Static and dynamic instabilities in a drop-rod system

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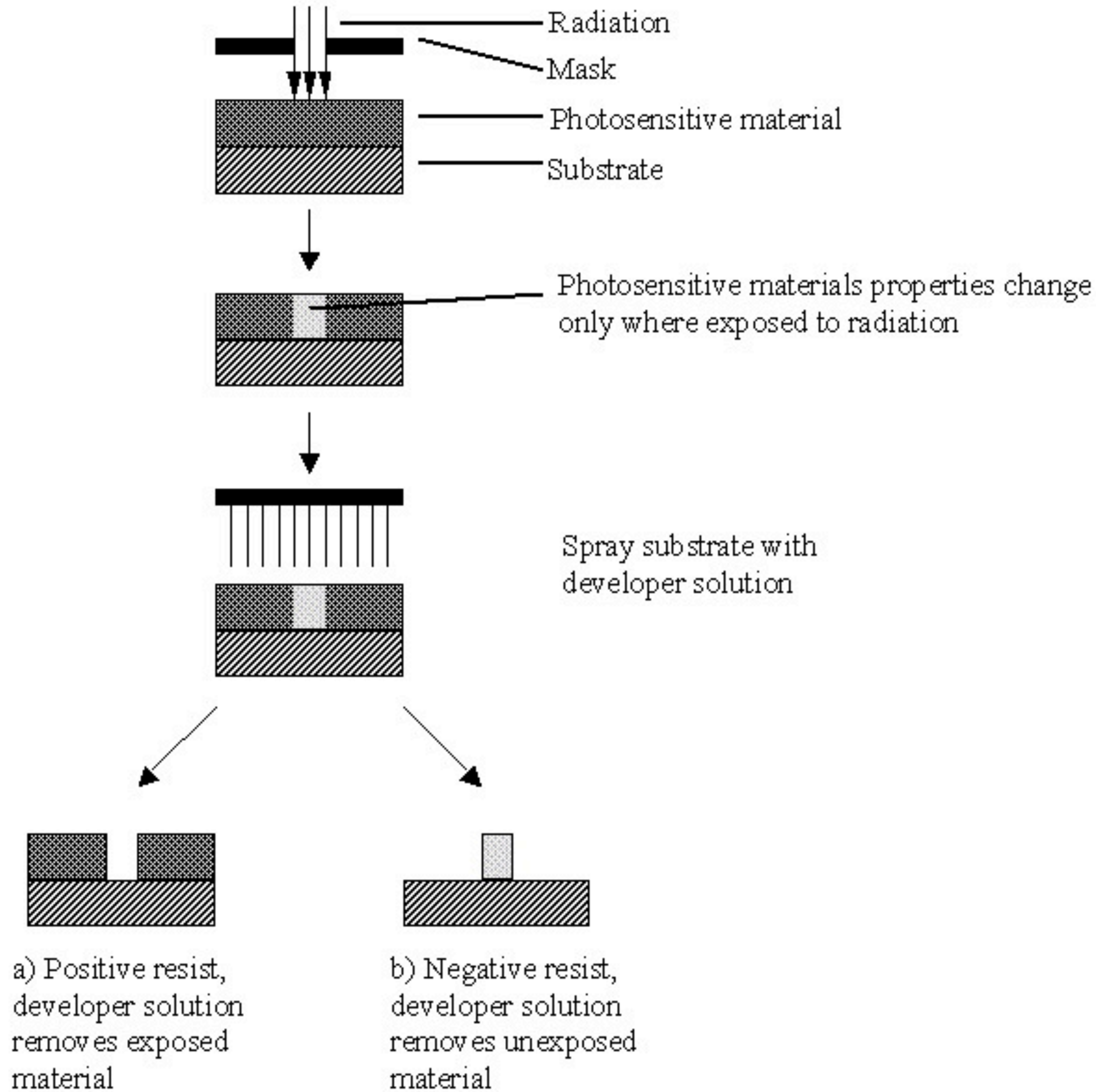
1 - Motivations

2 - Experiments

3 - Model

Nano-structures fabrication

Optical lithography

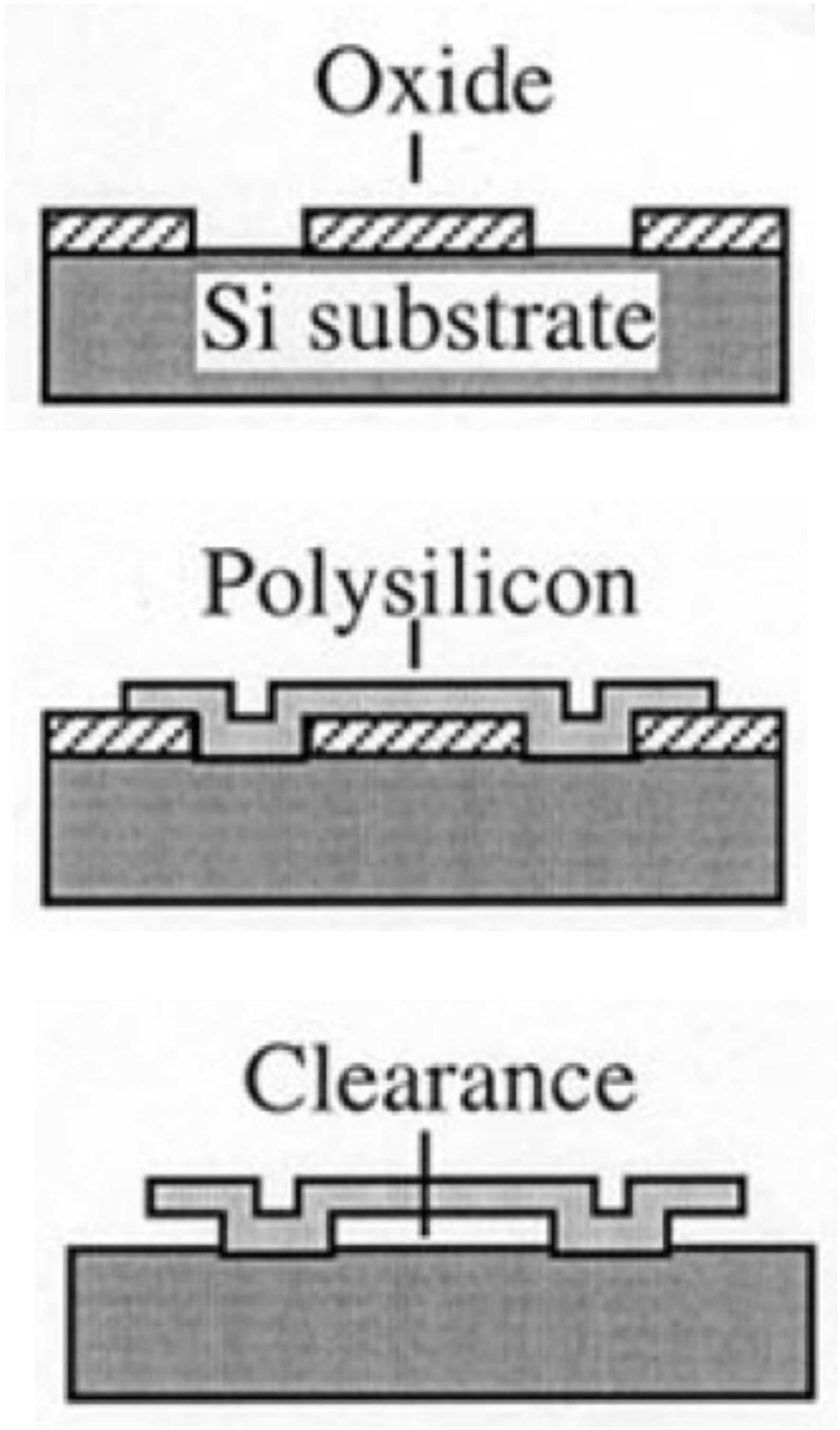


planar only

www.memsnet.org

Nano-structures fabrication

surface
micro-machining



expensive
limited shapes

Syms et al, *Journal of MEMS* (2003)

Nano-structures fabrication

melting of solder (tin)

rotated hinged joints

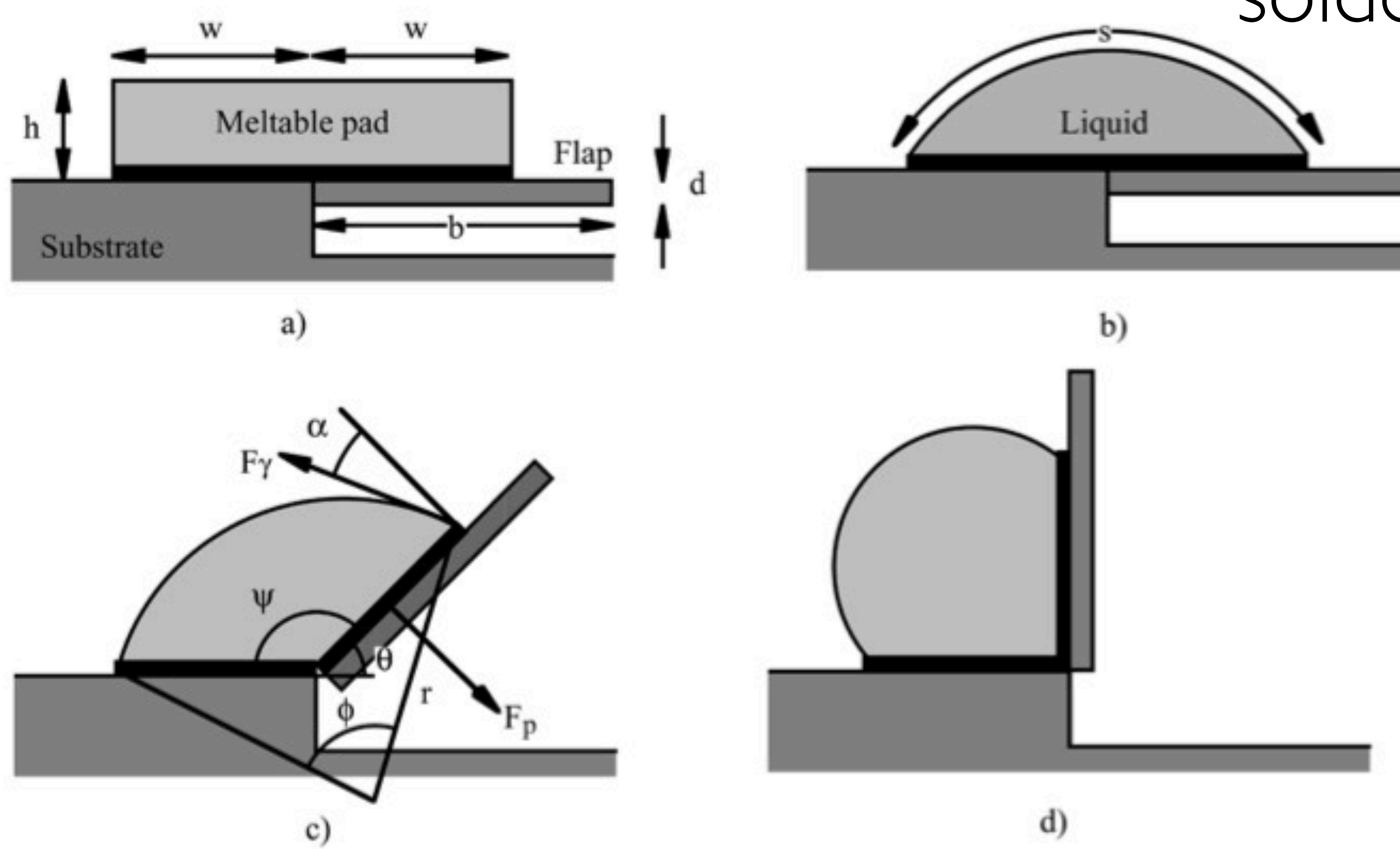


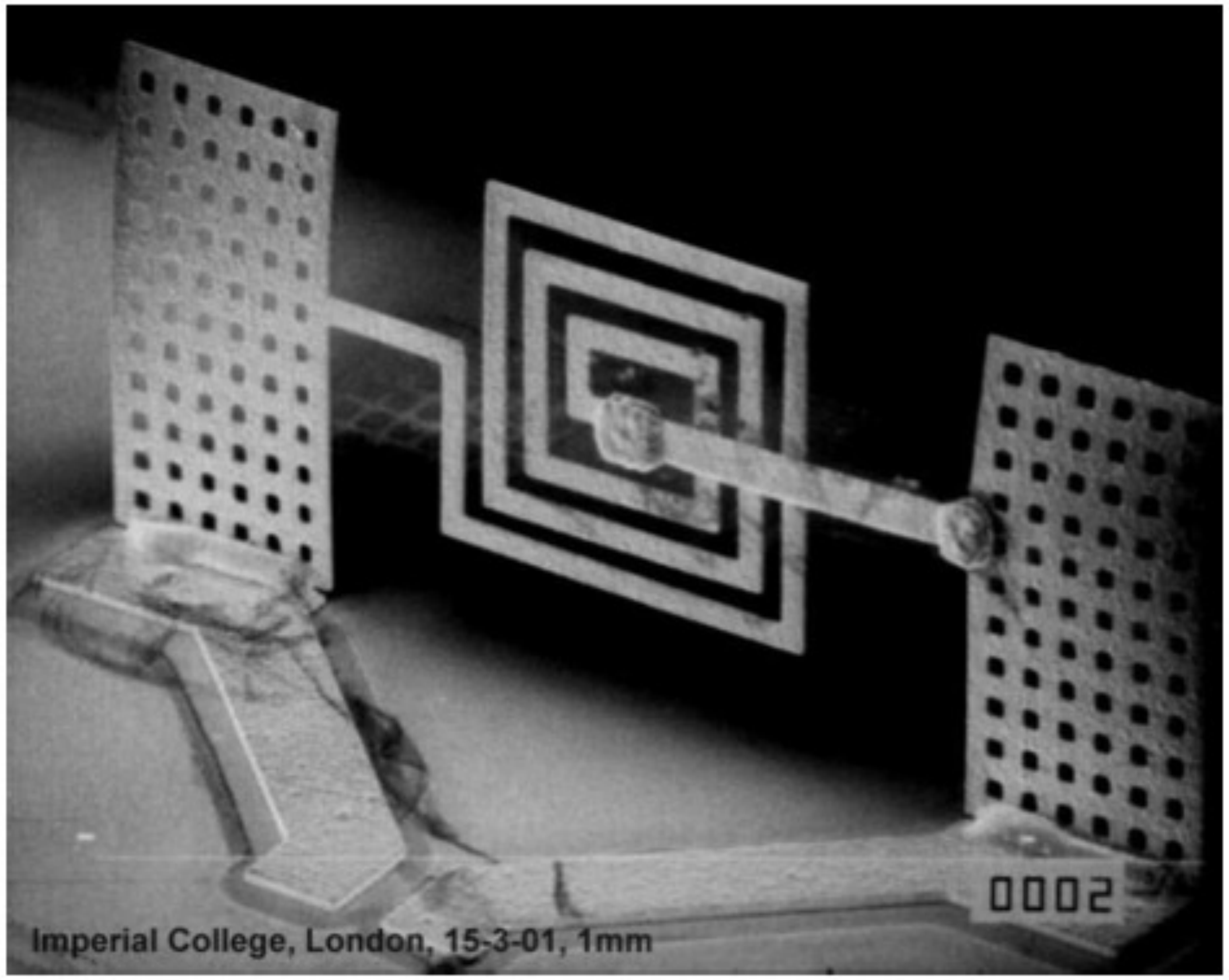
Fig. 3. Two-dimensional geometry for surface tension powered rotation.

