Climbing plants: how thick should their supports be?

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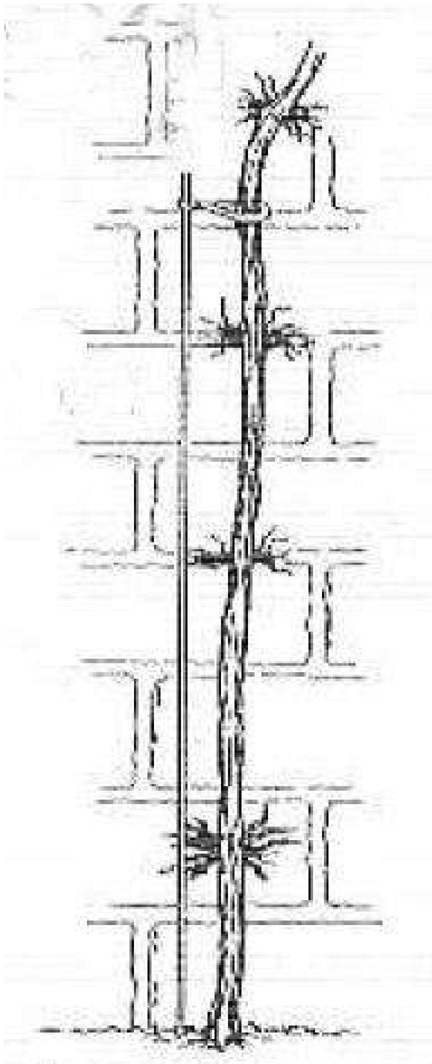
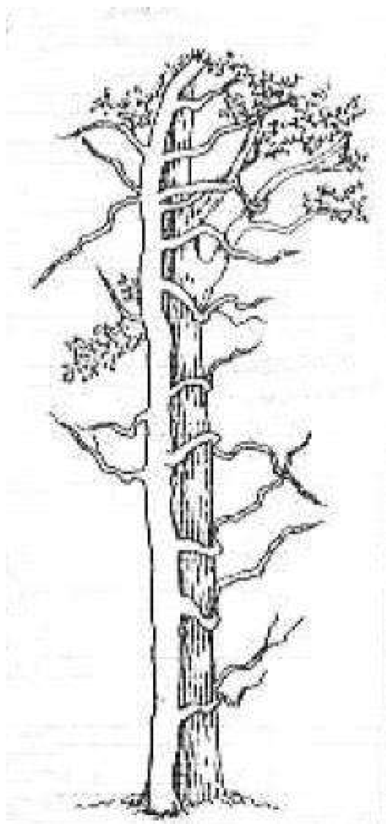
Morning Glory (Ipomoea purpurea) twining up a corn stalk

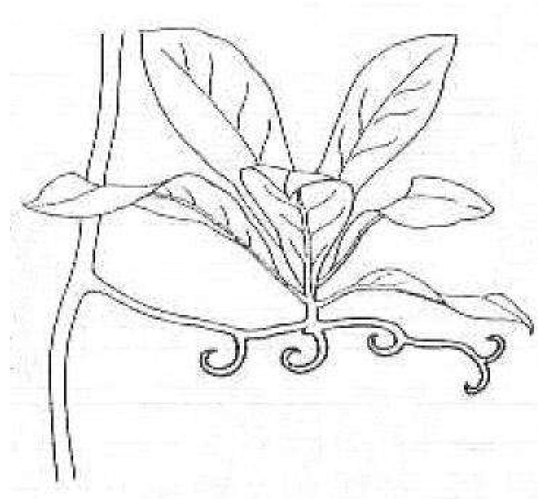
Different kinds of climbing plants



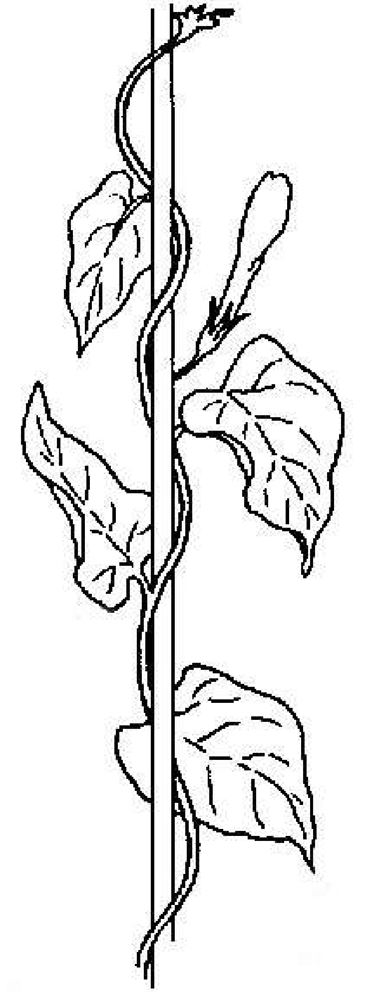
rooter



rooter



hooker



<u>twiner</u>

Twiners: some botanical facts

Goal : reach the canopy (the light).
Use as few structural tissues as possible.
Should be able to twine around different supports
(thick or not, slippery or not)
Evolution from self supporting to supported growth:
smaller stem diameter, more flexible

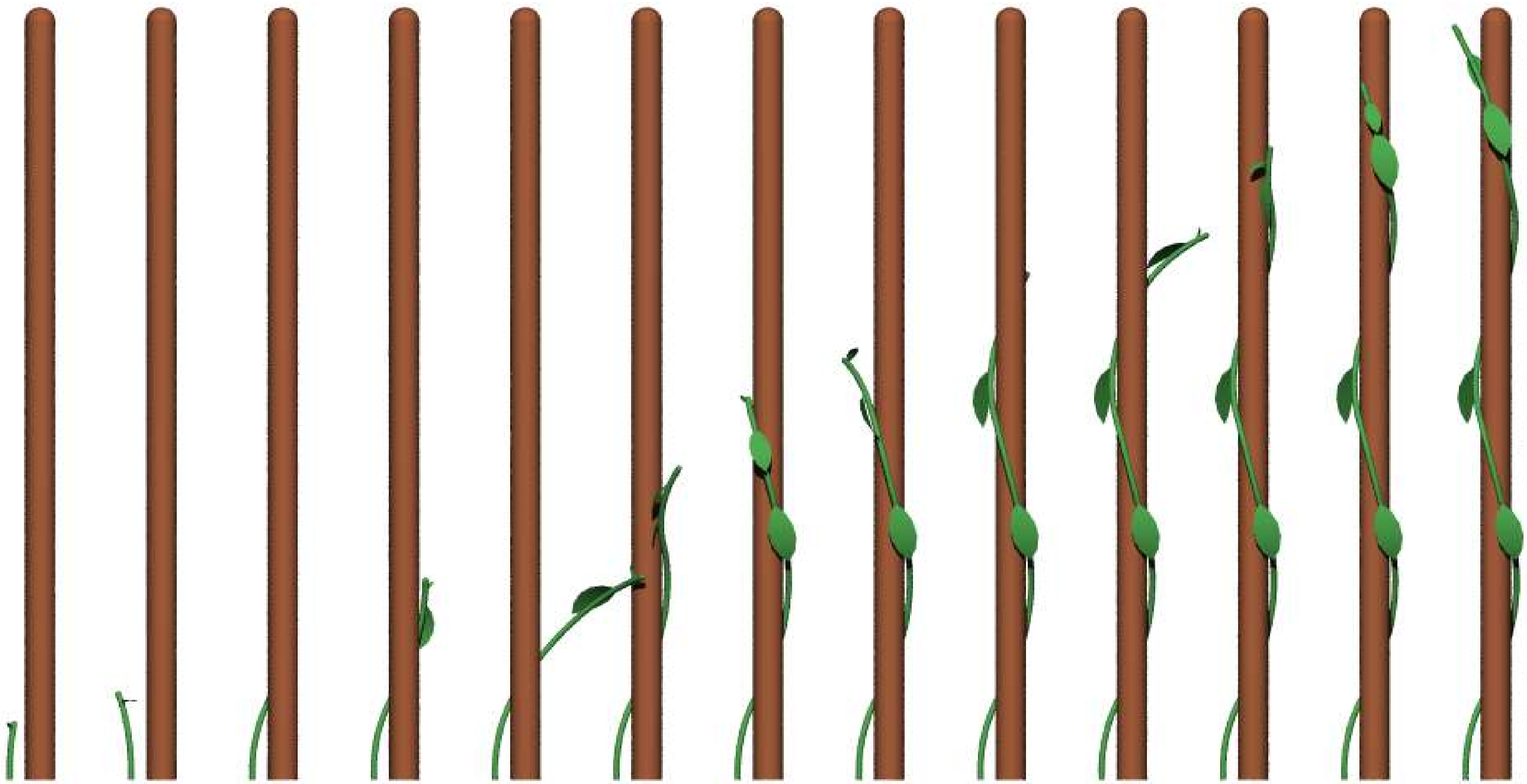
Typical growth speed: 1 cm / hour

Two different zones :

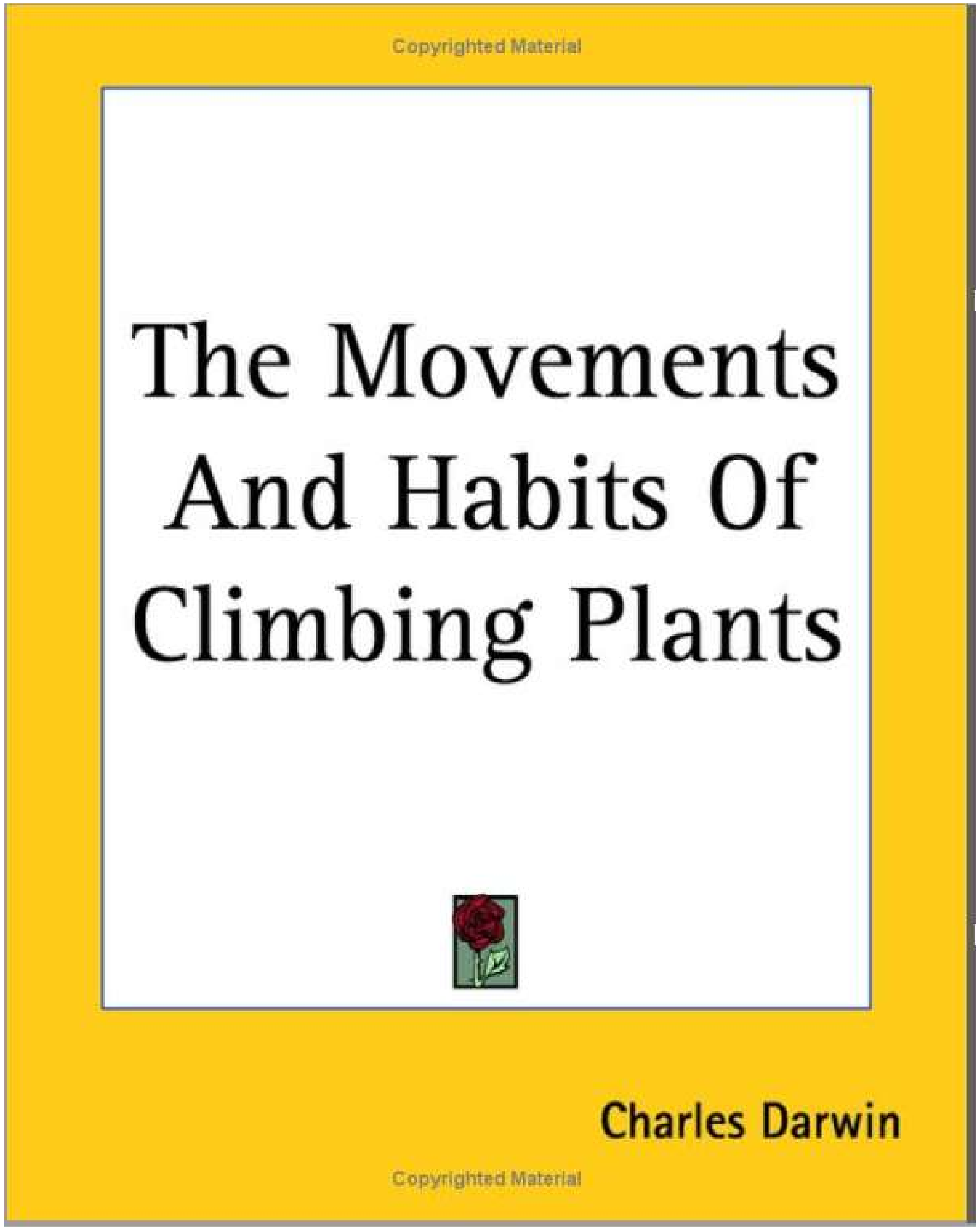
1- apex (search for support, goes around it)
2- lower part of stem (helix)

