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How large grains increase bulk friction in bi-disperse granular chute flows

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Abstract In this contribution, we apply contact dynamics discrete simulations to explore how the mechanical properties of simple bi-dimensional granular chute flows are affected by the existence of two grain sizes. Computing partial stress tensors for the phases of small and large grains, we show that the phase of large grain exhibits a much larger shear strength than the phase of small grains. This difference translates in terms of the flow internal friction: adopting the $\mu(I)$ dependence to describe the flow frictional properties, we establish that the flow mean friction coefficient increases with the volume fraction of large grains. Hence, while the presence of large grains may induce lubrication in 3D unconfined flows due to the self-channelisation and levées formation, the effect of large grains on the bulk properties is to decrease the flow mobility.

Keywords Size segregation · Friction · Stress state · Contact dynamics

1 Introduction

Size segregation occurs when a granular bed made of grains of different sizes is submitted to shear or shaking under gravity during a sufficiently long lapse of time: larger grains rise at the top, while smaller grains are trapped at the bottom. This behavior is robust; everyone has witnessed it either in nature—sorting of sand or pebbles in dunes or rivers—or simply at home in one's cereal box for instance [6,7,31,35]. Since raw material often comes in the shape of grains, size segregation is a constant concern for many industries: food industry, pharmaceutics, civil engineering, and for any other processes handling grains [2].

Size segregation may at first appear a simple mechanism. Indeed, the fact that smaller grains have a higher probability to fill in the gaps opening in the granular bed as a result of shear or shakes (as originally described in [36]) seems enough to account for the final sorting of the grains. It is however a very complex challenge to modeling. Obvious questions are for instance: what are the typical time scales? What are the feedbacks between segregation and flow dynamics? How to account for them in continuum modeling? And more specifically in this paper, how does the existence of two grain sizes affect the rheological properties of the flow?

Experiments are the privileged tool to evidence intricate phenomenologies involving different grain sizes, in free surface flows [1,4,11,27,28,45], shear cells [9,23], or in vibrating beds [8,20,37]. Precise measurements in granular systems are however uneasy, and only extremely well- controlled experiments may lead to a quantitative picture of the physics at play during segregation [9,15,16]. For this reason, discrete numerical simulation has proven a helpful tool to analyze the local mechanisms at play during segregation and to understand the structure of poly-disperse granular systems [5,17,22,33,38,40,46]. They are also a practical mean to inform continuum modeling [12–14,19,24,40].

In this contribution, we apply contact dynamics discrete simulations to explore how the mechanical properties of simple bi-dimensional granular chute flows are affected by the existence of two grain sizes. Computing partial stress tensors

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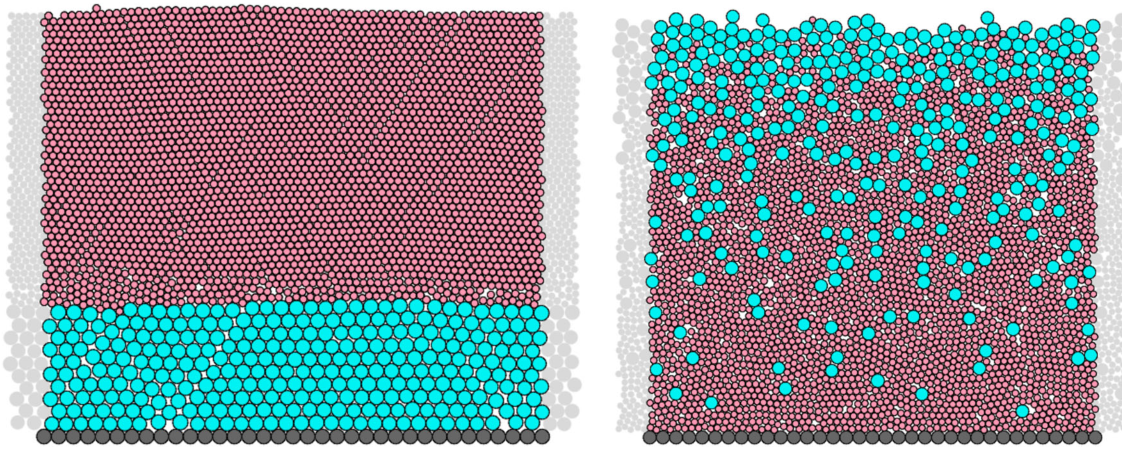