re-solidification

R. Syms, *Journal of MEMS* (1995)

Micro-Origamis

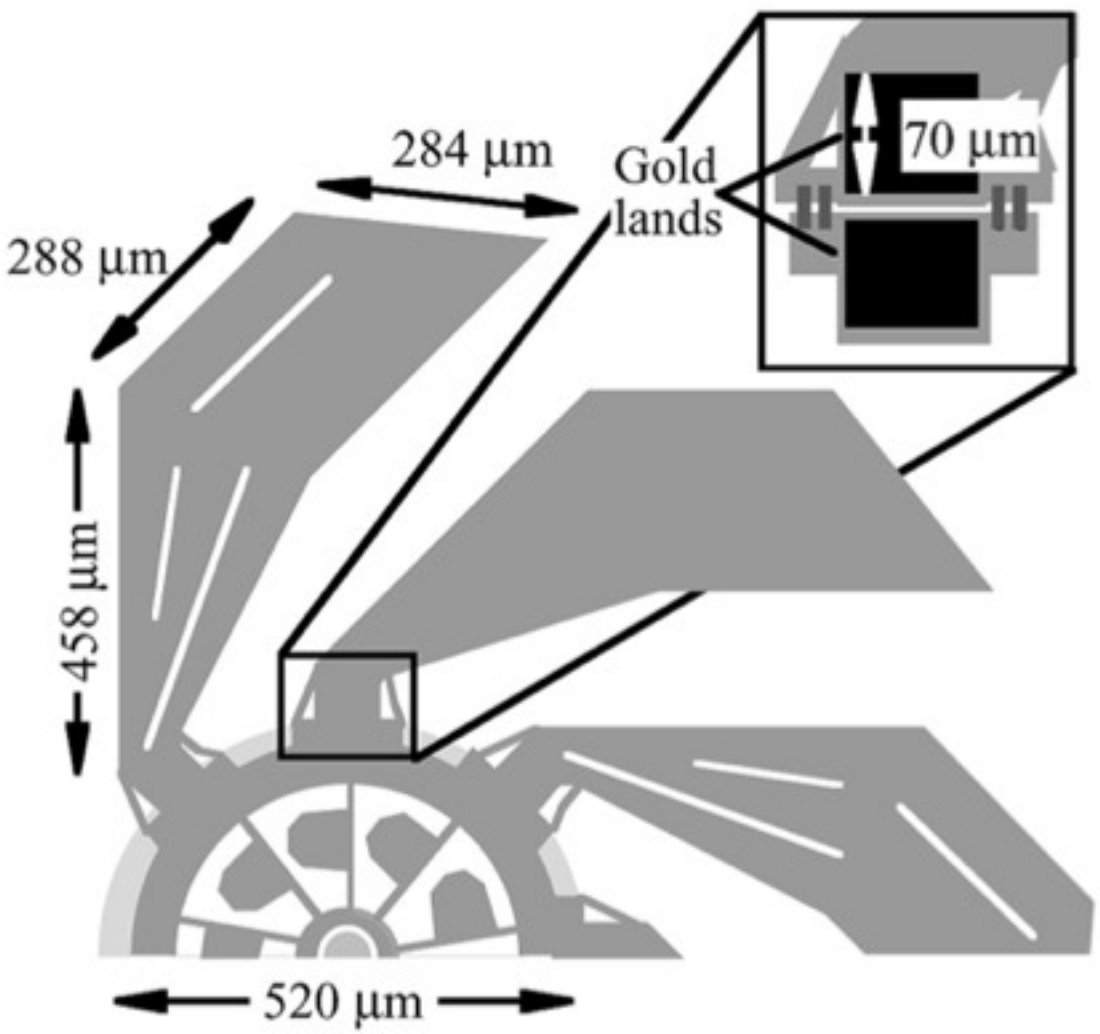
inductor has to be away from (metallic) substrate to prevent magnetic field loss



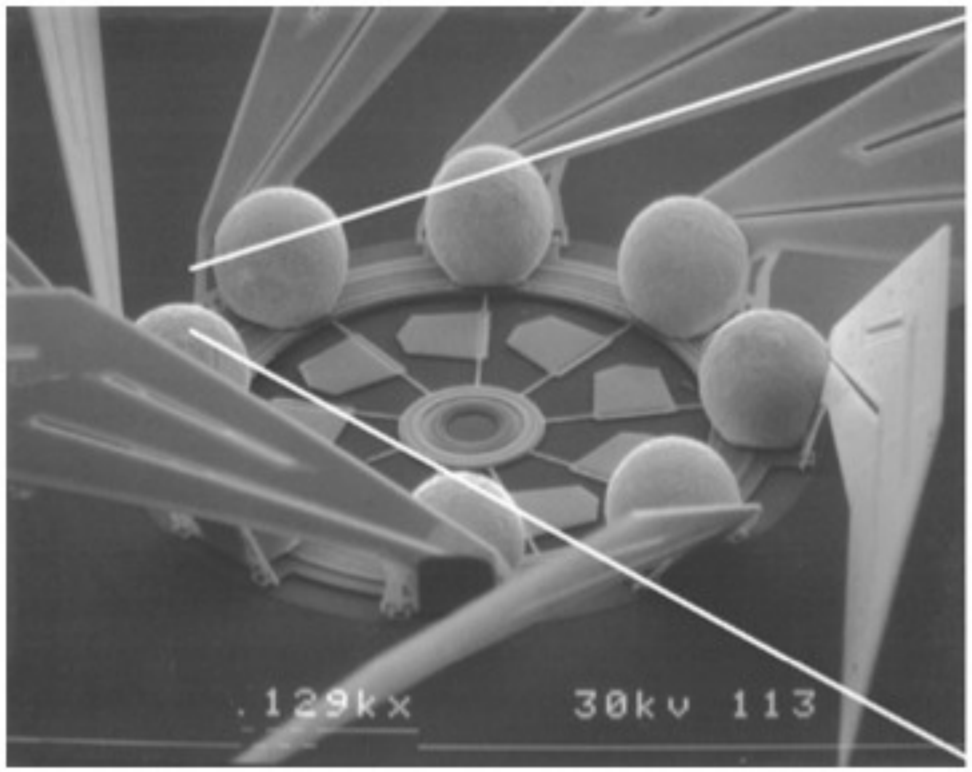
3D electrical components (here an inductor) assembled by surface tension

Dahlmann, *Electron Lett.* (2000)

Micro-Origamis



microfan with polysilicon
180 rpm
micro-fluidic systems

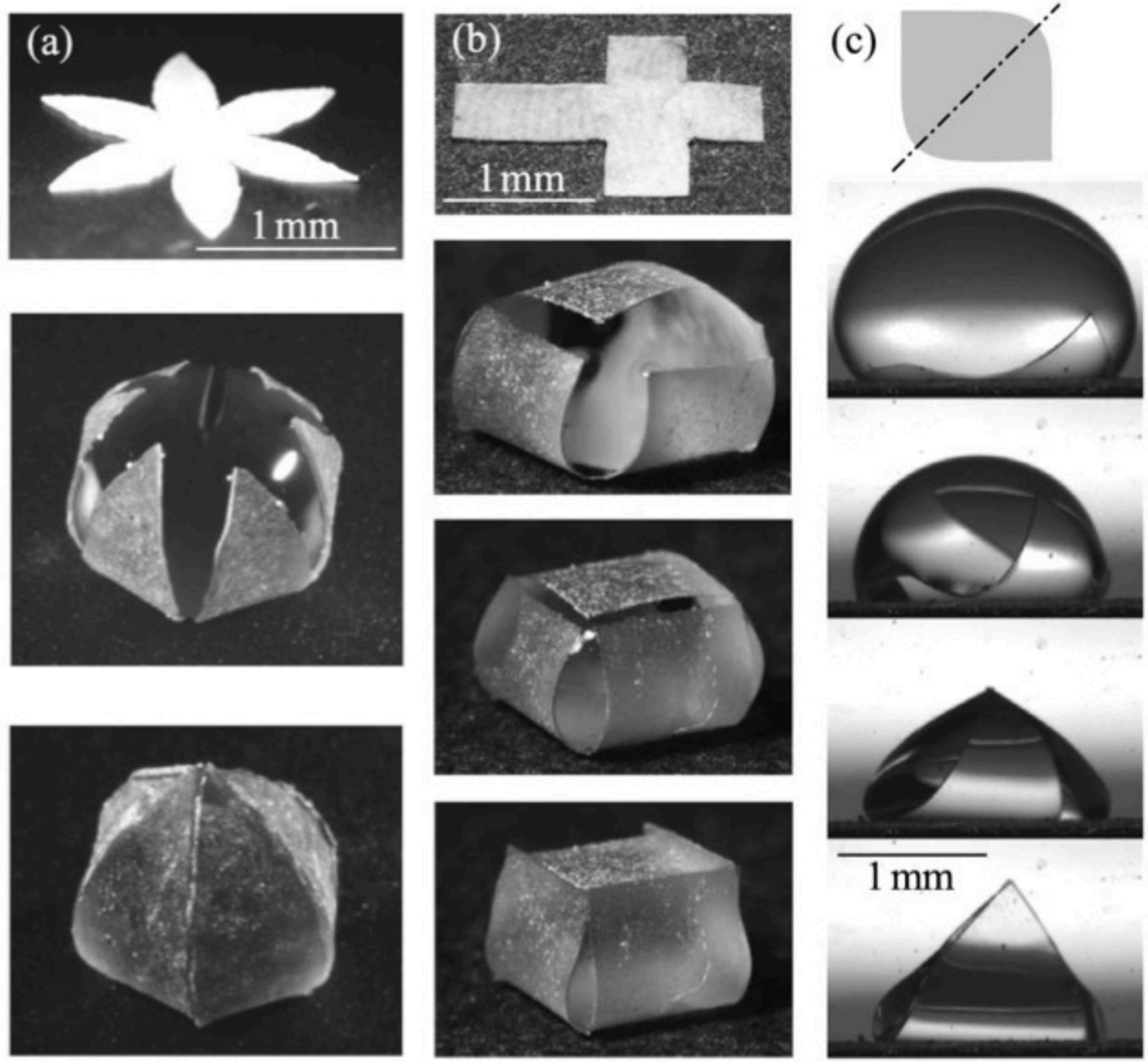


folding by surface tension of Pb:Sn solder spheres

Linderman et al, *Sens. Actuators* (2002)

Micro-Origamis

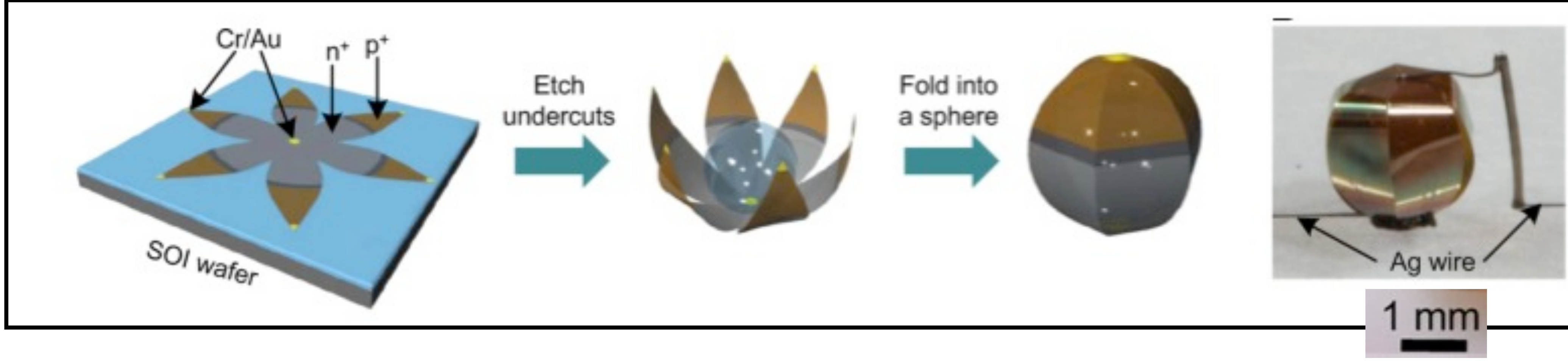
evaporation
water drop
on elastic surface



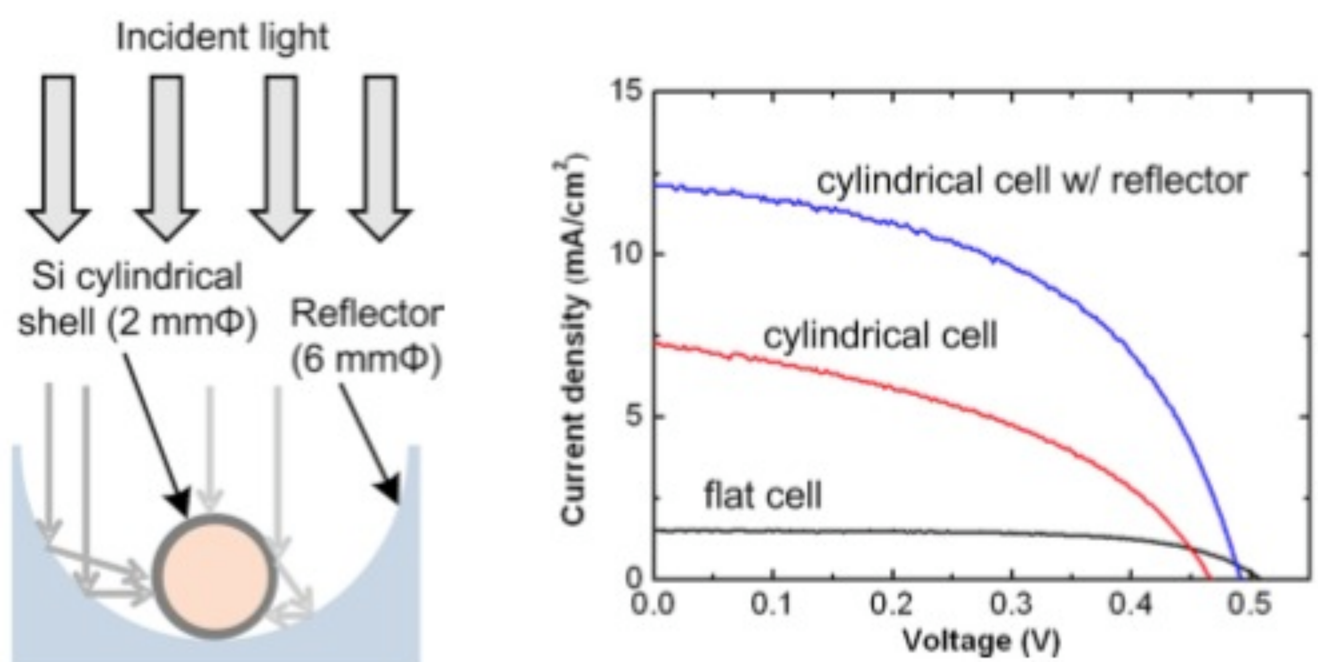
Py et al, *Phys. Rev. Lett.* (2007)

Micro-Origamis

3D photovoltaic devices



micro
solar
oven



Guo et al, *PNAS* (2009)

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Instant Origami

target $L = 7$ mm
drop $R = 1.5$ mm

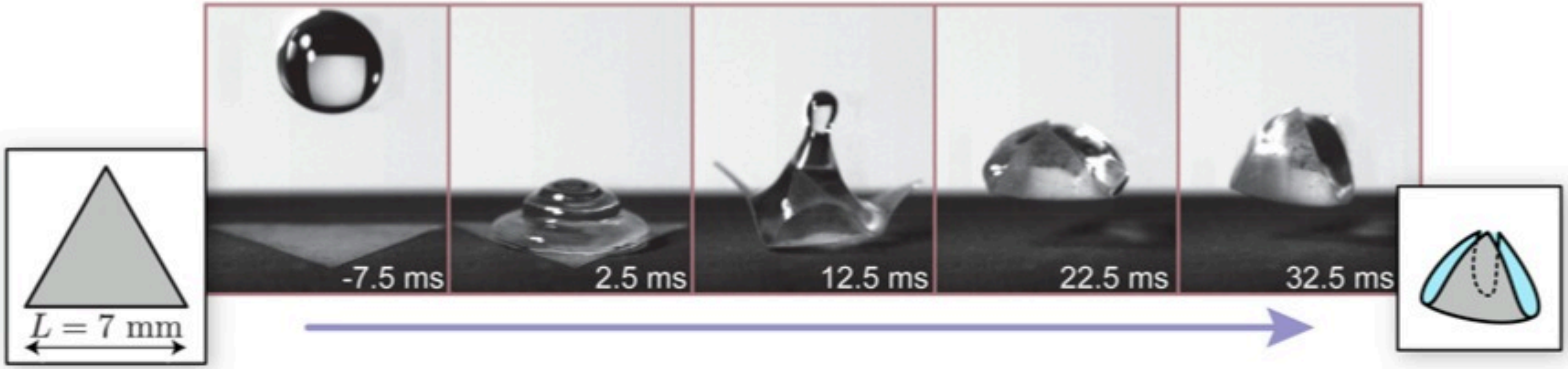
impacting speed $U = 0.53$ m/s

encapsulation in 20 ms

total video length 133 ms

Antkowiak et al, *PNAS* (2011)