video

Twining, step by step

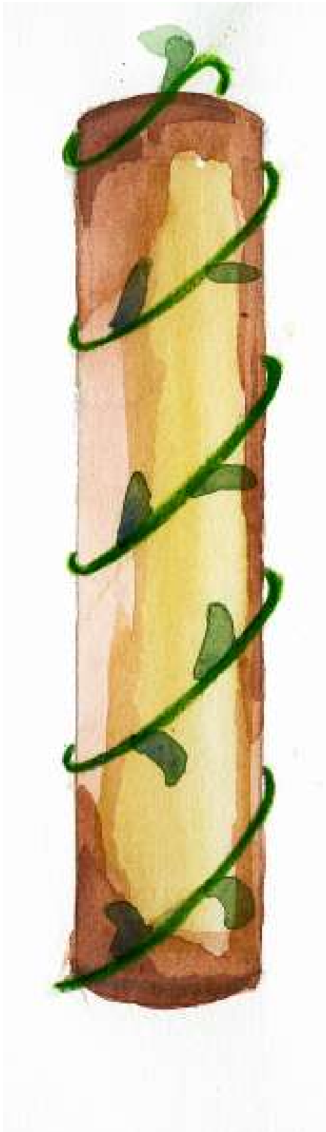


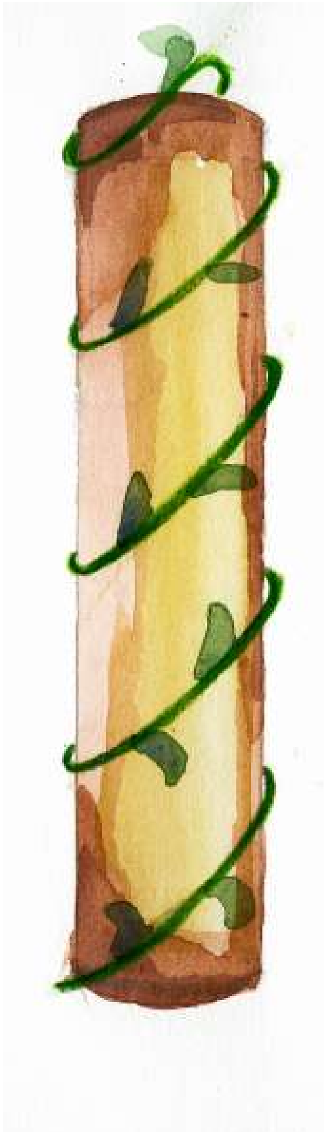
from Knut Arild Erstad www.ii.uib.no/~knute/ (artist view)



Publisher: Kessinger Publishing (2004)

"Most twining plants are adapted to ascend supports of moderate though of different thicknesses. "

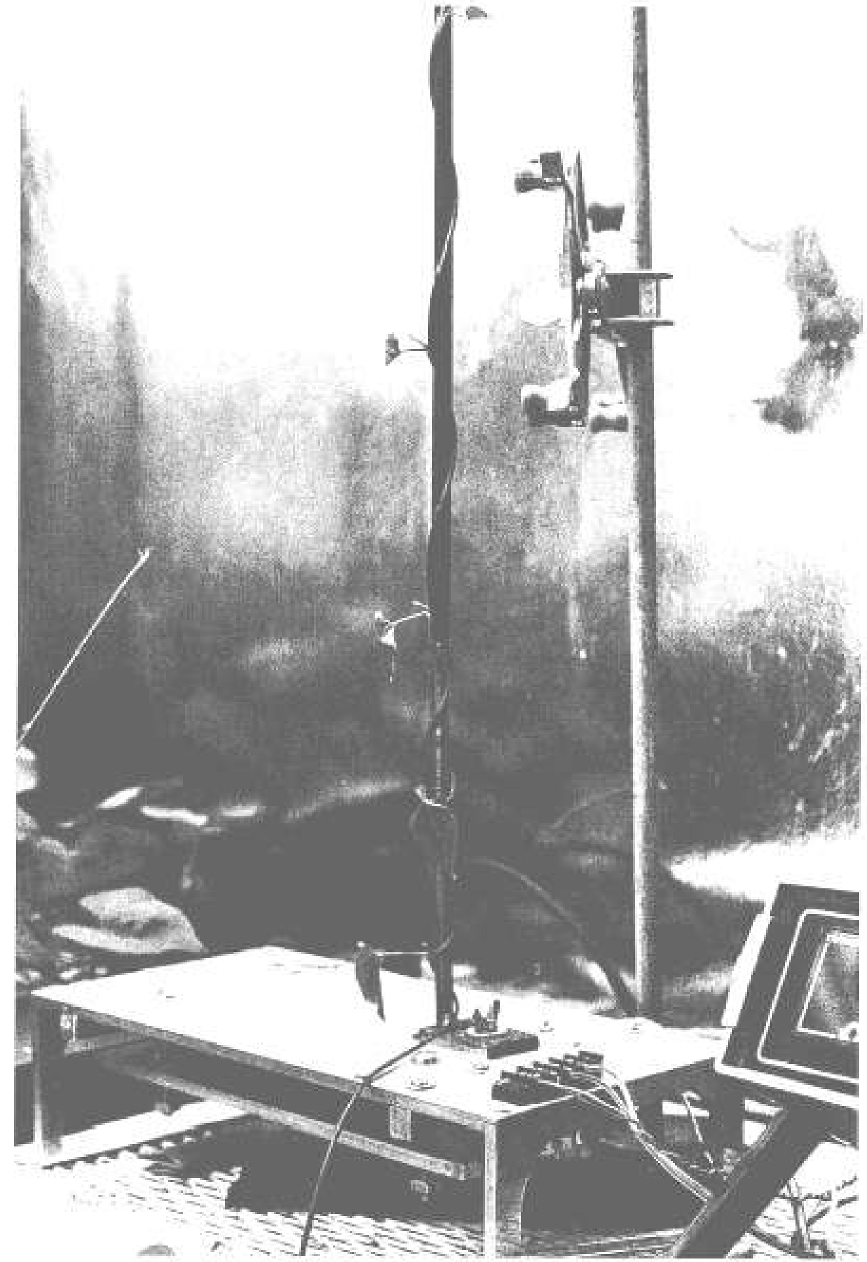




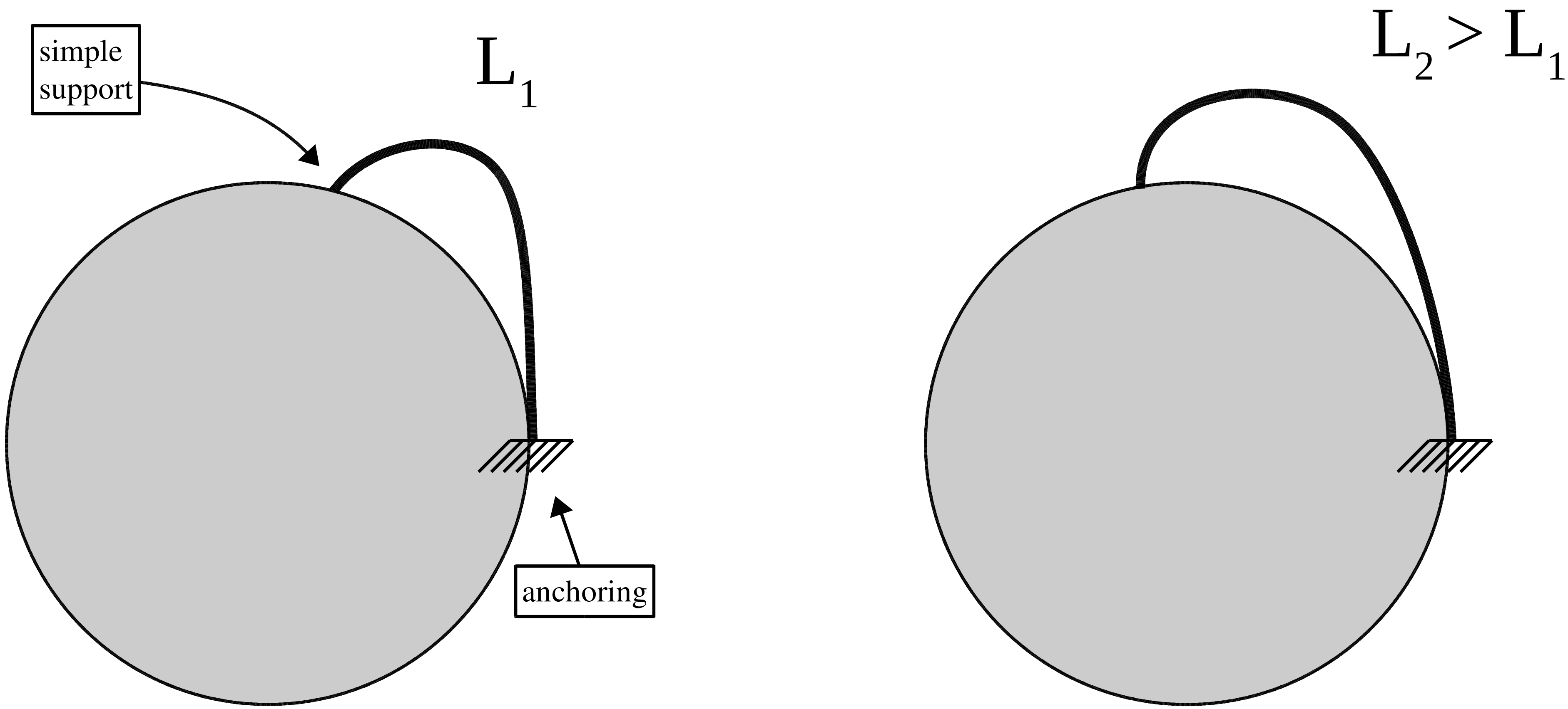
Mechanical experiments (W. Silk)

<u>Measurements</u>:
- geometrical parameters (on & off pole)
- contact pressure

<u>Results</u>:
- stem is in tension
- contact pressure >> weight
- uniform helix
- lower pitch on pole



