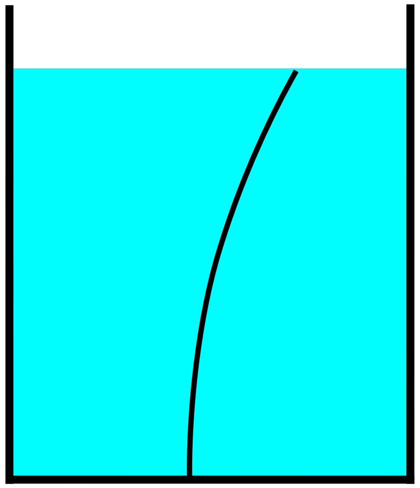
Tiges élastiques : deux exemples

Sébastien Neukirch (CNRS & UPMC Univ. Paris 6)

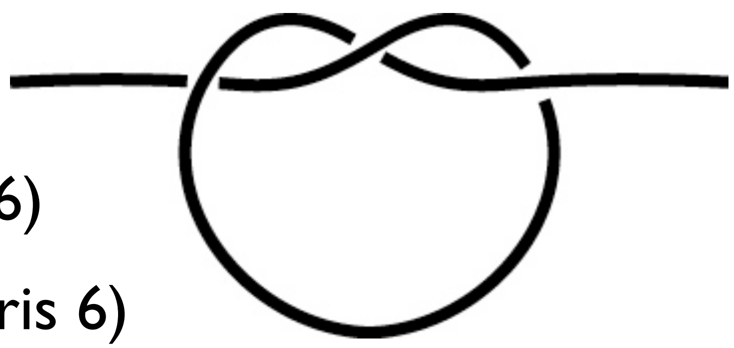
Elasto-capillarité

José Bico (ESPCI)
Benoît Roman (CNRS & ESPCI)



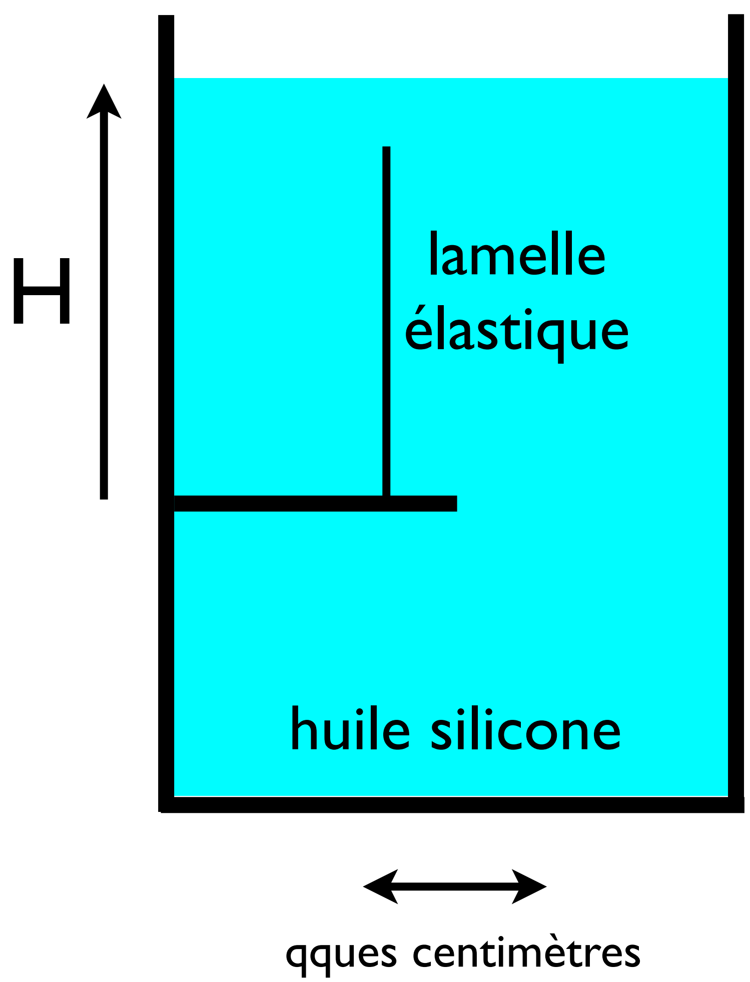
Noeuds élastiques

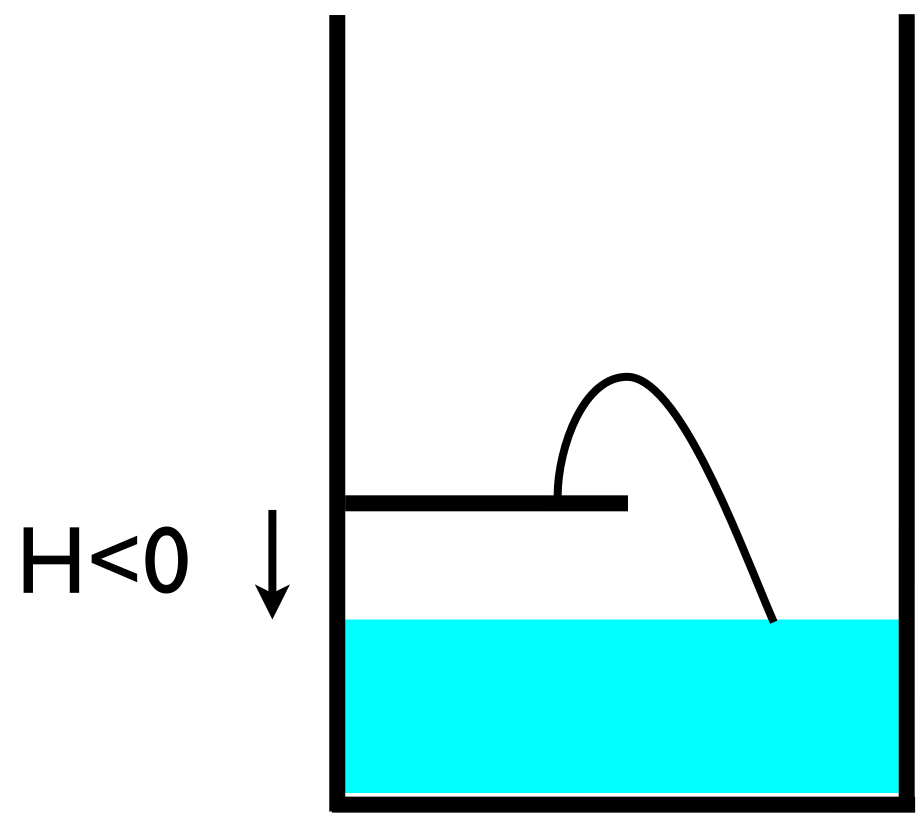
Nicolas Clauvelin (UPMC Univ Paris 6)
Basile Audoly (CNRS & UPMC Univ. Paris 6)



Elasto-capillarité : poils perçants

dispositif expérimental

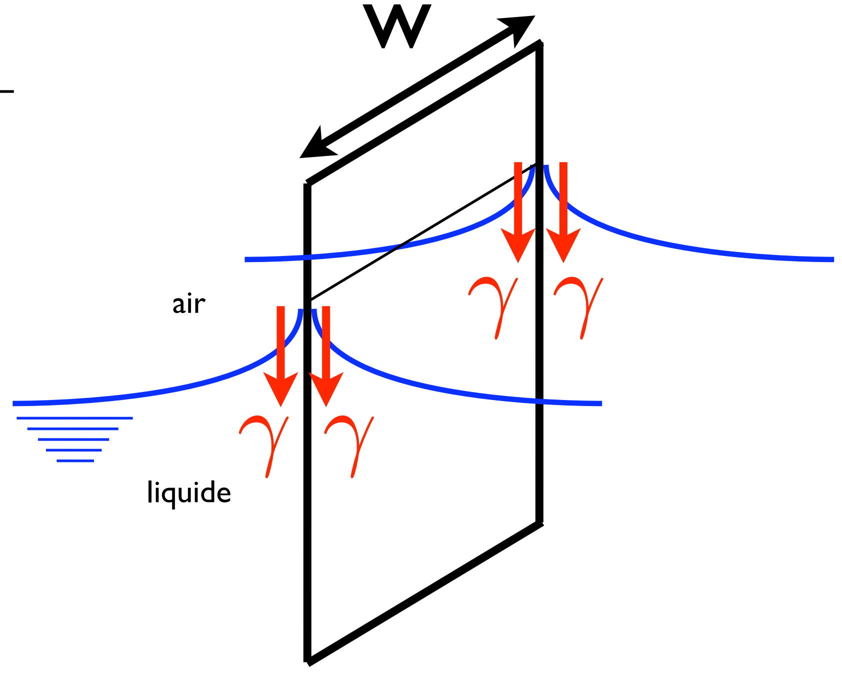




lamelle en polyester :
mouillage parfait avec l'huile

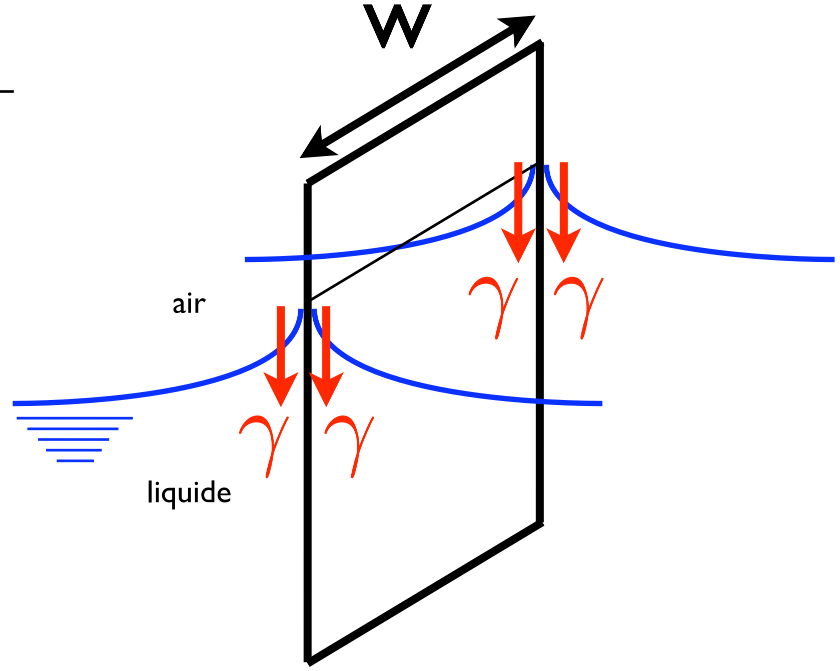
Tension de surface

$$F_{tot} = 2\,\gamma\,w$$

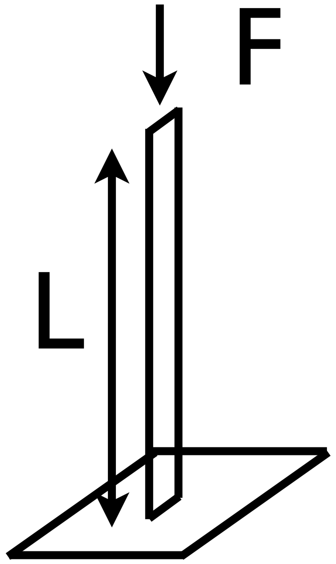


Tension de surface

$$F_{tot} = 2\,\gamma\,w$$



$$F_{euler} = \left(\frac{\pi}{2}\right)^2 \; \frac{EI}{L^2}$$



Tension de surface

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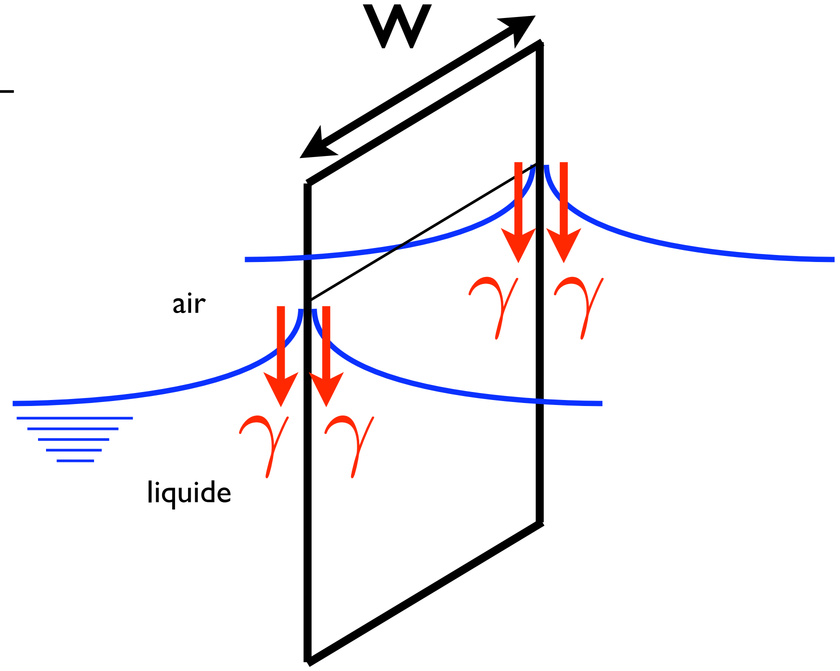

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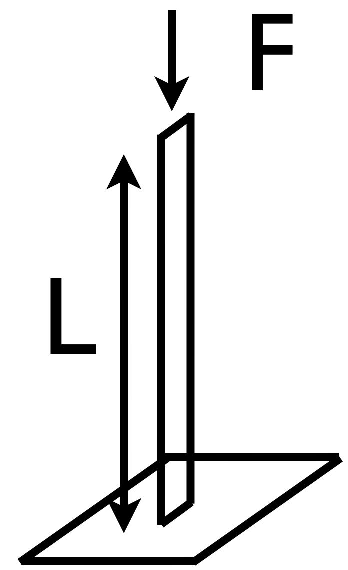

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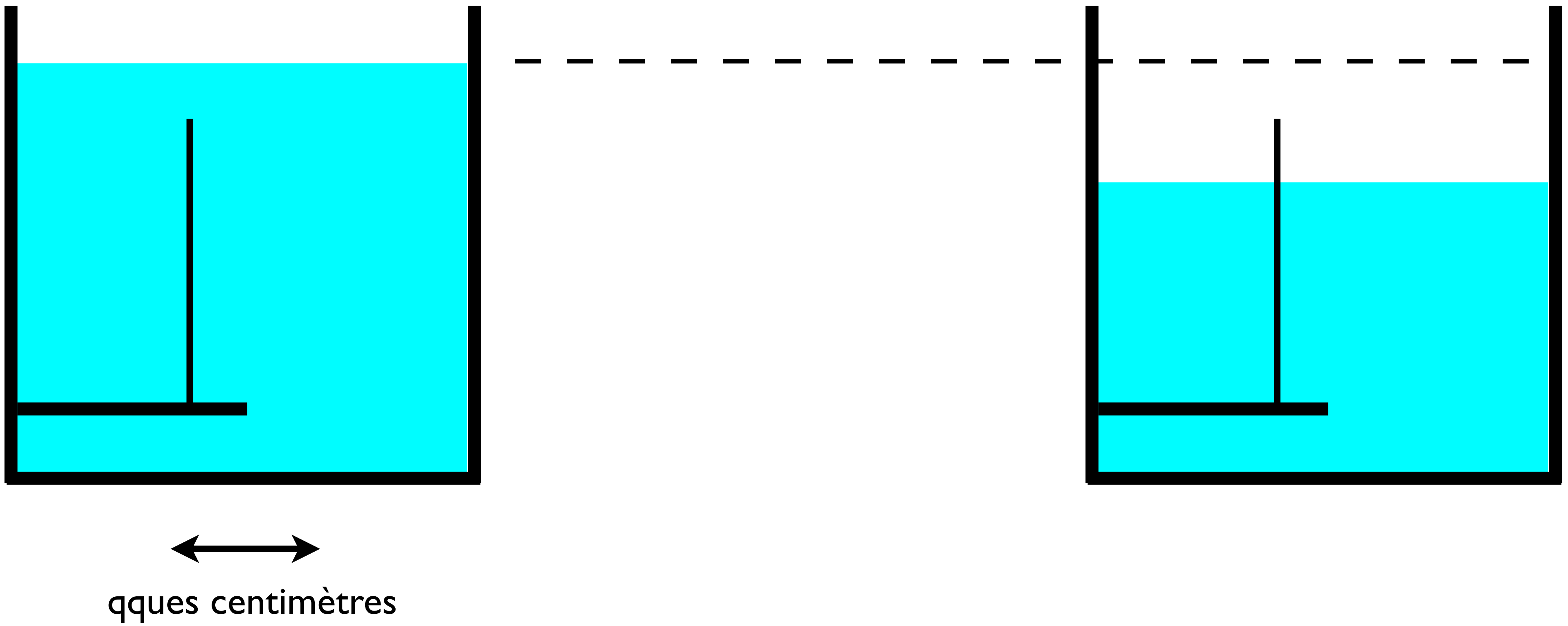
longueur critique $L_b = \frac{\pi}{2\sqrt{2}} \sqrt{\frac{EI}{\gamma w}}$