Instant Origami



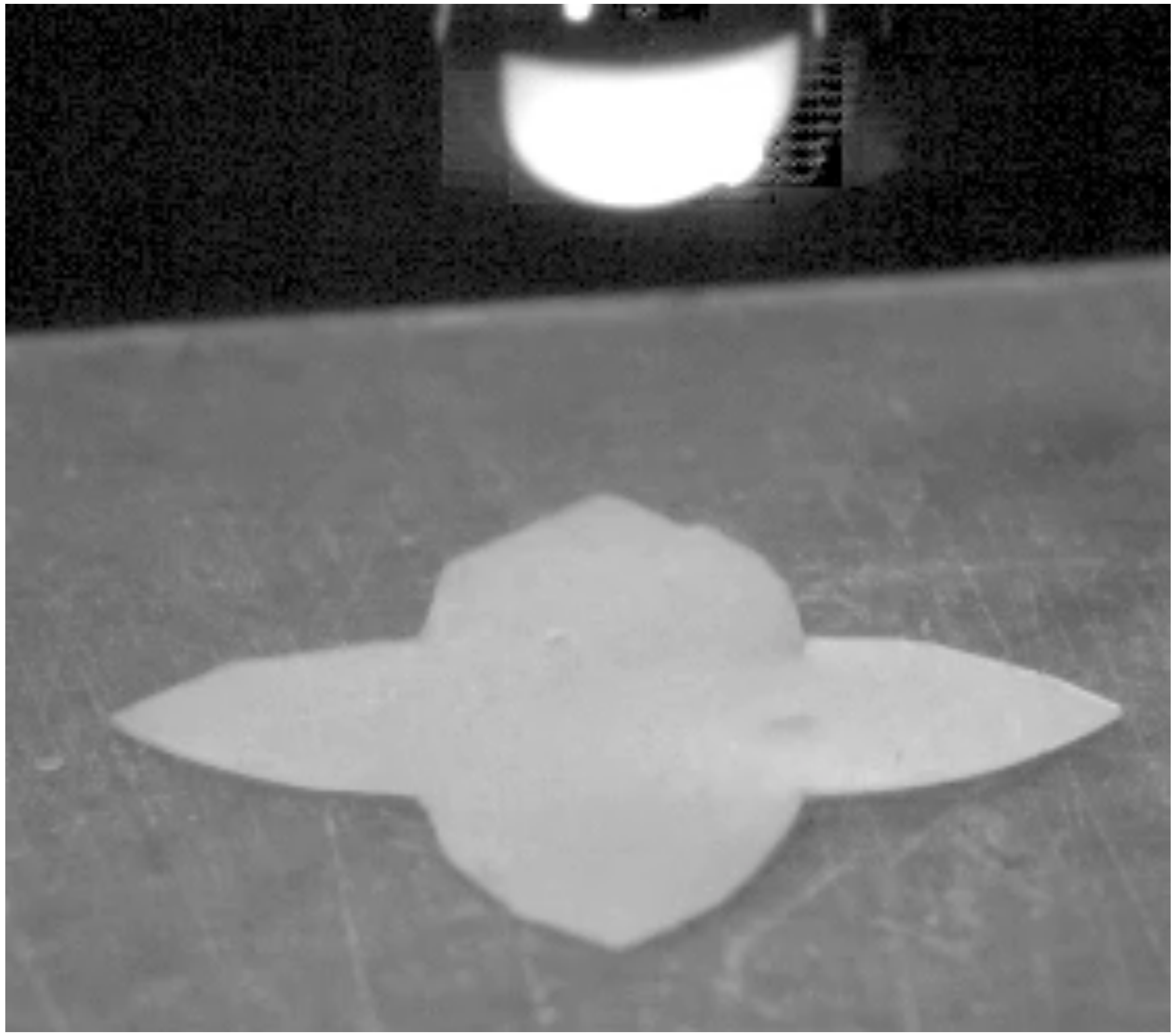
target $L = 7$ mm
drop $R = 1.5$ mm

impacting speed $U = 0.53$ m/s

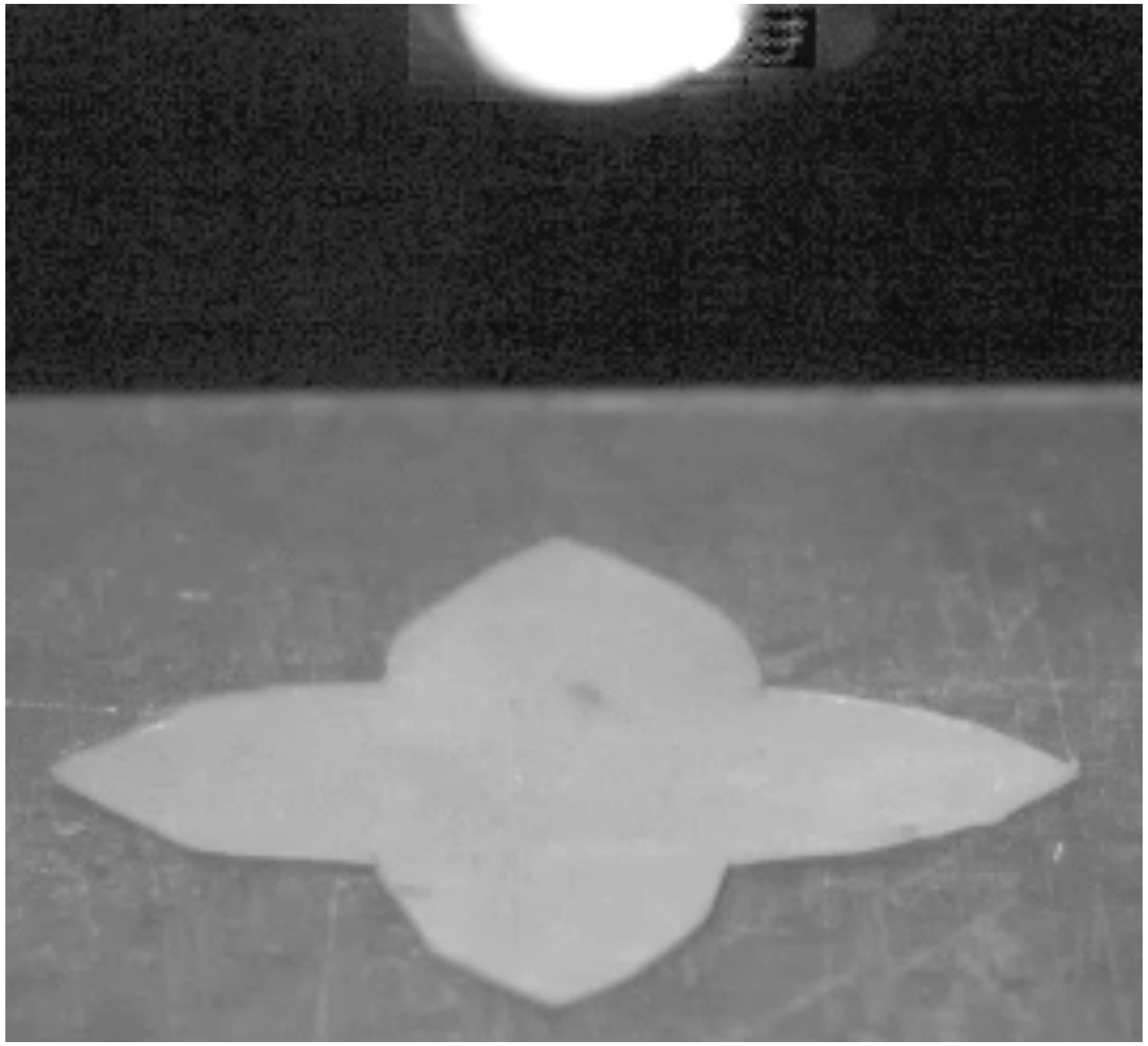
Antkowiak et al, *PNAS* (2011)

Instant Origami: final shape selection

same target $L = 10$ mm
same drop $R = 1.5$ mm



$Ua = 0.68$ m/s

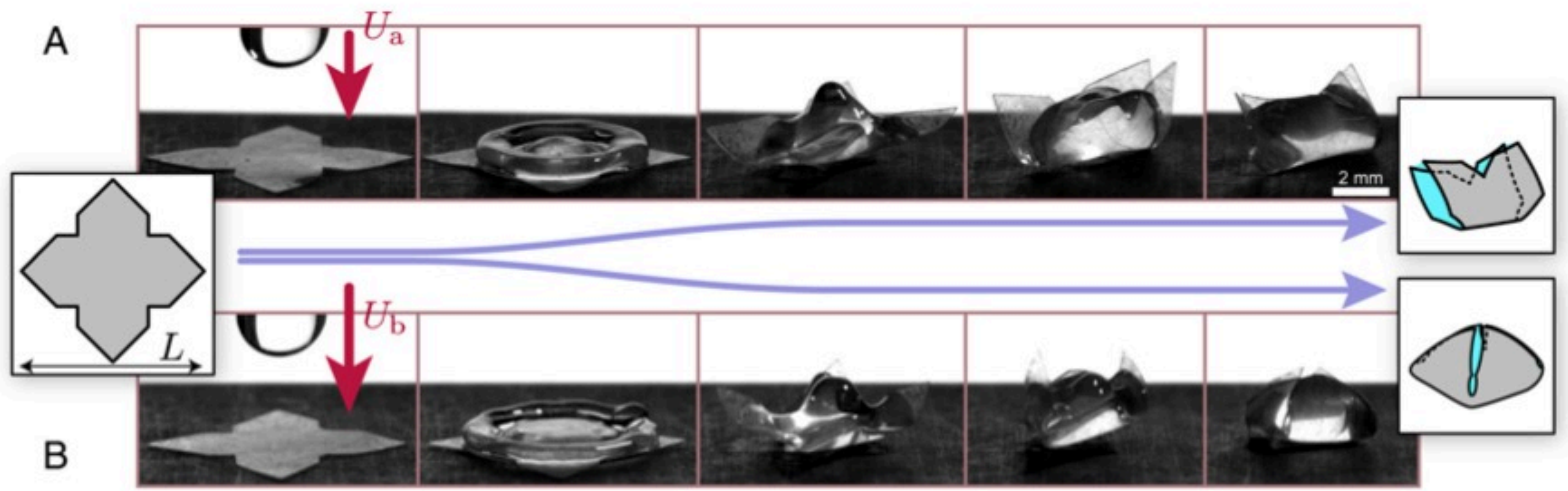


$Ua = 0.92$ m/s

Antkowiak et al, *PNAS* (2011)

Instant Origami: final shape selection

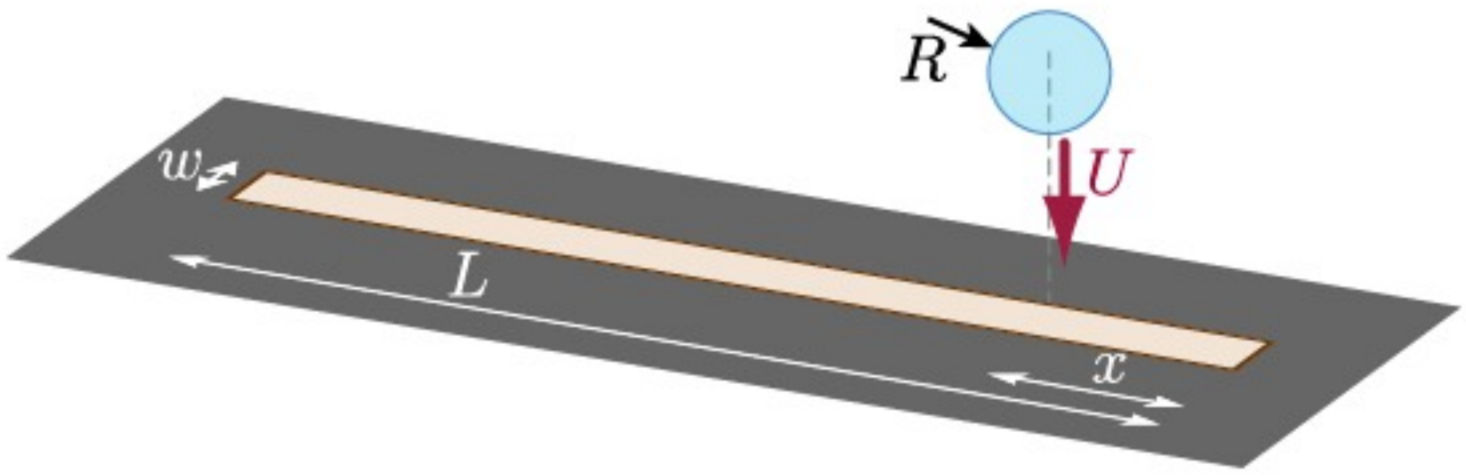
same target $L = 10$ mm
same drop $R = 1.5$ mm



different impacting speeds
$Ua = 0.68$ m/s
$Ub = 0.92$ m/s

Antkowiak et al, *PNAS* (2011)

Instant Origami: 1-D setup



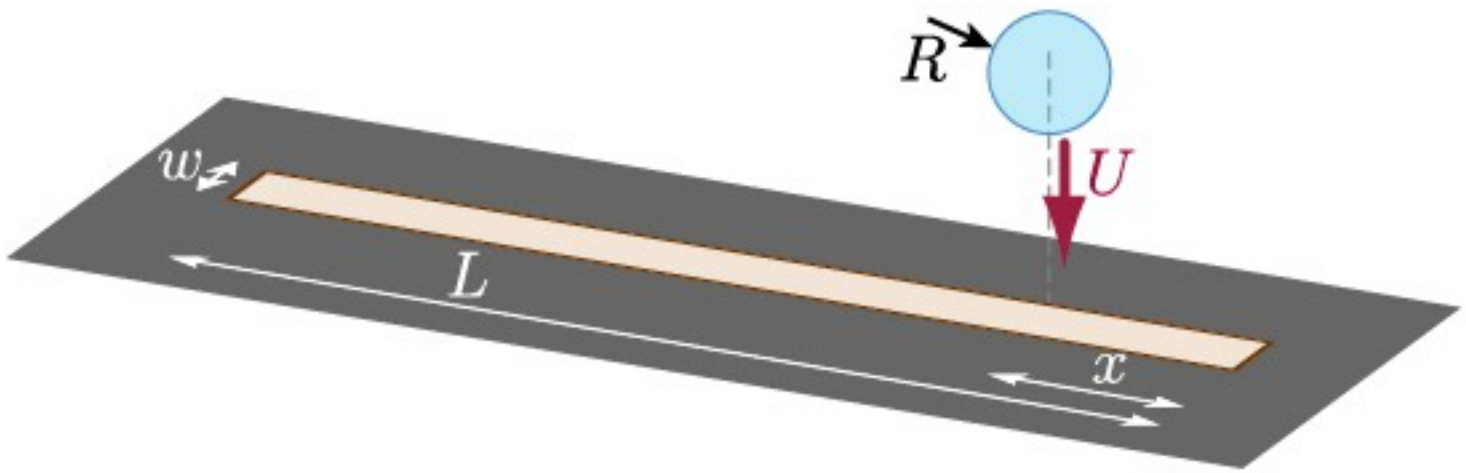
$R \sim 1.5\,\mathrm{mm}$

Weber number $\quad \mathrm{We} = \sqrt{\dfrac{\rho R U^2}{\gamma}}$ $\quad$ kinetic energy / surface energy