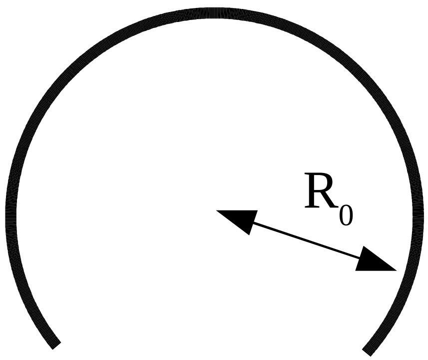
Our model: elastic beam with natural curvature in 2D



quasi-statically increase length

Naturally curved elastic beam in 2D

natural shape: arc of circle

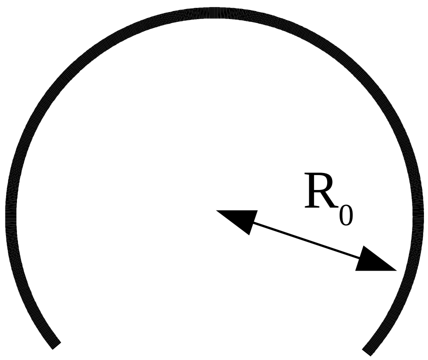


natural radius of curvature: R_0

intrinsic curvature: $\kappa_0 = \frac{1}{R_0}$

Naturally curved elastic beam in 2D

natural shape: arc of circle



natural radius of curvature: R_0

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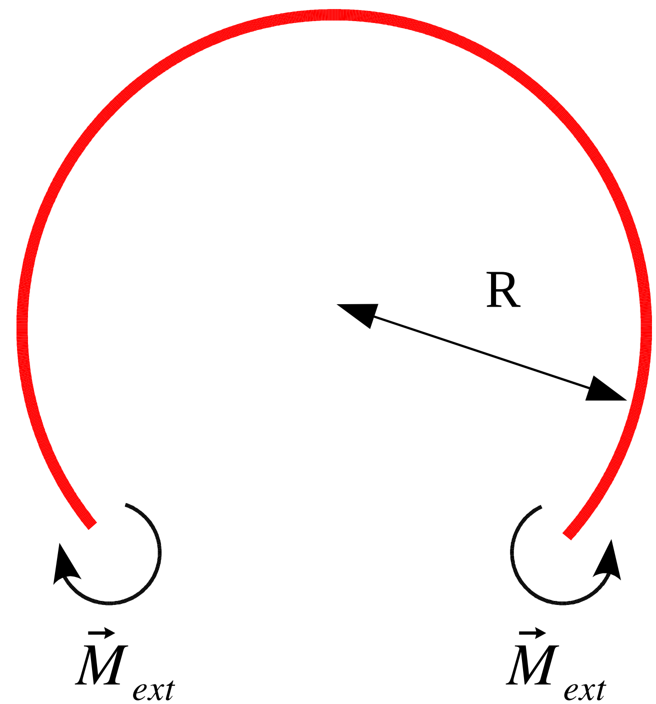
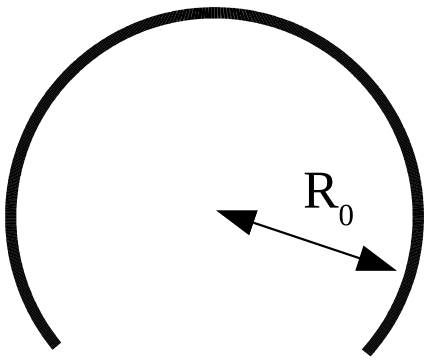
loaded shape



'loaded' curvature: $\kappa = \dfrac{1}{R}$

Naturally curved elastic beam in 2D

natural shape: arc of circle



natural radius of curvature: R_0

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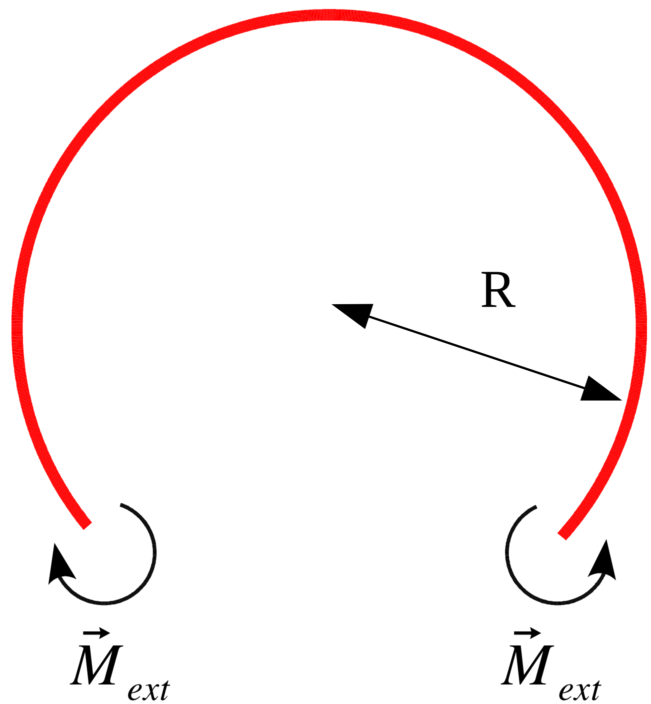


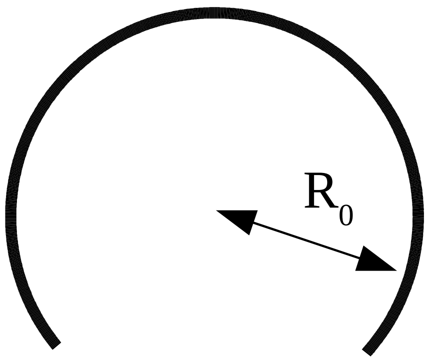
'loaded' curvature: $\kappa = \frac{1}{R}$

$$M_{ext} = EI \left(\kappa - \kappa_0\right) < 0$$

bending rigidity

Naturally curved elastic beam in 2D

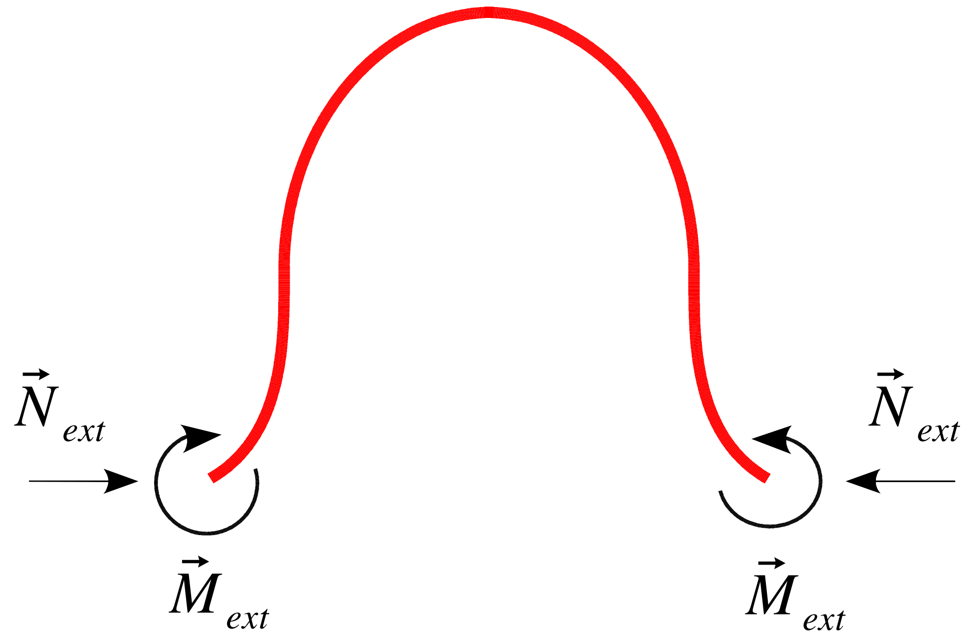
natural shape: arc of circle



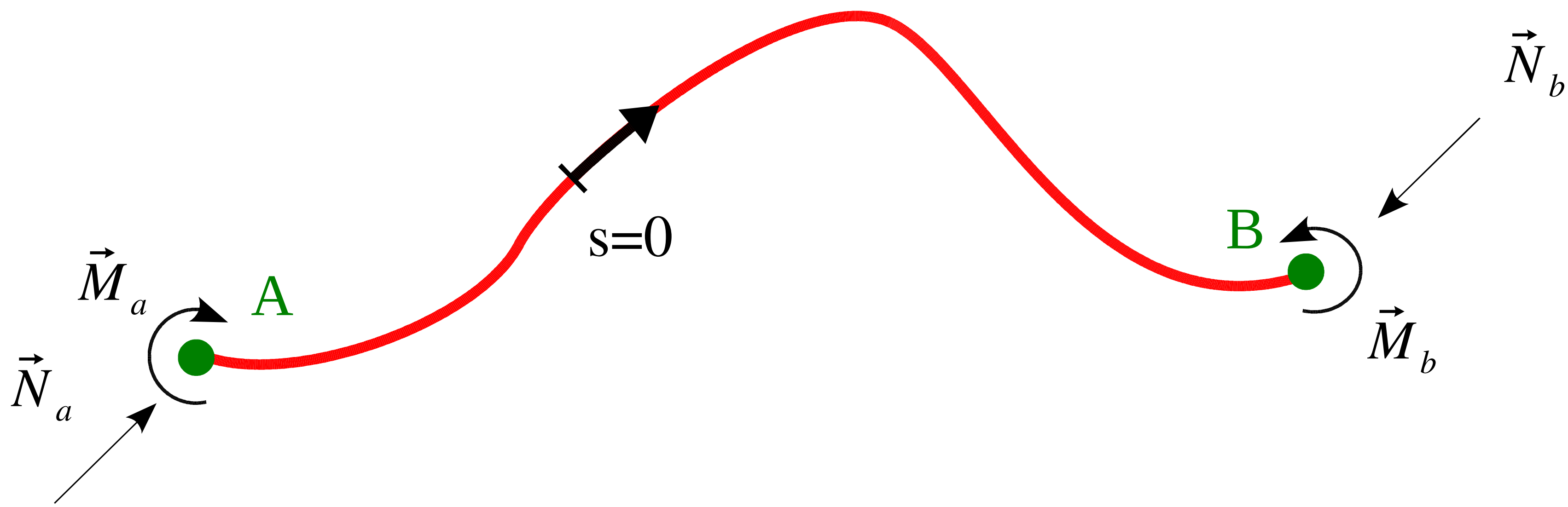
natural radius of curvature: R_0

intrinsic curvature: $\kappa_0 = \frac{1}{R_0}$

force and torque loading



(Kirchhoff) equilibrium equations



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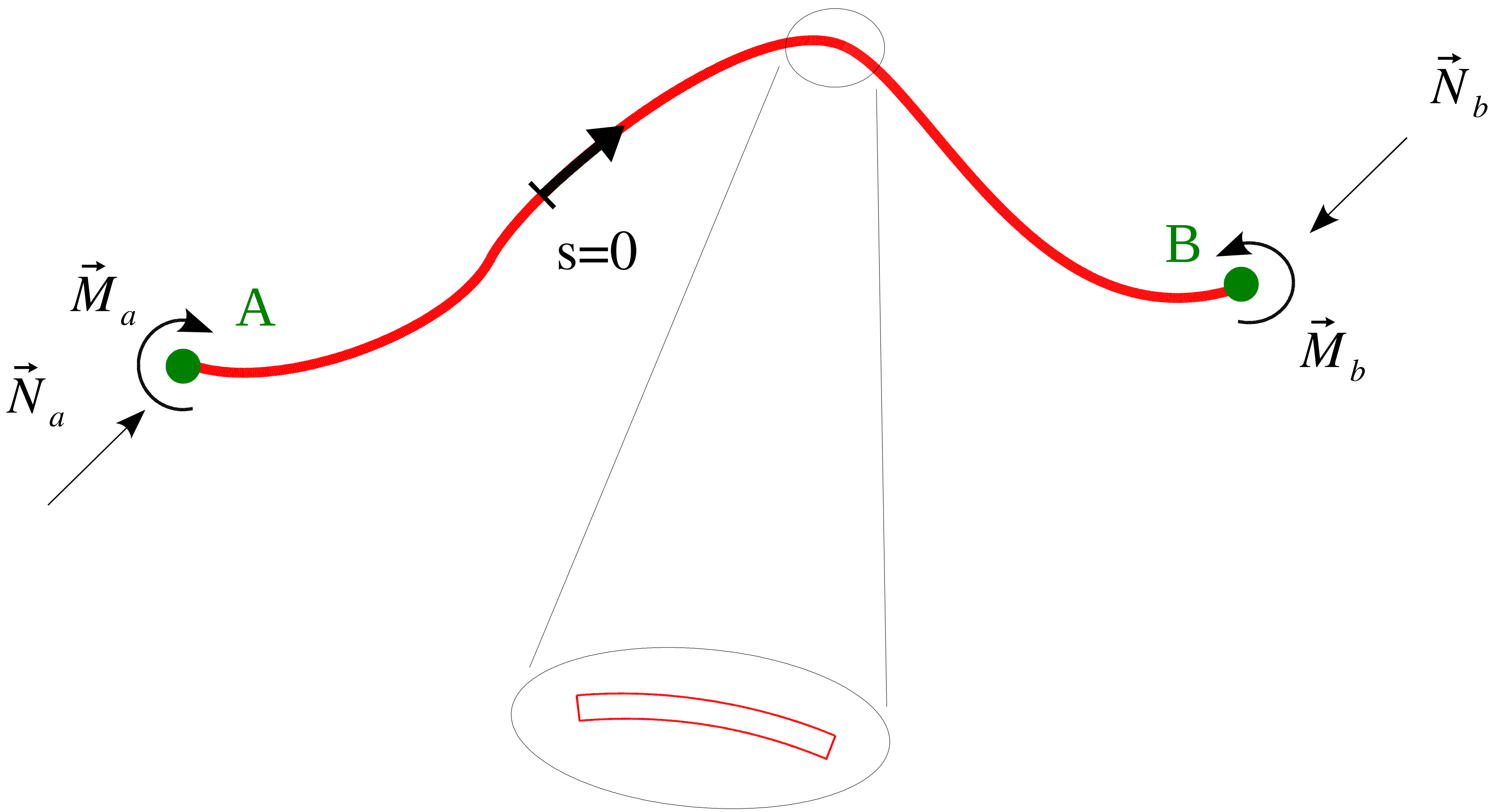

