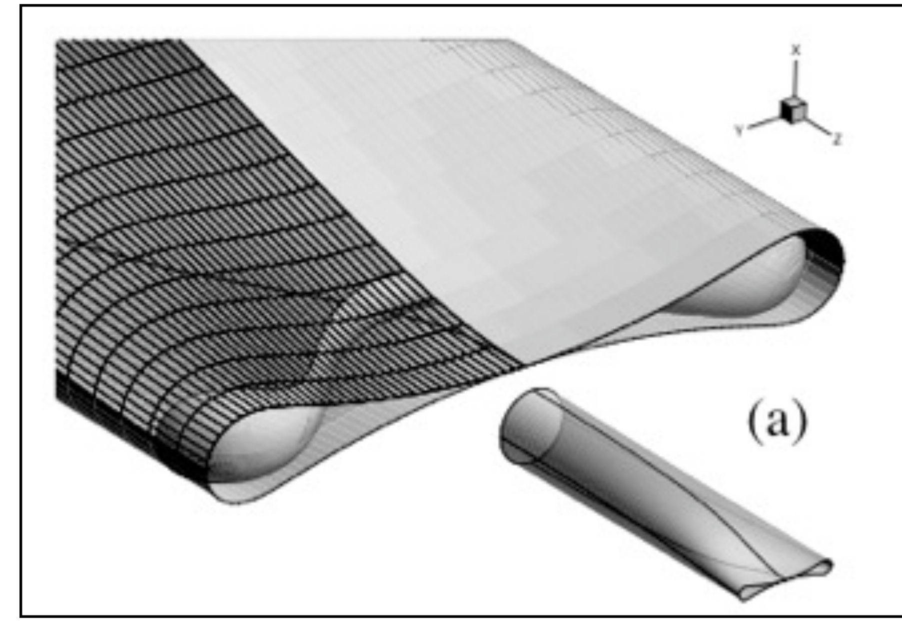
Bico et al., Nature (2004),
Kim & Mahadevan, JFM (2006),
Duprat et al., JFM (2011),
Cambeau et al., EPL (2011)

Mechanisms: Buckling

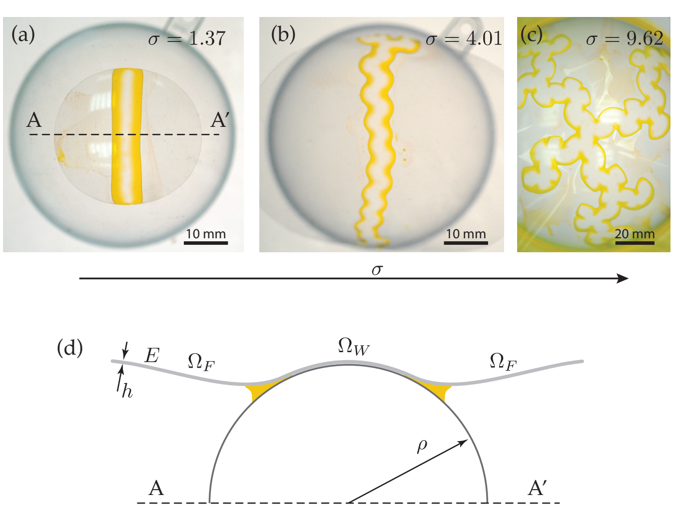
Capillary buckling

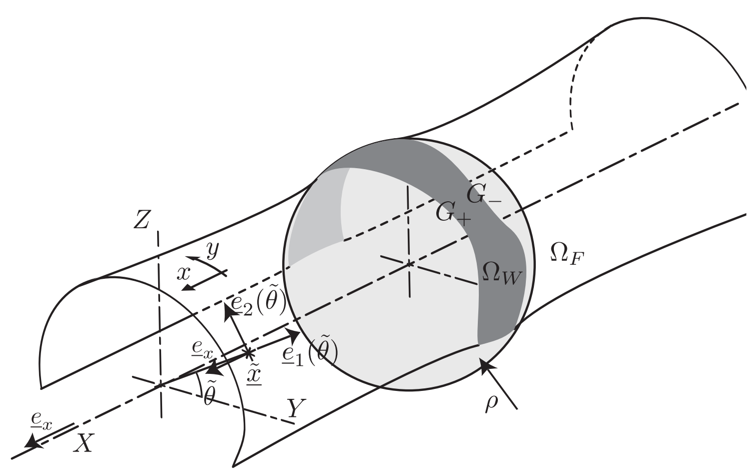


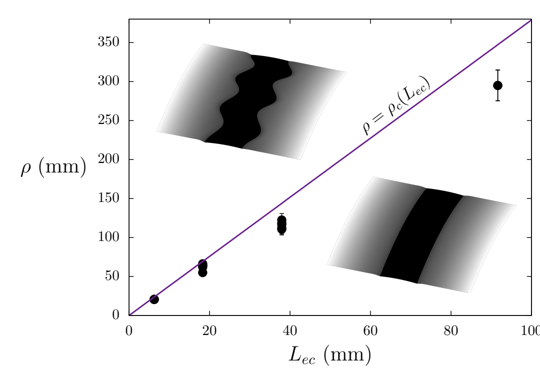


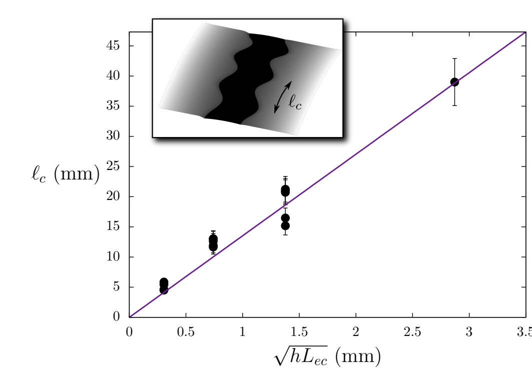


Vollrath & Edmonds, Nature (1989),
Heil, JFM (1999),
Lau et al., Nano Lett. (2003),
Neukirch et al., JMPS (2007)



