A numerical study of the collapse and spreading of a granular mass

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Abstract: Numerical simulations of the collapse of granular columns are performed. They are in good agreement with experimental results. We find that the collapse dynamics is mainly driven by free-fall. Simple basal friction is shown to be a good approximation for the dissipation. We give evidence of the key role played by the dynamics of mass sideways ejection. Accordingly, a scaling law for the runout distance is proposed.

1 INTRODUCTION

The flow of granular media has been the subject of numerous experimental and theoretical studies. However, until recently, the highly unsteady situation of a column of grains collapsing onto a horizontal plane due to gravity had not been investigated. Yet this situation is of particular interest for the geophysical issues of flow mobility and runout distance. Lately, severall experimental works have brought new insights in the runout distance problem (Lube *et al* 2004a; Lajeunesse *et al* 2004; Balmforth & Kerswell 2004). The experiments consist of releasing suddenly an initially confined column of granular material, of initial height H_0 and initial radius R_0, and let it spread freely onto a horizontal plane. The main outcome of these studies consists of scaling laws relating the runout distance $(R_\infty - R_0)$ to the initial geometry of the column both in quasi-2D (or planar) and axisymmetric configurations. When the initial aspect ratio of the column, $a = H_0/R_0$, is sufficiently large, the runout distance normalised by the initial radius of the column shows a power-law dependence on a. This power-law dependence is incompatible with a simple friction model, and suggests more complex dissipation mechanisms within the flow. However, no clear and comprehensive physical modelling of the collapse dynamics has been achieved yet.

We present here the results of 2D numerical simulations of the column collapse experiments using the Contact Dynamics algorithm (Moreau 1994). The scaling laws obtained for the runout distance are in good agreement with the experimental results. The dominating role of the free-fall dynamics over the spreading dynamics is established. Dissipation in the sideways flow is shown to be very well approximated by a simple basal friction. Investigating the process of mass ejection from the base of the column to the outflow, we establish its major influence on the overall dynamics. These results allow us to propose a new scaling law for the runout distance, compatible with a simple friction model, and providing a qualitative explanation for the behaviour of the collapsing columns.

Figure 1. Snapshot of the spreading process. The gray shade shows the velocity of the grains. White corresponds to zero velocity, while black indicates the maximum velocity.

2 NUMERICAL EXPERIMENT

The grains simulated are perfectly rigid disks. Their diameter d is uniformly distributed in a small interval such that $d_{min}/d_{max} = 2/3$. The grains interact through Coulombian friction, with a coefficient $\mu = 1$, and through collisions with a coefficient of restitution $\rho = 0.5$. The horizontal plane over which the grains are allowed to spread is perfectly smooth, and contacts between grains and plane have the same properties than contacts between grains only.

The initial columns are prepared by a random rain of grains between two vertical walls. The compacity of the packing is $c_0 \simeq 0.82$. The column initially has a radius R_0 and a height H_0, and $a = H_0/R_0$ is the initial aspect ratio. The total mass of grain is denoted m_0; m denotes the mass of each grain. At time $t = 0$, the vertical walls are removed, and the column collapses due to gravity. We measure the final radius of the deposit R_∞. The compacity of the final deposit is $c_\infty \simeq 0.78$, namely close to the initial compacity in spite of a slight loosening of the packing. We have carried out 25 simulations with a ranging between 0.21 and 17 and using between 1000 and 8000 grains.

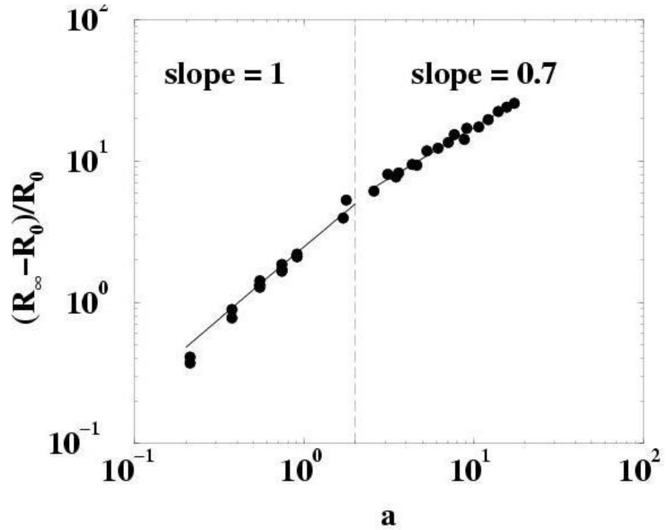

Figure 2. Normalised runout distance $(R_\infty - R_0)/R_0$ as a function of the initial aspect ratio a.