Fig. 1 Bi-disperse granular flow with a volume fraction of large beads $\Phi = 0.3$, in its initial state and after steady-state is reached and segregation occurred (slope $\theta = 22°$)

for the phases of small and large grains, we show that the phase of large grain exhibits a much larger shear strength than the phase of small grains. This difference translates in terms of the flow internal friction: adopting the $\mu(I)$ dependence to describe the flow frictional properties, we establish that the flow mean friction coefficient increases with the volume fraction of large grains. Hence, while the presence of large grains may induce lubrication in 3D unconfined flows due to the self-channelisation and levées formation, the effect of large grains on the bulk properties is to decrease the flow mobility.

2 Discrete simulations of bi-disperse granular flows

2.1 The contact dynamics method

This numerical experiment was performed using the contact dynamics algorithm [18,26]. An important advantage of this algorithm is its reliability to mimic the behavior of perfectly rigid grains in a multi-contact environment typical for dense flow regimes. Its efficiency for exploring poly-disperse granular packings is demonstrated in [43]. It is not our intention in this contribution to present the contact dynamics in its full complexity. We refer the reader to [32] for details on this aspect.

While at contact, grains interact through a friction force that allows for slip motion when the tangential force T reaches the frictional threshold set by the coefficient of friction μ and the compressive force N. In dense packings (as those simulated for this study), friction is very efficient at dissipating energy. In the following, the friction coefficient is set to $\mu = 0.5$ for all contacts, irrespective of the size of the grains involved.

Another efficient way by which granular flows dissipate their energy is through non-elastic collisions: the coefficient of energy restitution e is smaller than 1 so that grains bounce back with less momentum. We chose $e = 0.25$ for all contacts.

2.2 The numerical simulations

We simulate two-dimensional granular chute flows of slope θ, as described in [38]. Two grain sizes are considered. The larger grains—placed initially at the bottom of the granular layer—rise to the surface as the flow develops, as illustrated in Fig. 1. The periodic boundary conditions in the flow direction allow for infinite flow durations. This typical chute flow experiment is in its principle similar to those reported in [22,33,40].

We denote d_L and d_S the mean diameter of large and small grains, respectively, taken so that the size ratio is $d_L/d_S = 2$. The value of this ratio on the flow behavior is important: both the time scales of the segregation and the level of sorting (ie the saturation rate) are affected [9,33,40]. We do not investigate this aspect in the following. The choice $d_L/d_S = 2$ is motivated by earlier studies showing that it does favor segregation [9]. To prevent geometrical ordering, both large and small grains have diameters uniformly distributed around their mean value within a size interval of 8 %.

The width of the simulation cell is $70d_S$. The bottom is made of a row of large grains. In the initial state, a layer of large grains is overlaid by a layer of small grains, both deposited by random rain under gravity (Fig. 1). The volume fraction of large grains for the whole granular layer is denoted Φ, i.e. the ratio of the volume of large grains to the total volume of grains: $\Phi = V_L/(V_S + V_L)$. The volume fraction of small grains is thus $(1 - \Phi)$. The height of the granular bed in the initial state is $H \simeq 60d_S$ for all simulations, i.e. the flow volume is kept constant irrespective of the flow composition Φ. Both the slope θ and the volume fraction of large grains Φ are varied.

The numerical values used are $d_S = 5.10^{-3}$ m, $\rho = 1\,\mathrm{kg\,m^{-2}}$, $g = 9.8\,\mathrm{m\,s^{-2}}$.

2.3 General behavior

A typical time evolution is shown in Fig. 2 for a slope $\theta = 18°$ and a volume fraction of large grains $\Phi = 0.3$ (like shown in Fig. 1). The velocity (Fig. 2-a) first undergoes a sharp increase quickly checked and followed by a deceleration phase; it then slowly evolves towards its steady-state value. The existence of the initial peak can be related to the change of bottom slip velocity as segregation proceeds. Indeed, as larger grains rise in the bulk, the initial slip condition changes from the favorable large-grains-flowing-on-large-grains configuration to the less favorable small-grains-flowing-on-large-grains configuration [10]. As a result, the slip velocity suddenly drops. After this initial step, the evolution of the flow becomes smooth and reaches steady state.
For the same simulation, the vertical position of the center of mass of the large grains y_G is computed and is plotted in Fig. 2b. We observe the regular rise following an exponential dependence, as discussed in detail in [38]. From these graphs, segregation seems a relatively slow process. Here, the fact that the configuration is bi-dimensional is crucial. Indeed, geometrical rearrangements are harder in 2D than in 3D, and the probability of gaps opening in the neighborhood of a given grain is significantly diminished when suppressing one dimension. Nevertheless, we reach a state of equilibrium between segregation and diffusive remixing, for which neither the structure of the flow nor the velocity further evolves. This final state can thus be used to investigate how the co-existence of two grain sizes affects the mechanical properties of the flow.