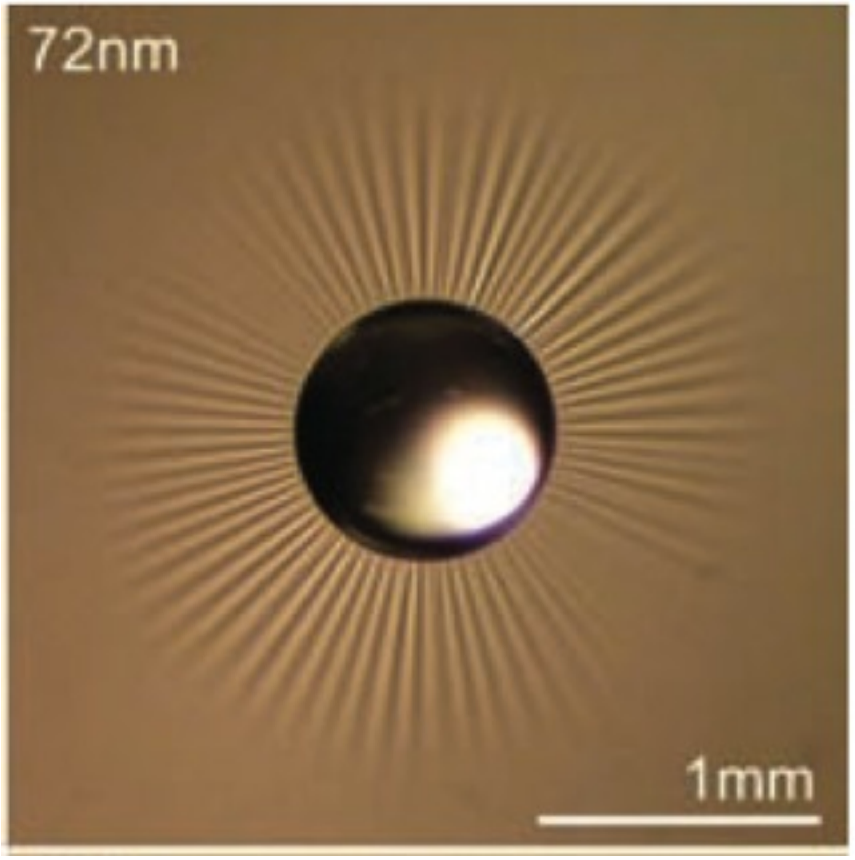


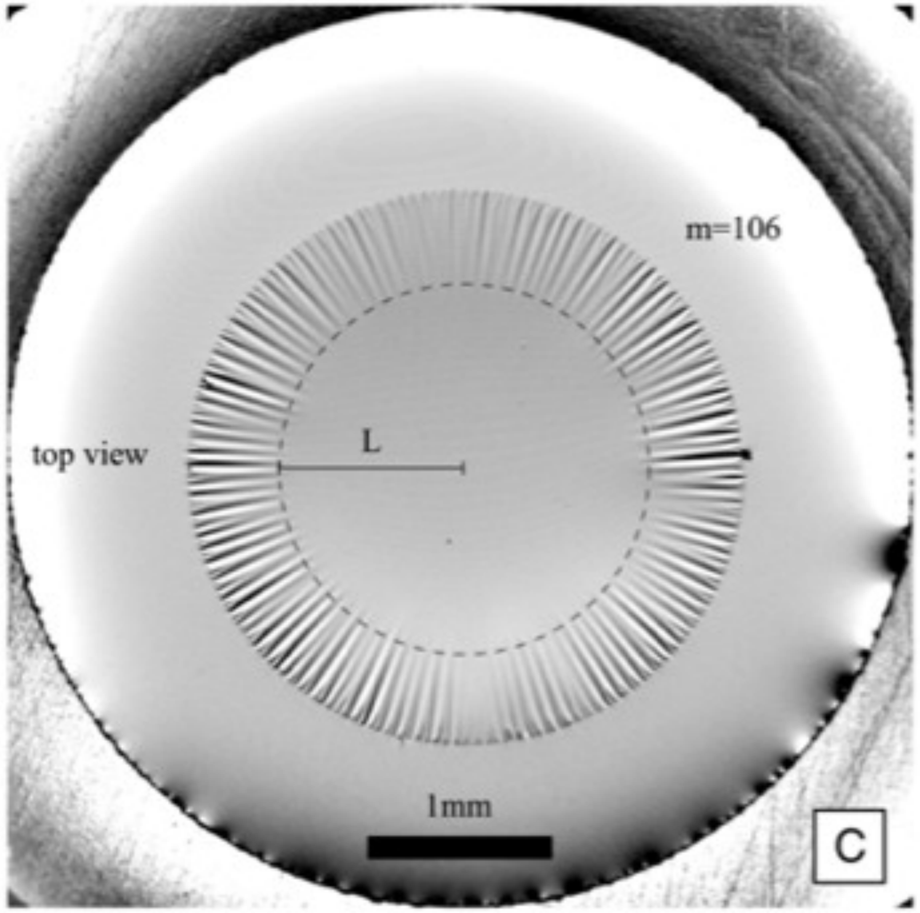




Hure & Audoly, *JMPS* (2012)