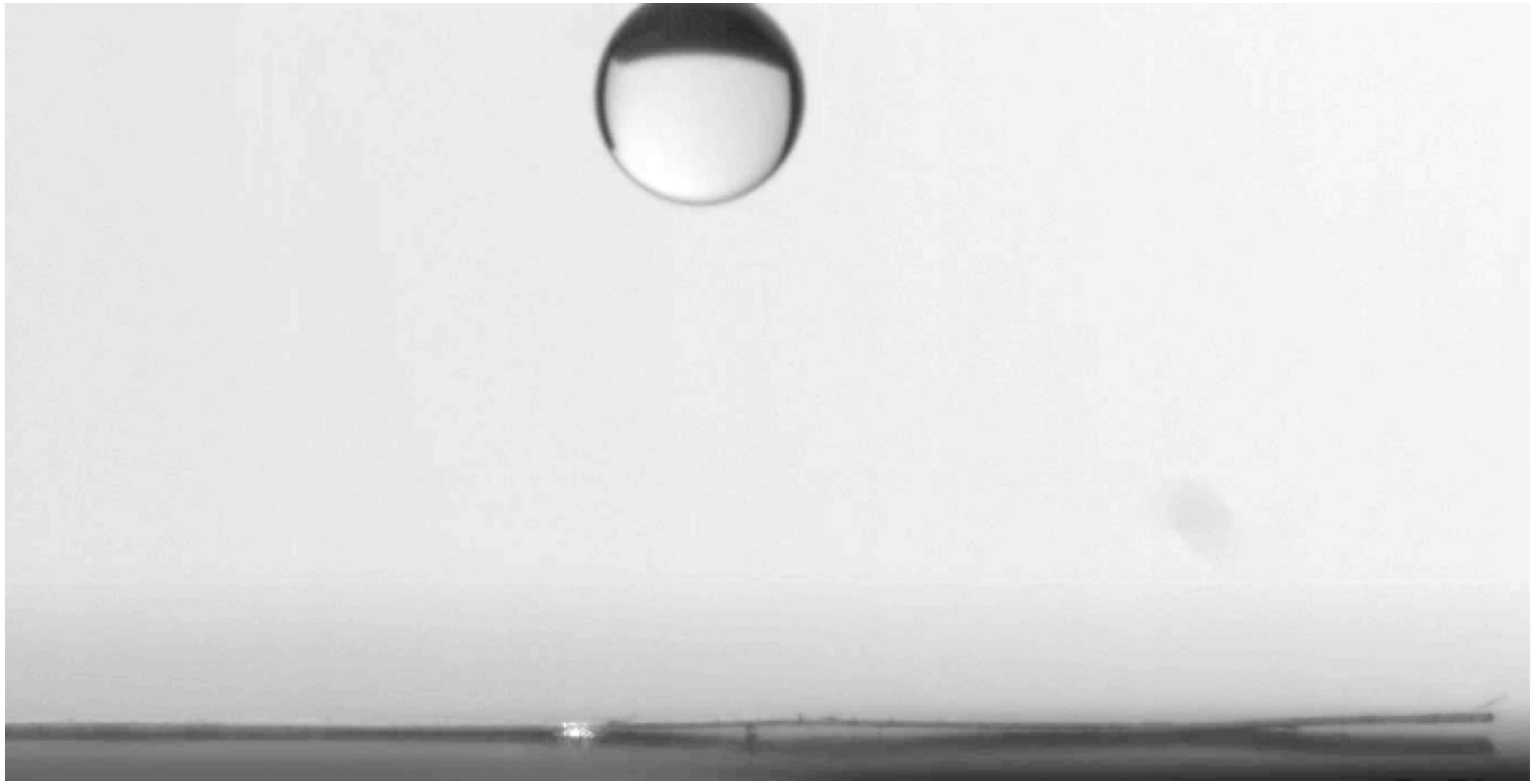
gravito-elastic length $\quad \ell_{\mathrm{eg}} = \left(\dfrac{EI}{\mu g}\right)^{1/3}$ $\quad$ bending rigidity / weight per length