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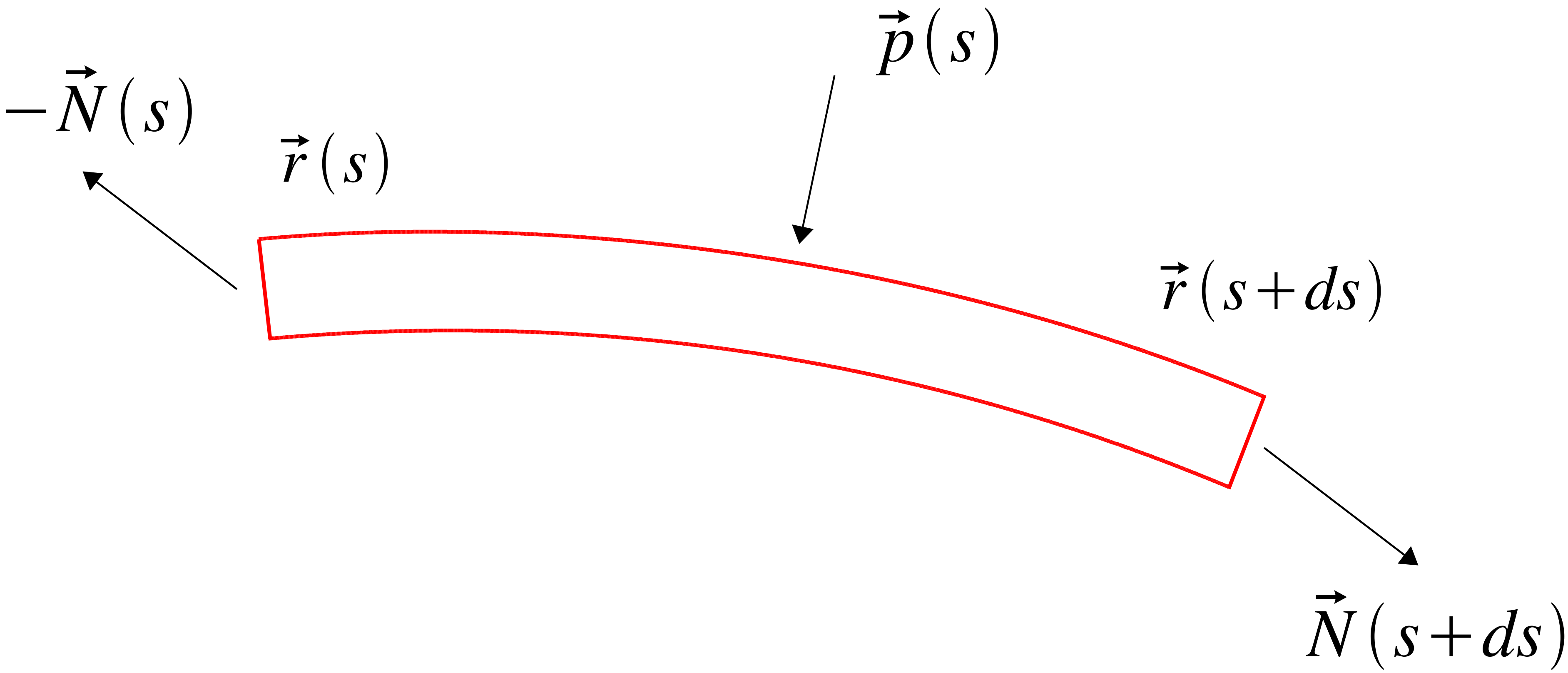
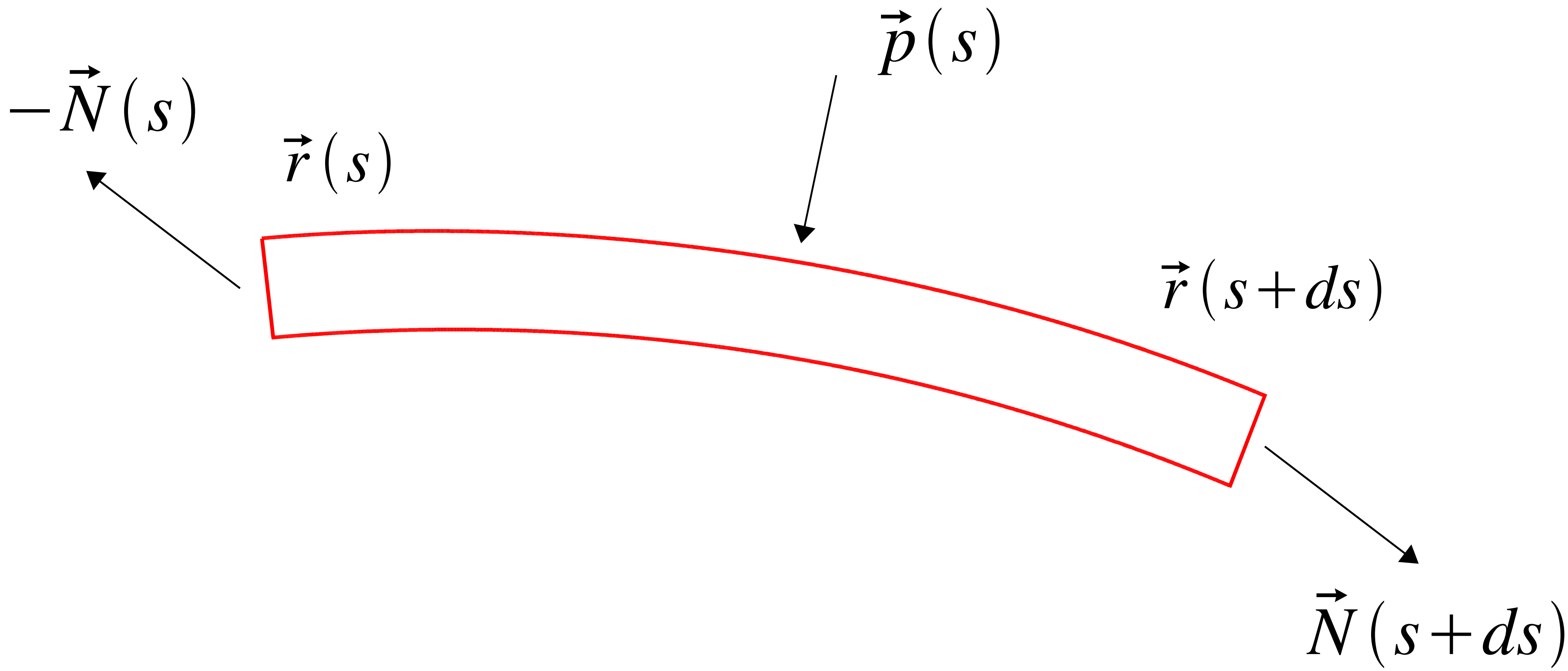