longueur élasto-capillaire

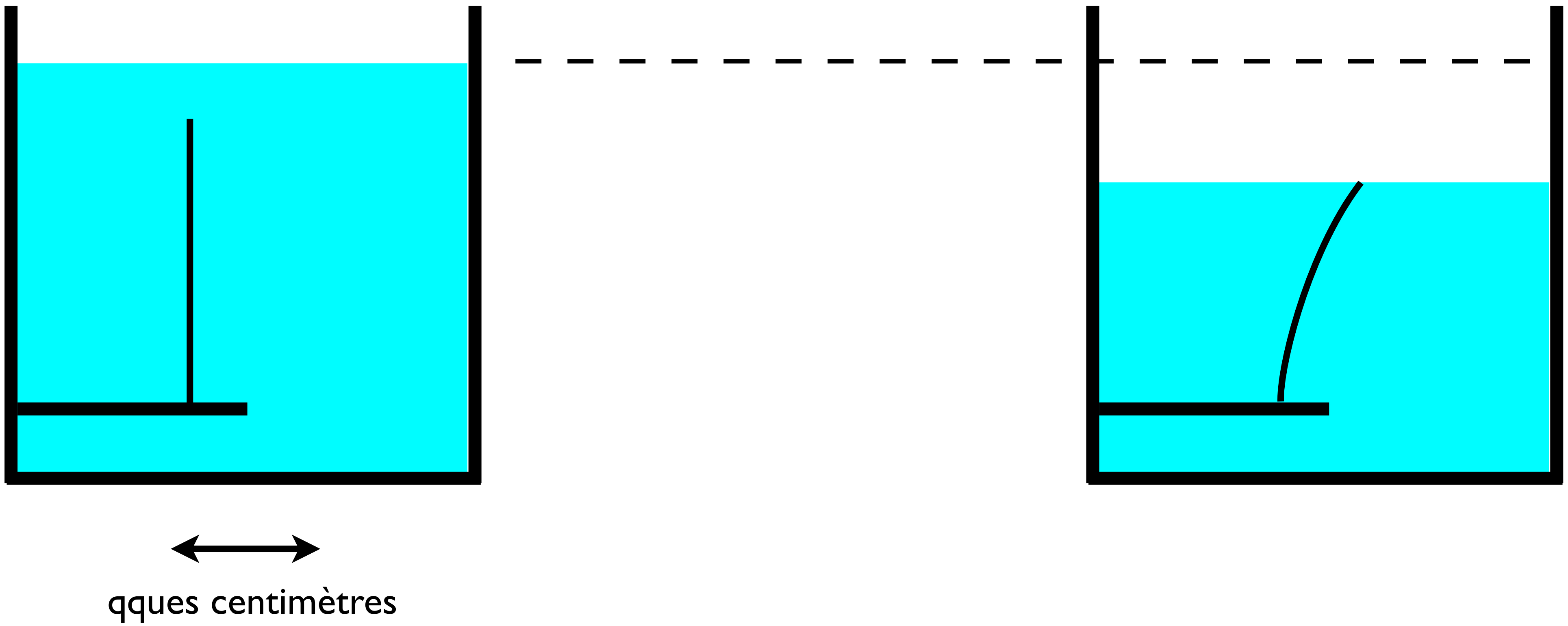
Flambage capillaire

lamelle courte $L < L_b$

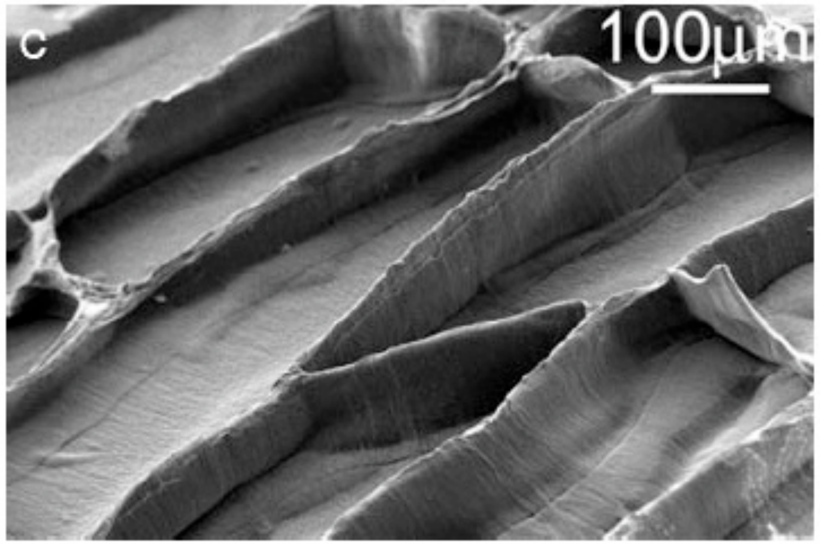


Flambage capillaire

lamelle longue $L > L_b$

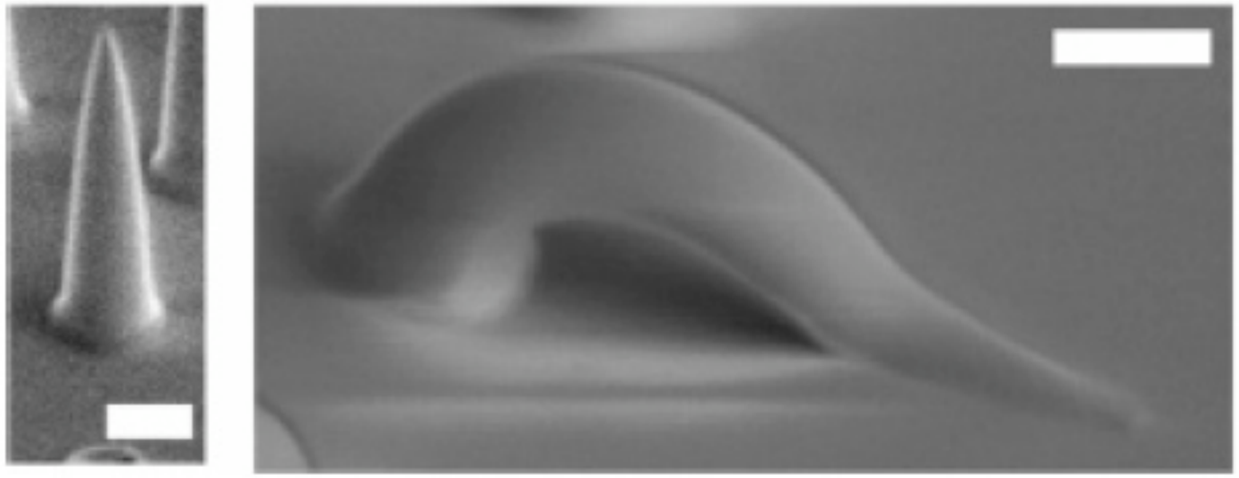


Sur le même sujet



nanotubes de carbone après évaporation

N. Chakrapani et al - Nature (2004)



plot de PDMS plié par ethanol

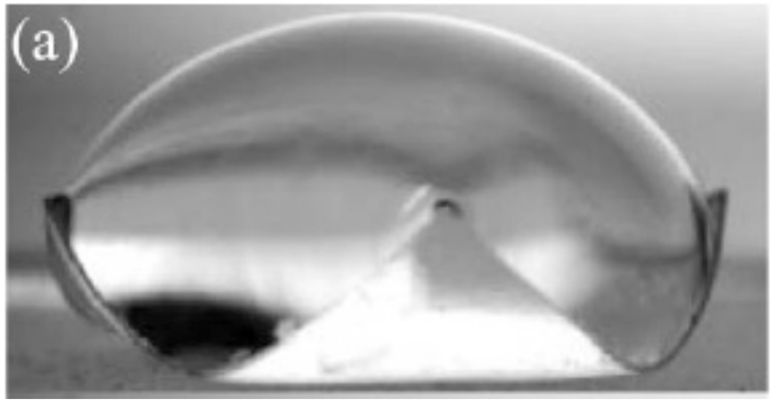
P. Roca-Cusachs et al - Langmuir (2005)

J. Bico et al - Nature (2004)

agglomération de fibres élastique et remonté capillaire

C. Py et al - Phys. Rev. Let. (2007)

pliage d'une plaque élastique par capillarité

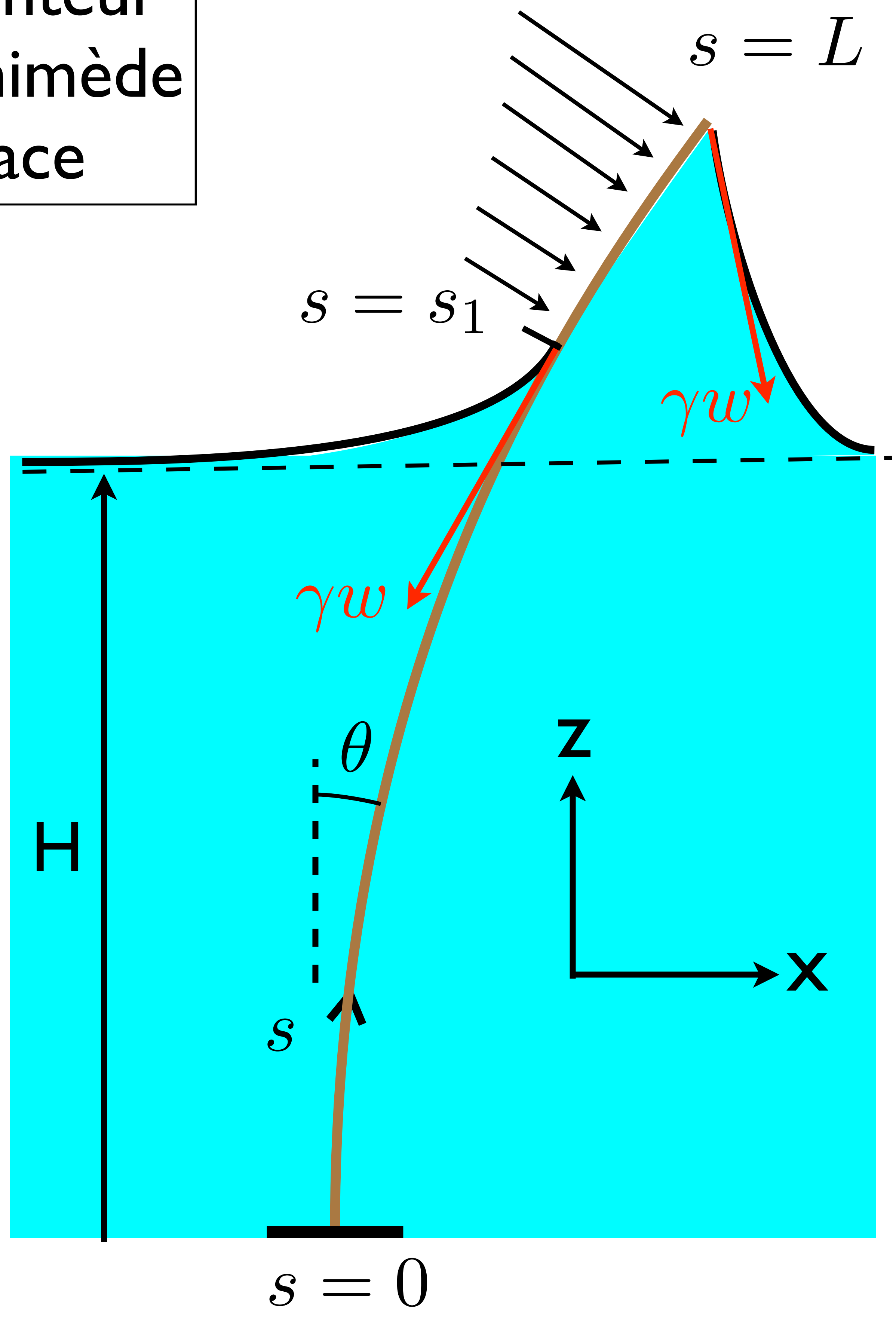


Modélisation

- pesanteur
- Archimède
- Laplace

$$N_x' = -P_x$$
$$N_z' = -P_z$$
$$M_y' = N_z \sin\theta - N_x \cos\theta$$
$$X' = \sin\theta$$
$$Z' = \cos\theta$$
$$\theta' = M_y/(EI)$$