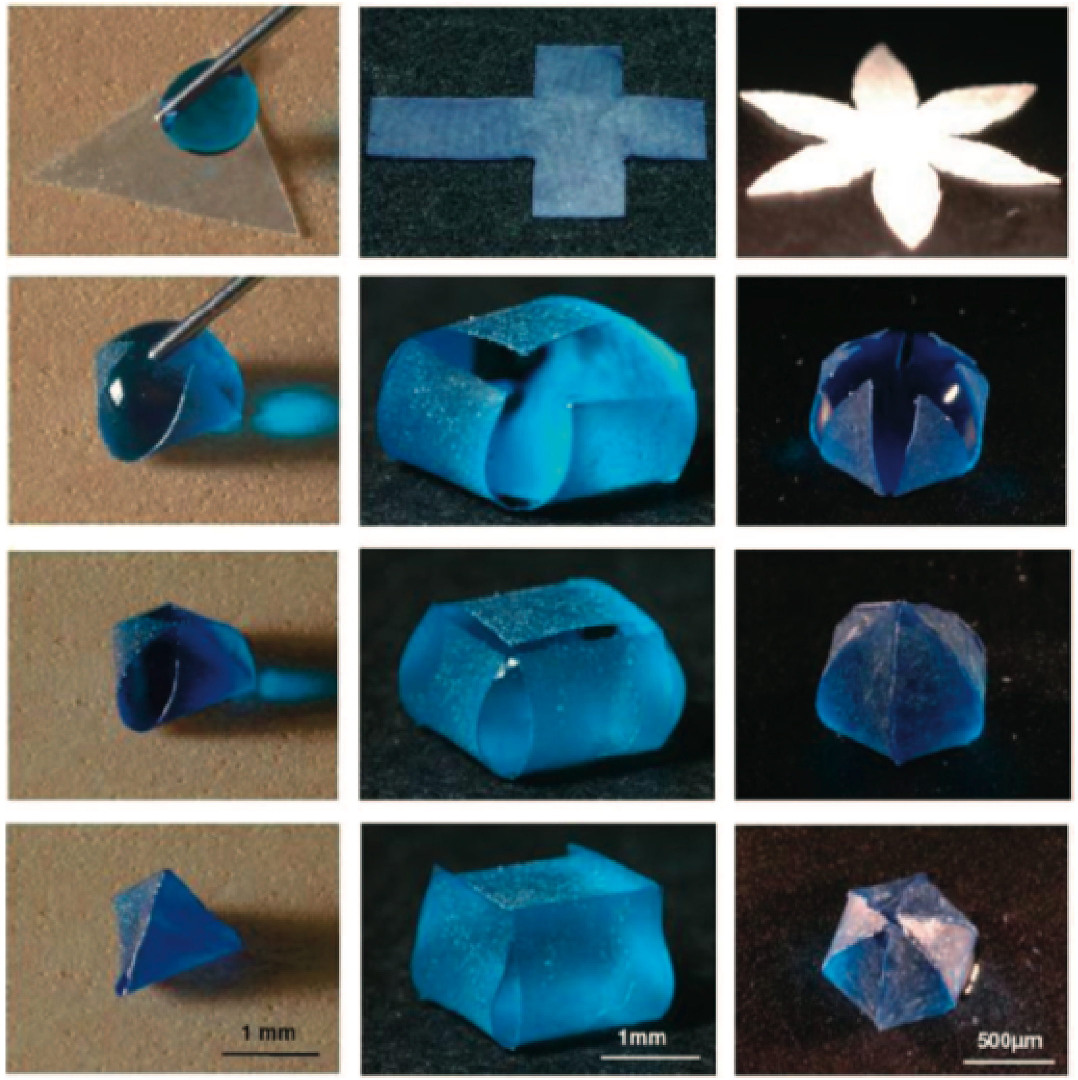
Mechanisms: Wrinkling

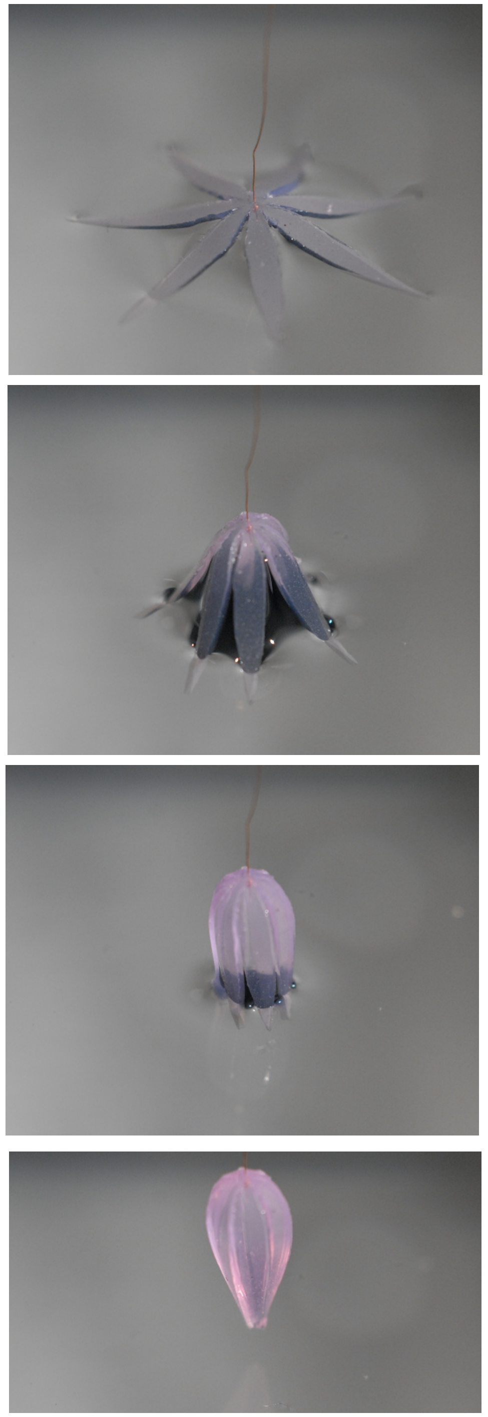




Huang et al., Science (2007), Pocivavsek et al., Science (2008), Hunt et al., Soft Matter (2012)