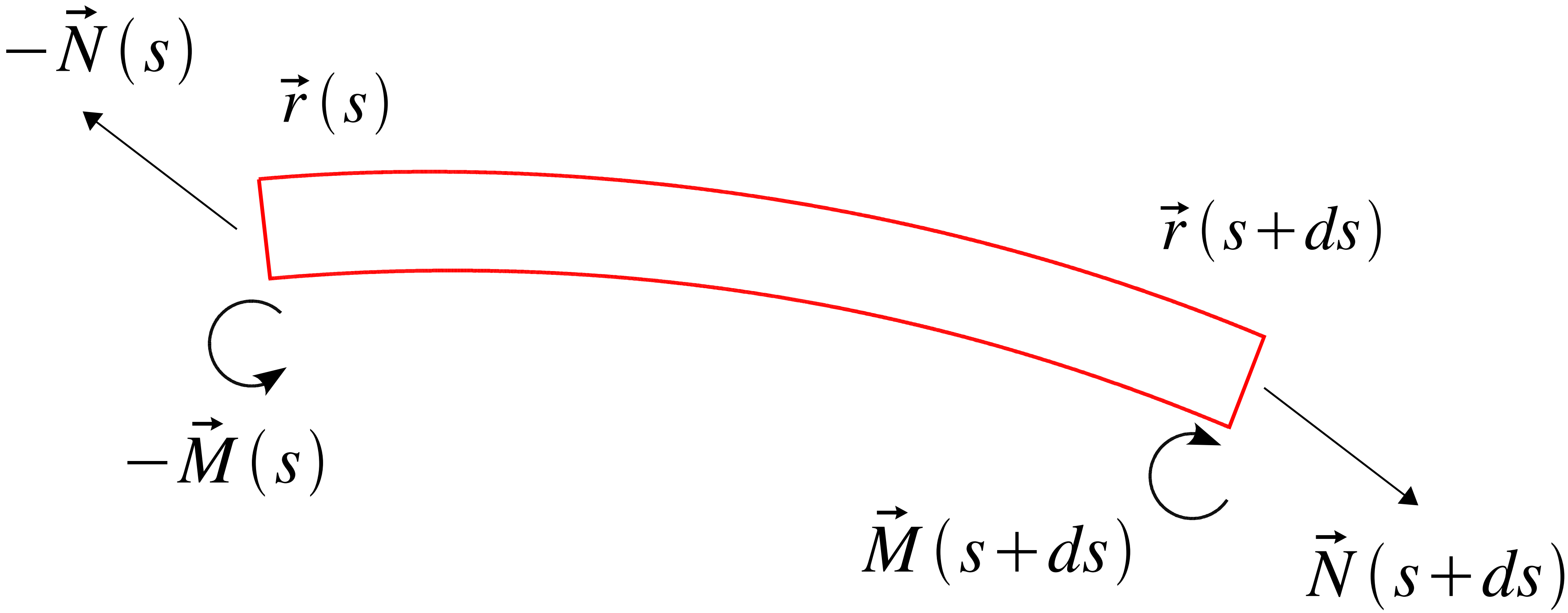
(Kirchhoff) equilibrium equations



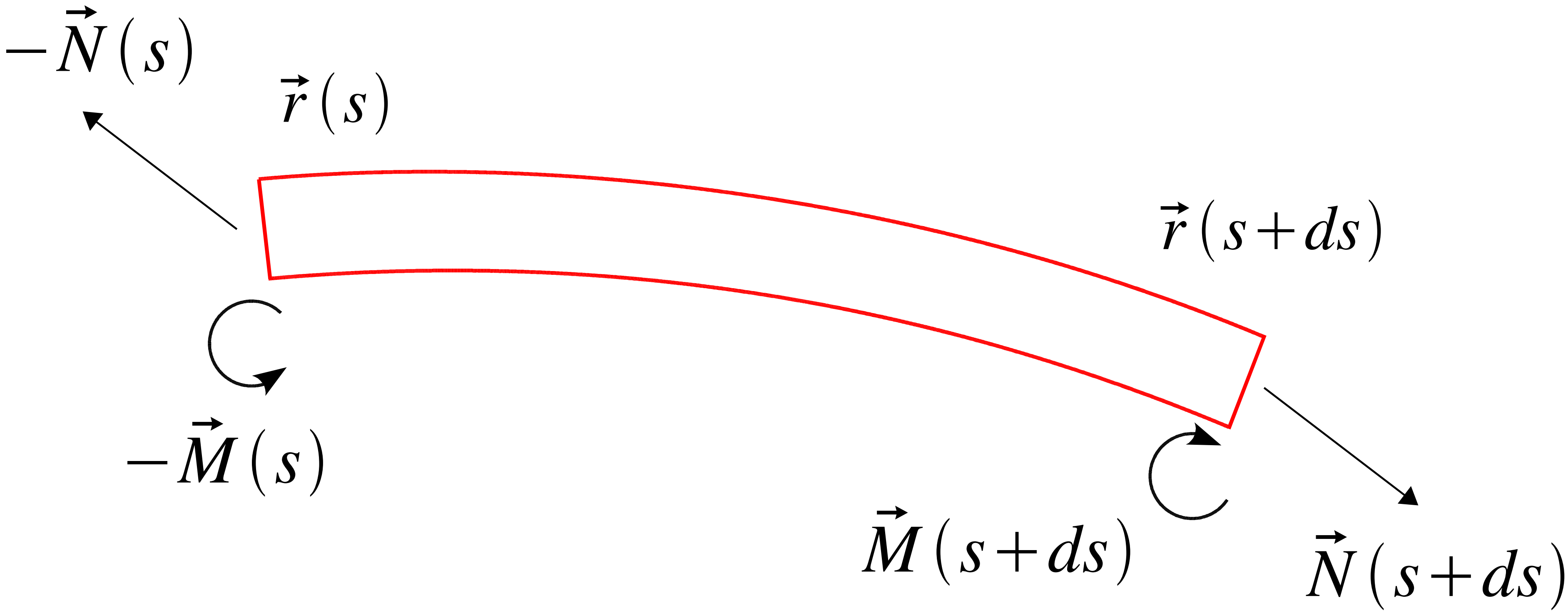
linear momentum: $\vec{N}(s+ds)-\vec{N}(s)+\vec{p}(s)\,ds=\vec{0}$

$$\vec{N}\,'(s)=-\vec{p}(s)$$

(Kirchhoff) equilibrium equations



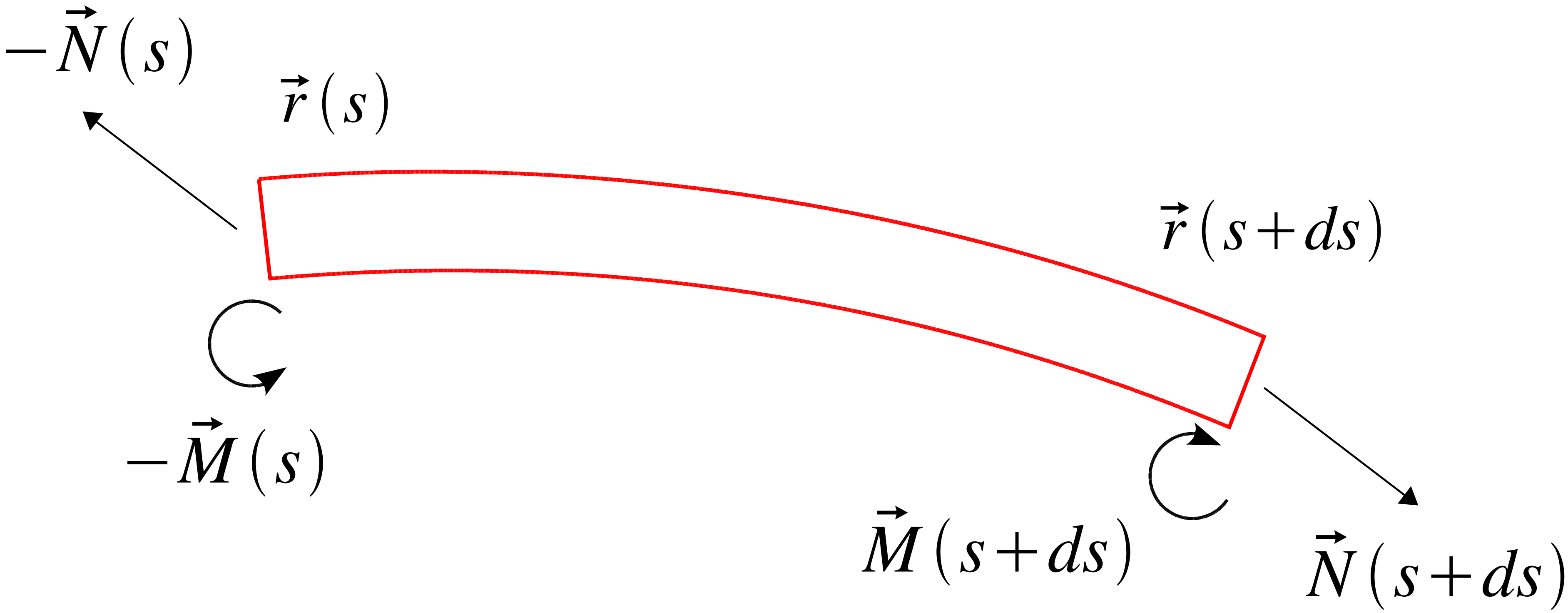
(Kirchhoff) equilibrium equations



angular momentum:

$$\vec{M}(s+ds)-\vec{M}(s)+torque(\vec{N}(s);\vec{N}(s+ds))=\vec{0}$$

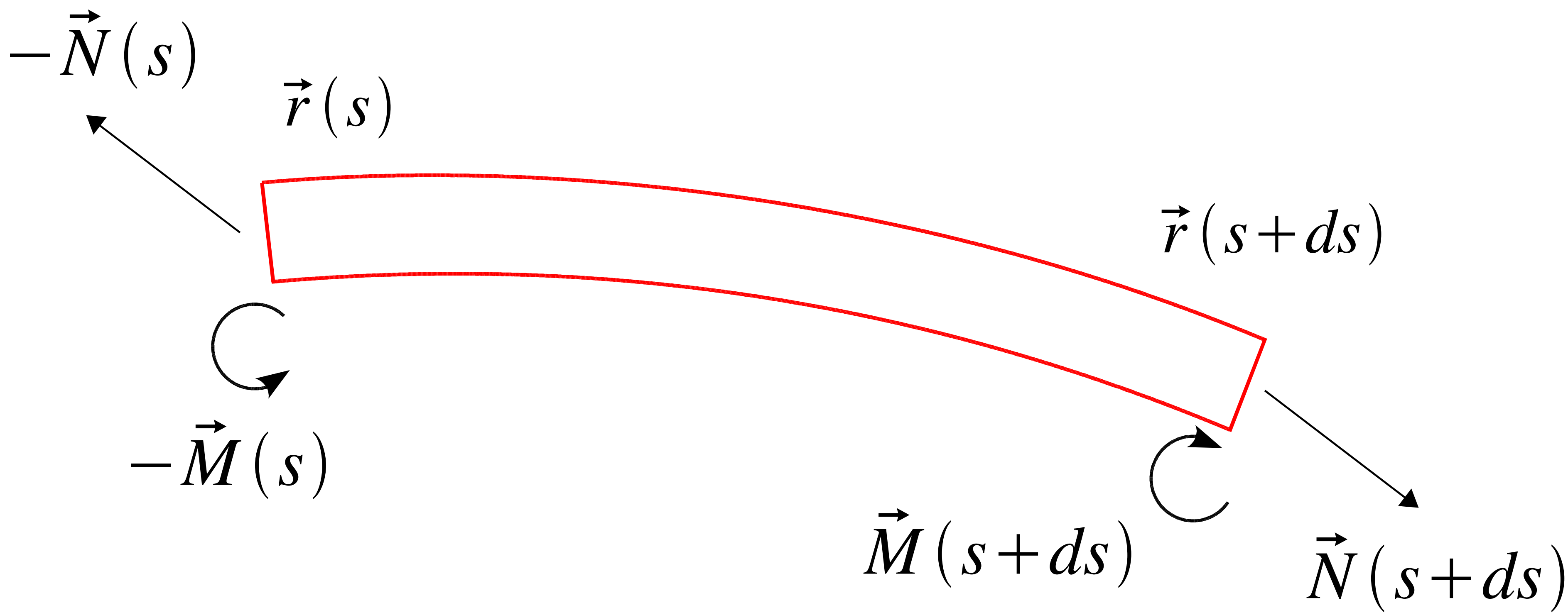
(Kirchhoff) equilibrium equations



angular momentum:

$$\vec{M}(s+ds)-\vec{M}(s)+\underbrace{torque(\vec{N}(s);\vec{N}(s+ds))}_{\left(\vec{r}(s+ds)-\vec{r}(s)\right)\times\vec{N}(s)}=\vec{0}$$

(Kirchhoff) equilibrium equations



angular momentum:

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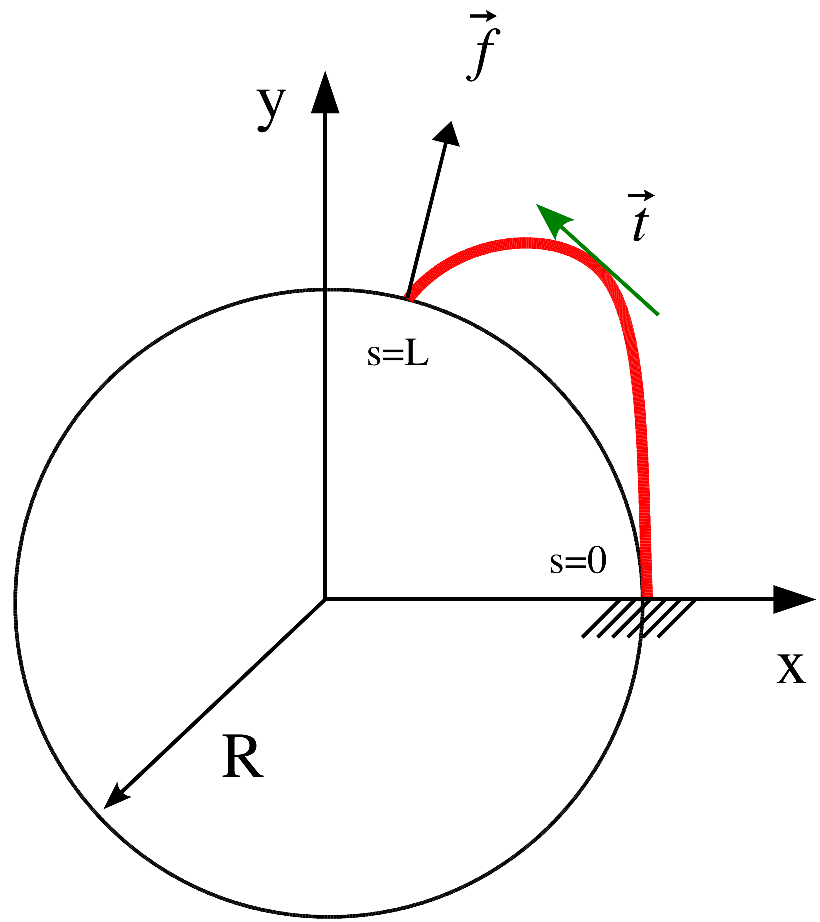
$$\boxed{\vec{M}\,'(s)+\vec{r}\,'(s)\times\vec{N}(s)=\vec{0}}$$

A model: Equilibrium of an elastic rod (Kirchhoff equations)

$$\begin{cases} \vec{N}' + \vec{p} = \vec{0} & : \quad \textit{force balance} \\ \vec{M}' + \vec{r}' \times \vec{N} = 0 & : \quad \textit{moment balance} \\ \vec{r}' = \vec{t} & : \quad \textit{tangent} \\ M_y = EI(\kappa - \kappa_0) & : \quad \textit{linear elasticity} \end{cases}$$

$$' \equiv \frac{d}{ds} \quad ; \quad (s : arclength)$$



Ordinary differential equations

A model: Equilibrium of an elastic rod (Kirchhoff equations)

$$\begin{cases} \vec{N}' + \vec{p} = \vec{0} & : \textit{force balance} \\ \vec{M}' + \vec{r} \times \vec{N} = 0 & : \textit{moment balance} \\ \vec{r}' = \vec{t} & : \textit{tangent} \\ M_y = EI(\kappa - \kappa_0) & : \textit{linear elasticity} \end{cases}$$