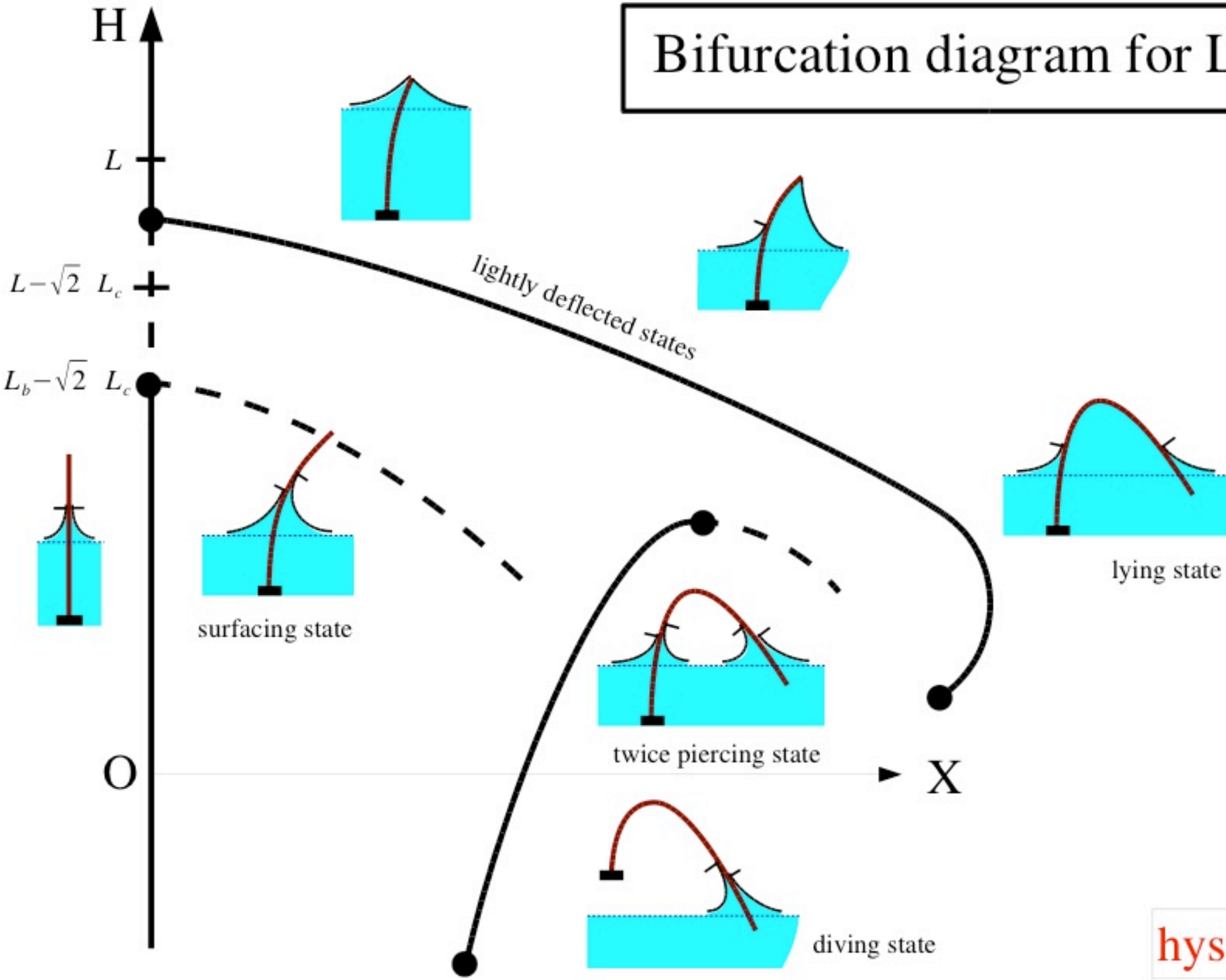
$$' \equiv \frac{d}{ds}$$



- équations de Kirchhoff
- sauts de force aux ménisques
- conditions aux bords

- suivi de courbes : AUTO 94

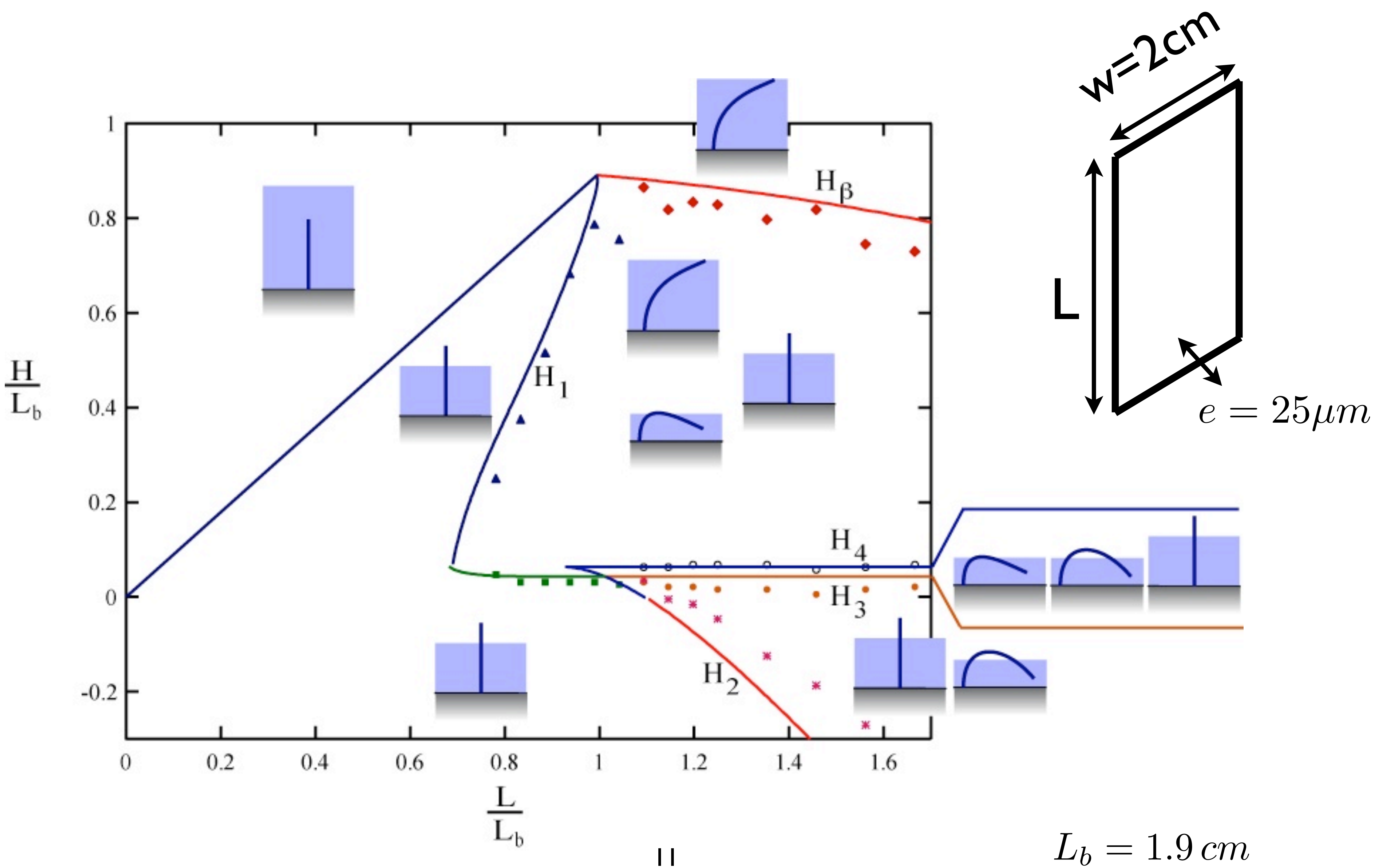
Bifurcation diagram for $L>L_b$



hysteresis

Comparaison théorie/expériences

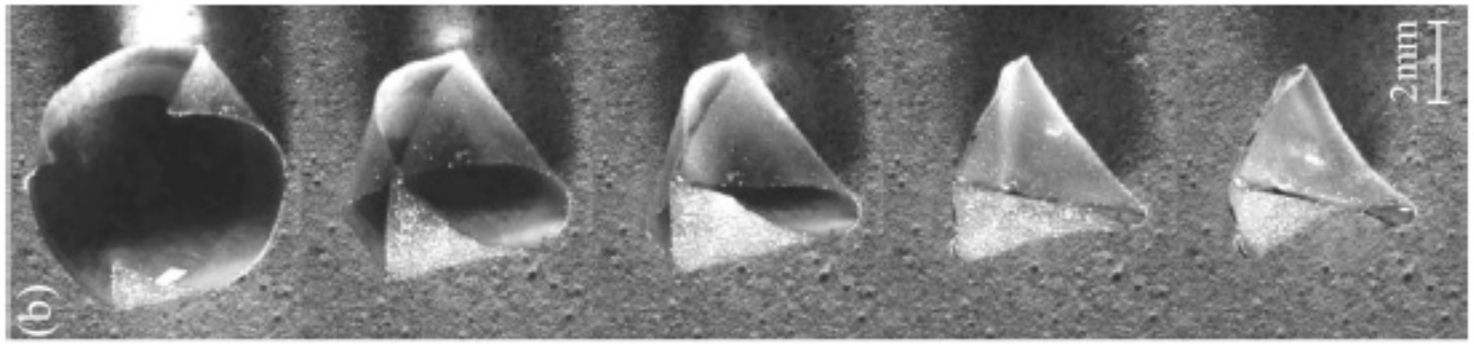
lamelle polyester



Origami capillaire

statique

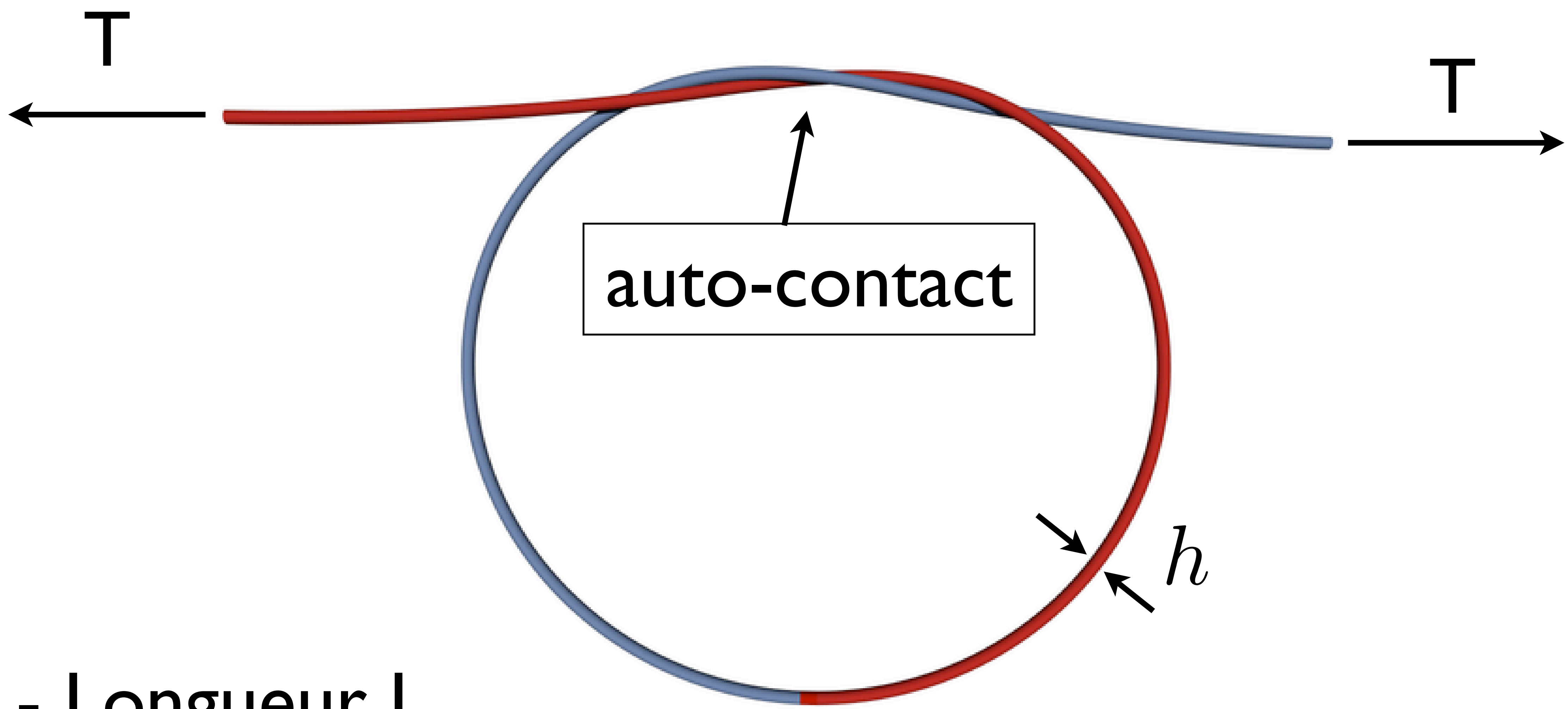
Py et al
Capillary origami
Phys. Rev. Lett. 2007



dynamique

M. Rivetti
A. Antkowiak

Noeuds élastiques



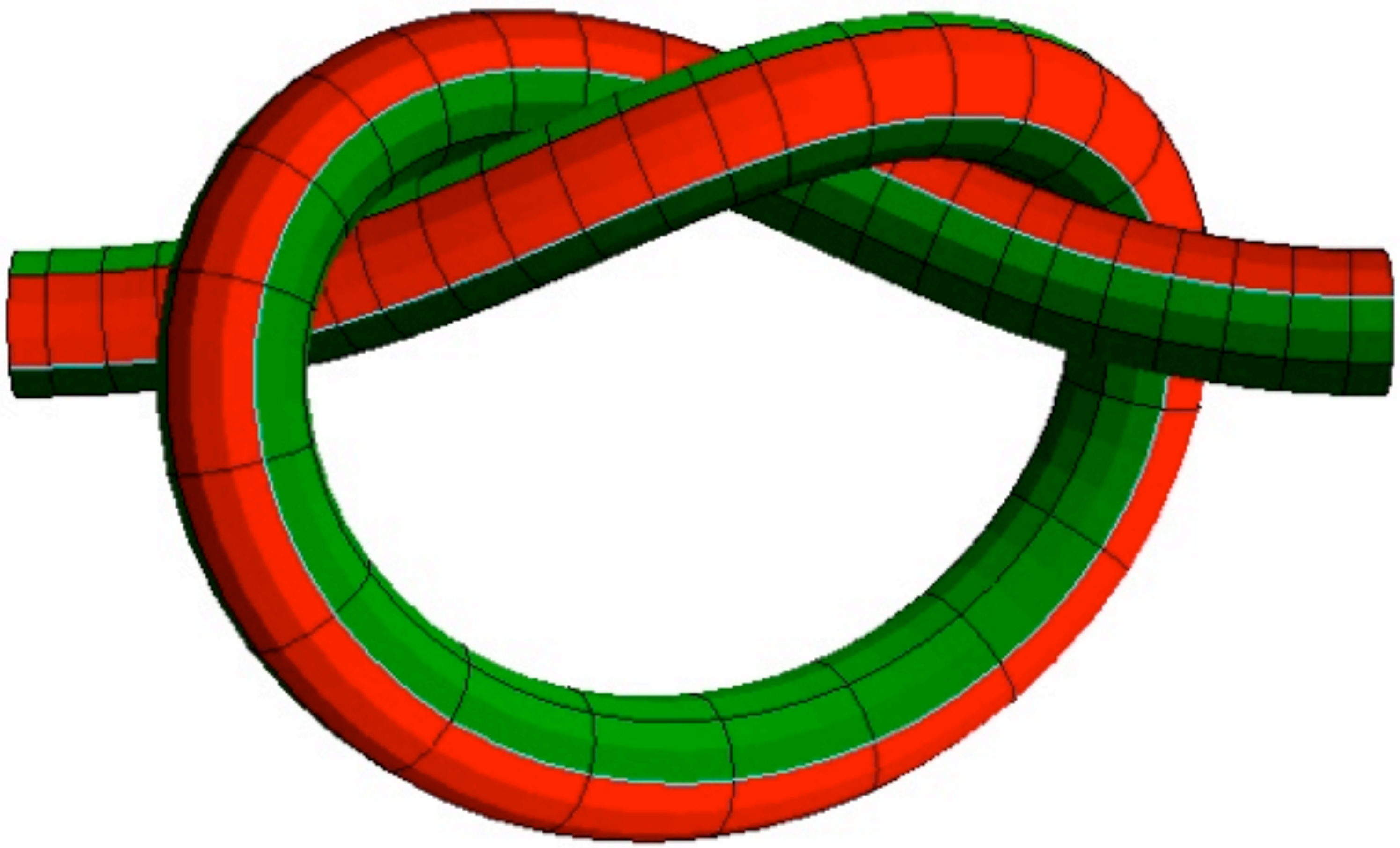
- Longueur L
- Section circulaire de rayon h
- Rigidité de flexion : E I
- Rigidité de torsion : G J