~ 3.6 mm

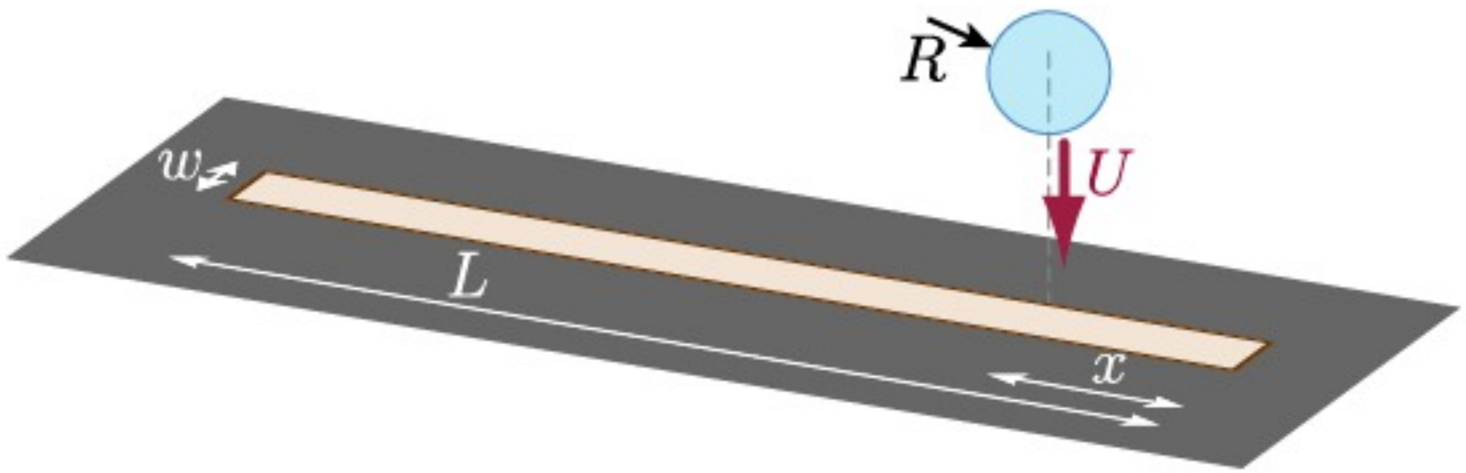
Instant Origami: 1-D setup



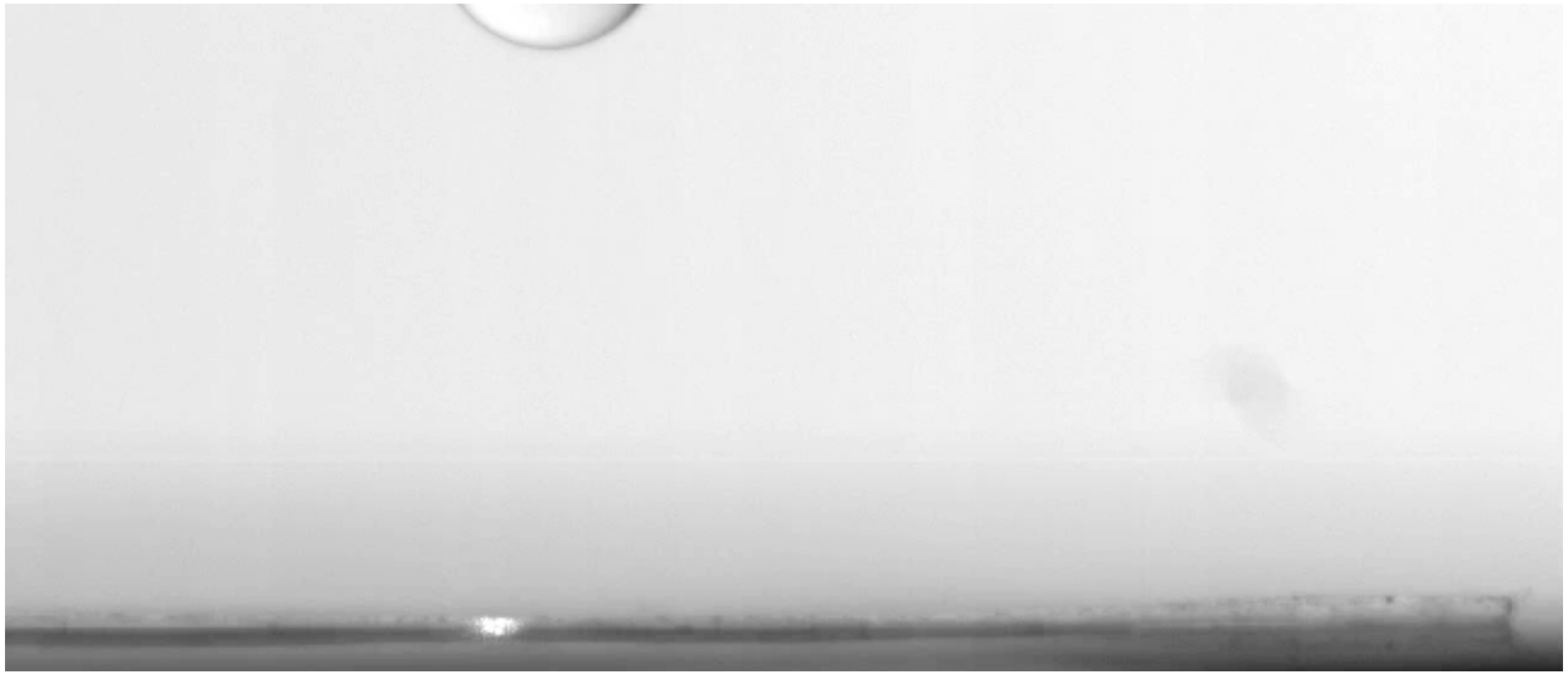
encapsulation



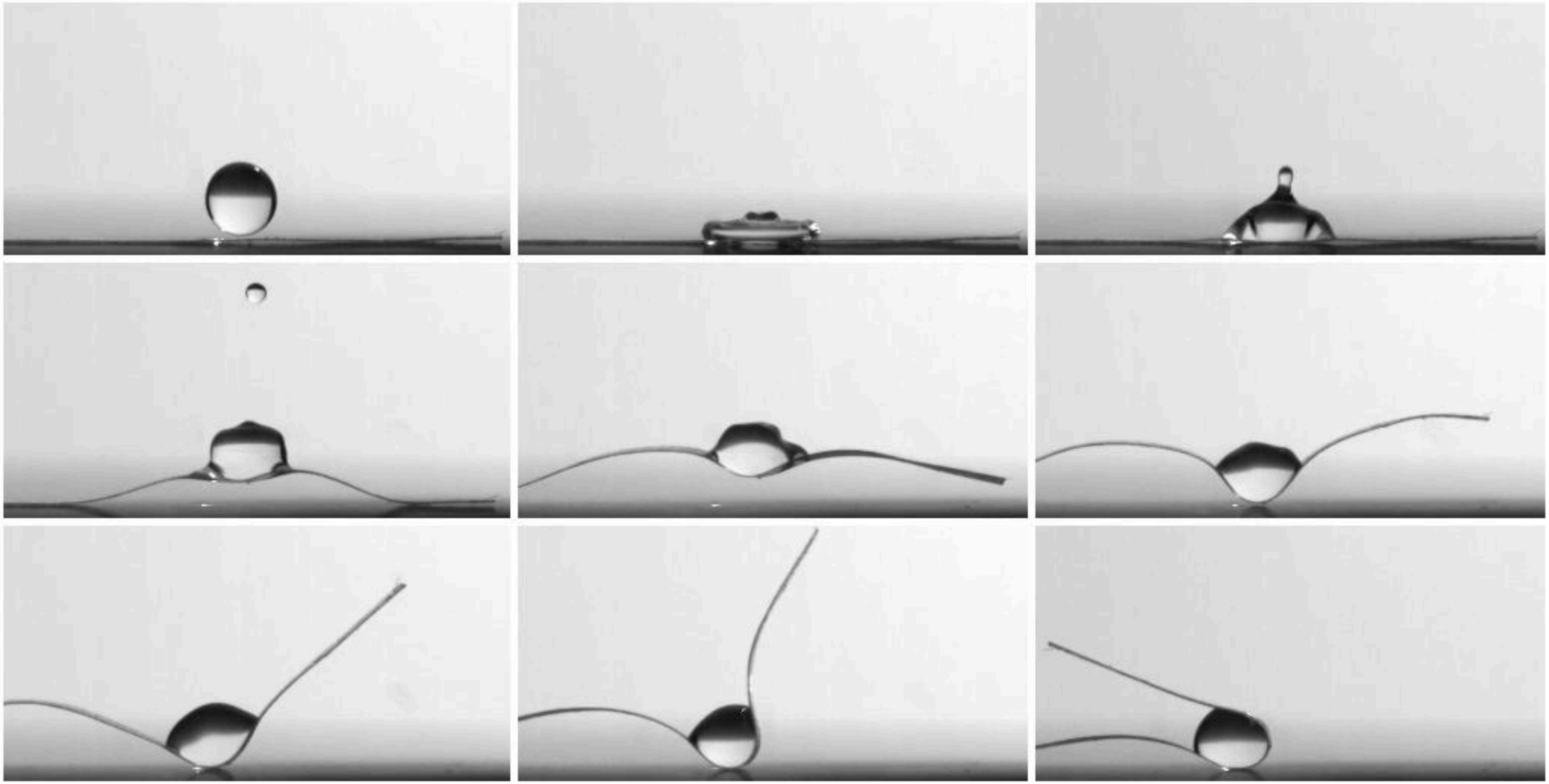
Instant Origami: 1-D setup



no encapsulation

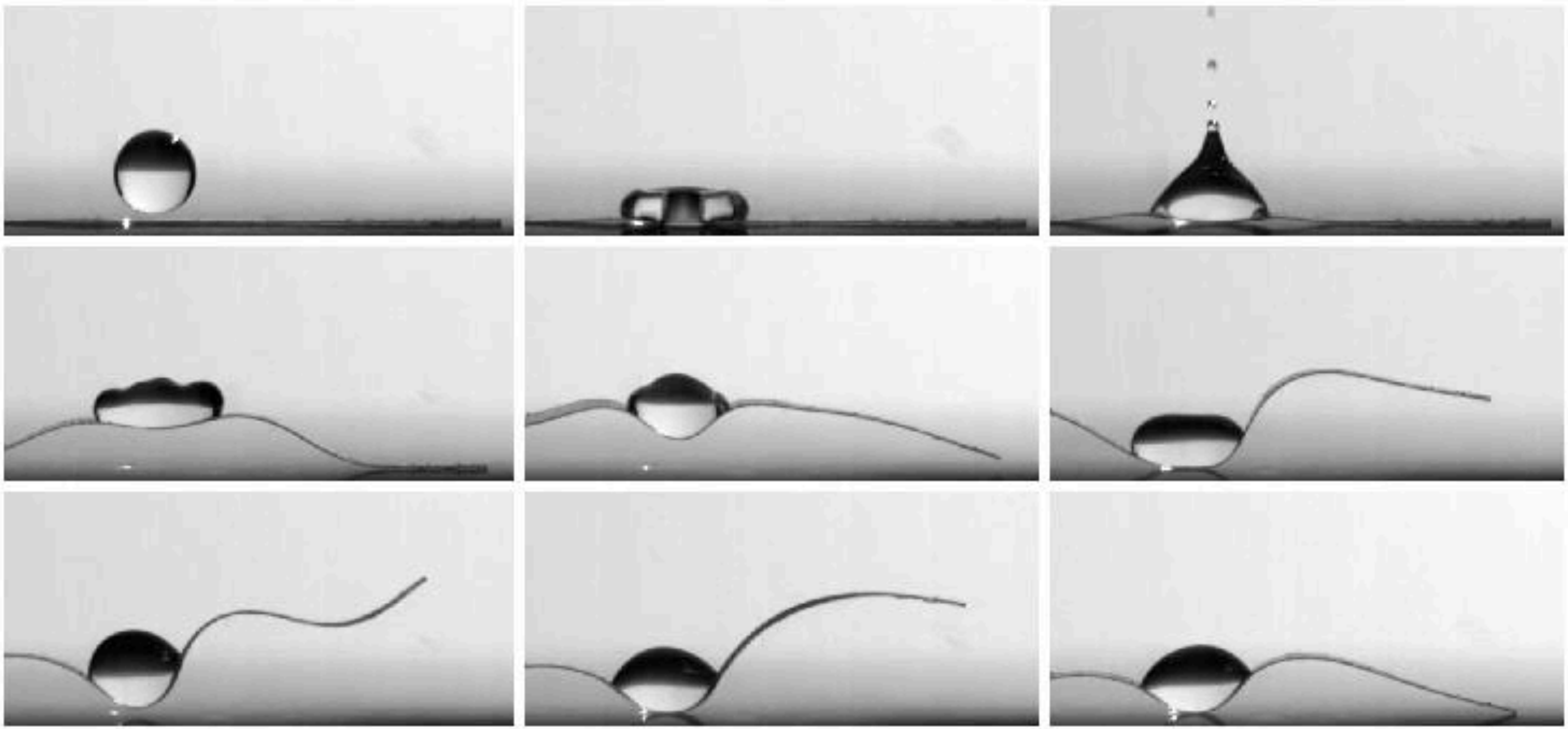


Instant Origami: 1-D setup



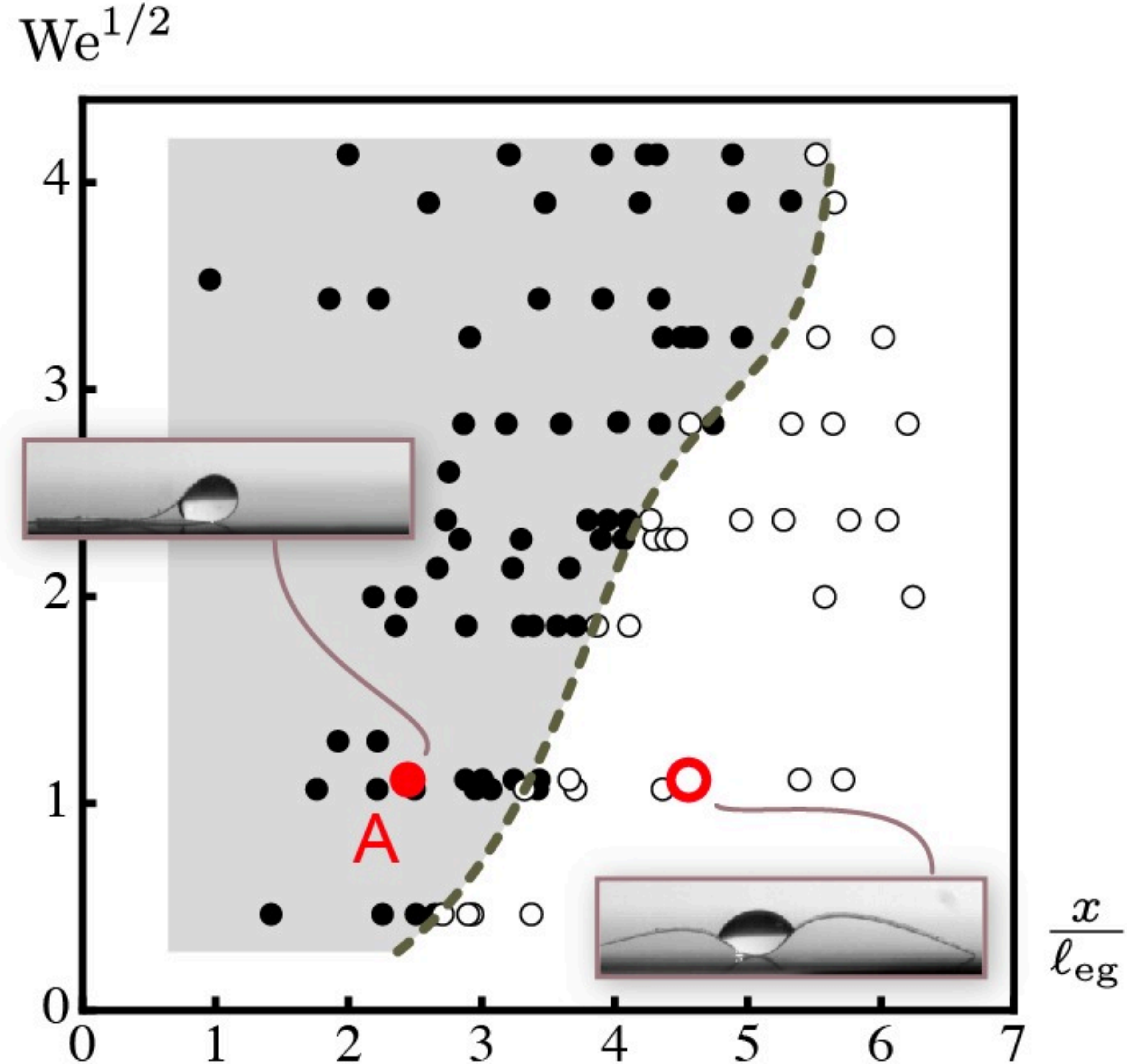
encapsulation

Instant Origami: 1-D setup



no encapsulation

Instant Origami: Exp. results



1 - Motivations

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Variational approach

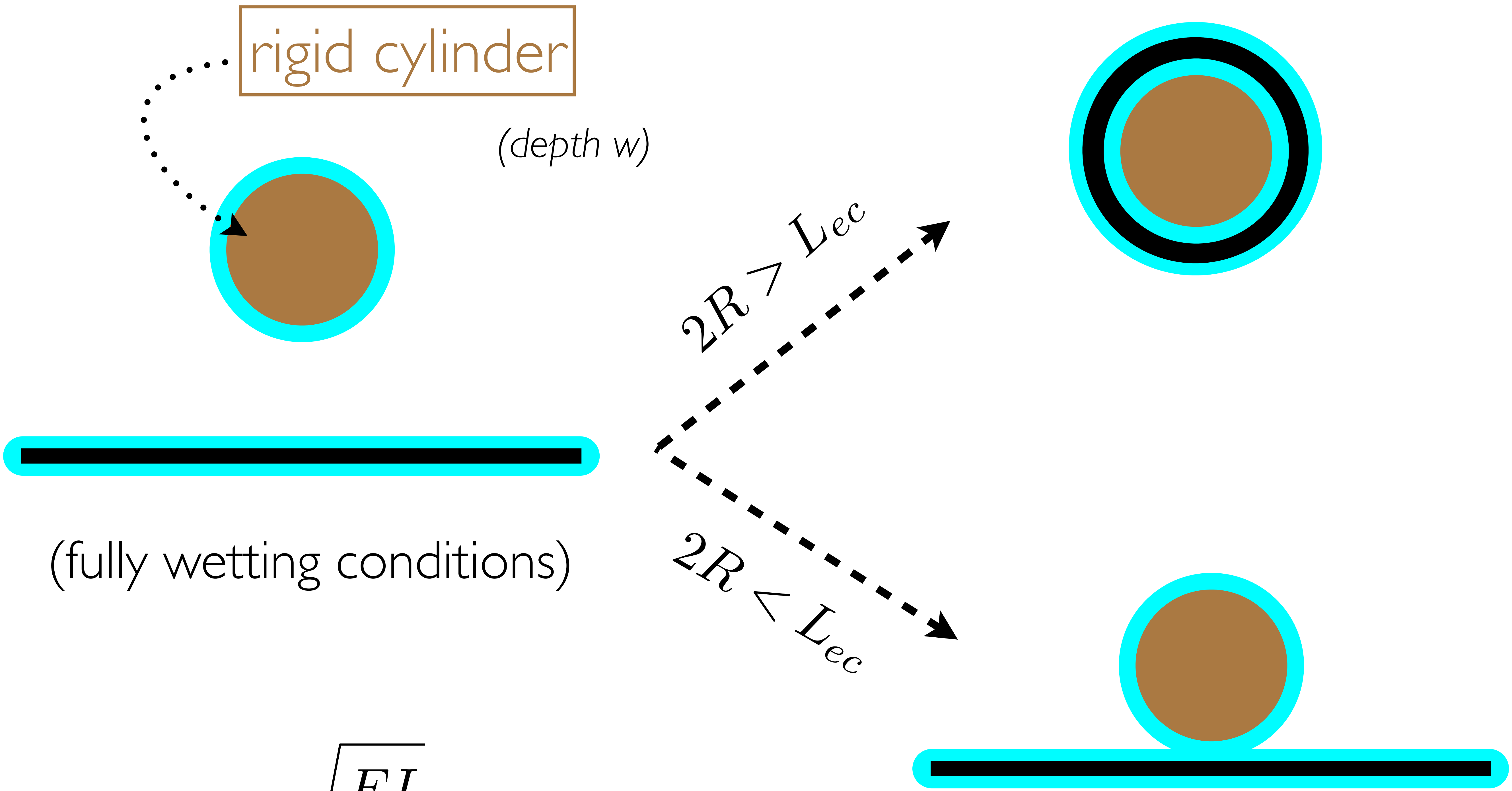
$$\mathcal{E}_{el} = \frac{1}{2}\int_0^L EI\,\kappa^2(s)\,ds$$ bending energy