3 SCALING LAW FOR THE RUNOUT

For each collapse, the runout distance is evaluated considering the course or the grains in contact with the main mass; any solitary grain escaping the collective motion and rolling away is not taken into account. The runout distance normalised by the initial radius of the column $(R_\infty - R_0)/R_0$ is plotted as a function of the aspect ratio a in Figure 2. Two different scalings can be observed depending on a, giving first a linear and then a power-law dependence:

$$\frac{R_\infty - R_0}{R_0} \simeq \begin{cases} 2.5\, a & \text{if } a < 2, \\ 3.25\, a^{0.7} & \text{if } a > 2. \end{cases} \tag{1}$$

These scalings are in very good agreement with the experimental results, which also show a linear and a power-law dependence depending on the aspect ratio a, irrespective of the planar or axisymmetric configuration of the experiment. The exponent found is $1/2$ in axisymmetric experiments, while it is $2/3$ in planar ones (Lube *et al* 2004a; Lube *et al* 2004b; Balmforth & Kerswell 2004). The exponent 0.7 found numerically is thus close to the experimental finding. The origin of the difference in the exponent of the scalings in planar and axisymmetric configuration has not been clarified yet. However, in both cases, the existence of a power-law with an exponent lower than 1 has the same implication: simple friction is no satisfactory description of the collapse dynamics. Indeed, if we suppose that the totality of the initial potential energy $E_0 = \frac{1}{2} m_0 g H_0$ of the column is dissipated by the work of friction forces over the runout distance, we can write:

$$\frac{1}{2} m_0 g H_0 = \mu_e m_0 g (R_\infty - R_0), \tag{2}$$

where μ_e is the effective coefficient of friction. This would lead to the s caling law for the runout distance

$$\frac{R_\infty - R_0}{R_0} = \frac{1}{2\mu_e} a,$$

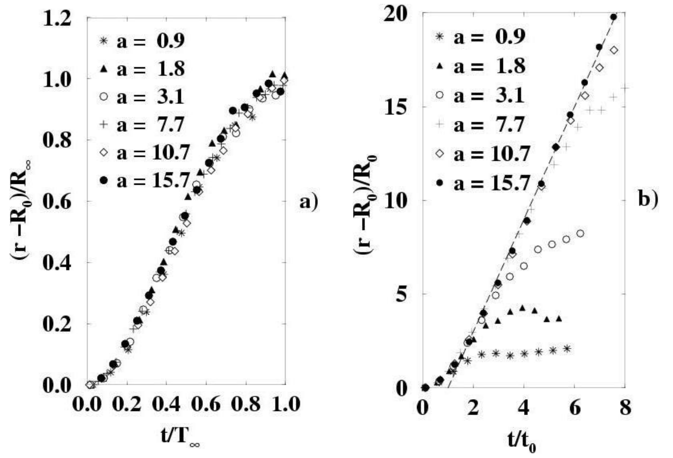

Figure 3. a) Normalised runout distance $(R_\infty - R_0)/R_\infty$ as a function of t/T_∞ and b) normalised runout distance $(R_\infty - R_0)/R_0$ as a function of t/t_0.