$$I = \frac{\pi h^4}{4}$$

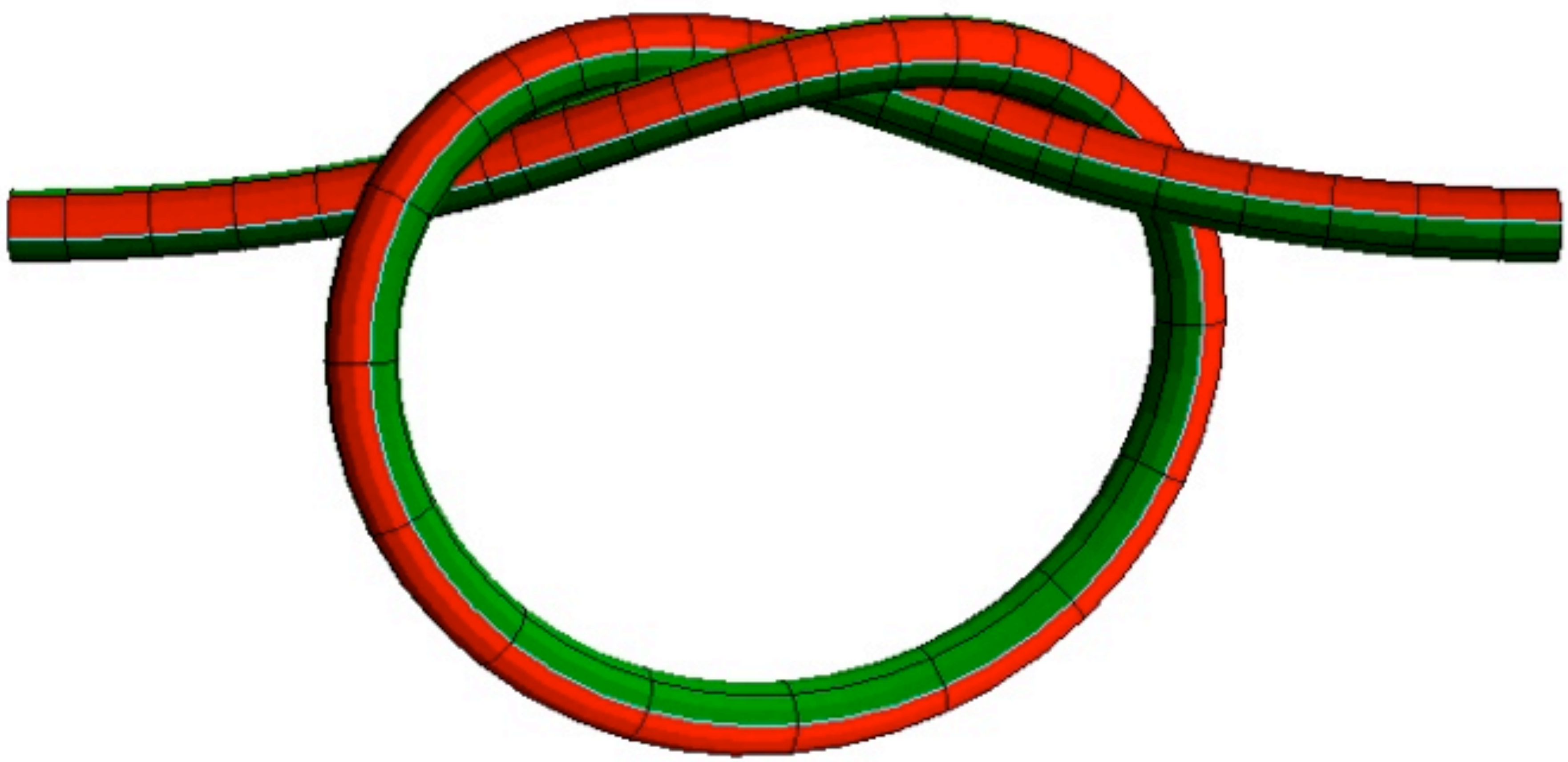
$$J = \frac{\pi h^4}{2}$$

E : module d'Young G : module de cisaillement

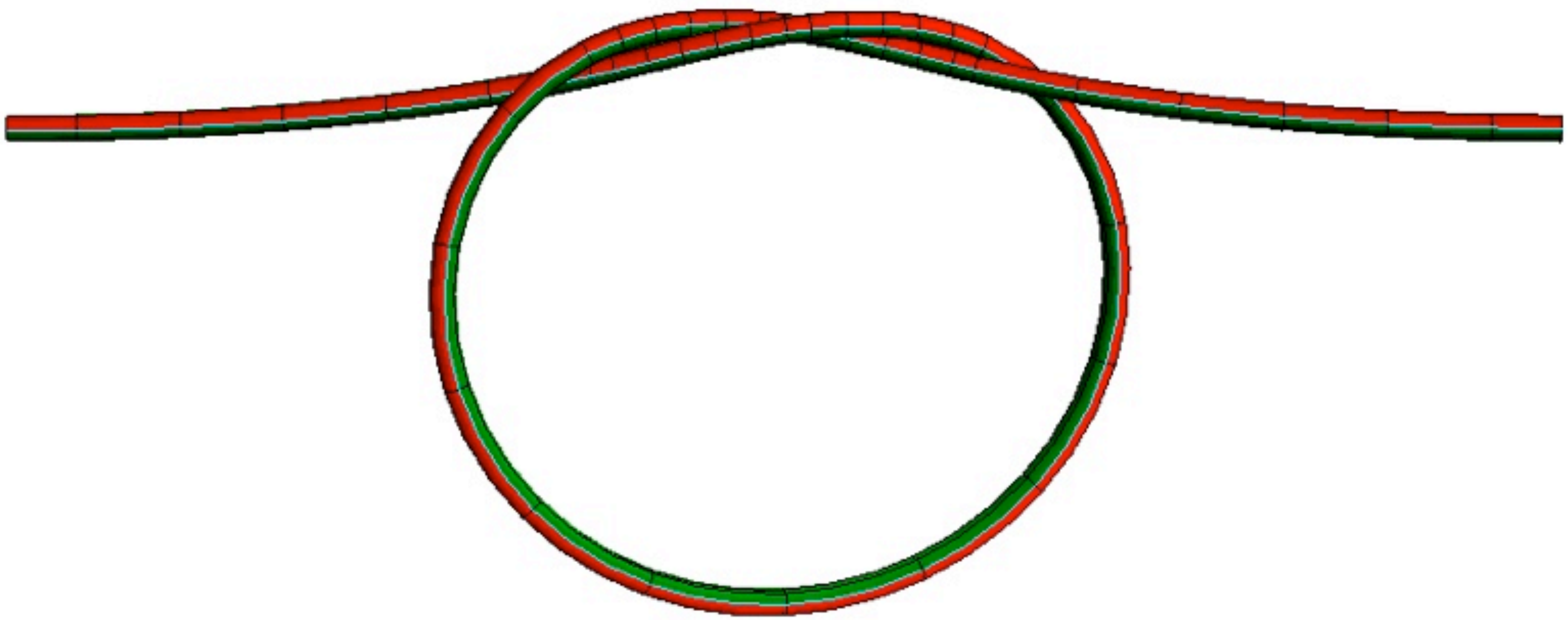
Making the rod thinner



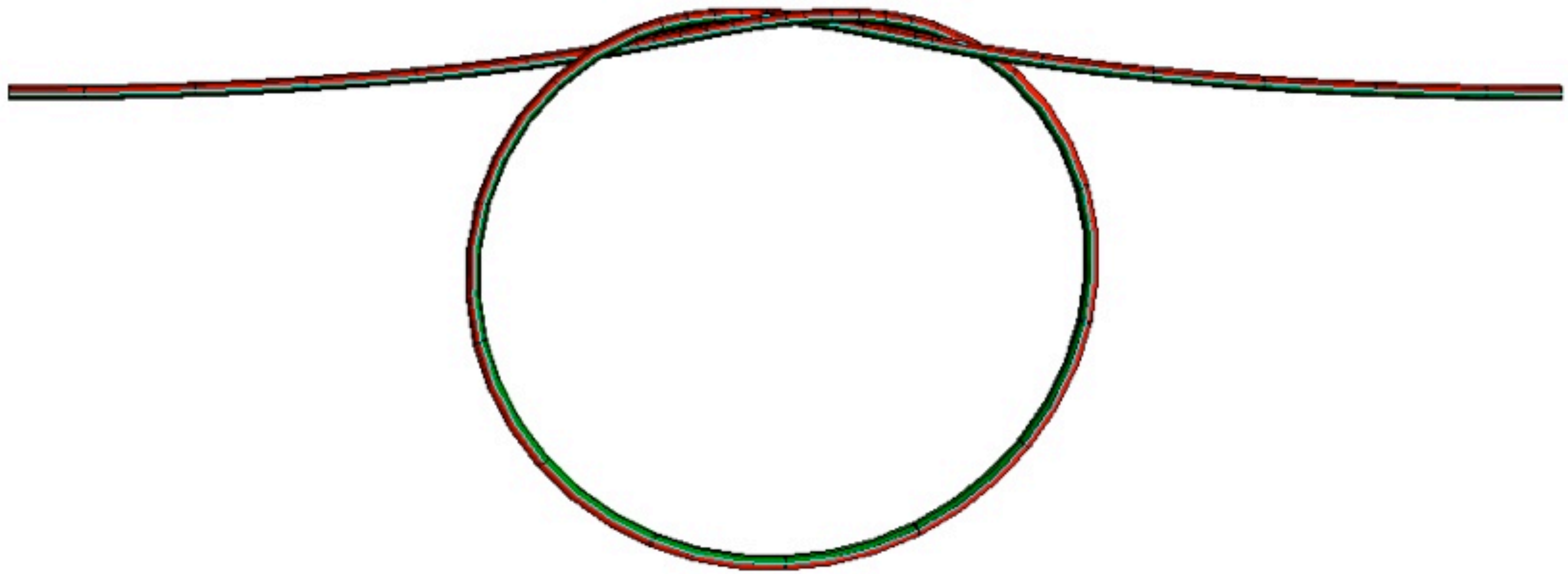
Making the rod thinner



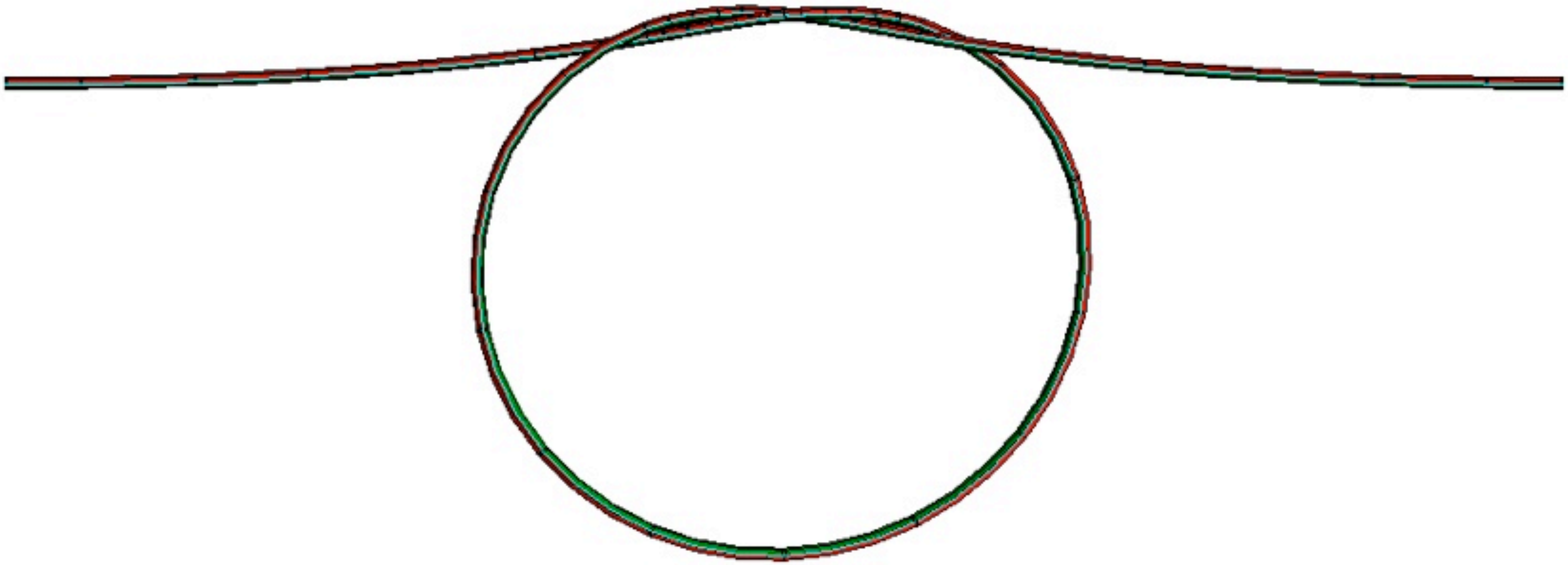
Making the rod thinner



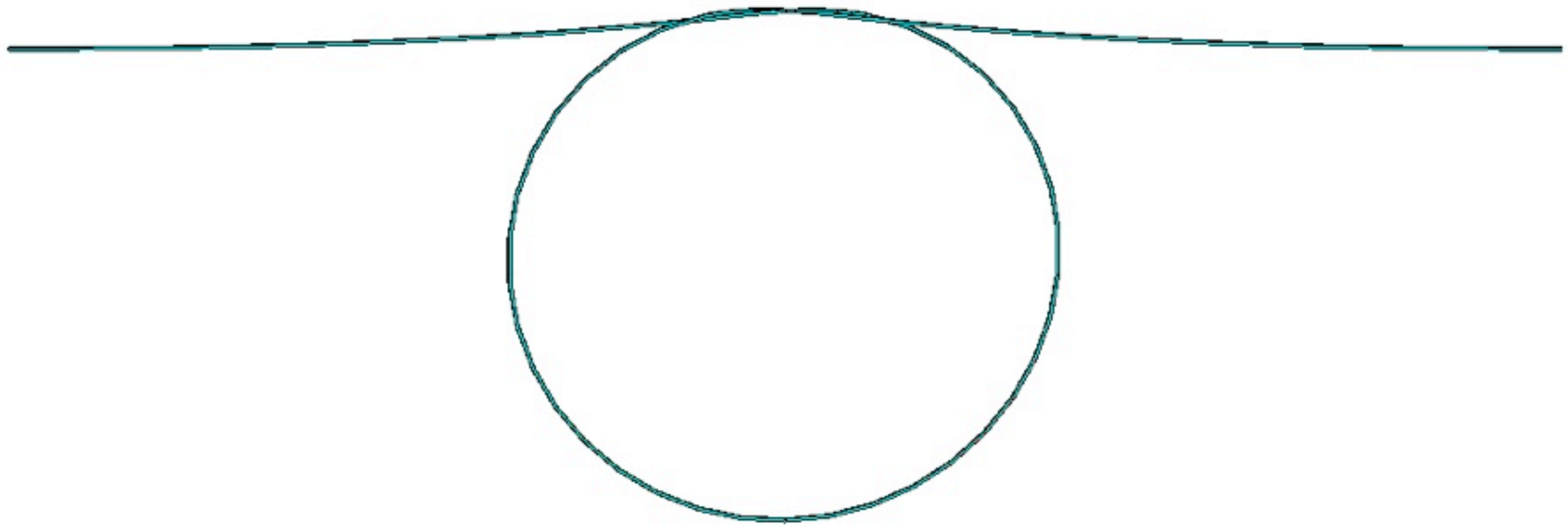
Making the rod thinner



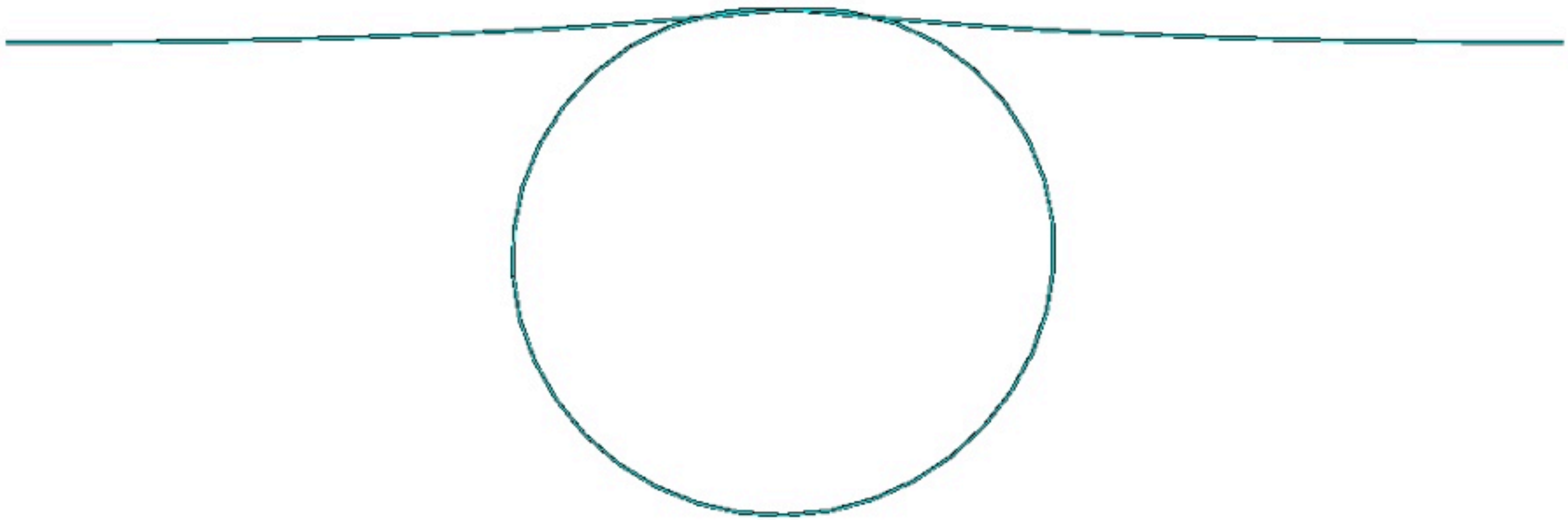
Making the rod thinner



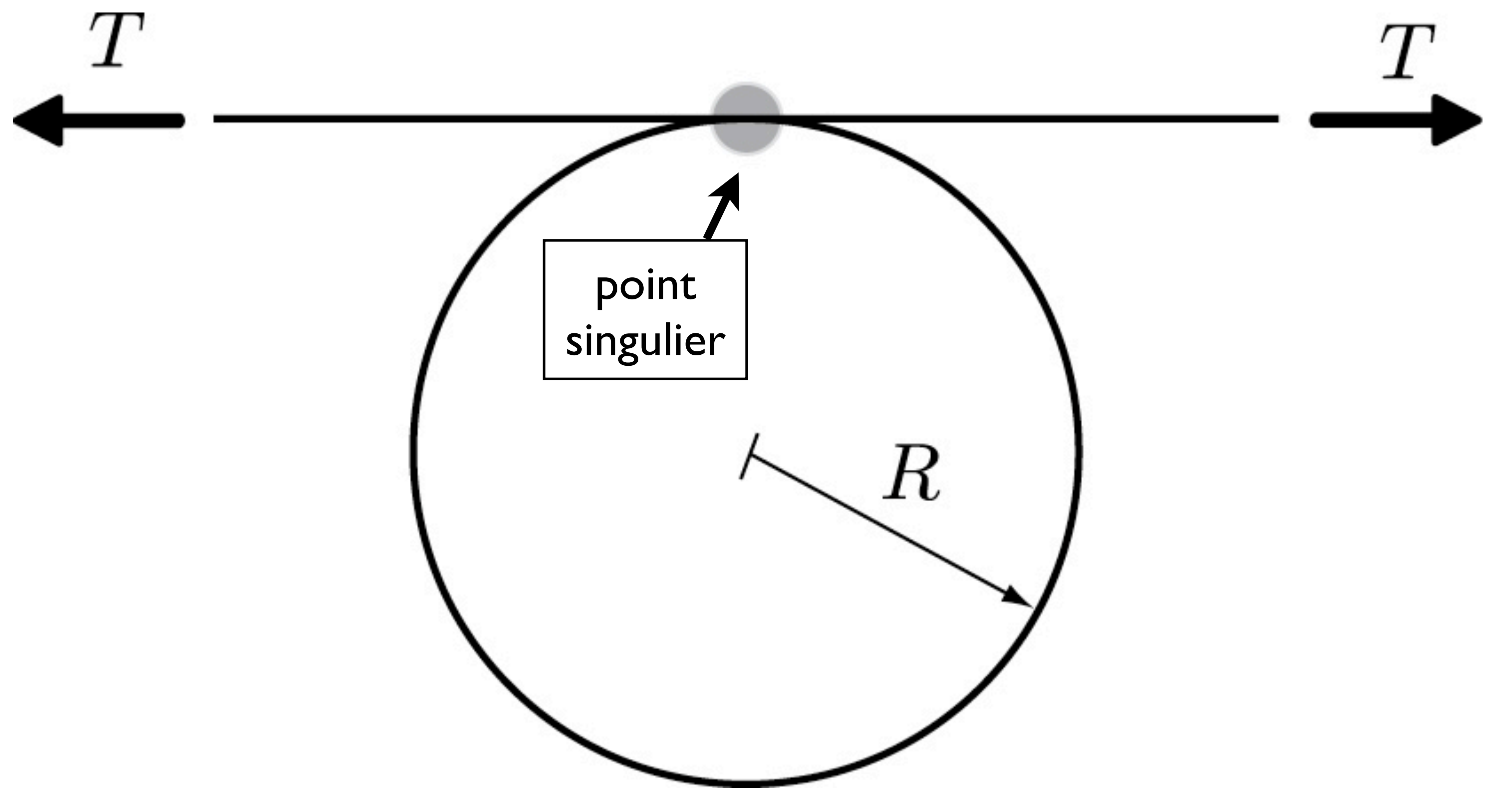
Making the rod thinner



Making the rod thinner



Limite d'épaisseur nulle

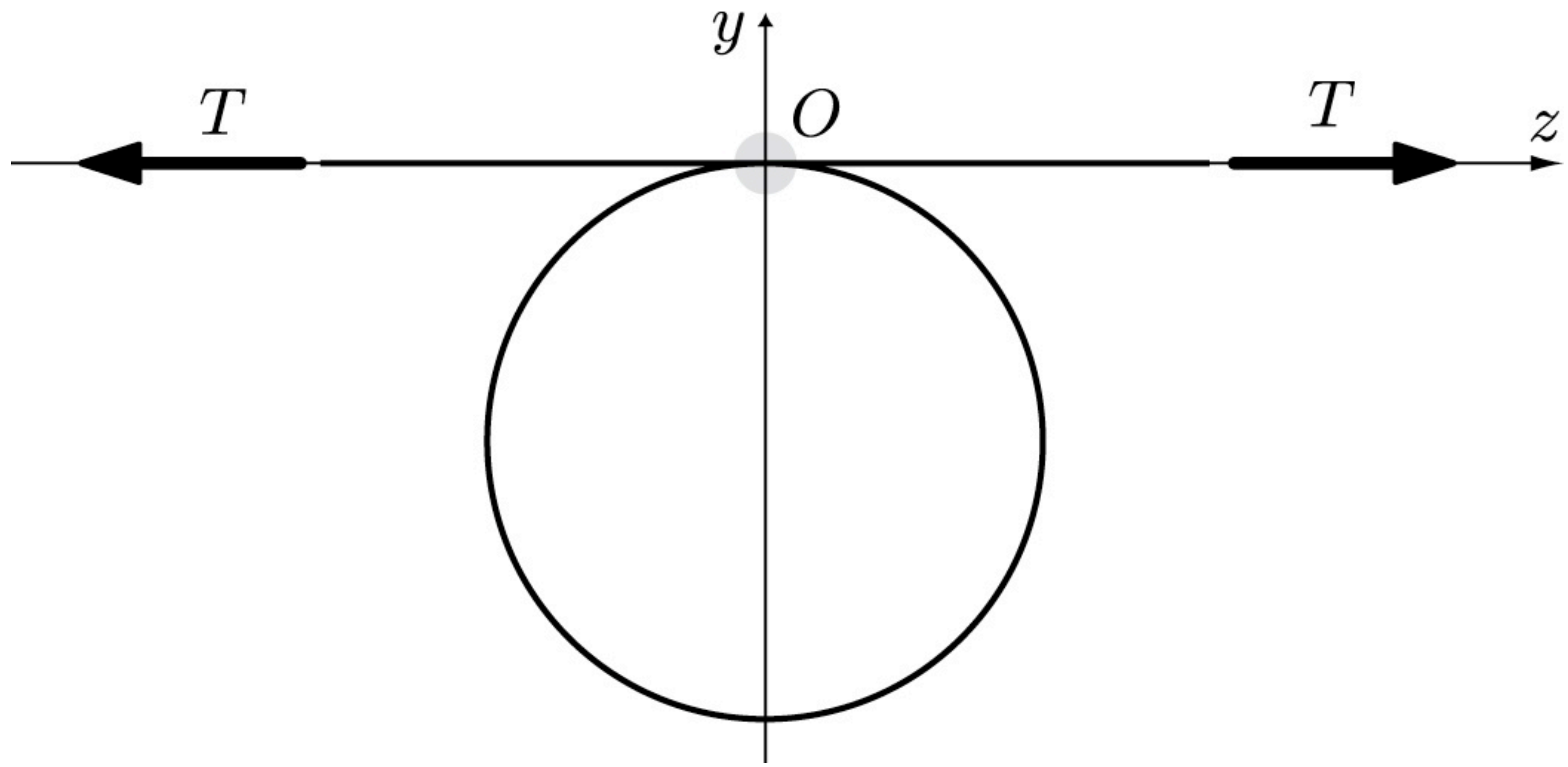


équilibre : $T = \dfrac{EI}{2R^2}$ Arai et al (1999)

tension T $\longrightarrow$ (point singulier) courbure $\dfrac{1}{R}$