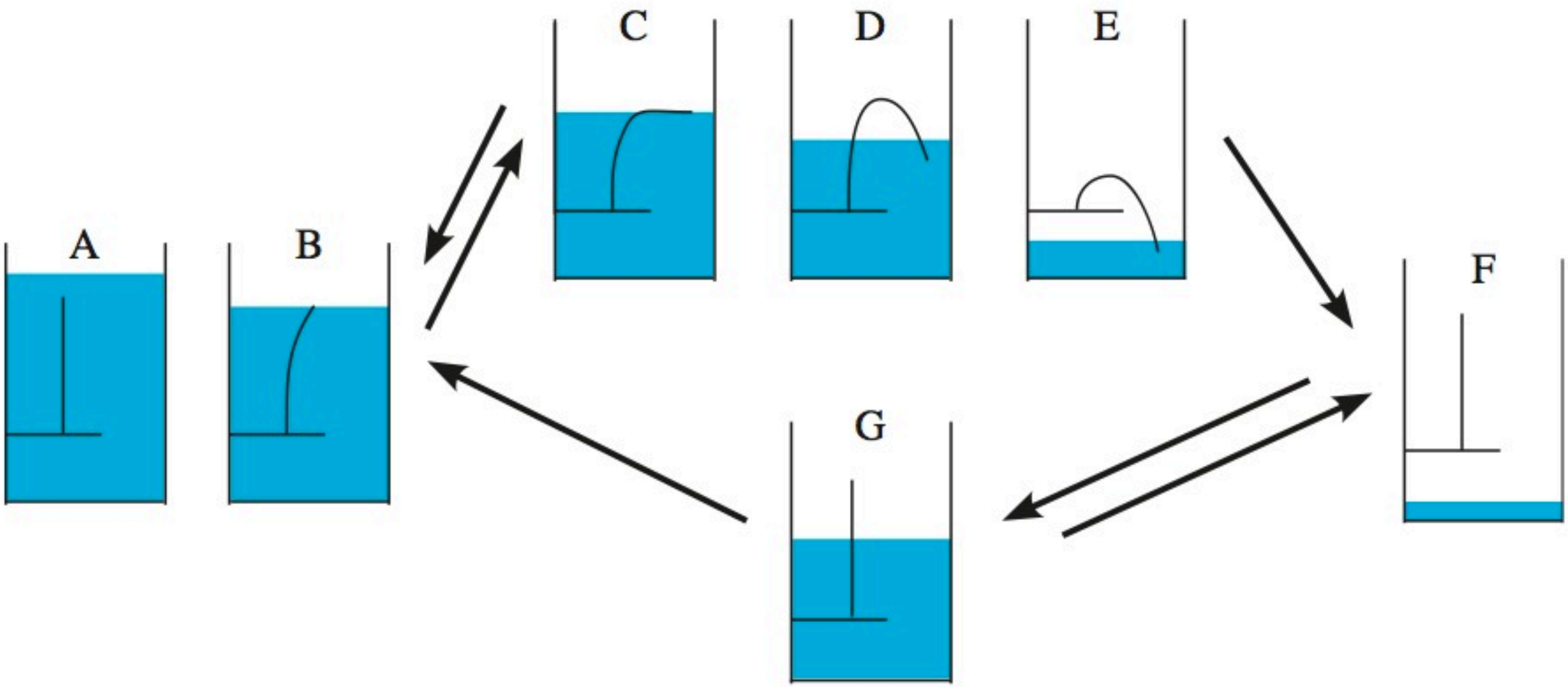
Mechanisms: Wrapping & Folding





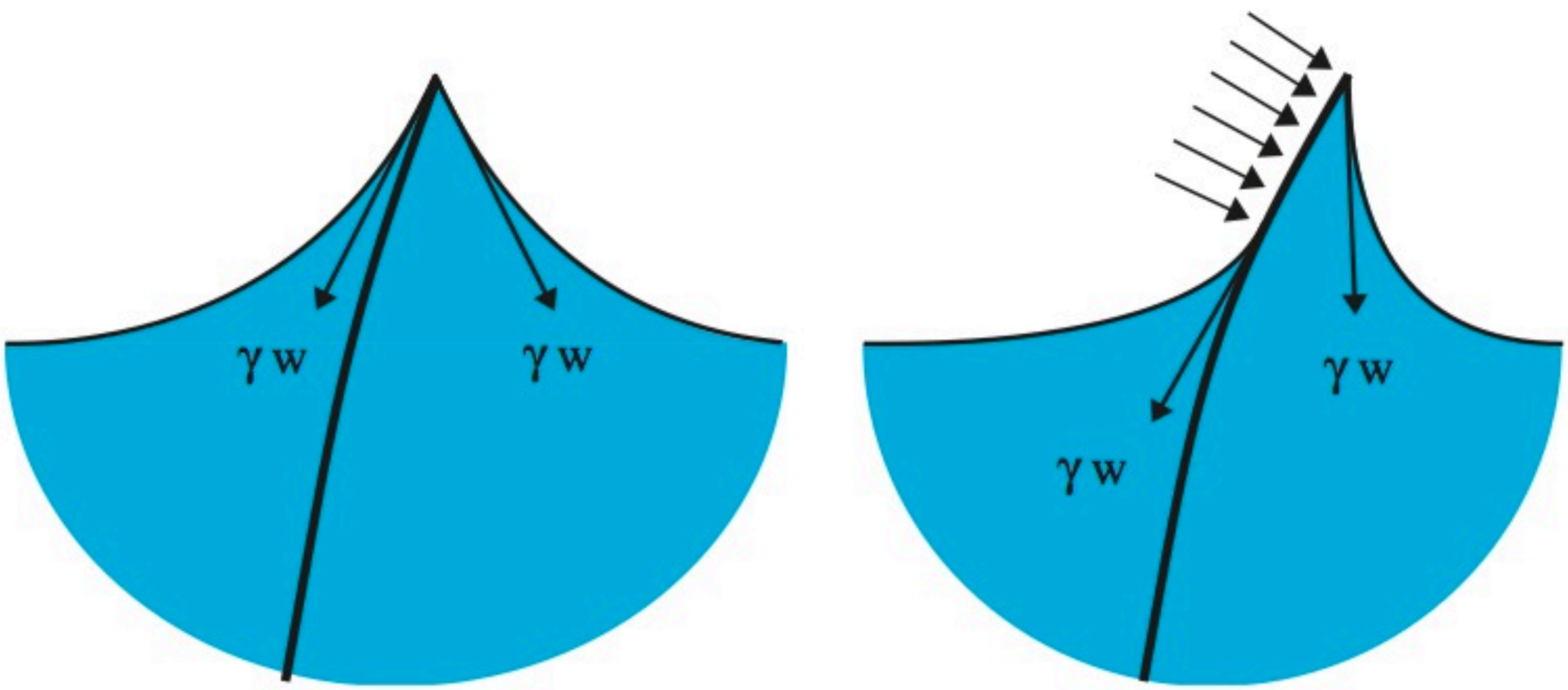
Syms et al., J. of MEMS (2003),
Py et al., PRL (2007),
Reis et al., Soft Matter (2010)