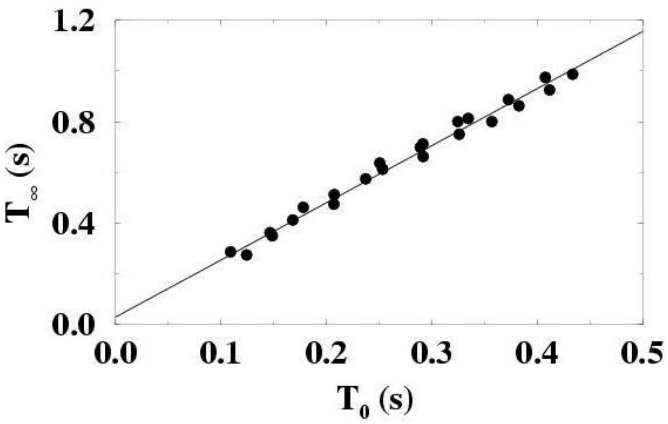

Figure 4. Duration of the collapse T_∞ (s.) as a function of the characteristic time $T_0 = (2H_0/g)^{1/2}$.

while we observe the scaling given in relation (1). We thus conclude that the friction model given in (2) does not apply.

4 THE FRONT PROPAGATION

The position $r - R_0$ of the flow front, normalised by the final runout R_∞, is displayed in Figure 3a for different values of the aspect ratio a as a function of the normalised time t/T_∞, where T_∞ is the total duration of the collapse. The plots nicely join in a single curve, showing first a period of acceleration, followed by a constant velocity regime and finally a slow deceleration leading the flow to a stop. The constant velocity regime is characterized when plotting the normalised front position $(r - R_0)/R_0$ as a function of the normalised time t/t_0, where $t_0 = (2R_0/g)^{1/2}$ (Figure 3b). When a is sufficiently large, the following relation appears to be a good approximation:

$$\frac{(r - R_0)}{R_0} \simeq 3\frac{t}{t_0} - 3$$

which can be rewritten in terms of the characteristic propagation velocity v_0:

$$r \propto v_0 t, \quad r > 2R_0, \tag{3}$$

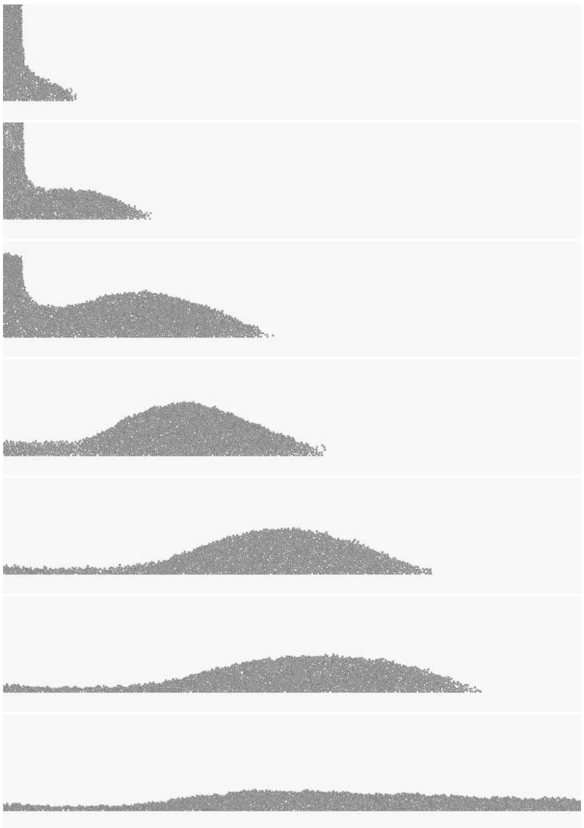

Figure 5. Snapshots of the spreading dynamics showing the process of mass sideways ejection for a very large aspect ratio.

where $v_0 = (2gR_0)^{1/2}$. The total duration of the collapse T_∞ is plotted in Figure 4 as a function of the free-fall time over the initial column height $T_0 = (2H_0/g)^{1/2}$. We observe a linear dependence $T_\infty \simeq 2.25T_0$, suggesting that the free-fall controls the spreading dynamics. Considering that the characteristic velocity of the flow front is $v_0 = (2gR_0)^{1/2}$, and that the typical time of spreading is $T_0 = (2H_0/g)^{1/2}$, we should expect the following scaling for the runout distance:

$$(R_\infty - R_0) \quad \propto \quad v_0 T_0,$$

$$\frac{(R_\infty - R_0)}{R_0} \quad \propto \quad a^{1/2},$$

as observed in axisymmetric experiments. The fact that we observe a different scaling in planar experiments (with an exponent $2/3$) suggests that the deceleration phase has a non-negligible contribution to the runout distance, less noticeable in axisymmetric configuration. Here, the dynamics of mass ejection seems to be a key aspect of the spreading dynamics. Indeed, while the columns is falling, grains are accelerated in free fall, so that the sideways flow is fed by an increasing flux of grains of increasing momentum. The flow must accommodate this addition of mass as can be seen clearly in Figure 5 for a large aspect ratio. This effect is expected to be more important in 2D than in axisymmetric configuration, for which the increase of the surface area of the flow is quadratic with the front position. The process of mass ejection is thus likely to influence the deceleration phase, and the spreading dynamics more generally.

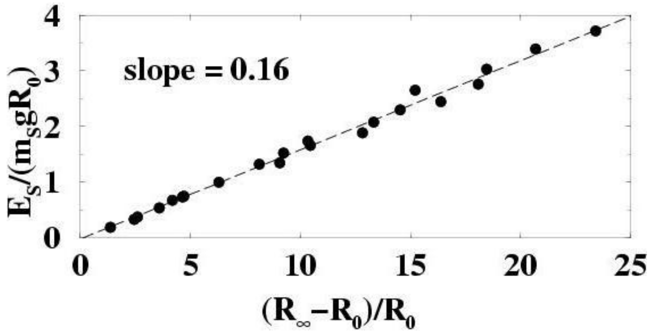

Figure 6. Normalised energy $E_S/(m_S g R_0)$ available for the flow as a function of the normalised runout distance $(R_\infty - R_0)/R_0$.

5 DISSIPATION IN THE FLOW

In order to check whether basal friction is a correct approximation of the dissipation process in the sideways flow, we mesure the mass of grains m_S and the energy E_S (potential and kinetic) taking part to the flow, namely crossing the initial radius vertical sections $-R_0$ and R_0. Assuming that the totality of the energy E_S is dissipated during the flow propagation, we compare for each value of a the energy E_S with the work of the mass m_S over the runout distance $R_\infty - R_0$. As shown in Figure 6, we obtain a linear relation, thus compatible with a simple friction:

$$E_S = \mu_e m_S g(R_\infty - R_0), \tag{4}$$

where μ_e is an effective coefficient of friction, found to be 0.16. Moreover the evolution of the energy E_S as a function of the initial potential energy E_0 shows a linear dependence (Figure 7). We can thus introduce an effective coefficient of restitution $\rho_S = 0.46$ so that the equation 4 can be rewritten:

$$\rho_S \frac{1}{2} m_0 g H_0 = \mu_e m_S g(R_\infty - R_0). \tag{5}$$

This equation is slightly different from the equation (1) in the account of the mass m_S flowing sideways. Again, the process of mass ejection at the base of the column appears to be dominating the spreading dynamics. In terms of runout distance, the equation (5) implies the following scaling

$$\frac{(R_\infty - R_0)}{R_0} \propto \frac{m_0}{m_S} a. \tag{6}$$

This scaling is discussed here after.

6 MASS EJECTION: A NEW SCALING

The process of mass ejection, illustrated in Figure 5, plays a non-trivial role in the flow dynamics. When plotting the proportion of mass taking part to the sideways flow m_S/m_0 as a function of the aspect ratio a (Figure 8), we observe a slow increase towards the

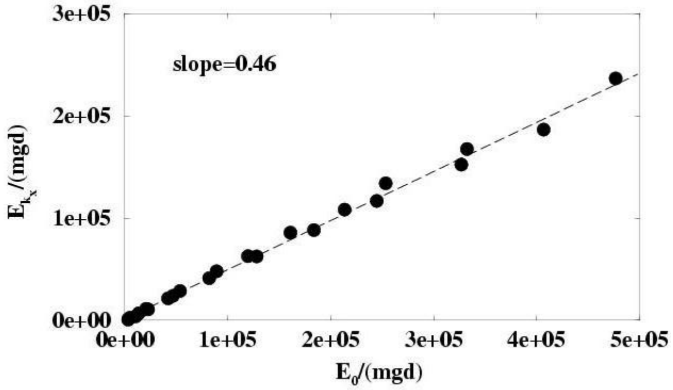

Figure 7. Normalised energy available for the flow E_S/mgd as a function of the normalised initial potential energy E_0/mgd.