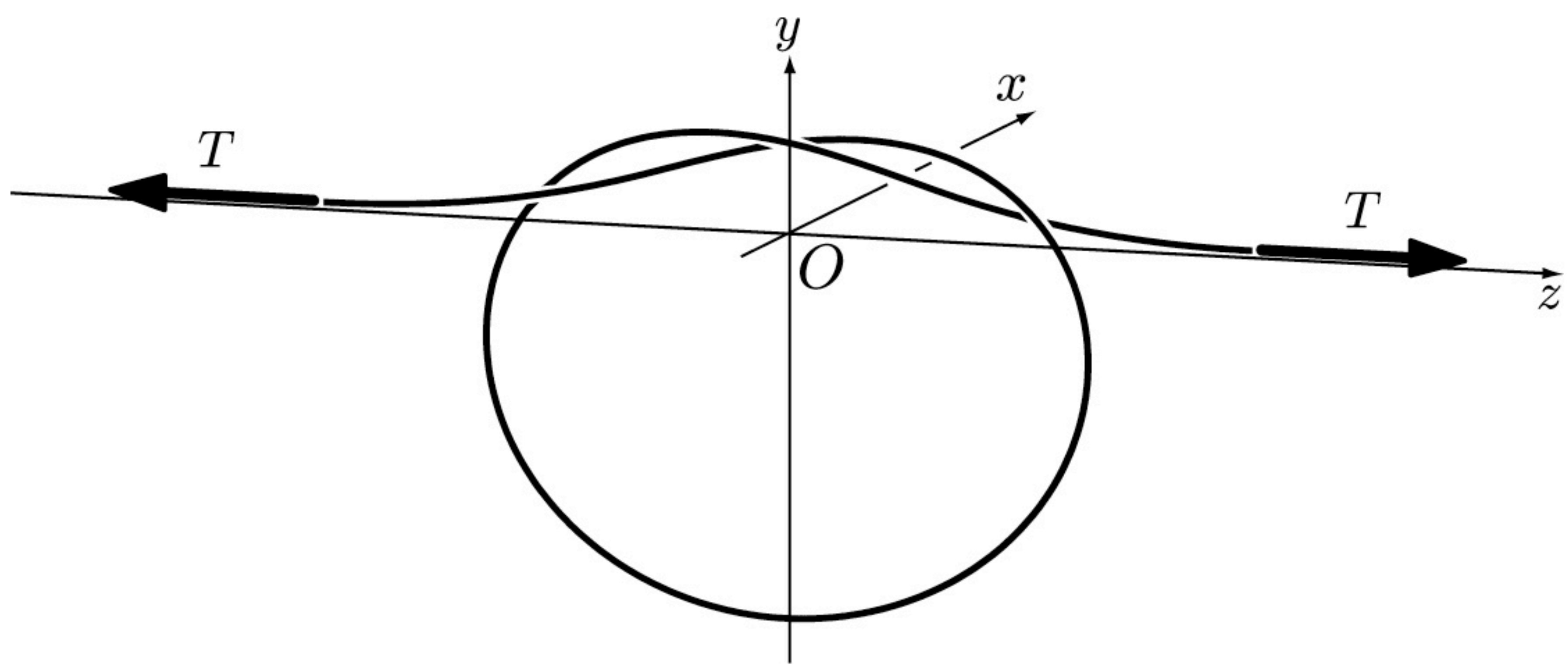
Faible épaisseur

$$\epsilon = 0$$
$$h = 0$$

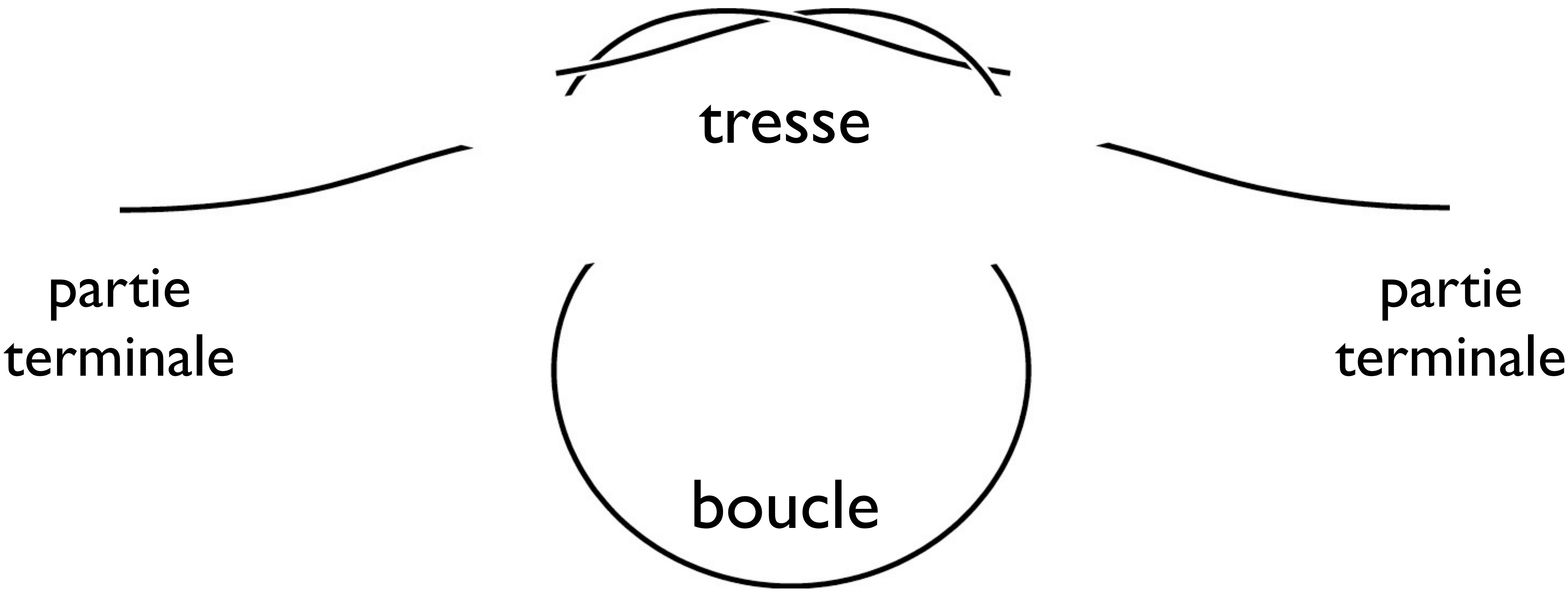


petit paramètre

$$\epsilon = \left(\frac{2h^2 T}{EI} \right)^{1/4} \ll 1$$



Dévelop. asymptotiques raccordés



petit paramètre : $\epsilon = \left(\frac{2h^2 T}{EI} \right)^{1/4} \ll 1$

Equations de Kirchhoff

$\vec{p}(s)$ pression ext.

$\vec{N}(s)$ force interne

$\vec{M}(s)$ moment interne

$\vec{R}(s)$ position

$\vec{t}(s)$ tangente

$$' \equiv \frac{d}{ds}$$

$$\vec{N}' = -\vec{p}$$ équil. forces

$$\vec{M}' = \vec{N} \times \vec{t}$$ équil. moments

$$\vec{R}' = \vec{t}$$ déf. tangente

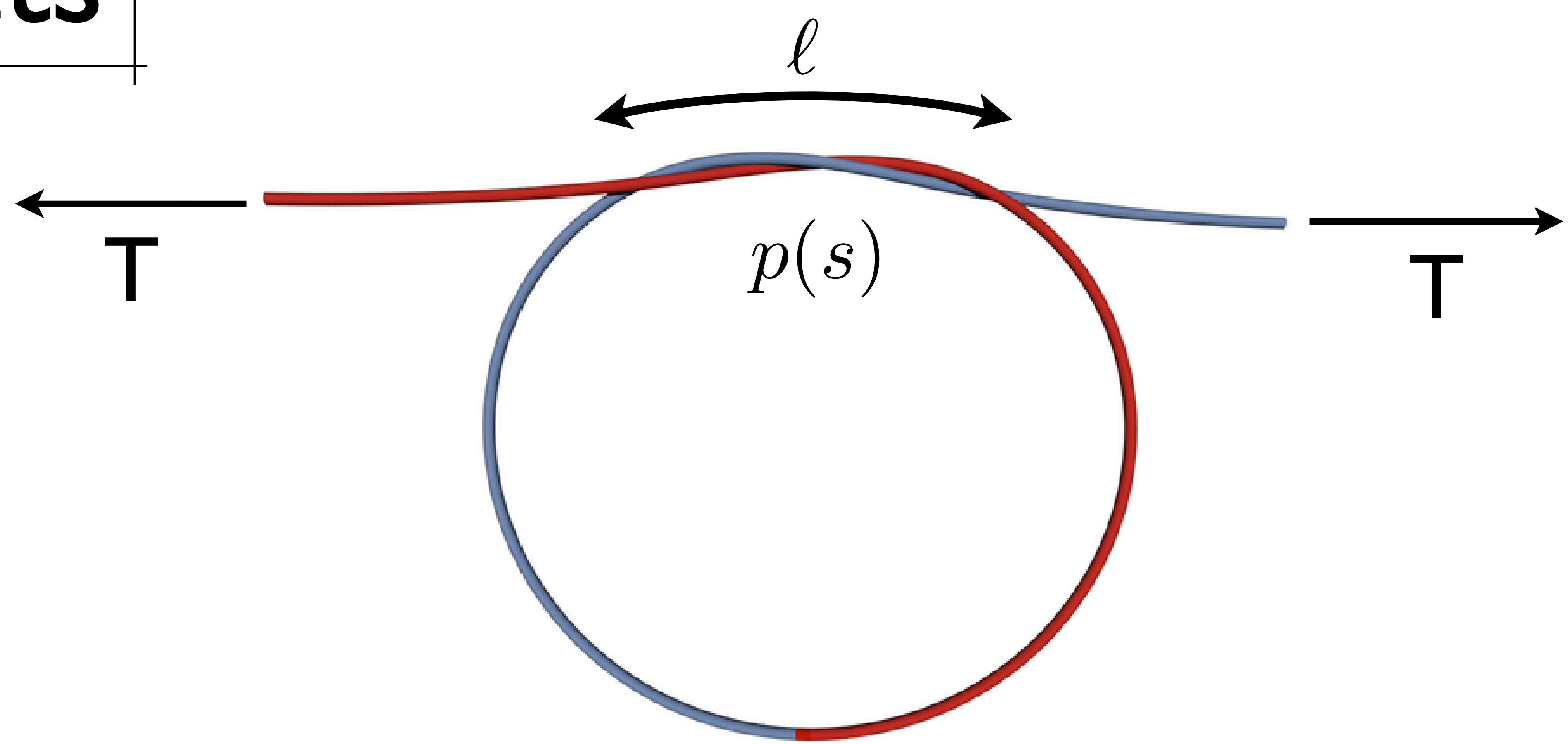
$$\vec{t}' = \frac{1}{EI} \vec{M} \times \vec{t}$$ déformation

loi de comportement :