Bending an elastic beam with surface tension



Neukirch, Bico, Roman JMPS 2007

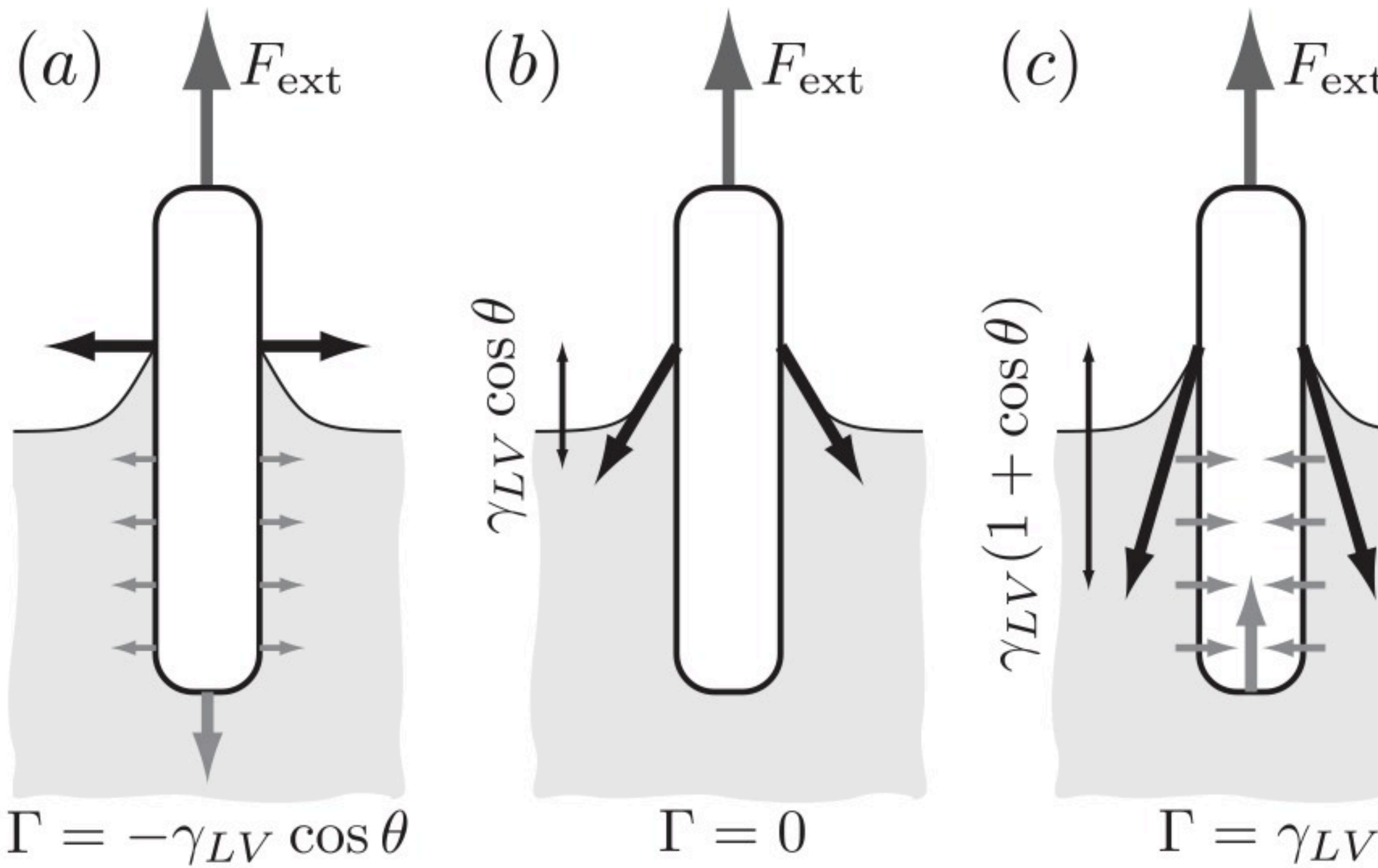
Bending an elastic beam with surface tension



Forces on the beam :

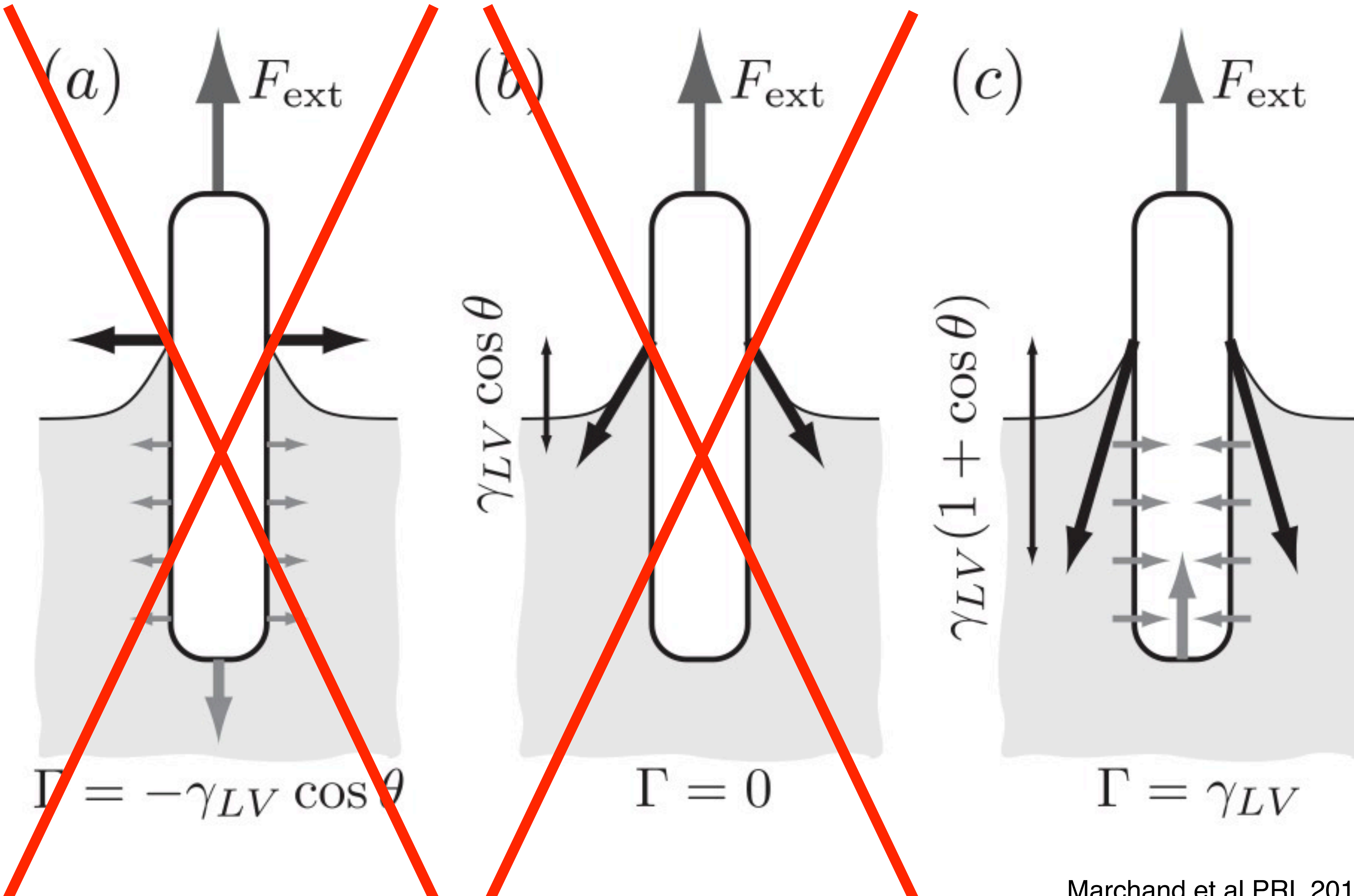
- hydrostatic pressure
- point force at meniscus

Direction of the force at meniscus ?