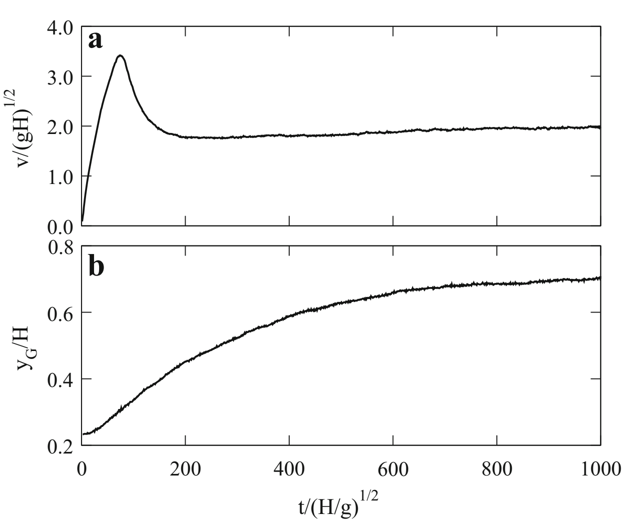

Fig. 2 **a** Mean flow velocity v (normalized by $\sqrt{gH}$) and **b** position of the center of mass of large grains y_G (normalized by H) as a function of time (normalized by $\sqrt{H/g}$) for the flow shown in Fig. 1, i.e. $\Phi = 0.3$ and $\theta = 22°$. The complete flow duration is 180 s

3 Different shear strengths for the phases of large and small grains

Segregation can be understood from a geometrical point of view, originally described by Savage and Lun [36] as the percolation of the smaller grains through the network of larger ones, or *random fluctuating sieve*. However, understanding segregation in terms of a mechanical process remains challenging. It was only recently that a lift force was effectively measured on an intruder moving in a granular packing, and the generalization of this result to rapid flows is not straightforward [16]. A fruitful hypothesis was proposed by Gray and Thornton [12] that allows for efficient continuum modeling of segregation in 3D configurations [19]: they postulate that smaller grains are screened from the mean pressure by the network of large grains, so that the lithostatic pressure gradient affects them in a lesser extent. An other hypothesis formulated by [5,17] is that kinematic pressure, namely the existence of velocity fluctuations, is at the origin of the segregation behavior. In both cases, the phases of small and large grains exhibit different mechanical properties while flowing, and these different mechanical properties induce the separation of the two phases. One can suspect that they also affect the mechanical properties of the segregated flow as a whole.
The fact that grains of different sizes contribute differently to the stress state of the mixture was shown numerically for poly-disperse granular packings sheared quasi-statically, in the absence of segregation. In this case, Voivret et al. [43] showed that the shear strength of the sample was essentially sustained by the class of larger grains. We show here that a similar conclusion holds for rapid segregating granular flows.
Using a micro-mechanical definition of the stress tensor [34,44], we try to analyze separately the mechanical properties of two subsets of contacts depending on their relation to grain size: the subset of contacts involving at least one large grains, and the subset of contacts involving at least one small grains. Note that the two subsets overlap, as they both include the mixed contacts (between a large and a small), so that the quantities computed cannot inform about the state of the whole system by simple addition. A rigorous definition of partial stresses would imply that mixed contacts are distributed between the stress tensors for the two phases of grains using a ponderation that respects additivity [44]. This issue is discussed in detail in [39]. This however does not change the conclusion of this section concerning the relative strength of the phases of large and small grains. We consider intrinsic stress tensors, that is computed over the volume of

either small or large grains, rather than over the whole volume as would be the case for partial stresses [44]. We thus compute the following stress tensors:

$$\boldsymbol{\sigma}_{L,S} = \frac{1}{V_{L,S}} \sum_{N_{L,S}} \boldsymbol{f} \otimes \boldsymbol{r}, \tag{1}$$

where $N_{L,S}$ is the number of contacts involving at least one large grains and at least one small grains, respectively, $V_{L,S}$ is the volume occupied by large and small grains, respectively, $\boldsymbol{f}$ is the force transmitted at contact, $\boldsymbol{r}$ is the vector joining the center of mass of the two grains in contact, $\otimes$ is the dyadic product. Accordingly, two stress tensors $\boldsymbol{\sigma}_L$ and $\boldsymbol{\sigma}_S$ are computed, reflecting the mechanical properties of the two different phases of grains. Intrinsic pressures p_L and p_S and intrinsic stress deviators q_L and q_S are computed from the eigen values λ_1^L and λ_2^L, and λ_1^S and λ_2^S, of the intrinsic stress tensors:

$$p_{L,S} = \frac{\lambda_1^{L,S} + \lambda_2^{L,S}}{2}, \quad q_{L,S} = \frac{\lambda_1^{L,S} - \lambda_2^{L,S}}{2}. \tag{2}$$

For each class of contacts, the ratios q_L/p_L and q_S/p_S measure the mechanical strength, or friction mobilization, of the corresponding subset of grains.

We consider systems with different composition, with a volume fraction of large grains $\Phi = 0.15, 0.30, 0.45, 0.60$, and 0.75, flowing at an angle $\theta = 23°$ and segregating large and small grains following the dynamics shown in Fig. 2. We divide each system in horizontal layers of thickness $3d_S$; the intrinsic stress tensors $\boldsymbol{\sigma}_L$ and $\boldsymbol{\sigma}_S$ are computed in each layer over successive time-intervals of 1.25 s, over the whole flow duration. Noteworthy, we have found that separating between the earlier and the later stages of segregation had no visible influence on the observations discussed hereafter. The resulting value of the ratios q_L/p_L and q_S/p_S is plotted as a function of the local value of the fraction of large grains ϕ at the corresponding layer's depth and corresponding time interval in Fig. 3. We observe that contacts involving larger grains sustain a shear stress significantly greater than those involving smaller grains for volume fraction for $\phi < 0.7$. For $0.7 < \phi < 0.95$, the behavior of the two phases becomes similar. For $\phi > 0.95$, the effect tends to reverse; this case corresponds to mostly isolated small grains trapped in layers of large grains, and is not necessarily relevant to the description of segregation. In general, this finding supports the idea that the presence of two grain sizes in the flow changes its mechanical behavior. Figure 3 specifically suggests that the presence of larger grains induces a larger shear strength, namely a larger internal friction. This aspect is investigated in the next section.

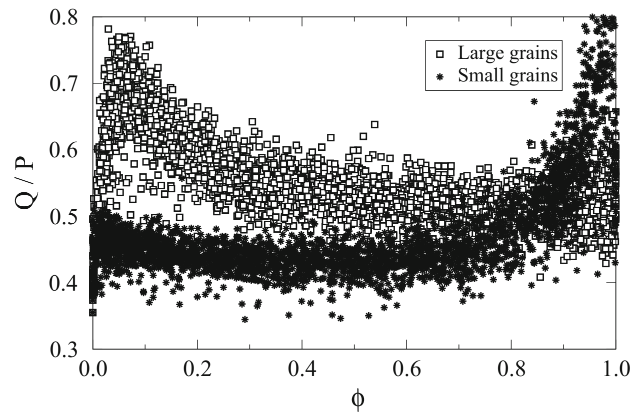

Fig. 3 Local value of the ratio of the stress deviator to the pressure for the phase of large grains q_L/p_L and the phase of small grains q_S/p_S as a function of the local volume fraction of large grains ϕ, computed in the course of time at different depths in the flow

4 Larger grains increase bulk frictional properties

The existence of different grain sizes in a granular mass may affect the flow characteristics due to 3D structuring like the formation of levées [3,6,19,27], due to the interaction with the bottom roughness [10,21], or, possibly, due to the modification of the bulk frictional properties as a result of the size dispersity. This last effect is however poorly constrained. While Rognon et al. [33] show that friction properties are affected by the grains size ratio and the volume fraction of large grain in bi-disperse granular chute flows (namely in systems very similar to those studied in this contribution), they conclude that this observation is difficult to disentangle from the bottom effects due to the different grain sizes used. On the other hand, Yohannes and Hill [46] find little influence of the packing composition in dense non-segregating bi-disperse sheared flows.