$$M_\kappa = EI\,\kappa$$ flexion κ

$$M_\tau = GJ\,\tau$$ torsion τ

Résultats



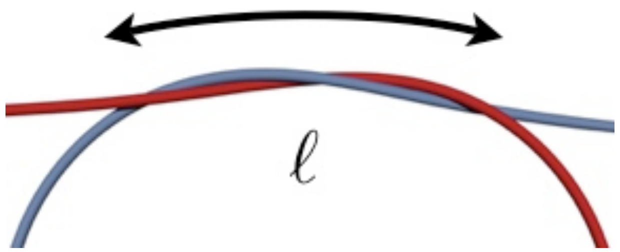
$$R = \sqrt{\frac{EI}{2T}}$$

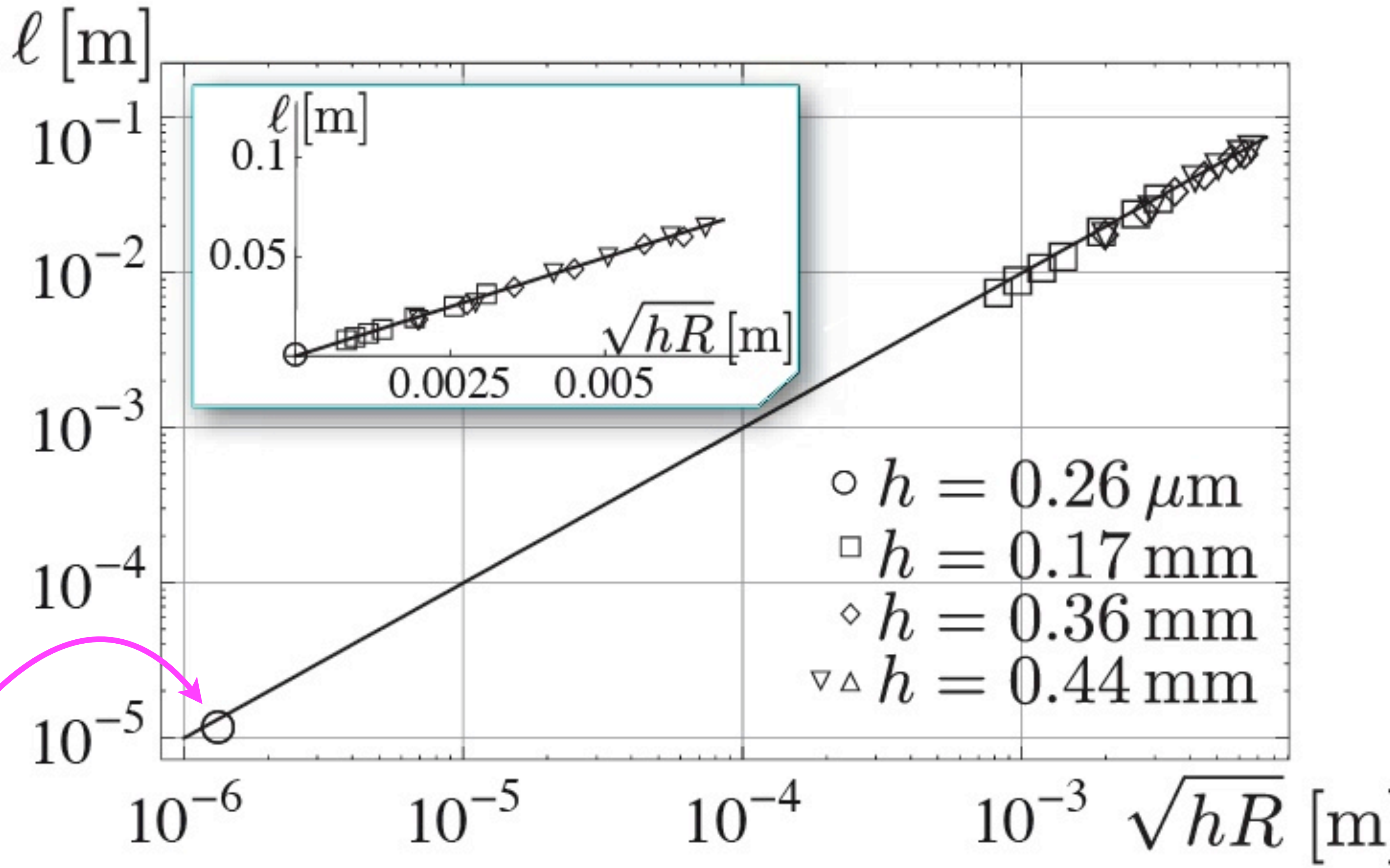
$$\ell = 9.91\, h^{1/2}\ (EI)^{1/4}\ T^{-1/4}$$

force de contact $p(s)$

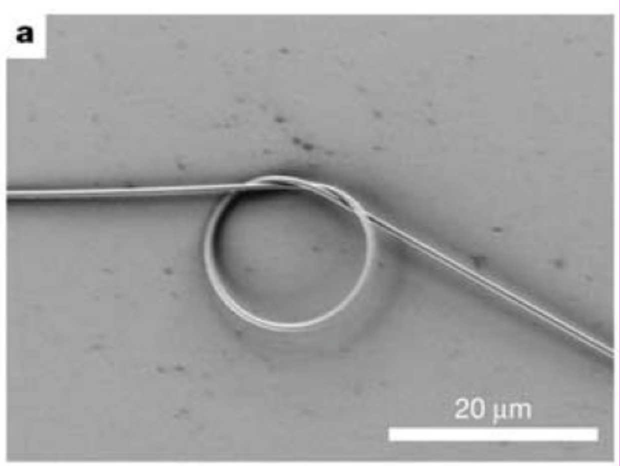
Force totale $P = \int_0^{\ell} p(s)ds = 0.82\, h^{-1/2}\ (EI)^{1/4}\ T^{3/4}$