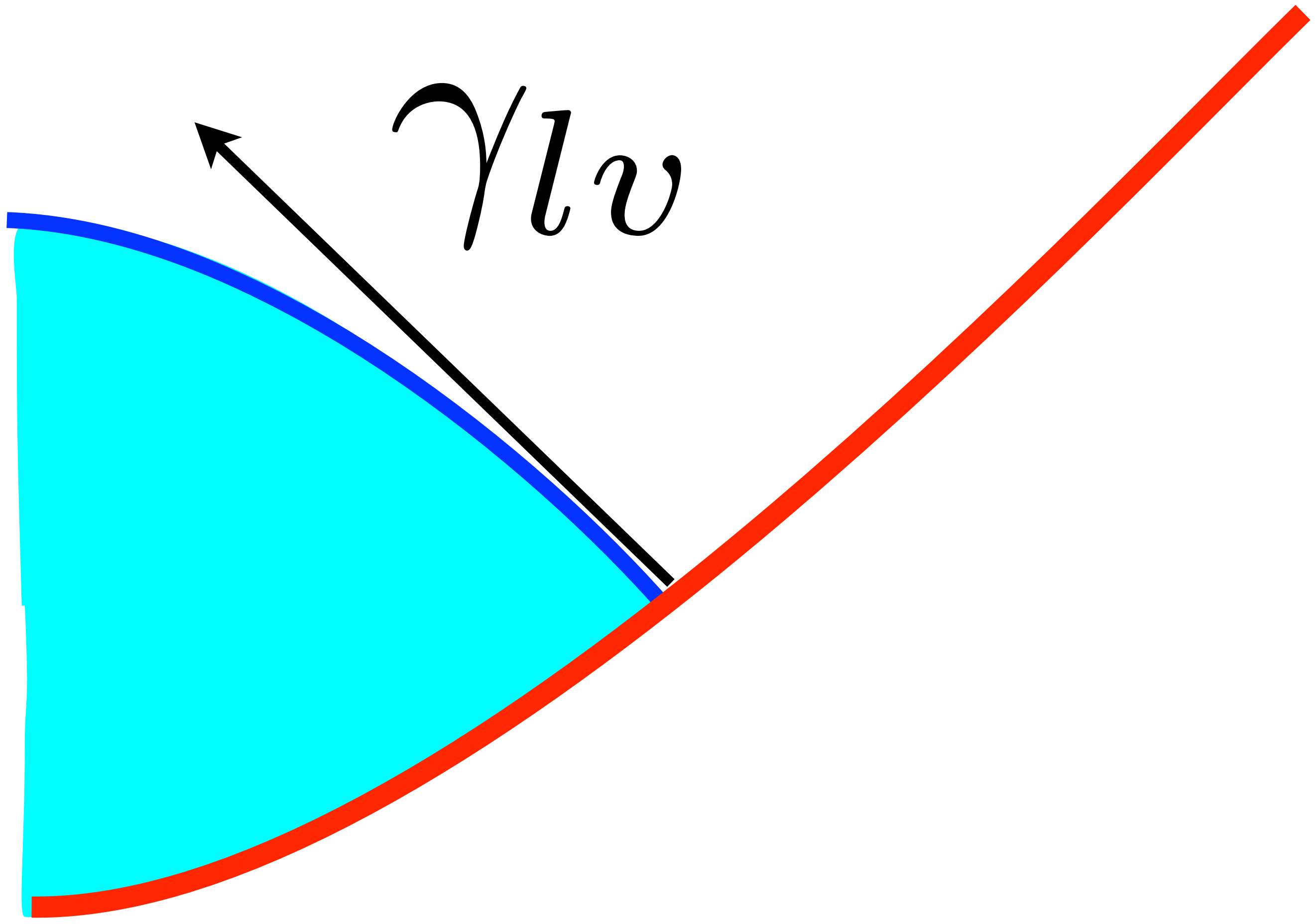
Marchand et al PRL 2012

Direction of the force at meniscus ?