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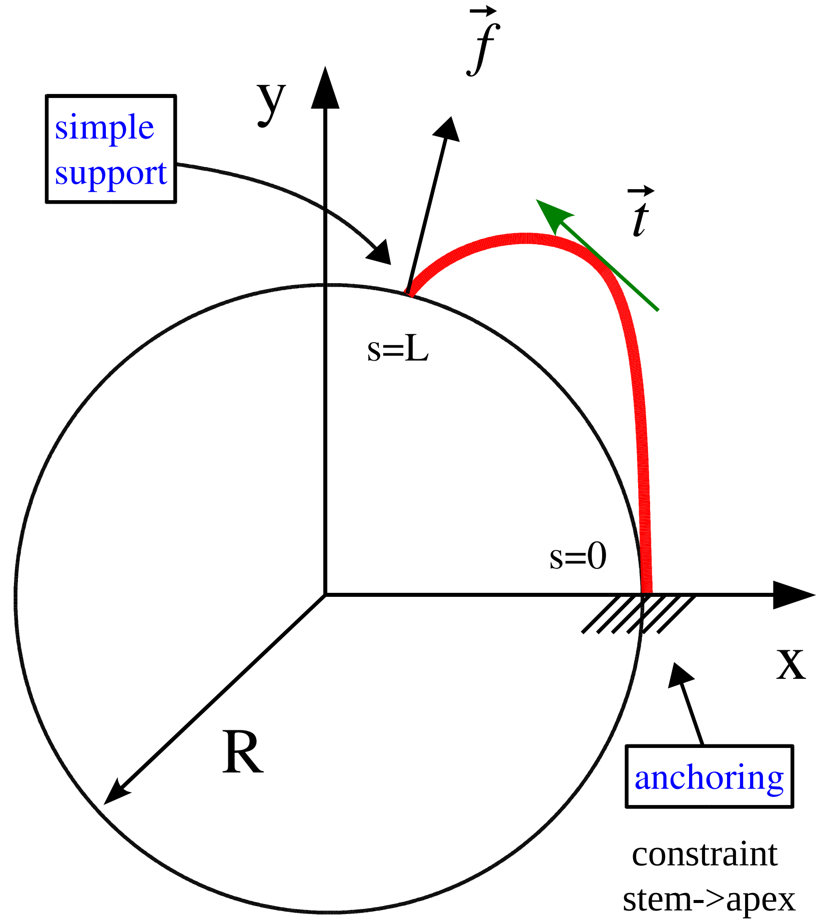


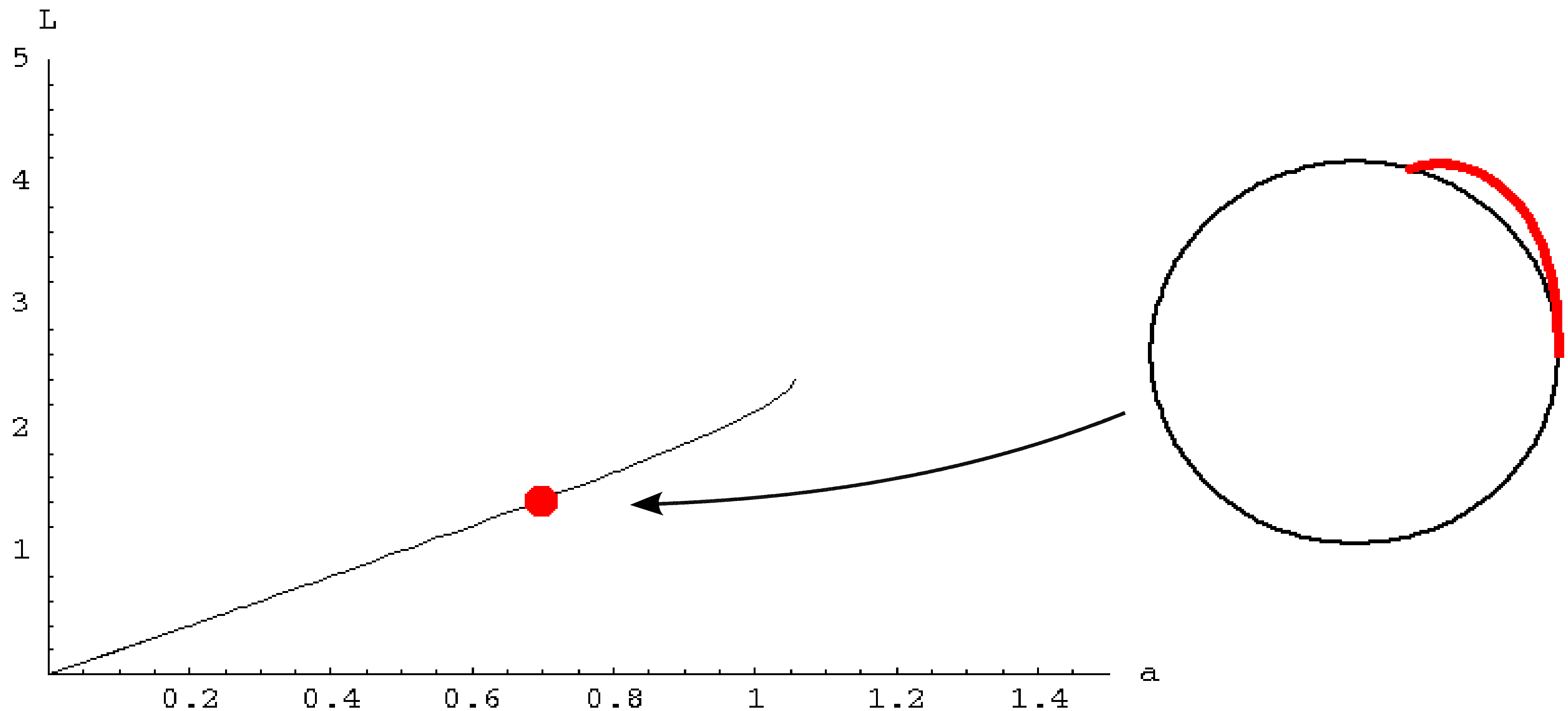
Ordinary differential equations

with boundary conditions:

$$s = 0 \quad : \quad anchoring \quad : \quad \vec{t}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$s = L \quad : \quad x(L)^2 + y(L)^2 = R^2 \quad \textbf{with} \quad \vec{N}(L) = \vec{f} \parallel \begin{pmatrix} x(L) \\ y(L) \end{pmatrix} \quad \textbf{also} \quad \vec{M}(L) = 0$$

Numerical continuation of solutions : bifurcation diagram K = R / R0 = 3



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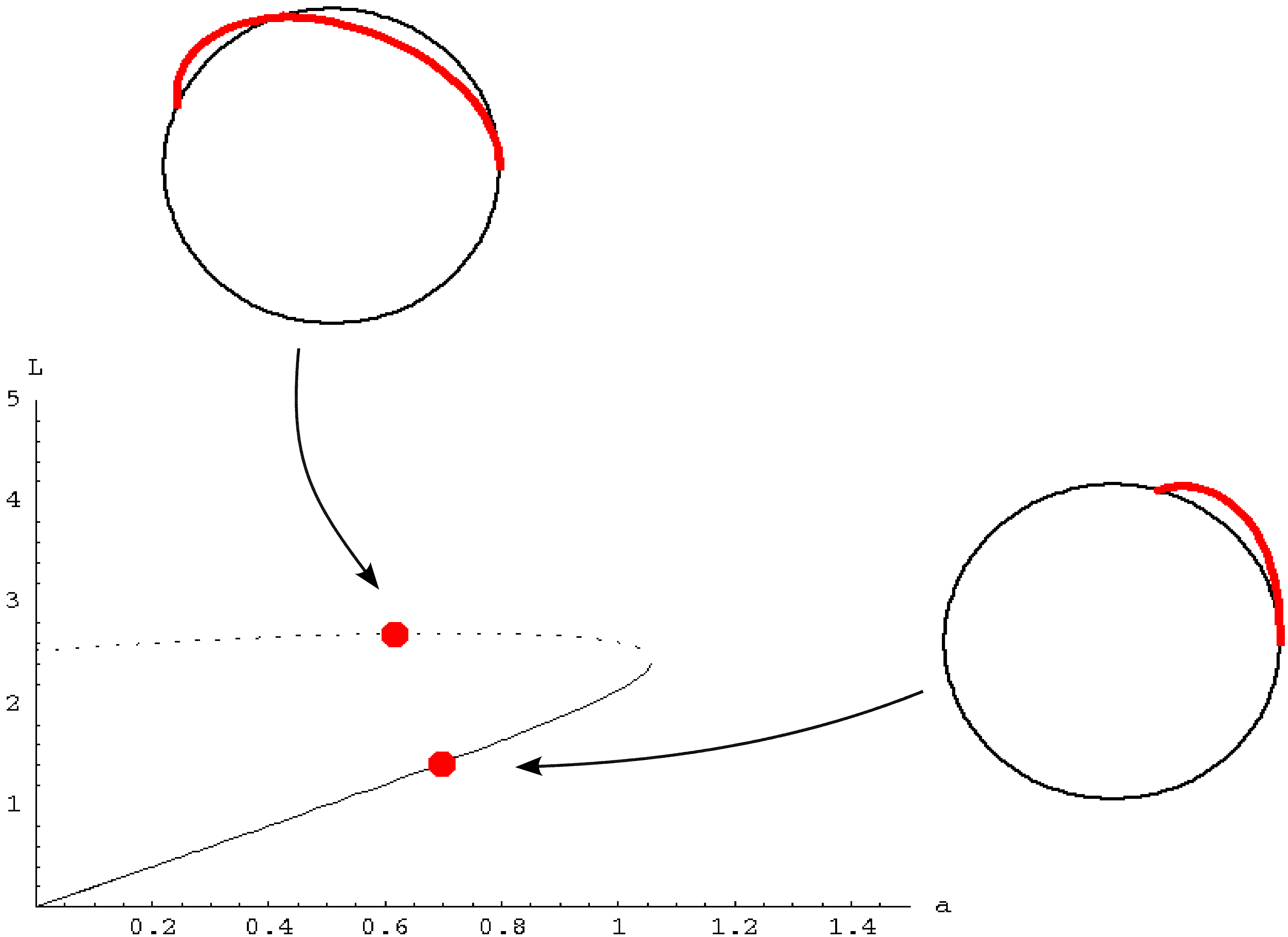