$\mathcal{E}_\gamma$ surface energy

$\mathcal{E}_g$ gravitational energy

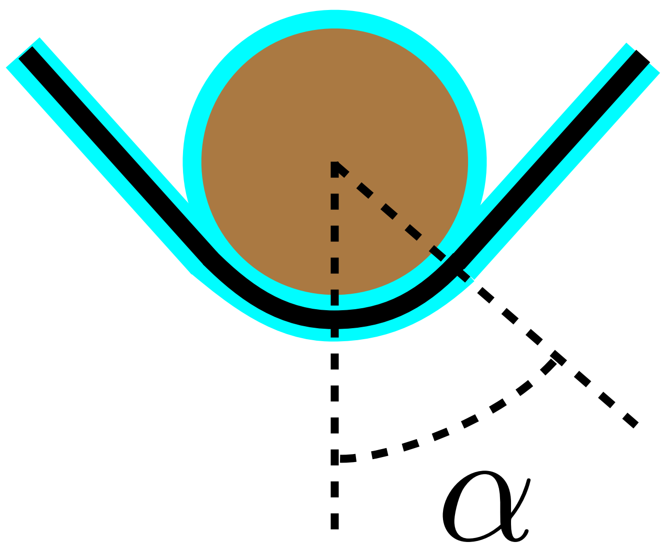
+ constraints

Elasto-capillary length



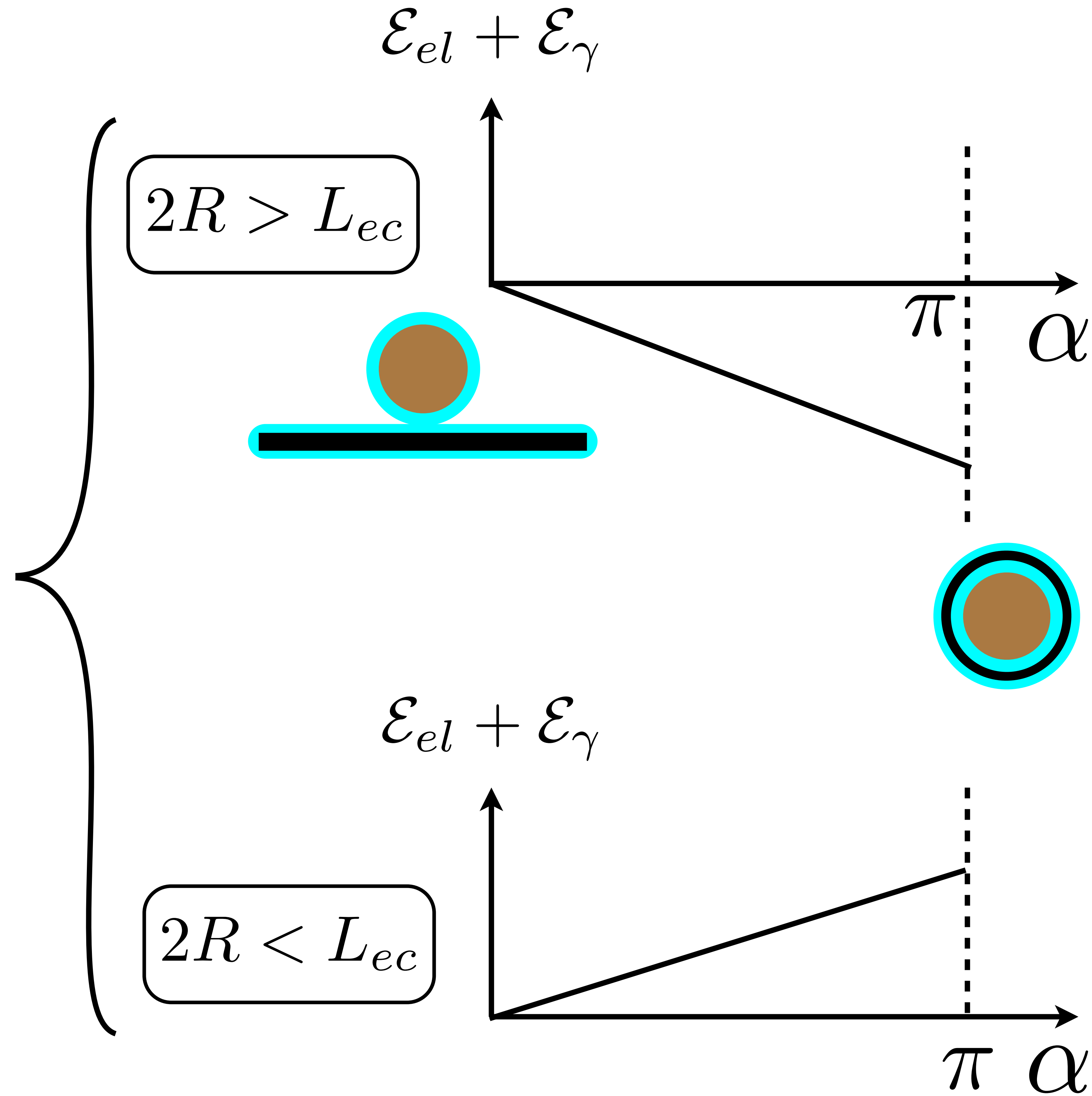
$$L_{ec} = \sqrt{\frac{EI}{\gamma w}}$$

Elasto-capillary wrapping

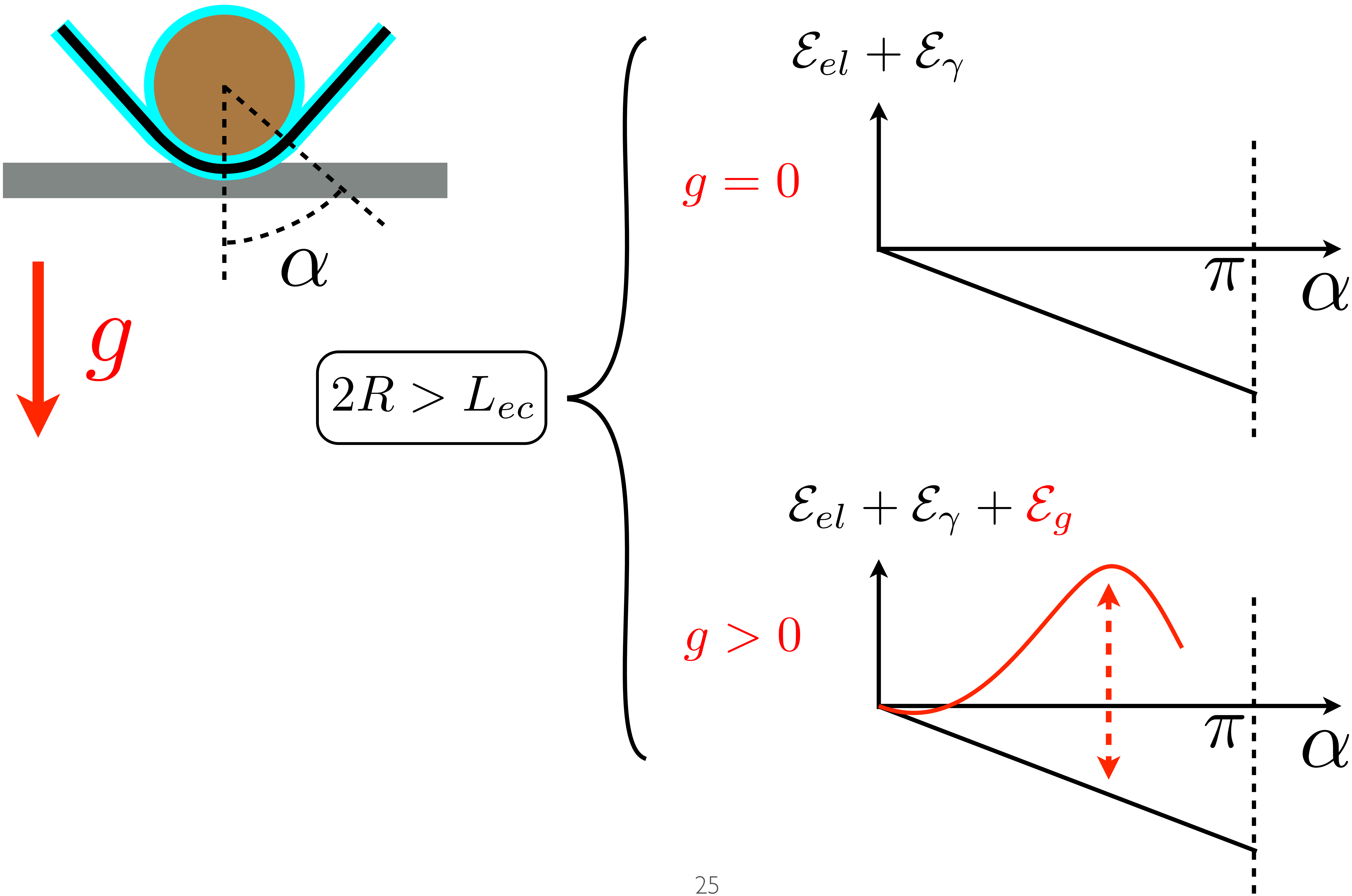


$$\mathcal{E} = \mathcal{E}_{el} + \mathcal{E}_{\gamma}$$

$$= 4\gamma w R \alpha \left[\frac{L_{ec}^2}{(2R)^2} - 1 \right]$$



Weight as energy barrier

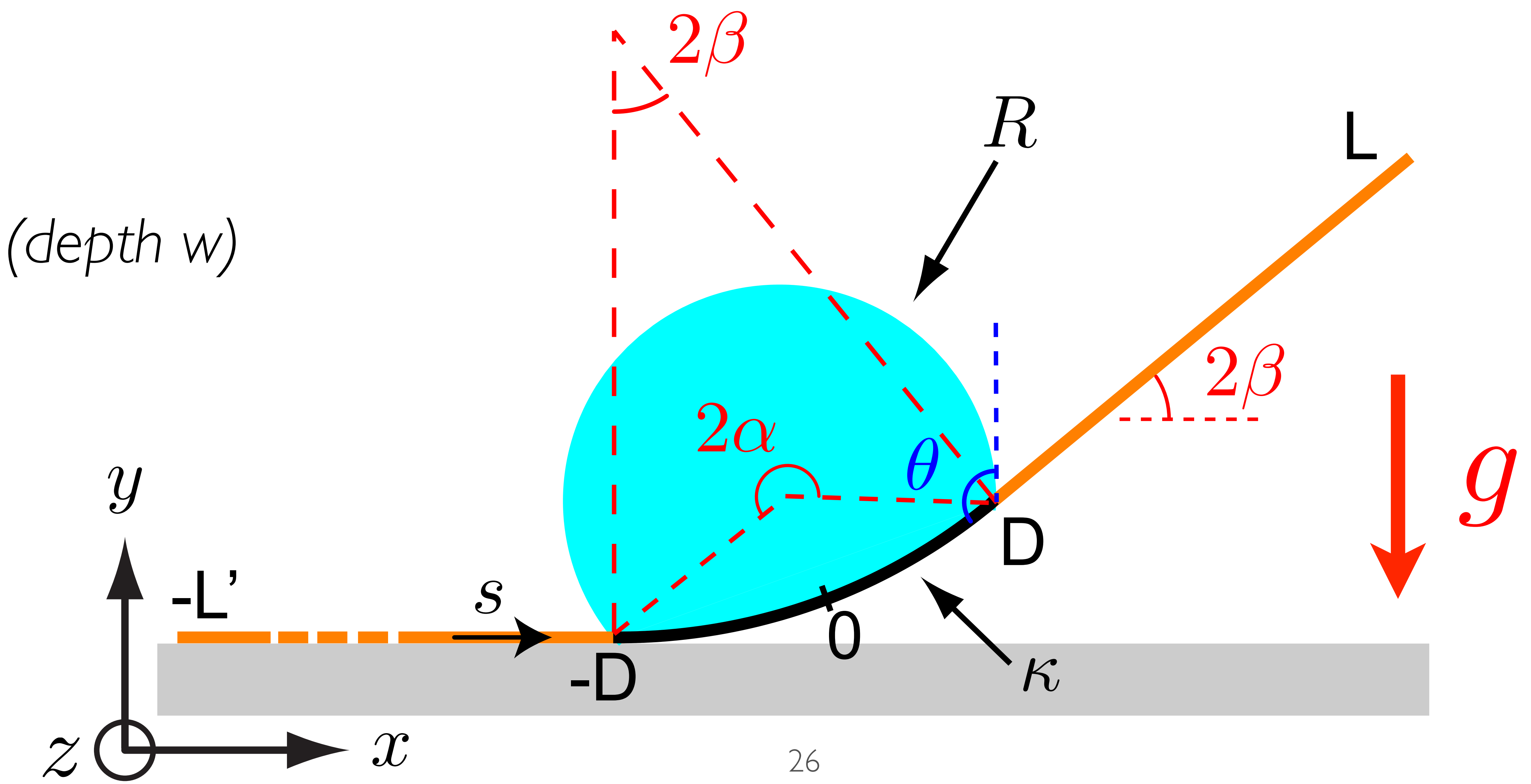


Discrete `two elements' model

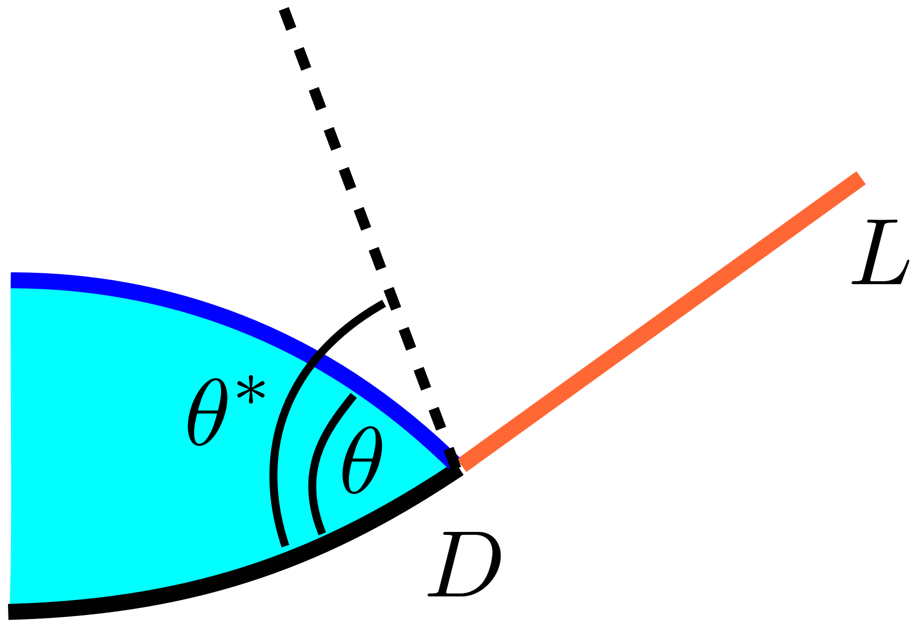
$s \in (-D; D)$ circular arc

$s \in (D; L)$ straight (heavy) beam

drop : no weight (circular interface)

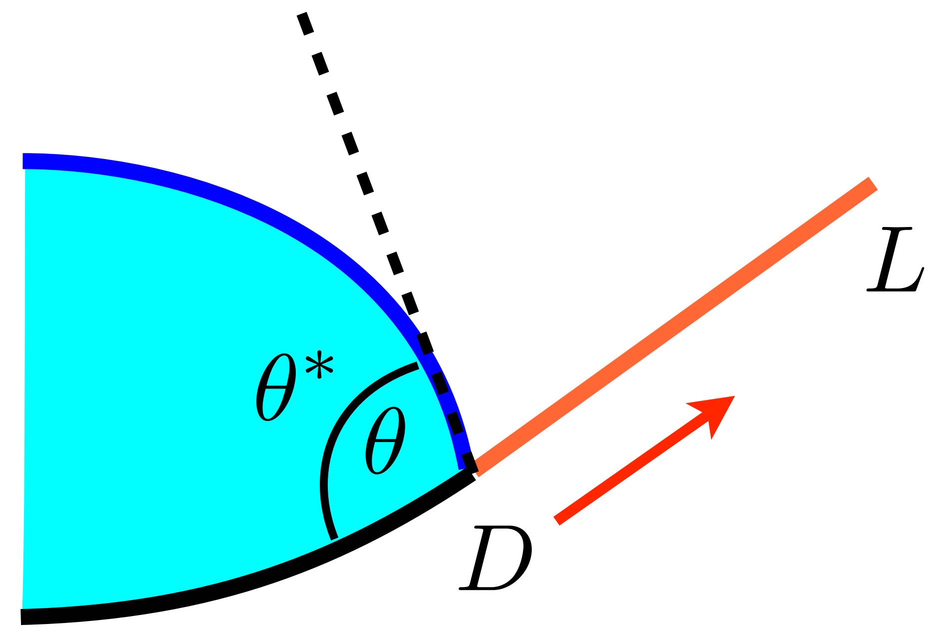


Contact line pinning



$$\theta < \theta^*$$

pinning: D fixed

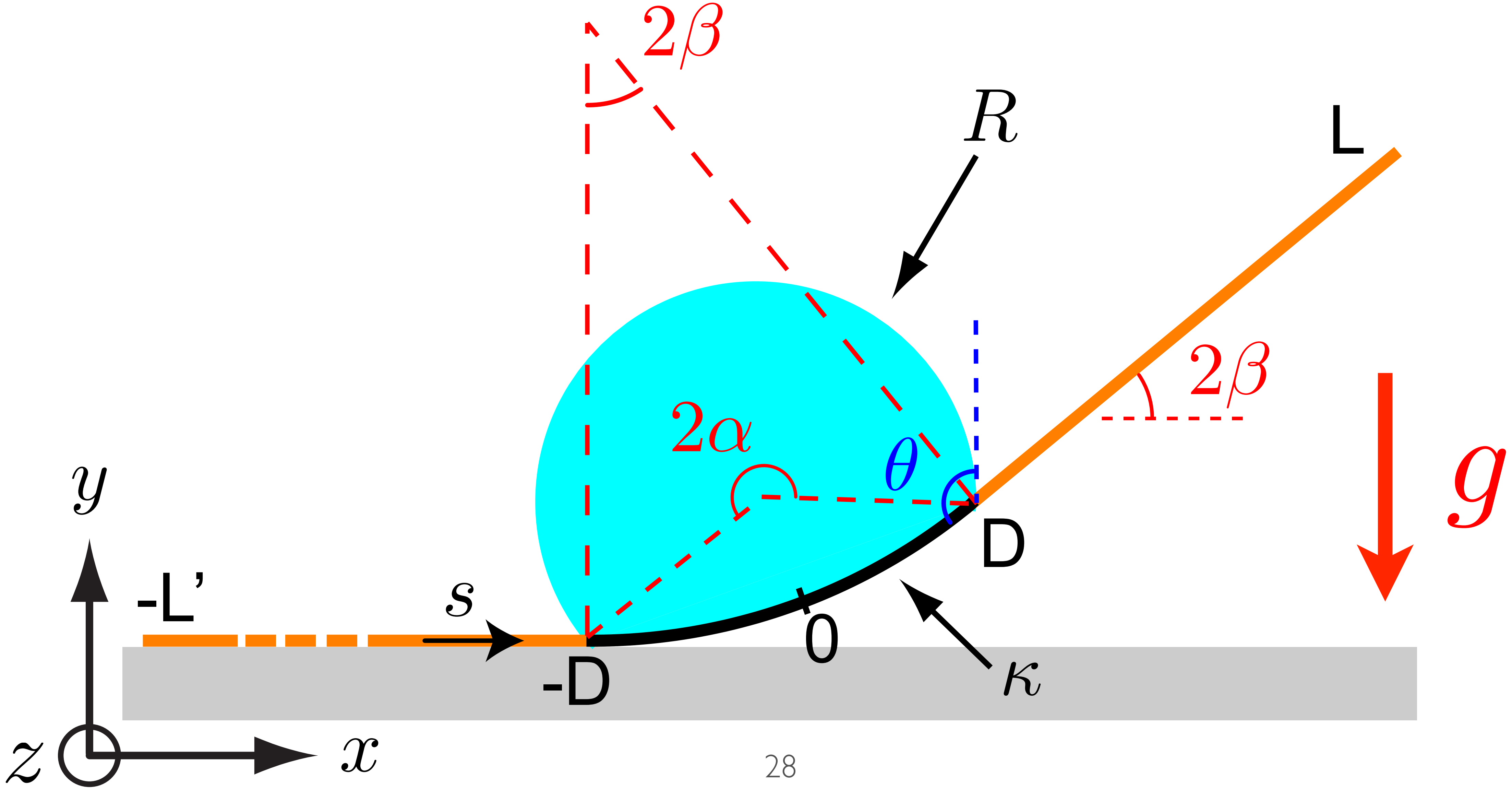


$$\theta = \theta^*$$

advancing: D increases

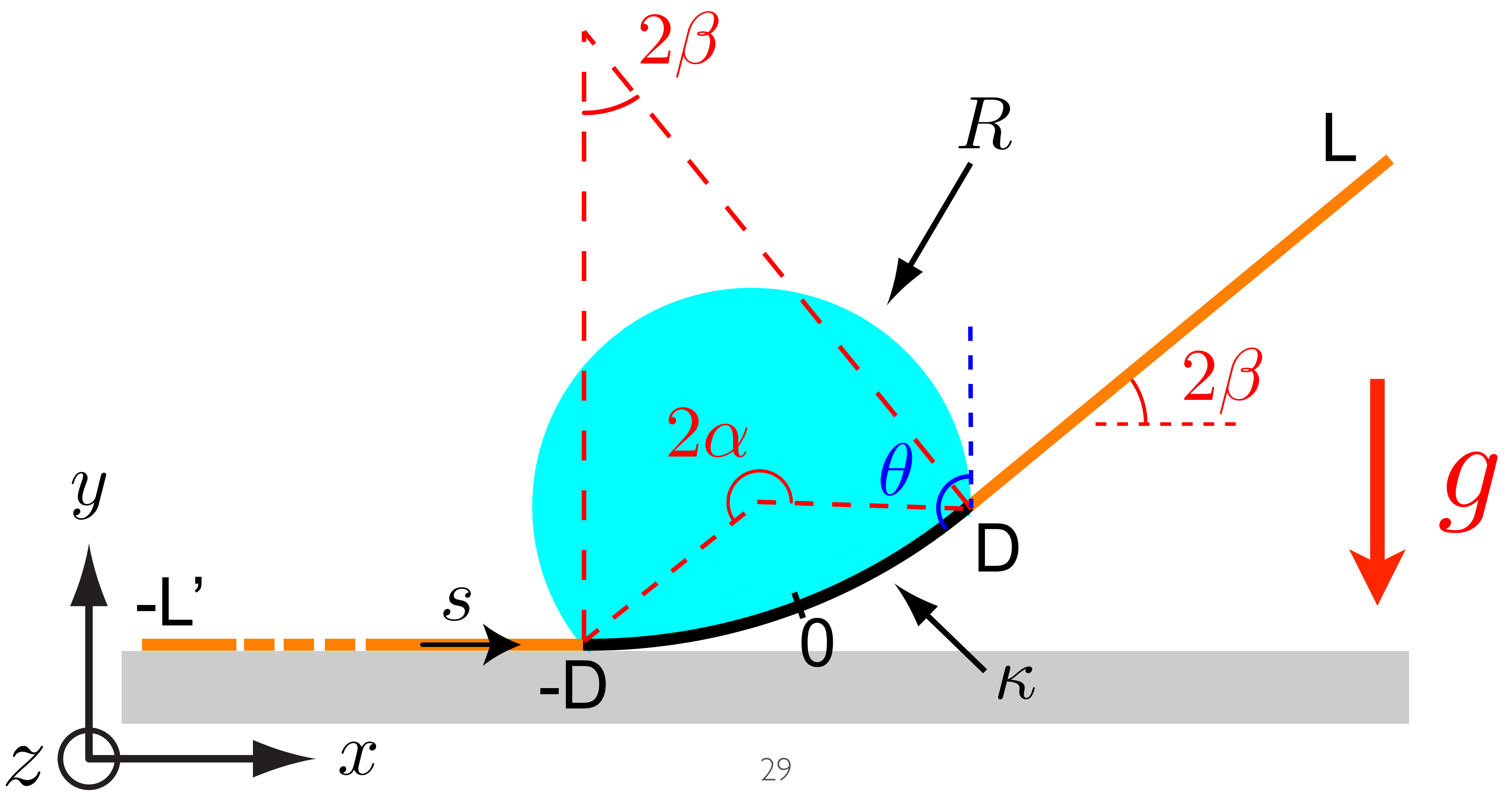
Bending energy

$$\mathcal{E}_{\mathrm{el}} = \frac{1}{2}\int_{-D}^{D} EI\,\kappa^2(s)\,\mathrm{d}s = EI\,\kappa^2\,D$$