Expériences



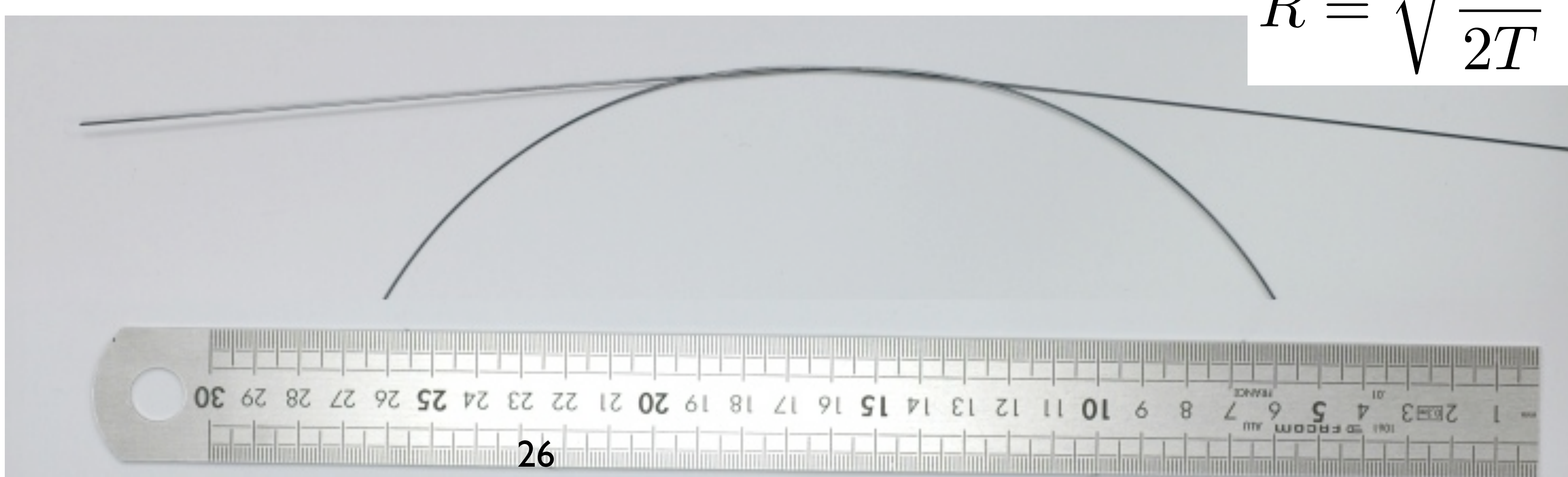


Tong et al., Nature 2003

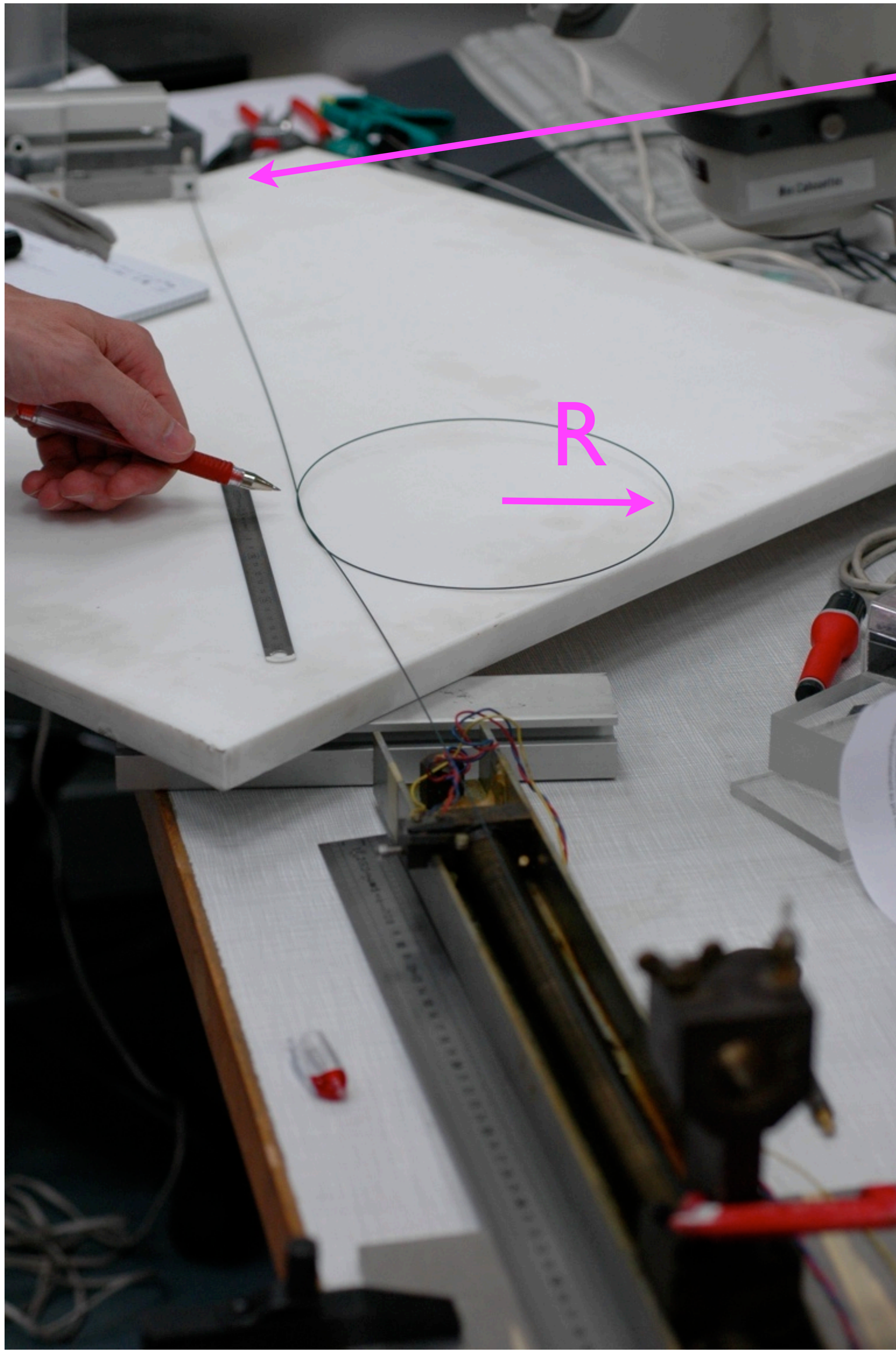


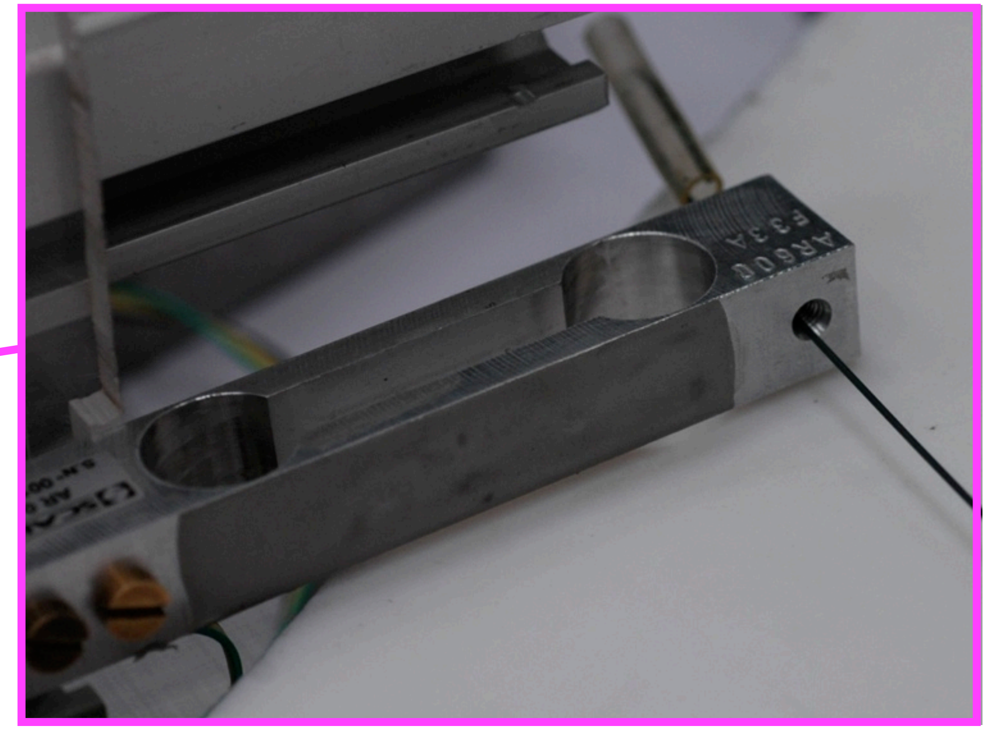
fil en silice
h=1/2 micron

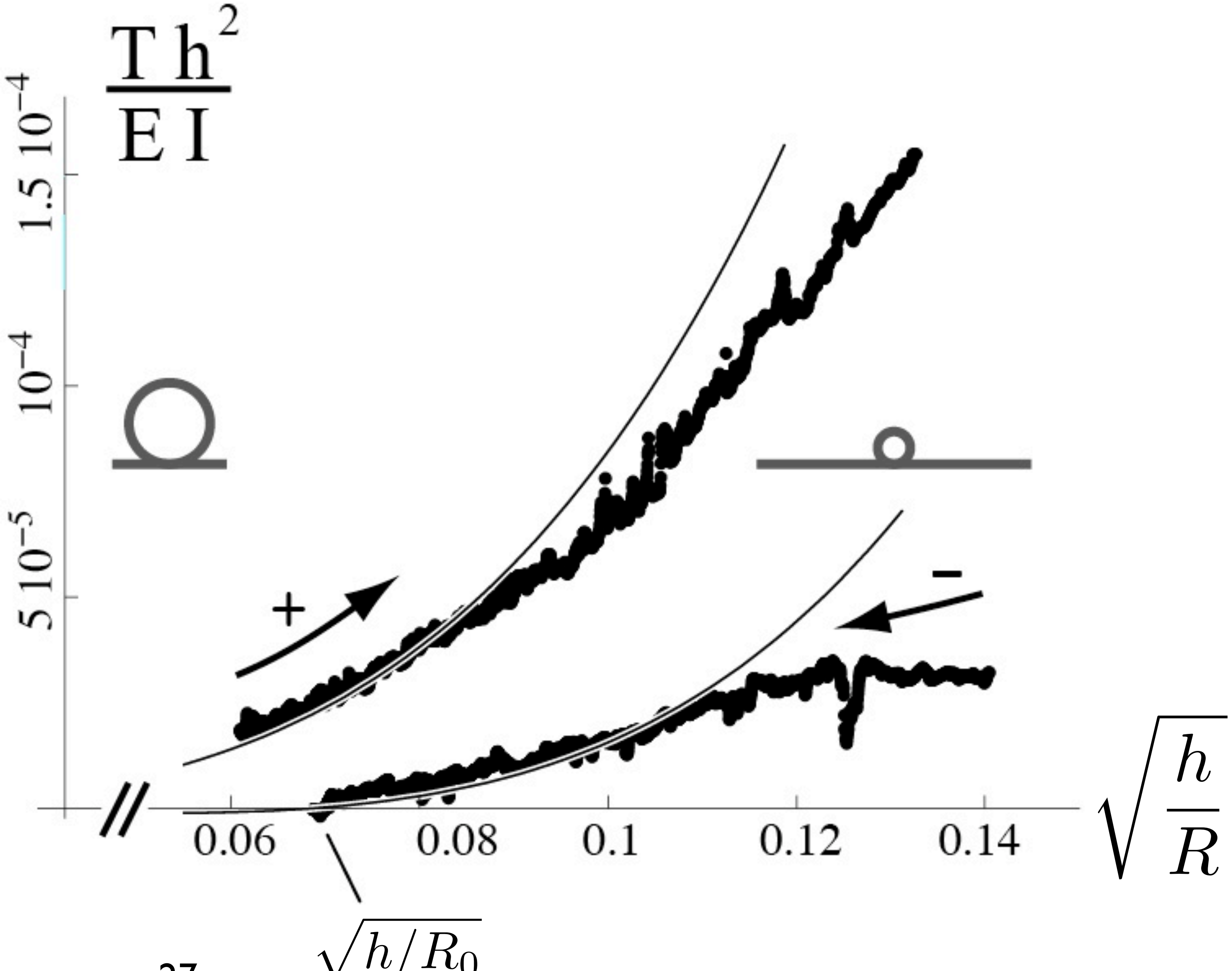
$$R = \sqrt{\frac{EI}{2T}}$$



Expériences





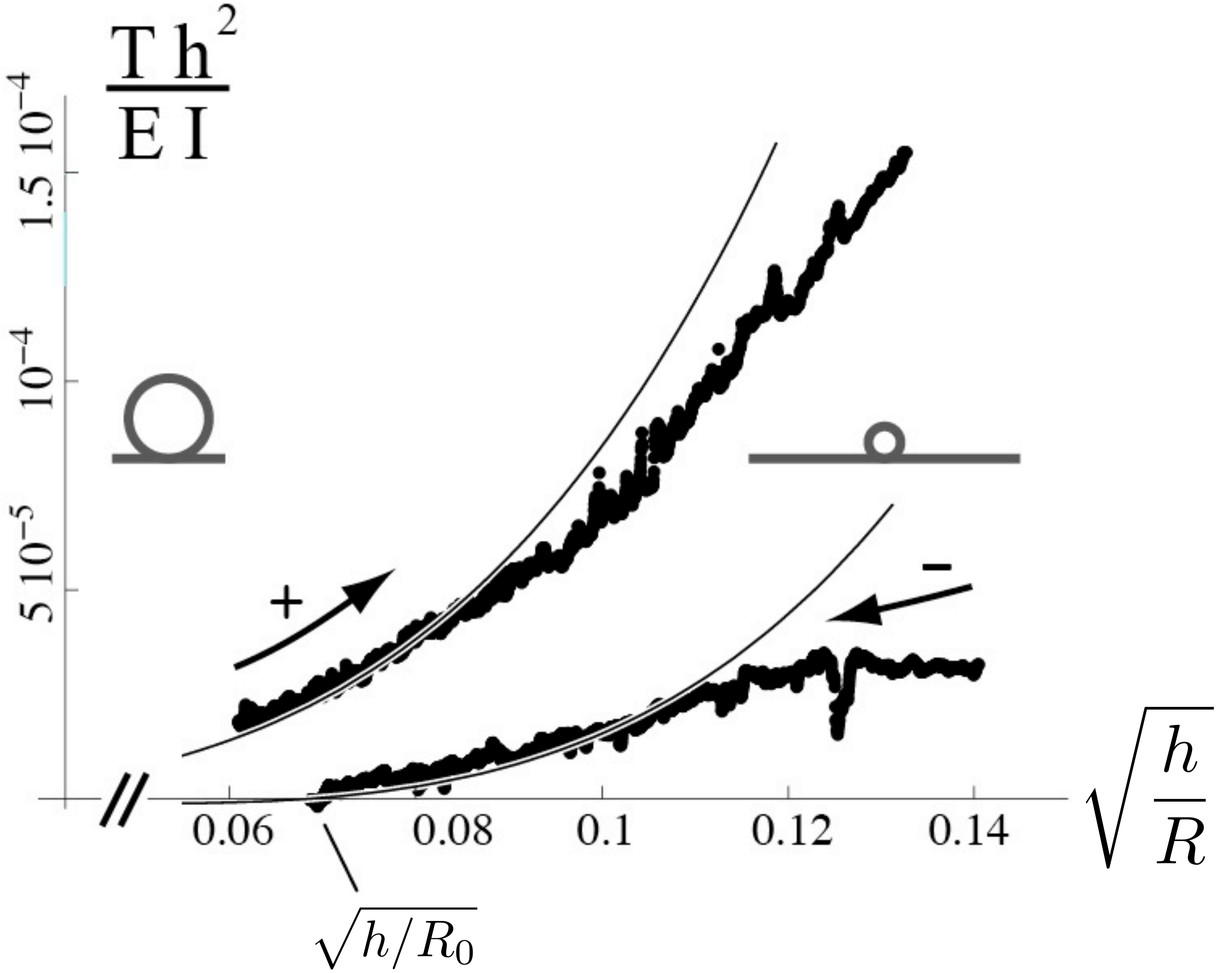


Expériences

sans frottement

$$T = \frac{1}{2}\frac{EI}{R^2}$$

$$\Rightarrow \frac{Th^2}{EI} = \frac{1}{2}\frac{h^2}{R^2}$$



avec frottement $\left|\frac{Th^2}{EI} - \frac{1}{2}\frac{h^2}{R^2}\right| \leq \mu\, P = 0.49\,\mu \left(\frac{h}{R}\right)^{3/2}$

si $T = 0$: glissement jusqu'à $R = R_0$ tel que : $\mu = 1.02\sqrt{\frac{h}{R_0}}$

Fin

www.lmm.jussieu.fr/~neukirch

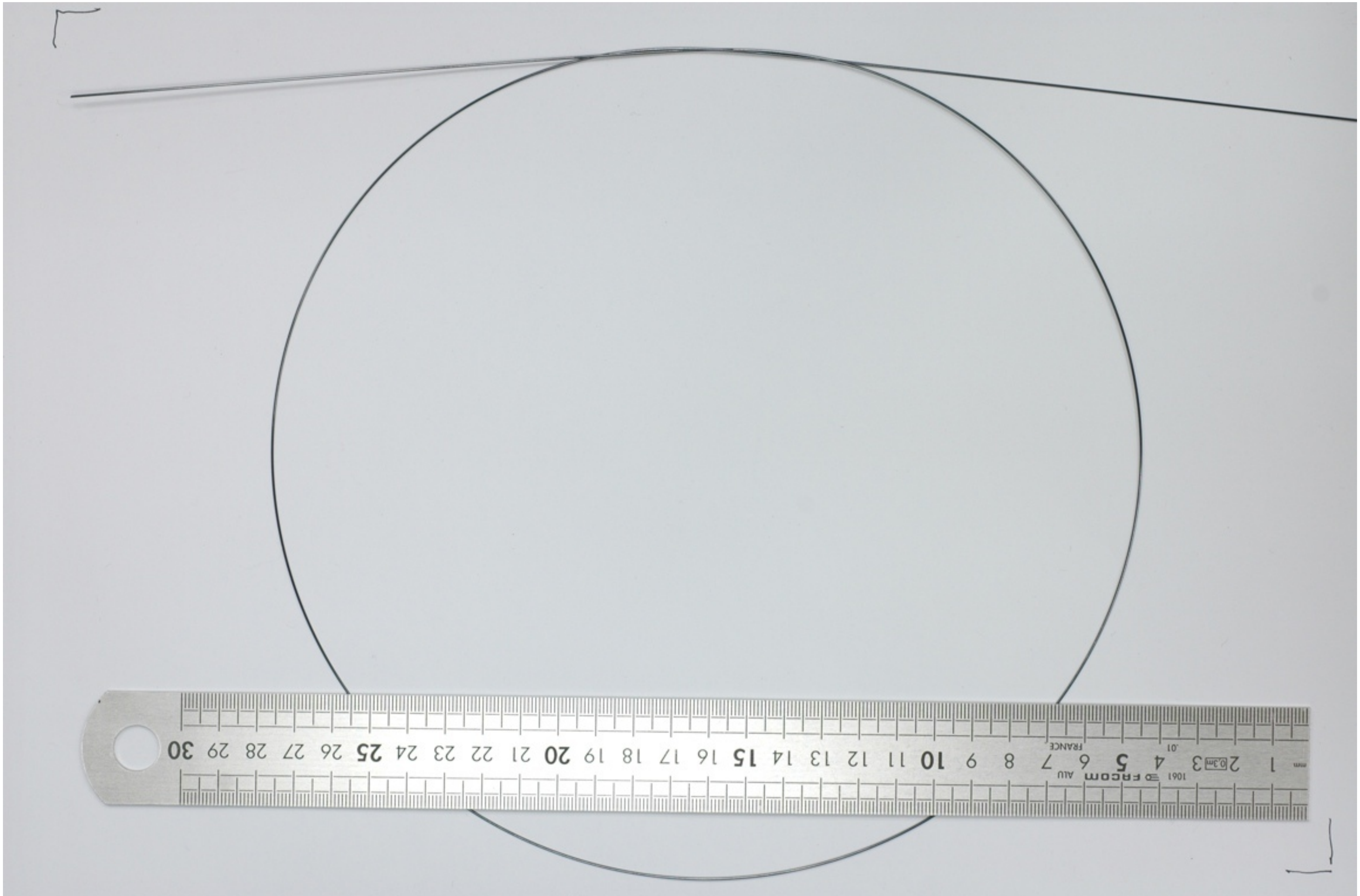
S. Neukirch, B. Roman, B. de Gaudemaris, and J. Bico. *Journal of the Mechanics and Physics of Solids*, 55 (2007) 1212–1235.

B. Audoly, N. Clauvelin, and S. Neukirch. *Physical Review Letters*, 99 (2007) 164301.

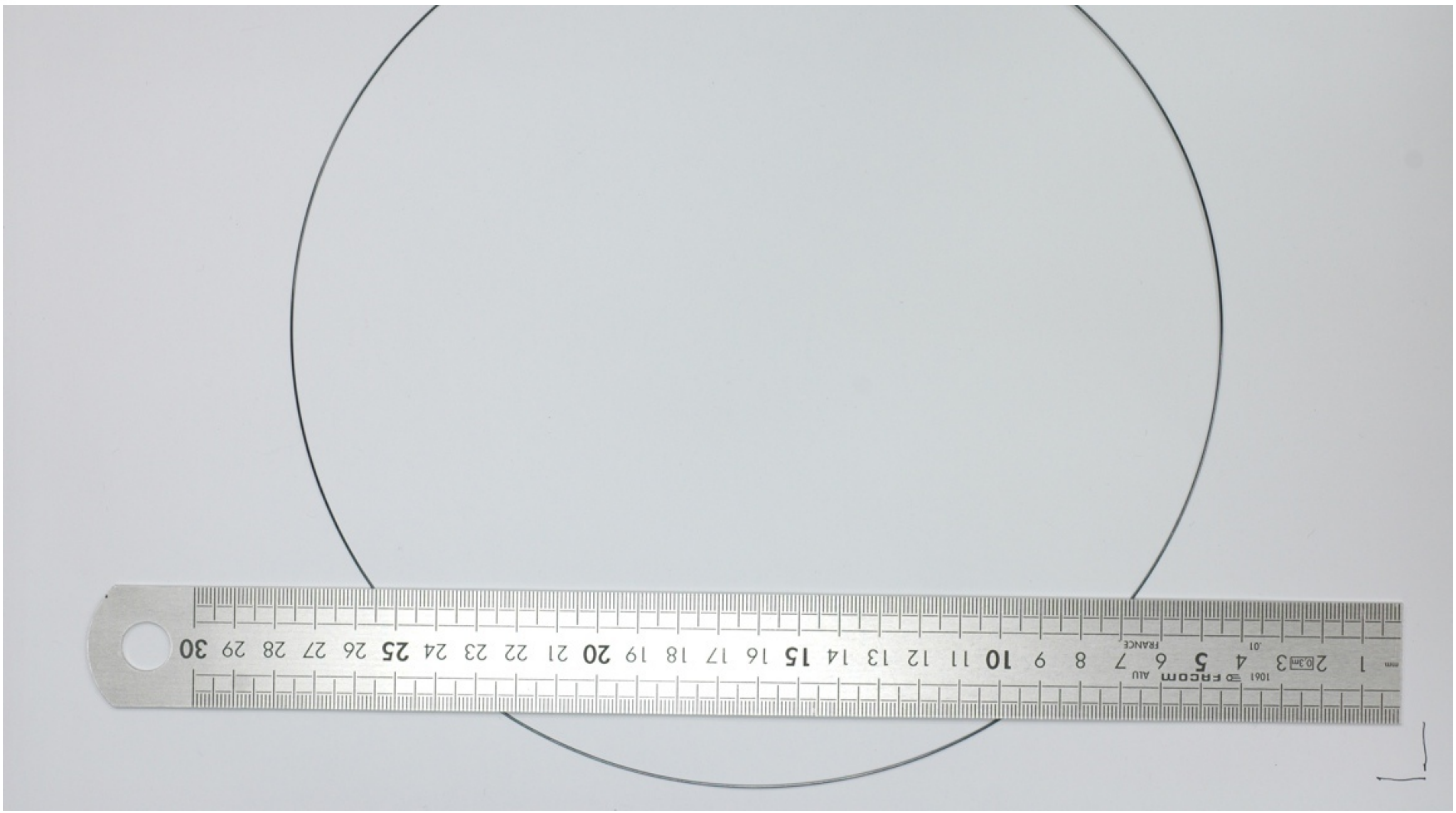
N. Clauvelin, B. Audoly, and S. Neukirch. *Journal of the Mechanics and Physics of Solids*, 57 (2009) 1623–1656.