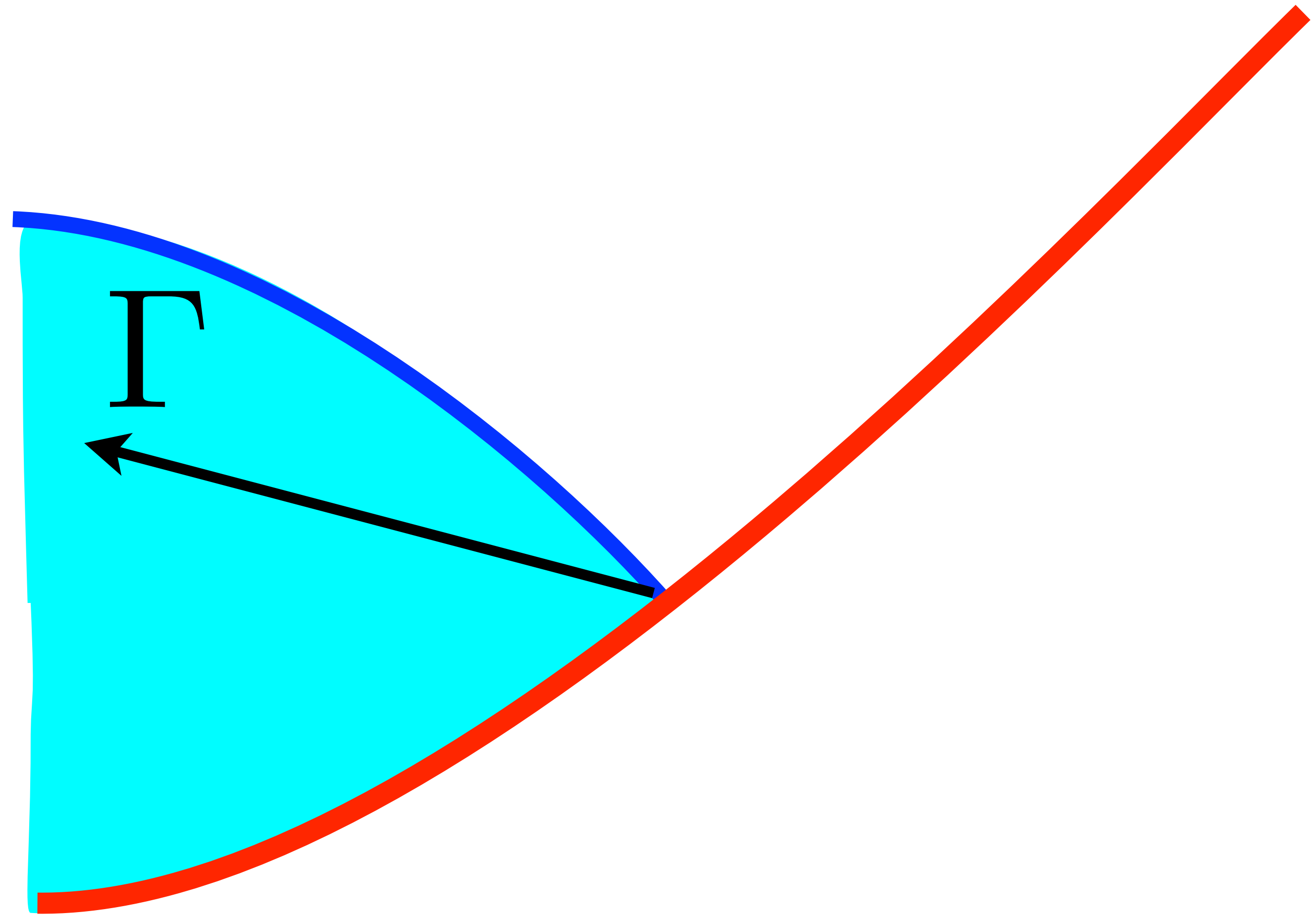


Marchand et al PRL 2012

Question



Question



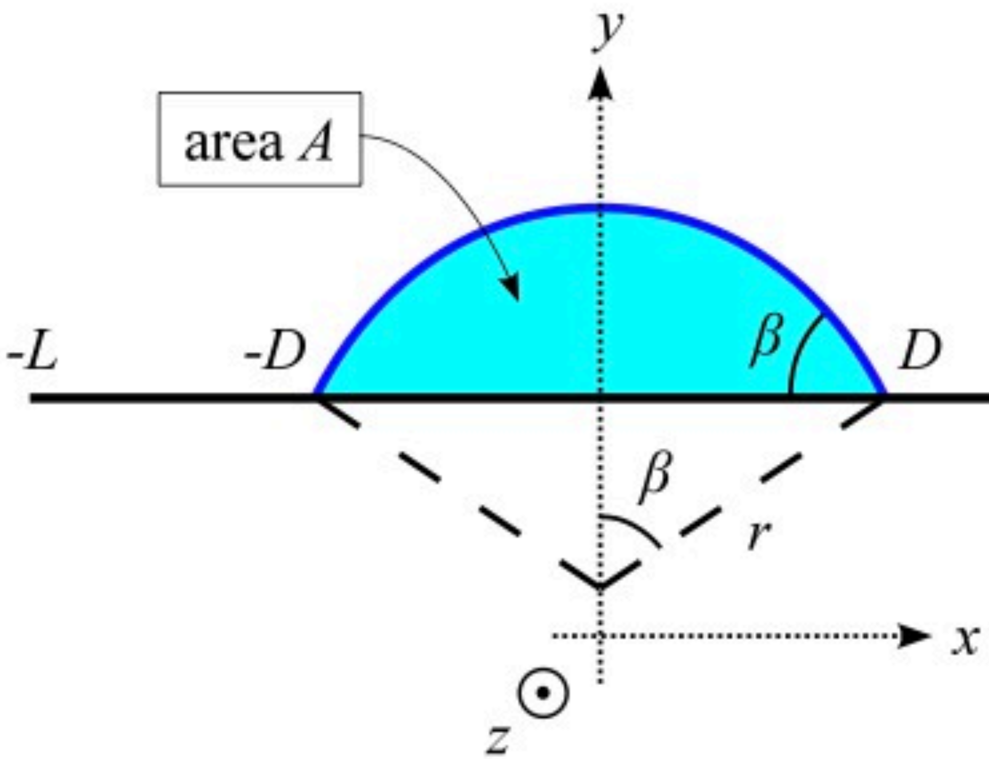
Goal :
compute the forces
via
an energy approach

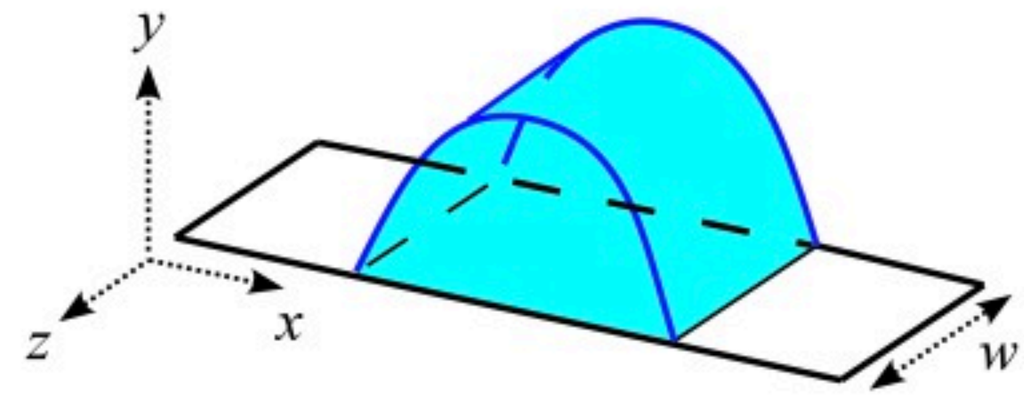
Variational approach

1- write potential energy of the system
2- minimize the energy => stable equilibrium

Advantages on the direct (force) approach:
- elegant, self-contained, powerfull
- only deals with energies :
  no need to postulate where the forces are.

Variational approach





energy

$$E(\beta, r, D) = \sum_i \gamma_i \, A_i \qquad \text{with} \qquad i = lv, sv, sl$$