Bi-disperse granular flows reach stationary regime for a given interval of slope angles θ, as their mono-disperse counterparts [30]. Within this interval of slope angles, their internal coefficient of friction can be estimated: $\mu = \tan\theta$. Generalizing the $\mu(I)$ approach to the case of bi-disperse flows as proposed in [33,41,46] allows for the characterisation of the influence of the flow composition on the frictional properties. The $\mu(I)$ approach relates the internal friction of a mono-sized granular system to the inertial number I defined as

$$I = \frac{d\dot{\gamma}}{\sqrt{P/\rho}},$$

where d is the grain diameter, $\dot{\gamma}$ is the shear rate, P the pressure, and ρ the density [25]. The relation takes the following form:

$$\mu = \tan(\theta) = \mu_s - \frac{\mu_d - \mu_s}{\frac{I_0}{I} + 1}, \tag{3}$$

where μ_S, μ_d, and I_0 describe the frictional properties of the flow and its dependence on the dynamics. In the case of bi-disperse flows, a generalized inertial number is introduced instead, taking into account the mean grain size $\langle d \rangle = \Phi d_L + (1-\Phi) d_S$:

$$I_\Phi = \frac{\langle d \rangle \dot{\gamma}}{\sqrt{P/\rho}} = (\Phi d_L + (1-\Phi) d_S) \frac{\dot{\gamma}}{\sqrt{P/\rho}}.$$

Varying the volume fraction of large beads Φ and the slope θ, the influence of the flow composition on the parameters of the $\mu(I_\Phi)$ dependence can thus be evidenced. Therefore, series of simulations are performed varying θ from 22° to 26°, for the following values of the volume fraction of large beads $\Phi = 0, 0.06, 0.15, 0.30, 0.45, 0.60, 0.75$, and 0.90, keeping the height of the flow to $H \simeq 60 d_S$ irrespective of Φ. This range of slopes coincides with stationary flow regime for all the values of Φ considered, for $H \simeq 60 d_S$. For each simulation, the steady-state bottom condition is made of layers of small grains flowing on a fixed layer of large grains with a zero slip velocity, and does not influence the analysis of the bulk properties. We compute the steady-state velocity V, the mean pressure P, and the flow density ρ over the whole flow volume. Assuming a Bagnold velocity profile [38], the mean value of I_Φ is determined for each flow:

$$I_\Phi = (\Phi d_L + (1-\Phi) d_S) \frac{\frac{5}{3} V}{H \sqrt{P/\rho}}.$$

The corresponding dependence of μ on I_Φ is plotted in Fig. 4. Although the influence of Φ is weak for small values of Φ,

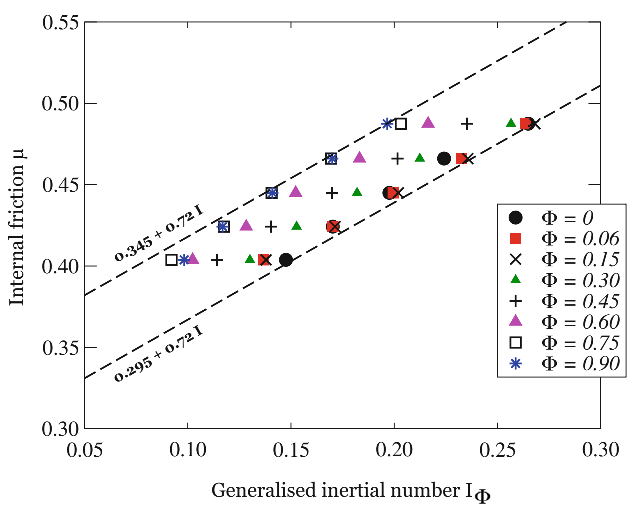

Fig. 4 Coefficient of friction μ of the flows as a function of the generalized inertial number I_Φ for different flows with volume fraction of large beads Φ varying from 0 to 0.9 and slope θ varying between 22° and 26°. The *dotted lines* show affine approximations of the $\mu(I_\Phi)$ dependence describing the increase of the static friction from 0.295 to 0.345 with Φ

a very clear trend emerges: increasing Φ leads to larger friction. Considering an affine relation between μ and I_Φ rather than the more sophisticated relation (3), our simulations show that increasing Φ between 0 and 0.9 increases the static coefficient of friction from 0.295 to 0.345, namely a significant 17 % increase.