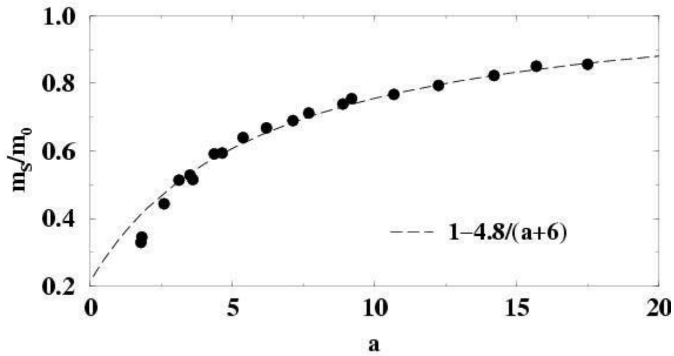

Figure 8. Mass of grains ejected sideways m_S normalised by the initial column mass m_0 as a function of the aspect ratio a.

limit case $m_S/m_0 = 1$. This evolution is well approximated by the form

$$\frac{m_S}{m_0} = 1 - f(a), \tag{7}$$

where $f(a) = 4.8/(a+6)$ is a phenomenological fit. No physical argument for the form of the function f is to be discussed here. Let just appreciate that it satisfies $f(a) \to 0$ when $a \to \infty$.
Equations 6 and 7 suggest for the runout distance a new scaling of the form:

$$\frac{(R_\infty - R_0)}{R_0} \propto \frac{a}{1 - f(a)}. \tag{8}$$

This new scaling is plotted in Figure 9 together with the power-law one; we observe that both are equally acceptable. For large values of a, the two scalings diverge. The newly suggested approximation tends to behave like a, namely as in a simple friction case, accordingly to our analysis of the energy dissipation.

7 CONCLUSION

The numerical simulations of the granular column collapse are in very good agreement with the experimental findings, in particular concerning the runout distance. Analysing the collapse dynamics we find that it is mainly driven by free fall. An accurate study of the energy transfer shows that simple basal friction is a good approximation of the dissipation. We give evidence of the key role played by the process of mass ejection sideways, and propose, accordingly,

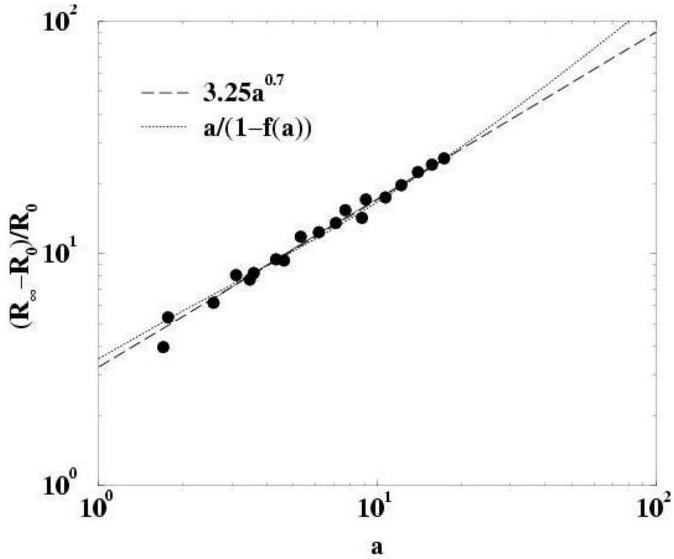

Figure 9. Normalised runout distance as a function of the aspect ratio a, with the power-law scaling (dashed line) and the empirical fit $a/(1-f(a))$ (dotted line).

a new scaling for the runout distance. These results suggest that the understanding of the runout dependence on the column geometry requires a clear formulation of the ejection dynamics at its base. Beyond the frictional properties of the material, we show that the flow characteristics strongly depend on the early dynamics of the collapse. This dependence should be of importance when transposed in the complex case of natural granular flows.

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