$$\min\ E(\beta, r, D) \Rightarrow$$

Young-Dupré relation $\gamma_{sl} - \gamma_{sv} + \gamma_{lv} \cos\beta = 0$

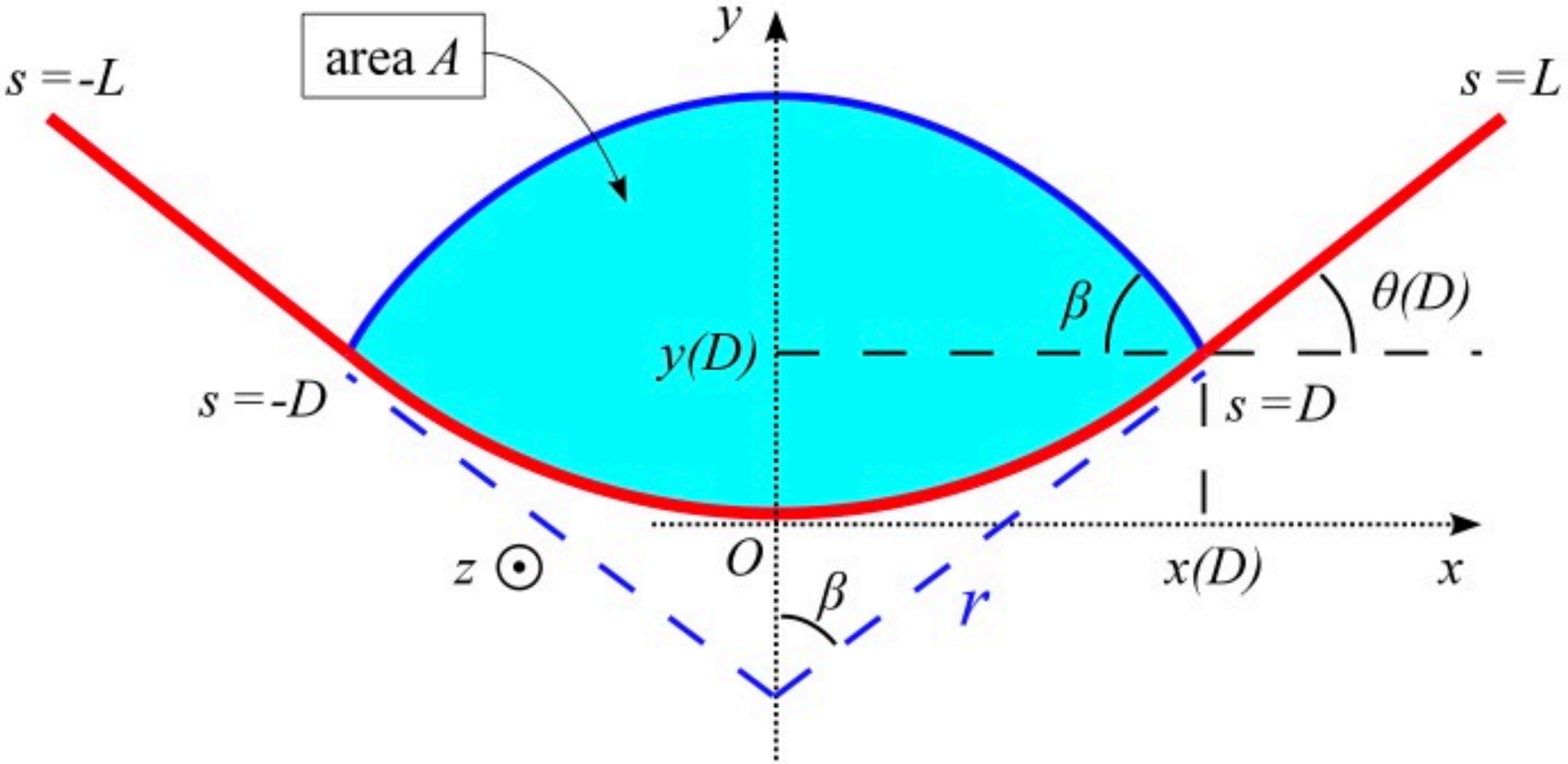
Laplace pressure $P = \dfrac{\gamma_{lv}}{r}$

Gauss 1830

Variational approach

one never sees forces
one only sees their effect (deformation)

Variational approach



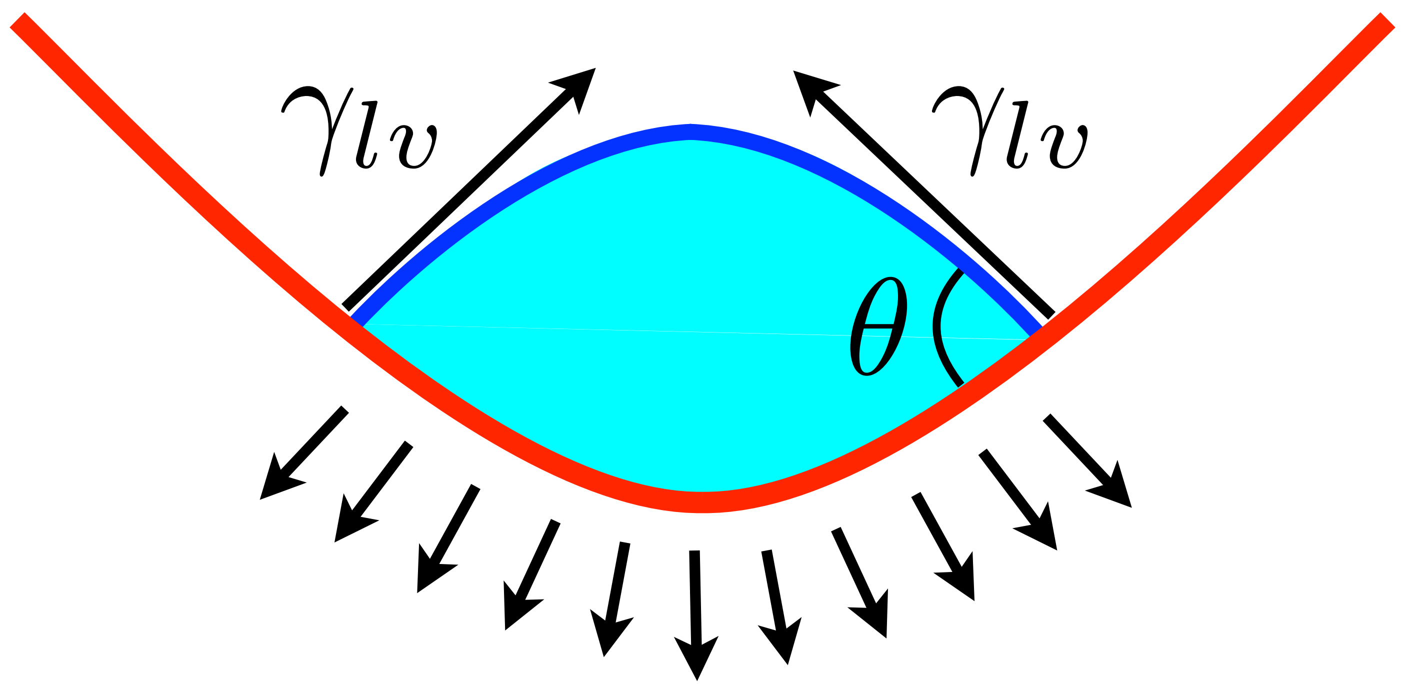
energy

$$E = \sum_i \gamma_i \, A_i + \frac{1}{2} Y I \int_{-L}^{L} \left[\theta'(s)\right]^2 \mathrm{d}s$$

interfaces bending

$$E = E(\beta, r, D, x(s), y(s), \theta(s))$$

Variational approach



minimization $\delta E = 0 \;\Rightarrow$

Young-Dupré relation: $\gamma_{sl} - \gamma_{sv} + \gamma_{lv}\cos\theta = 0$

Equilibrium equations for the elastic beam with:

- Laplace pressure
- localized forces at triple points *directed along the meniscus*

thank you