5 Conclusion

Discrete numerical simulations are a helpful tool to study size segregation in granular flows. The fact that contact properties are entirely controlled ensures that the phenomena observed result solely from grain size effects, and are not related to different surface properties or selective wear (as reported in [29,42]). The stress state can be measured accurately within the two phases of grains. In the chute flows simulated in this contribution, the analysis of the stress state reveals a greater shear strength in the phase of larger grains. Because long flow durations are made possible by periodic boundary conditions, we can characterize the final segregated state precisely. Adopting the $\mu(I)$ approach to describe the rheology of the flow depending on its composition, we show that increasing the volume fraction of larger grains in the flow increases its frictional properties. While the presence of large grains may induce lubrication in 3D unconfined flows due to the self-channelisation and levées formation, the effect of large grains on the bulk properties is to decrease the flow mobility.

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References

1. Berton G, Delannay R, Richard P, Taberlet N, Valance A (2003) Two-dimensional inclined chute flows: transverse motion and segregation. Phys Rev E 68:051303
2. Bridgewater J, Foo WS, Stephens DJ (1985) Particle mixing and segregation in failure zones—theory and experiment. Powder Technol 41:147–158
3. Deboeuf S, Lajeunesse E, Dauchot O, Andreotti B (2005) Flow rule, self-channelization, and levees in unconfined granular flows. Phys Rev Lett 97–15:158303
4. Degaetano M, Lacaze L, Phillips JC (2013) The influence of localised size reorganisation on short-duration bidispersed granular flows. Eur Phys J E 36–4:1–9
5. Fan Y, Hill KM (2011) Phase transitions in shear-induced segregation of granular materials. Phys Rev Lett 106:218301
6. Félix G, Thomas N (2004) Relation between dry granular flow regimes and morphology of deposits: formation of leves in pyroclastic deposits. Earth Planet Sci Lett 221:197–213
7. Frey P, Church M (2009) How river beds move. Science 325:1509–1510
8. Garzo V, Reyes FV (2012) Segregation of an intruder in a heated granular dense gas. Phys Rev E 85:021308