H. O. Kirchner and S. Neukirch. *Journal of the Mechanical Behavior of Biomedical Materials*, 3 (2010) 121–123.

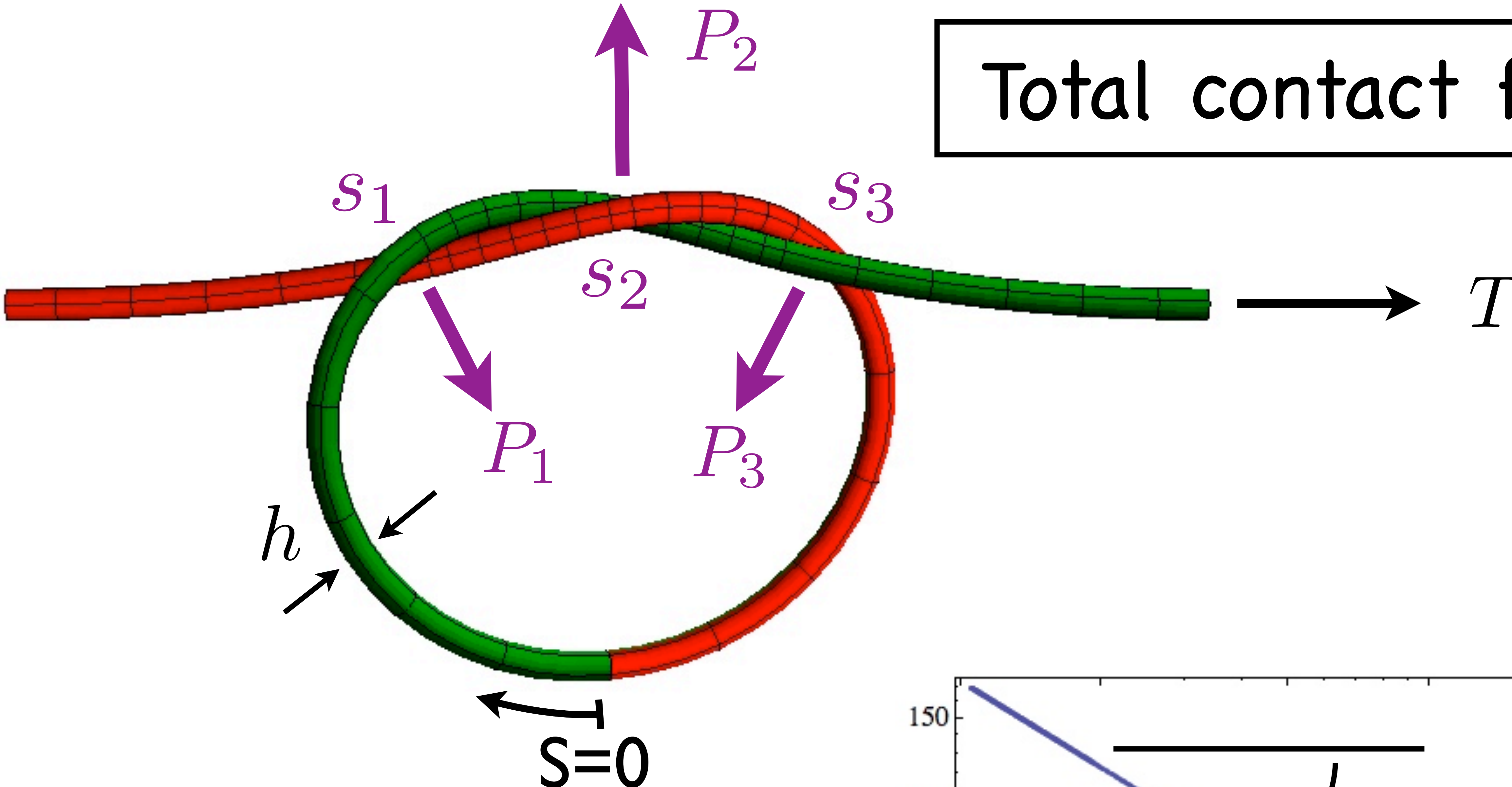
Braid : contact topology

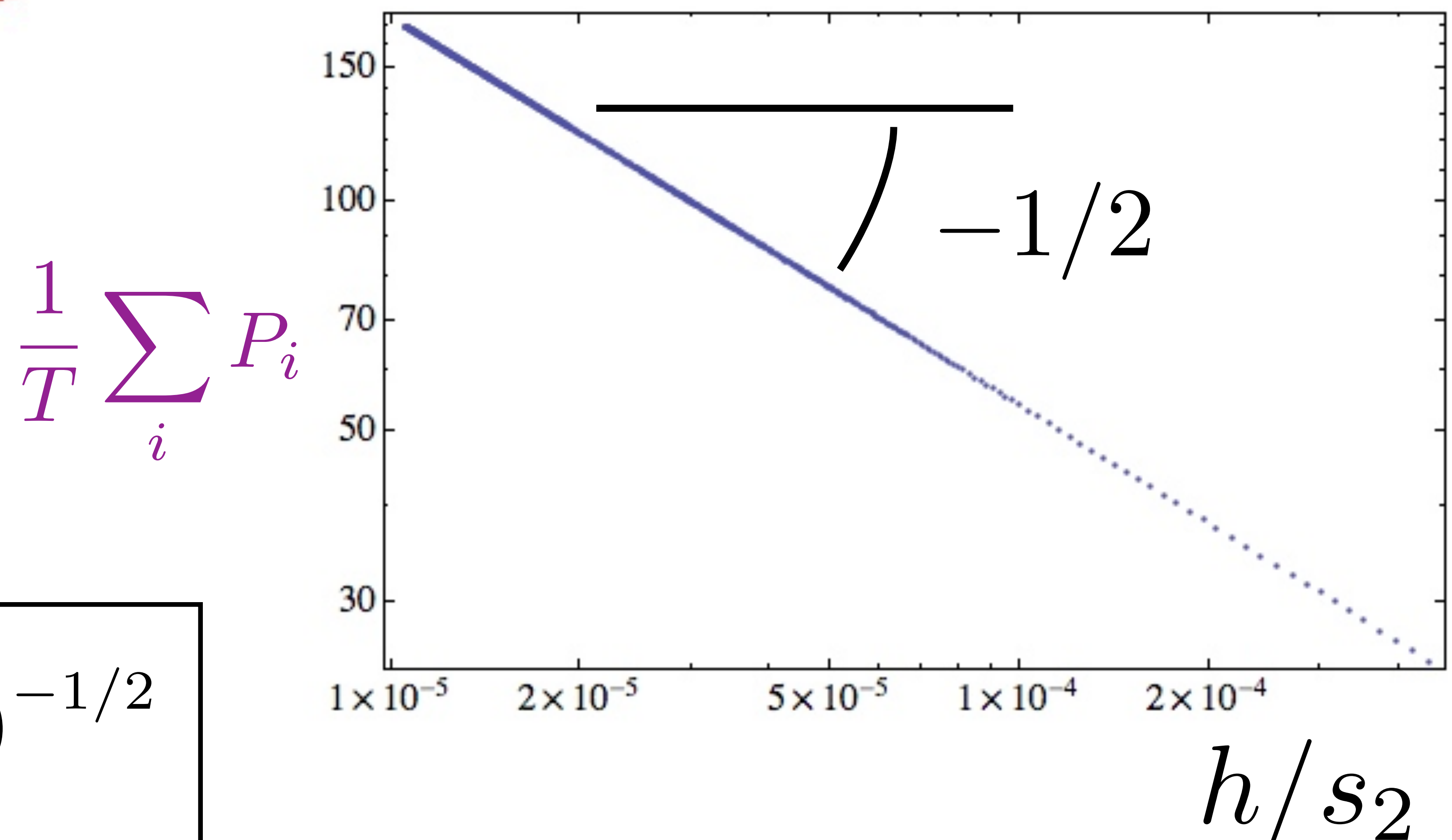


Braid : contact topology



Total contact force

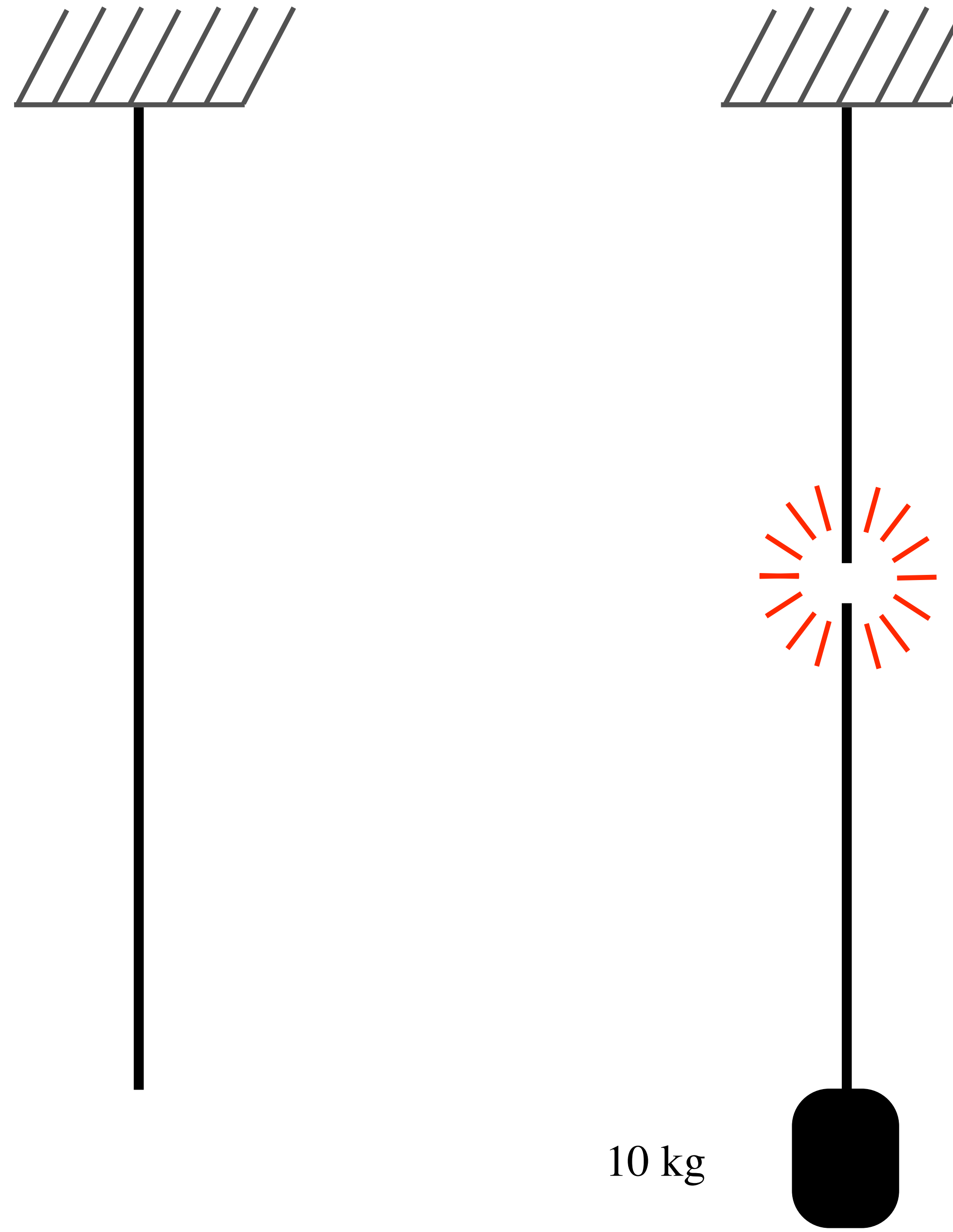




$$\frac{1}{T}\sum_i P_i \simeq 0.55\,(h/s_2)^{-1/2}$$

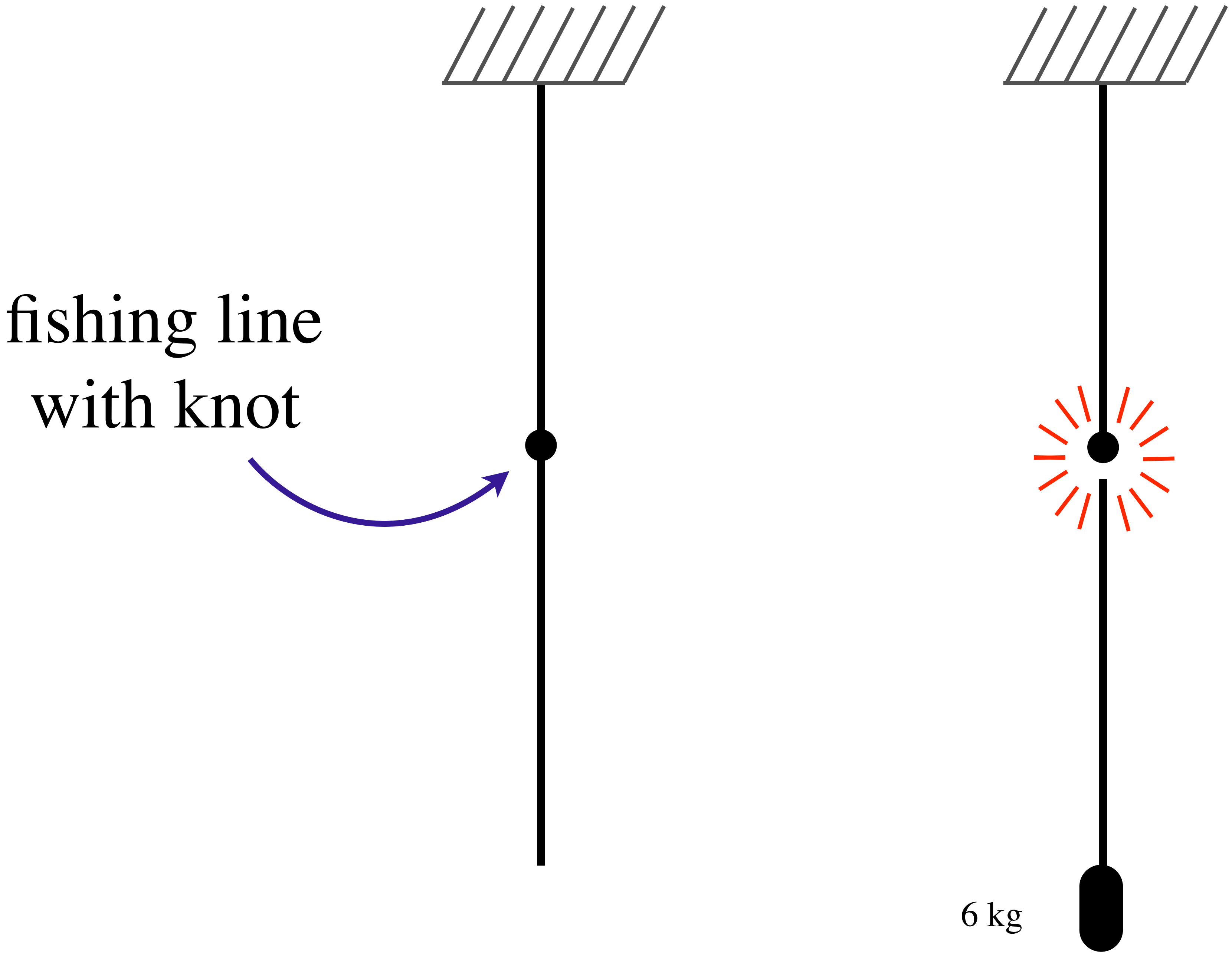
Tensile strength of a wire

fishing line



Stasiak et al, *Science* (1999)

Tensile strength of a wire



Stasiak et al, *Science* (1999)

Knots are everywhere

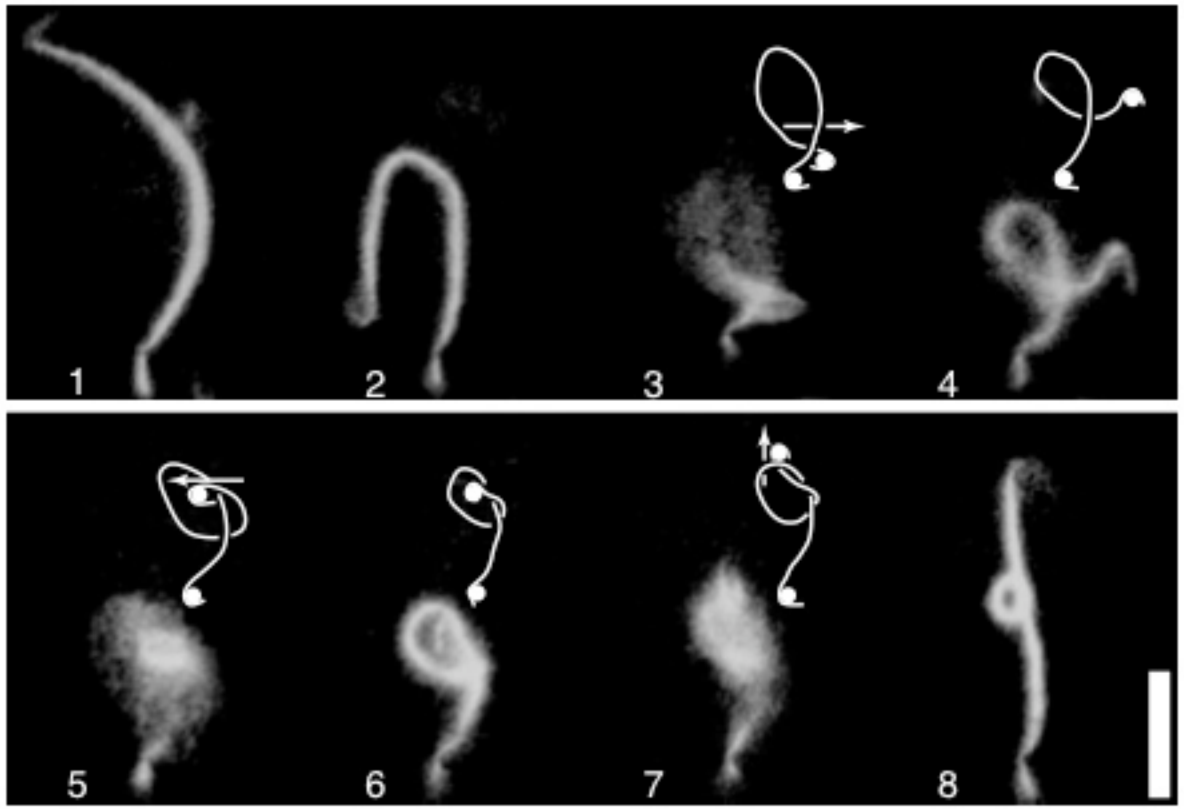
Long enough polymers are (almost) certainly knotted

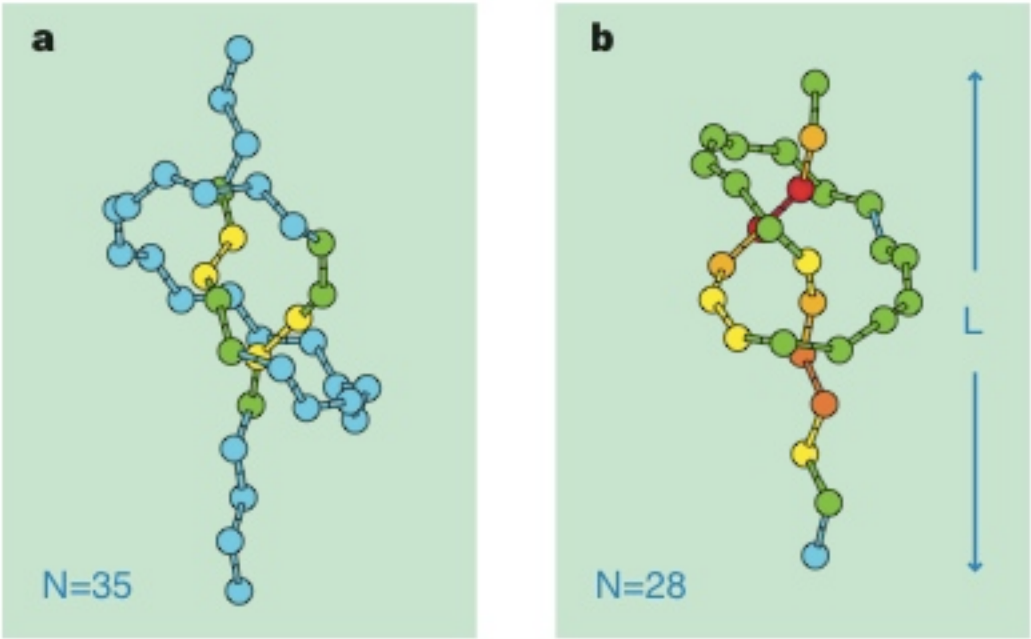
Sumners+Whittington, J. Phys. A : Math. Gen. 1988

273 knotted proteins in the ProteinDataBank (1%)

Single molecule experiment with knotted F-Actin filaments

Arai et al, *Nature* (1999)





Ab-initio molecular simulations for alcane molecule (C10H22)

Saitta et al, *Nature* (1999)