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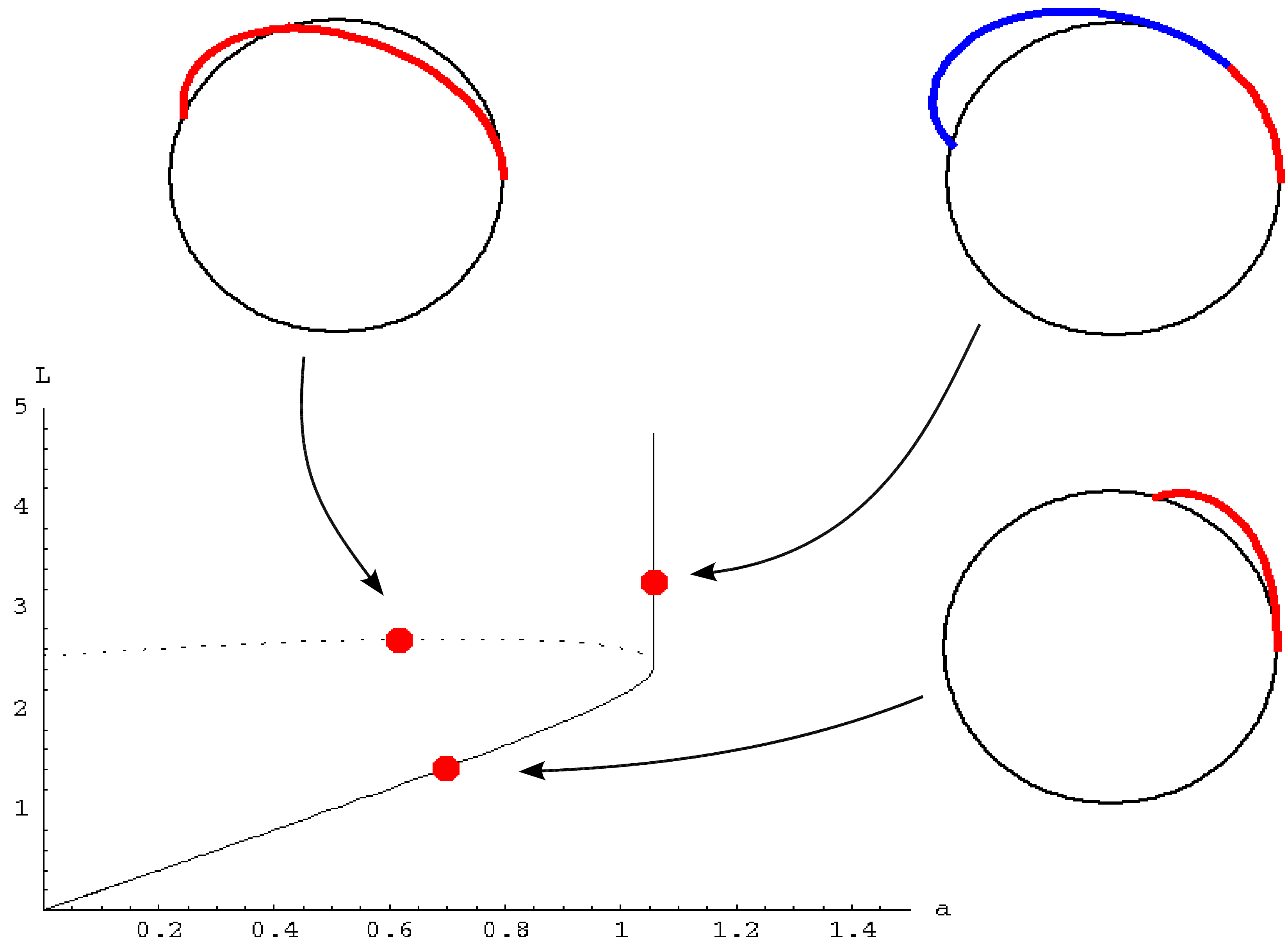
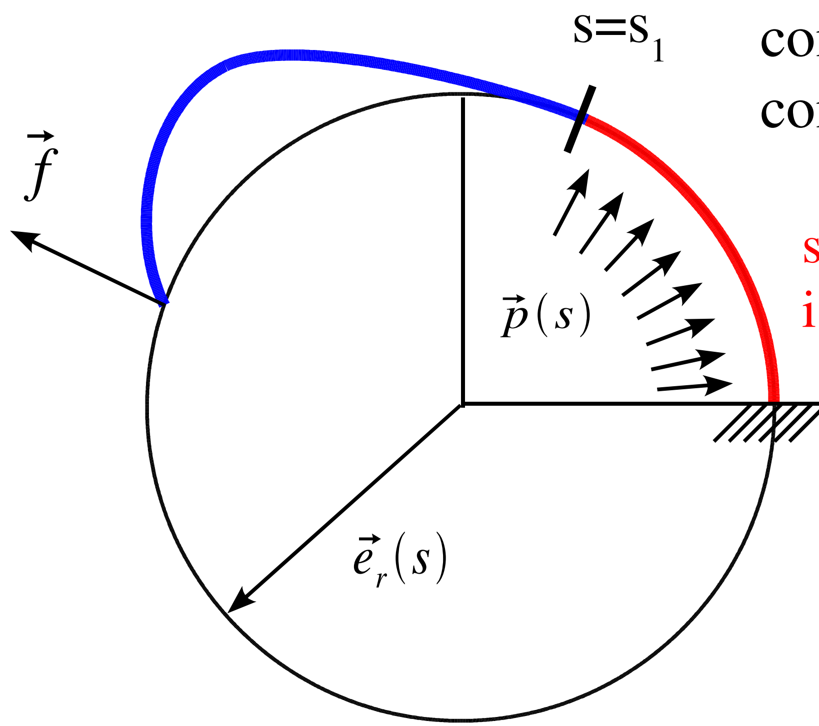


Configurations with continuous contact

tension: $\vec{N}\cdot\vec{t}=p\,R>0$

$$M=EI\left(\frac{1}{R}-\frac{1}{R_0}\right)<0$$

$$\vec{p}(s)=\frac{EI}{R^2}\left(\frac{1}{R_0}-\frac{1}{R}\right)\vec{e}_r(s)$$



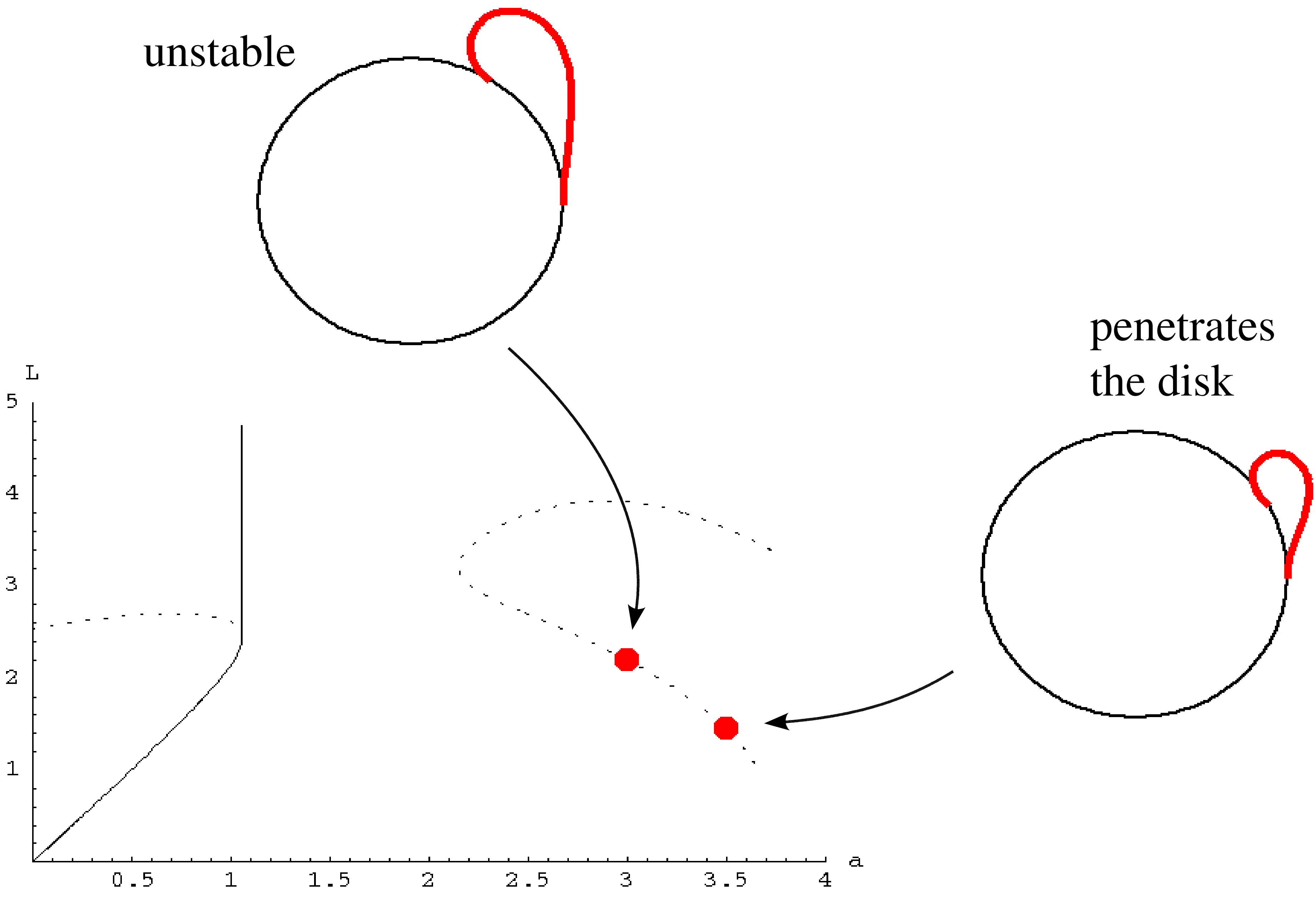
continuous contact

solution is uniform

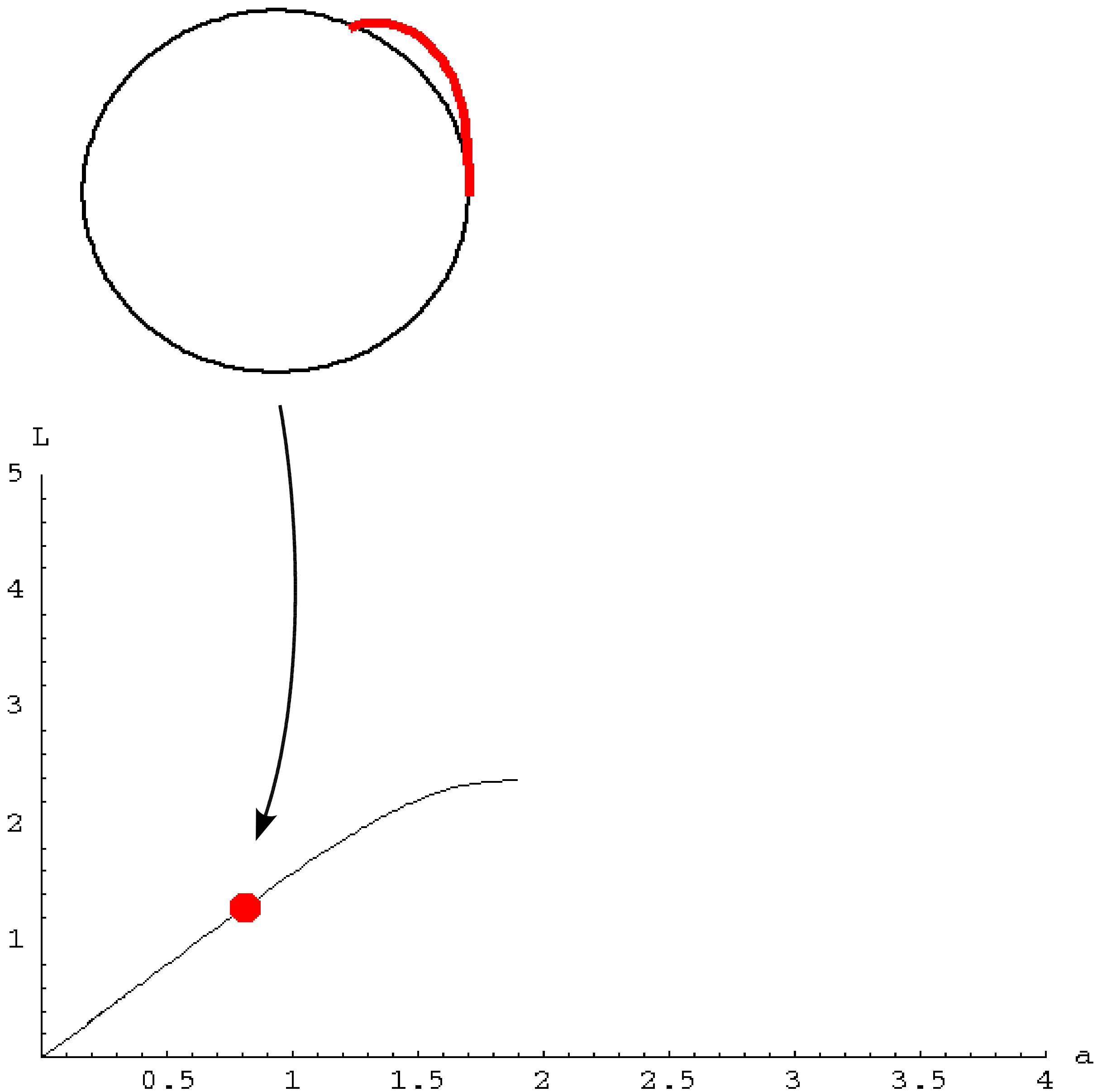
The continuous part can be lengthen arbitrarily