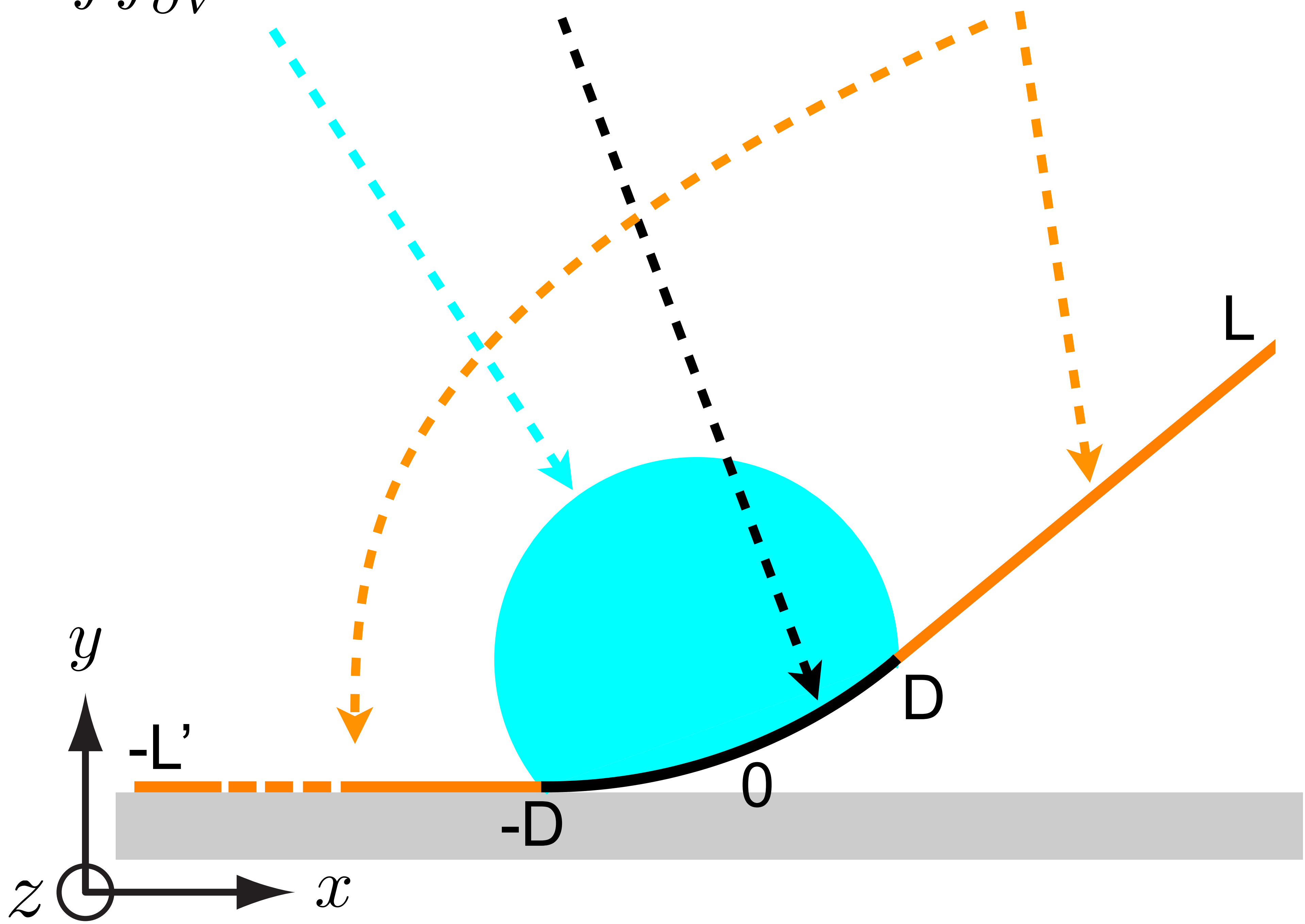
Gravitational energy

$$\frac{\mathcal{E}_{\mathrm{g}}}{\rho g w h} = \frac{1}{\kappa}\left(2D - \frac{\sin(2\kappa D)}{\kappa}\right) + \frac{1-\cos(2\kappa D)}{\kappa}(L-D) + \frac{1}{2}(L-D)^2\sin(2\kappa D)$$



Surface energy

$$\mathcal{E}_\gamma = 0.87 \iint_{\partial V} \gamma \,\mathrm{d}A + 2\gamma_{sl} D\, w + \gamma_{sv}(L + L' - 2D)w$$



Surface energy

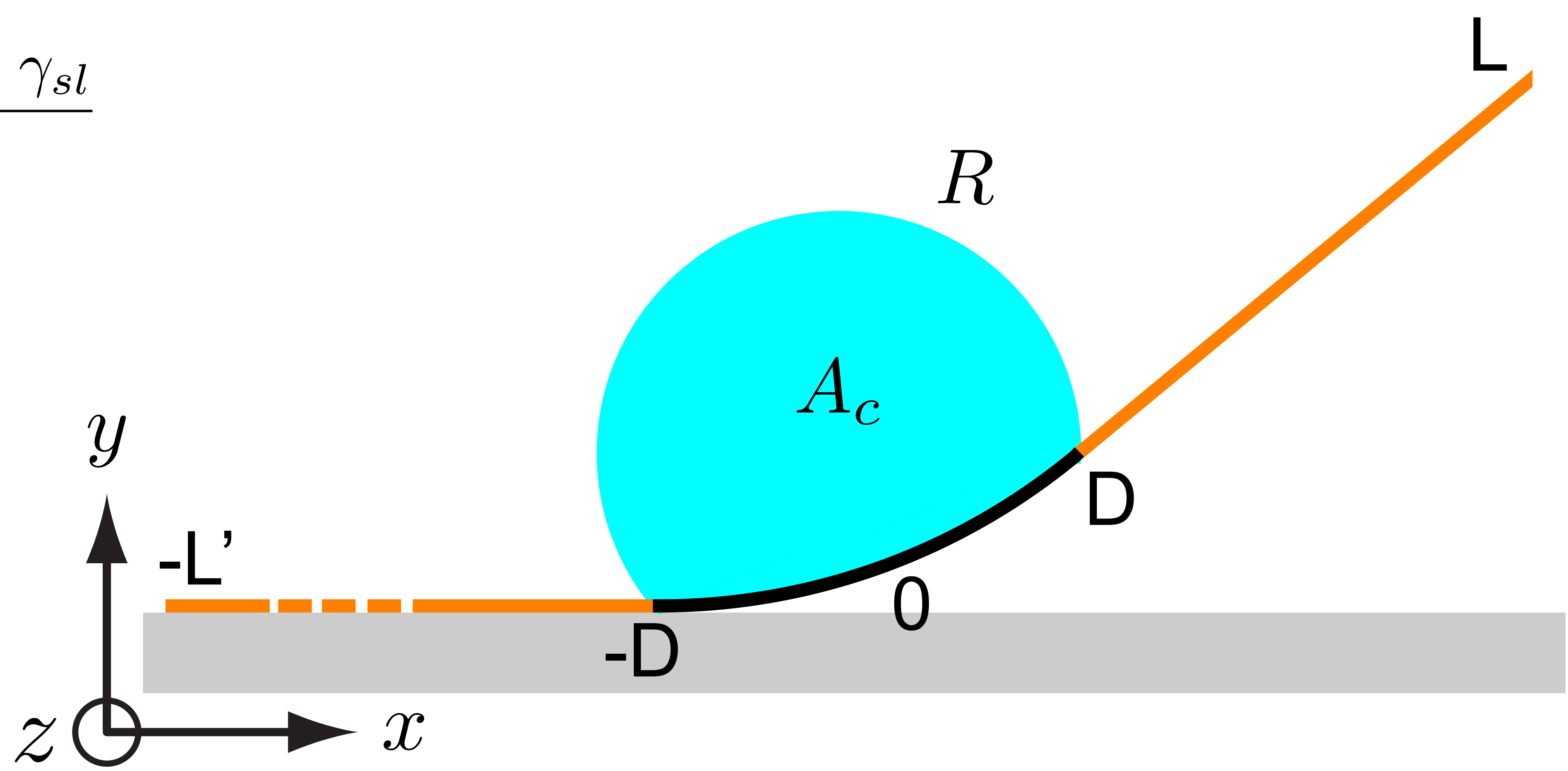
$$\mathcal{E}_\gamma = 0.87 \iint_{\partial V} \gamma \, \mathrm{d}A + 2\gamma_{sl} D\, w + \gamma_{sv}(L + L' - 2D)w$$

$$\mathcal{E}_\gamma = 0.87\, \gamma\, (2\alpha\, R\, w + 2A_c) - 2\gamma D\, w \cos\theta_Y + \gamma_{sv}(L + L')\, w$$

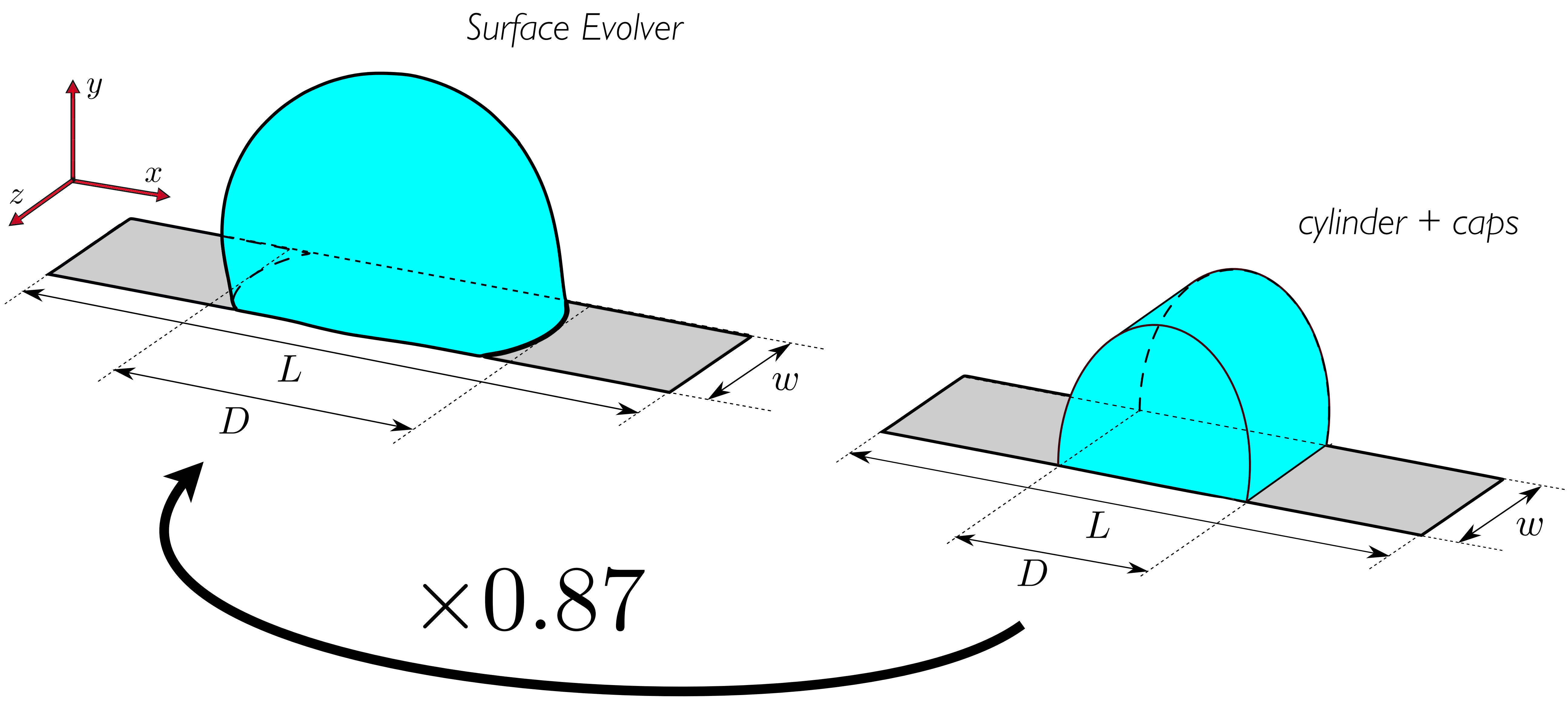
with

$$\cos\theta_Y = \frac{\gamma_{sv} - \gamma_{sl}}{\gamma}$$

(depth w)



Surface correction factor



Constraints

$$\mathcal{E}_{\rm el} + \mathcal{E}_{\rm g} + \mathcal{E}_\gamma = \mathcal{E}(\alpha, \beta, R, \kappa, D)$$

5 variables

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kinematics

$$\begin{cases} (1/\kappa)\sin\beta = R\sin\alpha \\ \kappa D = \beta \end{cases}$$

conserved volume

$$\frac{V_0}{w} = \frac{1}{\kappa^2}\left(\beta - \frac{\sin 2\beta}{2}\right) + R^2\left(\alpha - \frac{\sin 2\alpha}{2}\right) = A_c$$

contact line pinning

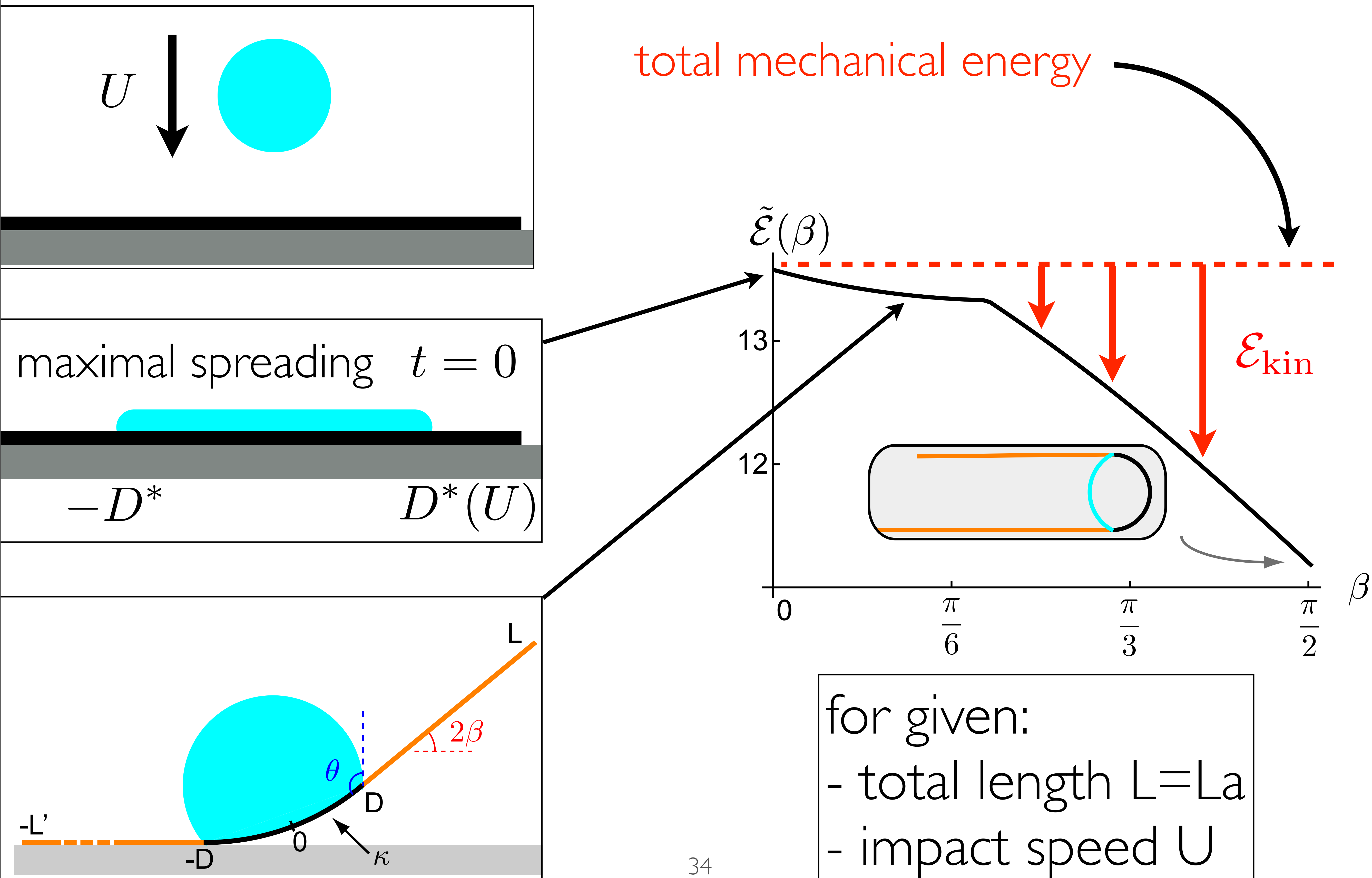
either $D = D^*$

or $\theta = \theta^*$

4 constraints

$$\mathcal{E}_{\rm el} + \mathcal{E}_{\rm g} + \mathcal{E}_\gamma = \mathcal{E}(\cancel{\alpha}, \beta, \cancel{R}, \cancel{\kappa}, \cancel{D})$$