9. Golick LA, Daniels KE (2009) Mixing and segregation rates in sheared granular materials. Phys Rev E 80:042301
10. Goujon C, Dalloz-Dubrujeaud B, Thomas N (2003) Monodisperse dry granular flows on inclined planes: role of roughness. Eur Phys J E 11:147–157
11. Goujon C, Dalloz-Dubrujeaud B, Thomas N (2007) Bidisperse granular avalanches on inclined planes: a rich variety of behaviors. Eur Phys J E 23:199–215
12. Gray JMNT, Thornton AR (2005) A theory for particle size segregation in shallow granular free-surface flows. Proc R Soc A 461:1447–1473
13. Gray JMNT, Chugunov VA (2006) Particle-size segregation and diffusive remixing in shallow granular avalanches. J Fluid Mech 569:365–398
14. Gray JMNT, Kokelaar BP (2010) Large particle segregation, transport and accumulation in granular free-surface flows. J Fluid Mech 652:105–137
15. Guillard F, Forterre Y, Pouliquen O (2013) Depth-independent drag force induced by stirring in granular media. Phys Rev Lett 110:138303
16. Guillard F, Forterre Y, Pouliquen O (2014) Lift forces in granular media. Phys Fluids 26:043301
17. Hill KM, Tan DS (2014) Segregation in dense sheared flows: gravity, temperature gradients, and stress partitioning. J Fluid Mech 756:54–88
18. Jean M, Moreau J-J (1992) Unilaterality and dry friction in the dynamics of rigid bodies collections. In Curnier A (ed) Proceedings of contact mechanics international symposium, pp 31–48
19. Johnson CG, Kokelaar BP, Iverson RM, Logan M, LaHusen RG, Gray JMNT (2012) Grain-size segregation and levee formation in geophysical mass flows. J Geophys Res 117:F01032
20. Kudrolli A (2004) Size separation in vibrated granular matter. Rep Prog Phys 67:209–247
21. Linares-Guerrero E, Goujon C, Zenit R (2007) Increased mobility of bidisperse granular avalanches. J Fluid Mech 593:475–504
22. Marks B, Rognon P, Einav I (2012) Grainsize dynamics of polydisperse granular segregation down inclined planes. J Fluid Mech 690:499511
23. May LBH, Golick LA, Phillips KC, Shearer M, Daniels KE (2010) Shear-driven size segregation of granular materials: modeling and experiment. Phys Rev E 81:051301
24. Meruane C, Tamburrino A, Roche O (2012) Dynamics of dense granular flows of small-and-large-grain mixtures in an ambient fluid. Phys Rev E 86:026311
25. Midi G (2004) On dense granular flows. Eur Phys J E 14–4:341–365
26. Moreau JJ (1994) Some numerical methods in multibody dynamics: application to granular materials. Eur J Mech A Solids 13–4:93–114
27. Moro F, Faug T, Bellot H, Ousset F (2010) Large mobility of dry snow avalanches: insights from small-scale laboratory tests on granular avalanches of bidisperse materials. Cold Reg Sci Technol 62:55–66
28. Phillips JC, Hogg AJ, Kerswell RR, Thomas NH (2006) Enhanced mobility of granular mixtures of fine and coarse particles. Earth Planet Sci Lett 246:466–480
29. Pouliquen O, Delour J, Savage SB (1997) Fingering in granular chute flows. Nature 386:816–817
30. Pouliquen O (1999) Scaling laws in granular flows down rough inclined planes. Phys Fluids 11:542
31. Powell DM (1998) Patterns and processes of sediment sorting in gravel-bed rivers. Prog Phys Geogr 22–1:1–32
32. Radjai F, Dubois F (2011) Discrete numerical modeling of granular materials. In: Radjai F, Dubois F (eds) Wiley-Iste publishers, ISBN 978-1-84821-260-2
33. Rognon PG, Roux J-N, Naaim M, Chevoir F (2007) Dense flows of bidisperse assemblies of disks down an inclined plane. Phys Fluids 19:058101-1
34. Rothenburg L, Bathurst RJ (1989) Analytical study of induced anisotropy in idealized granular materials. Geotechnique 39:601
35. Rowley PJ, Kokelaar P, Menzies M, Waltham D (2011) Shear-derived mixing in dense granular flows. J Sediment Res 81:874–884
36. Savage SB, Lun CKK (1988) Particle size segregation in inclined chute flow of dry cohesionless granular solids. J Fluid Mech 189:311–335
37. Schröter M, Ulrich S, Kreft J, Swift JB, Swinney HL (2006) Mechanisms in the size segregation of a binary granular mixture. Phys Rev E 74:011307
38. Staron L, Phillips JC (2014) Segregation time-scales in bi-disperse granular flows. Phys Fluids 26(3):033302
39. Staron L, Phillips JC (2015) Stress partition and micro-structure in size-segregating granular flow. Phys Rev E 92:022210
40. Thornton AR, Weinhart T, Luding S, Bokhove O (2012) Modeling of particle size segregation: calibration using the discrete particle method. Int J Mod Phys C 23:1240014
41. Tripathi A, Khakhar DV (2011) Rheology of binary granular mixtures in the dense flow regime. Phys Fluids 23:113302
42. Ulrich S, Schröter M, Swinney HL (2007) Influence of friction on granular segregation. Phys Rev E 76:042301
43. Voivret C, Radjai F, Delenne J-Y, El Youssoufi MS (2009) Multiscale force networks in highly polydisperse granular media. Phys Rev Lett 102:178001
44. Weinhart T, Luding S, Thornton AR (2013) From discrete particles to continuum fields in mixtures. AIP Conf Proc 1542:1202–1205
45. Wiederseiner S, Andreini N, Épely-Chauvin G, Moser G, Monnereau M, Gray JMNT, Ancey C (2011) Experimental investigation into segregating granular flows down chutes. Phys Fluids 23:013301
46. Yohannes B, Hill KM (2010) Rheology of dense granular mixtures: particle-size distributions, boundary conditions, and collisional time scales. Phys Rev E 82:061301
47. Zuriguel I, Gray JMNT, Peixinho J, Mullin T (2006) Pattern selection by a granular wave in a rotating drum. Phys Rev E 73:061302