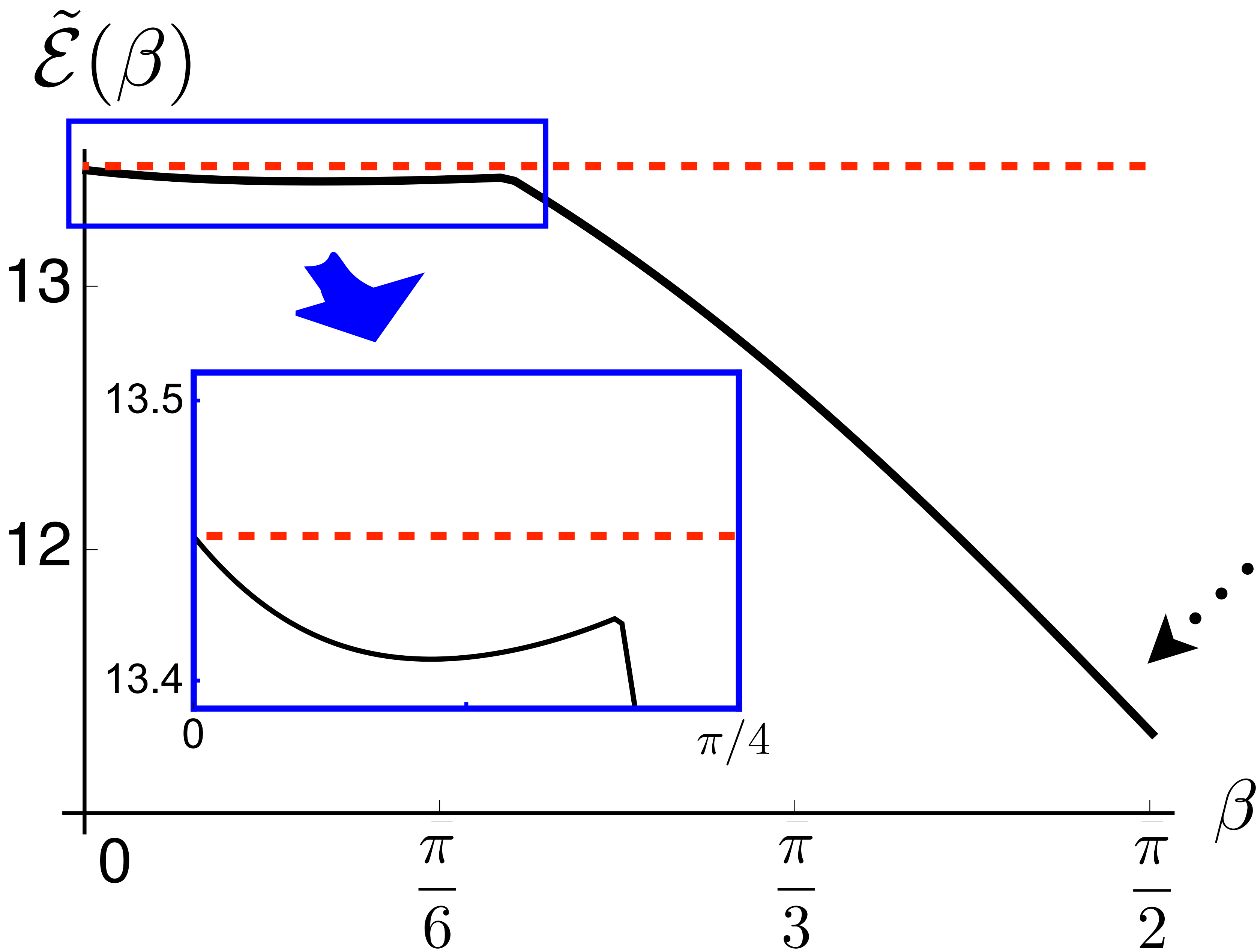
Results



for given:
- total length L=La
- impact speed U

Results

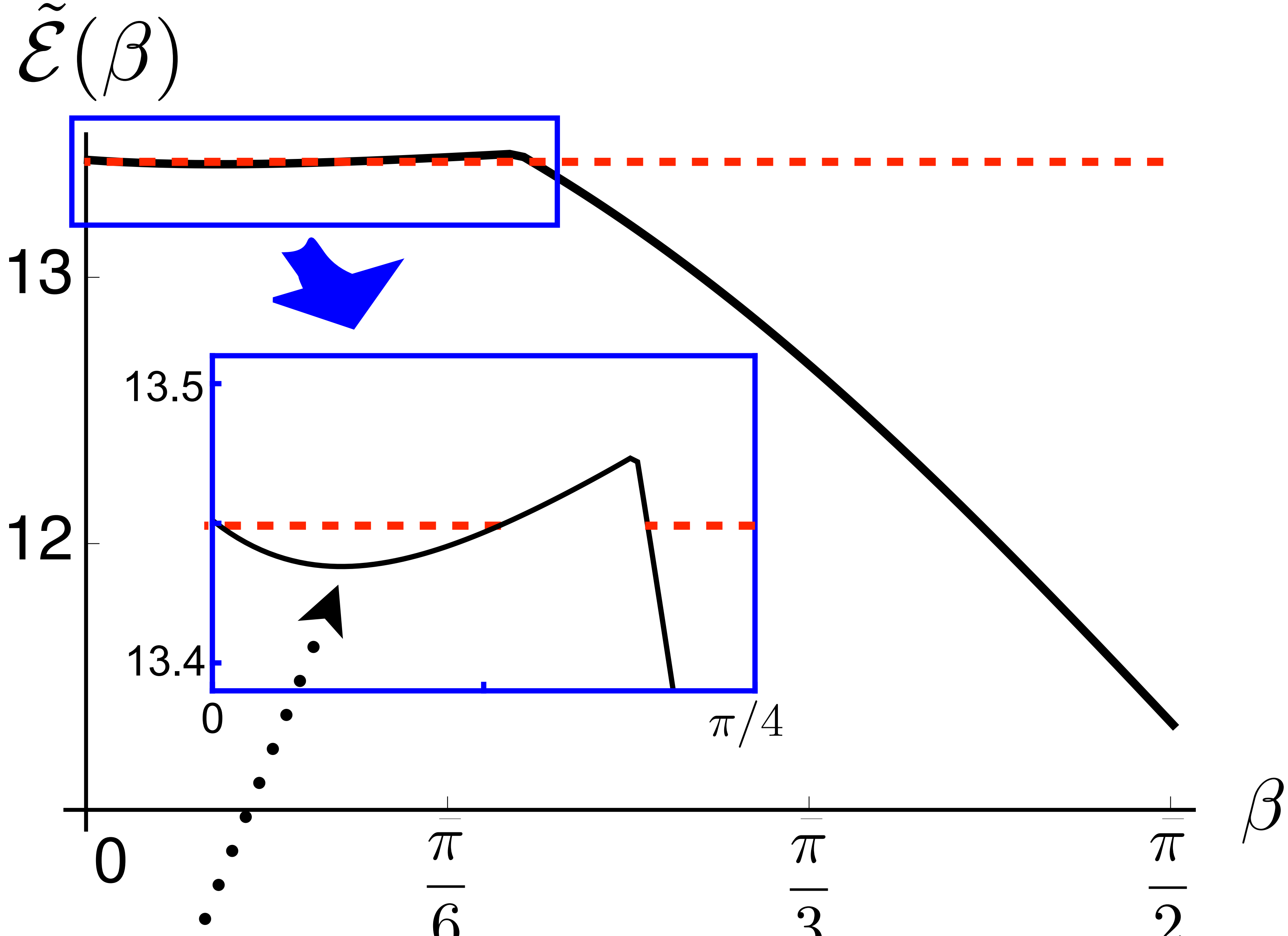
same impact speed U
longer length: Lb > La



final state
is still
encapsulated

Results

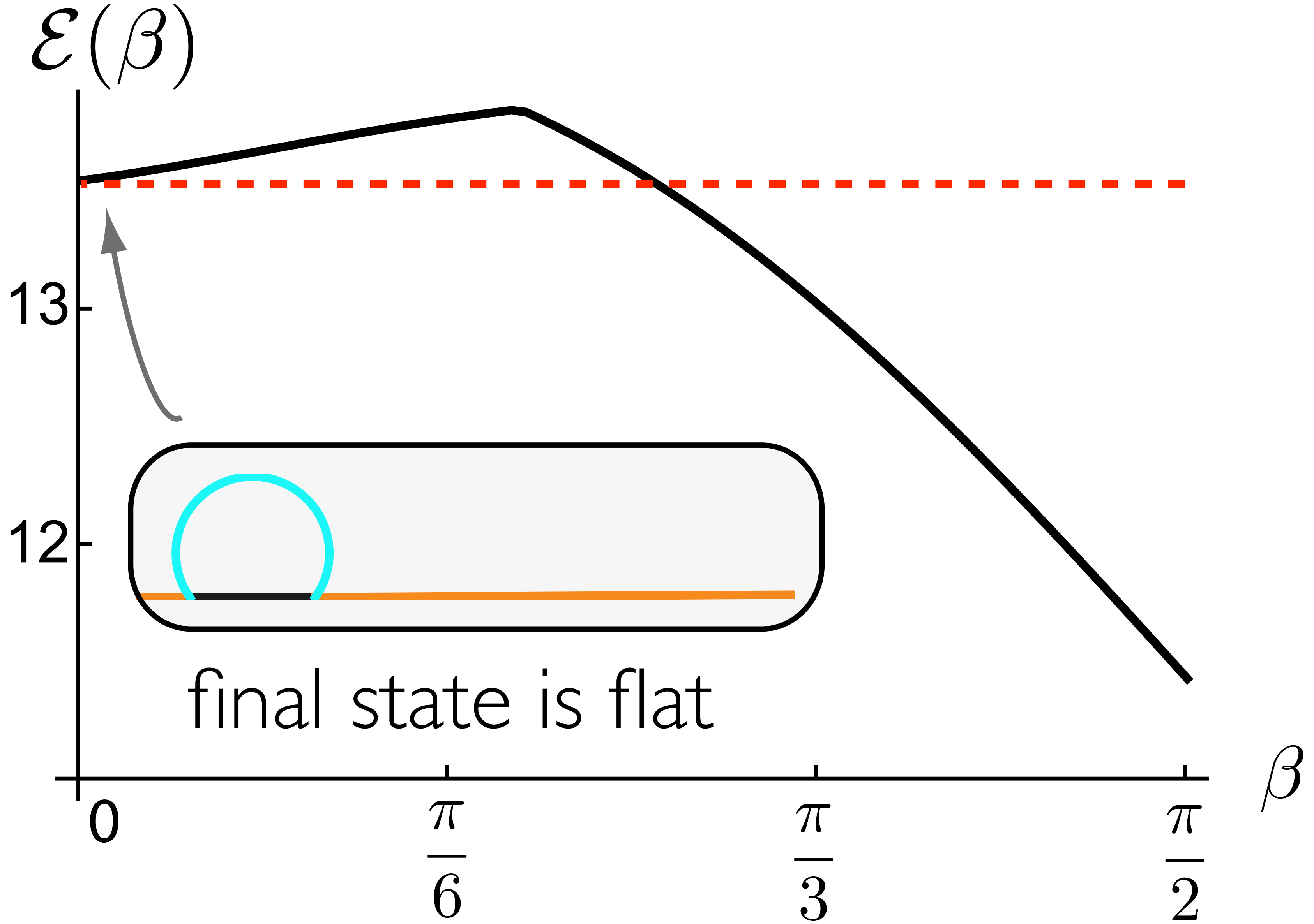
same impact speed U
longer length: Lc > Lb > La



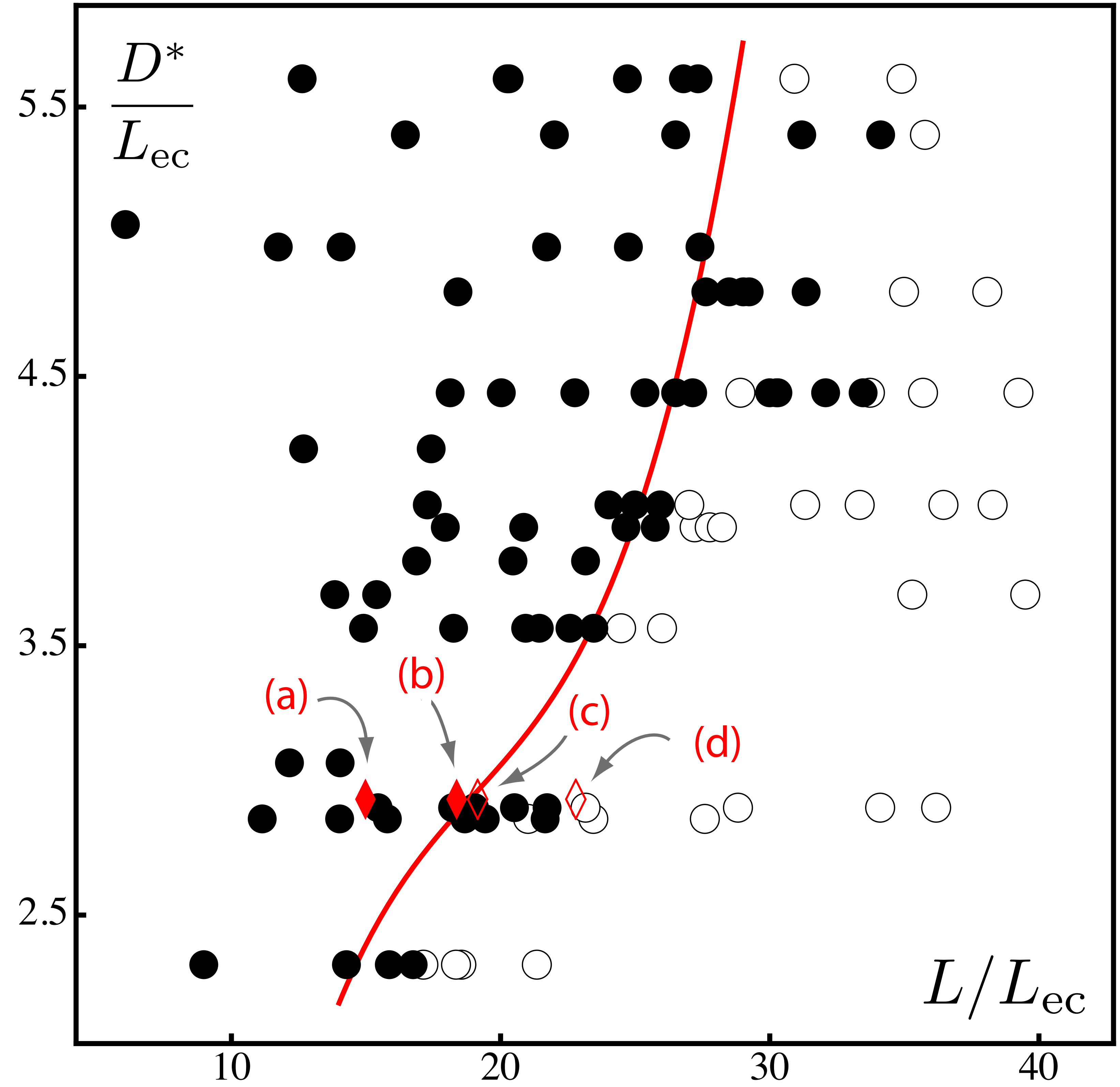
final state is open

Results

same impact speed U
longer length: Ld > Lc > Lb > La



Results



with experimentally measured:

$\theta_Y = 110^\circ$

$\theta^* = 150^\circ$

$L_{ec} = 0.55$ mm

$L_{eg} = 3.6$ mm