These configurations correspond to climbing cases

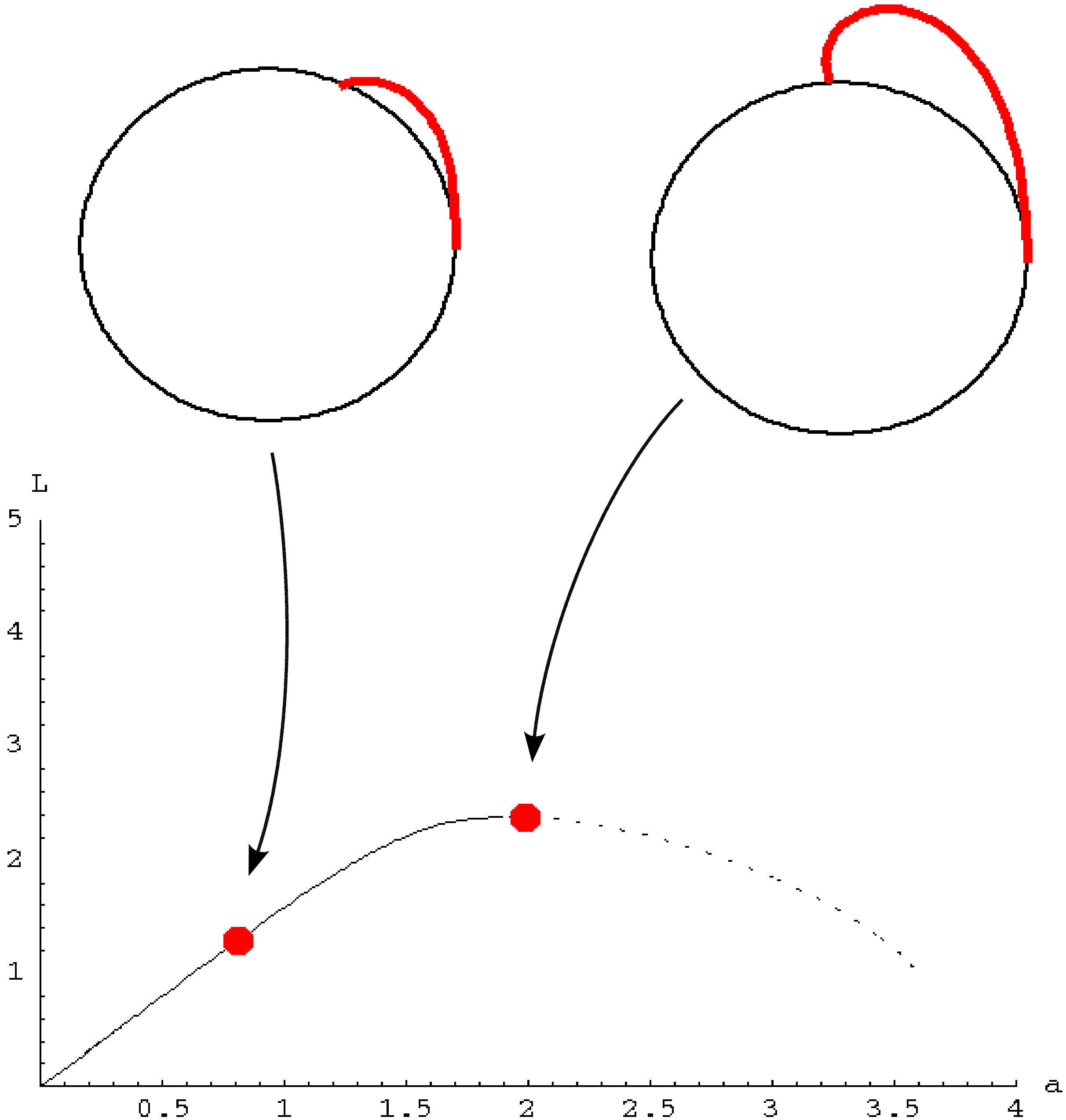
Numerical continuation of solutions : bifurcation diagram K = R / R0 = 3



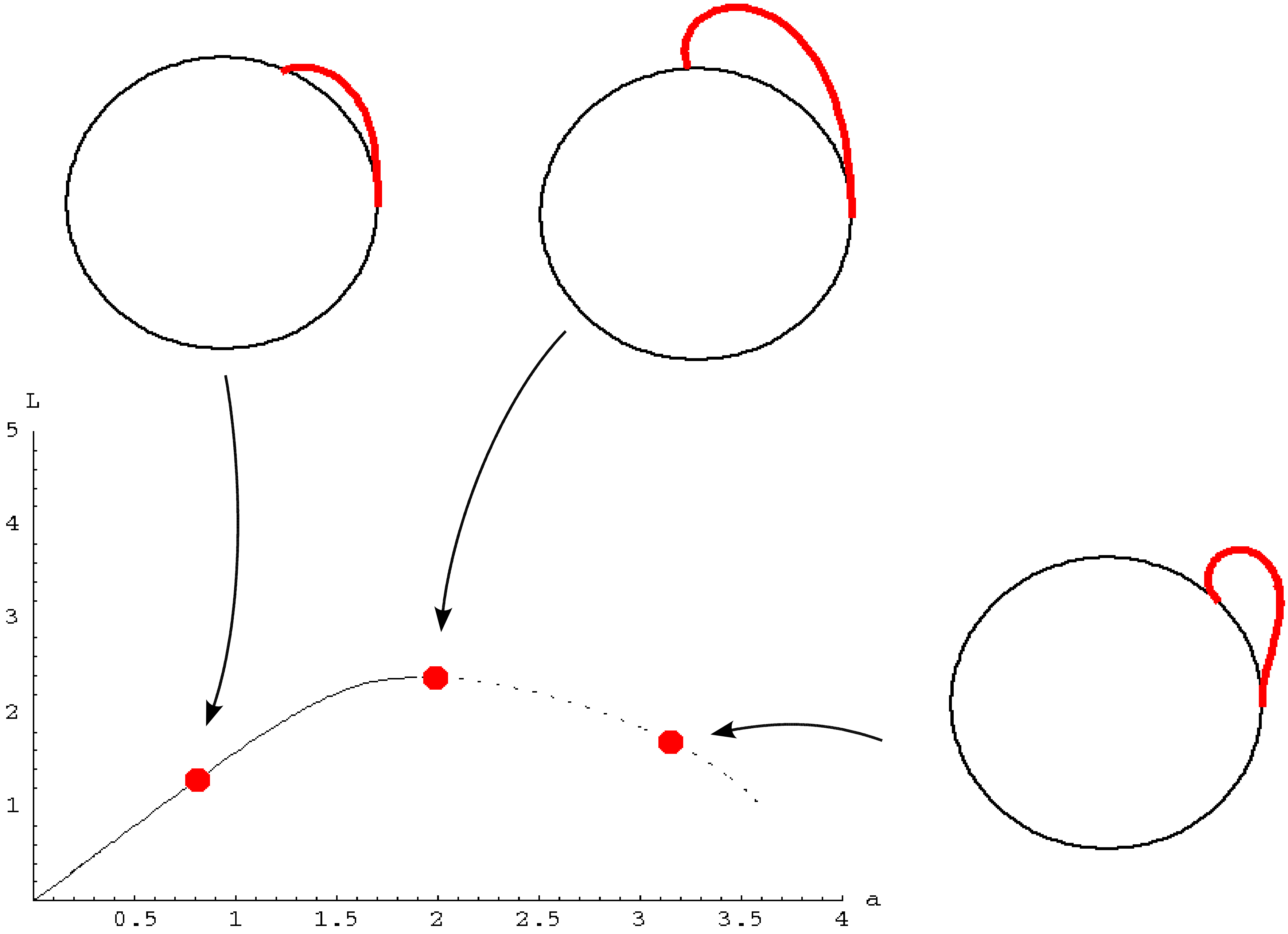
Numerical continuation of solutions : bifurcation diagram K = R / R0 = 3.5



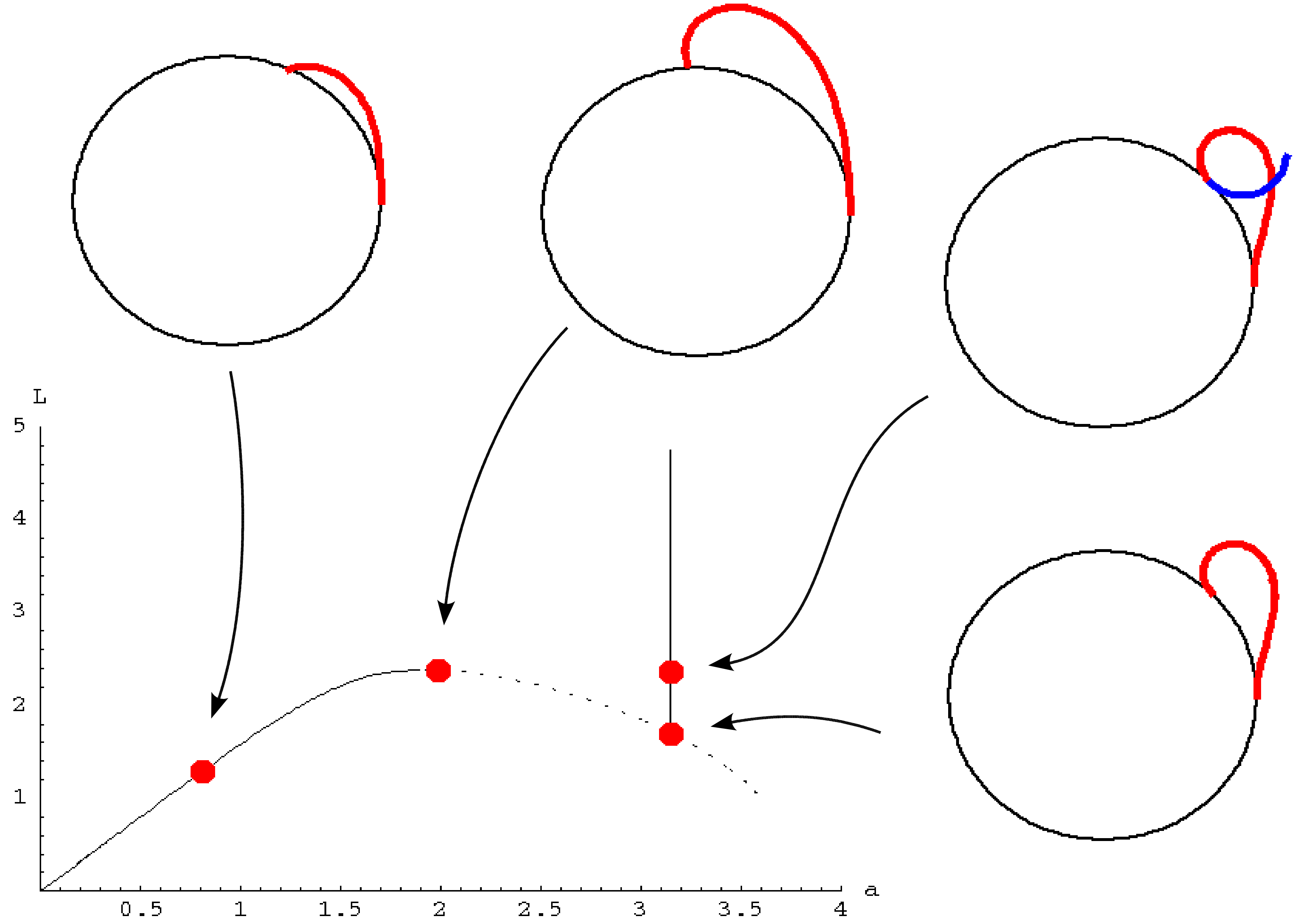
Numerical continuation of solutions : bifurcation diagram K = R / R0 = 3.5



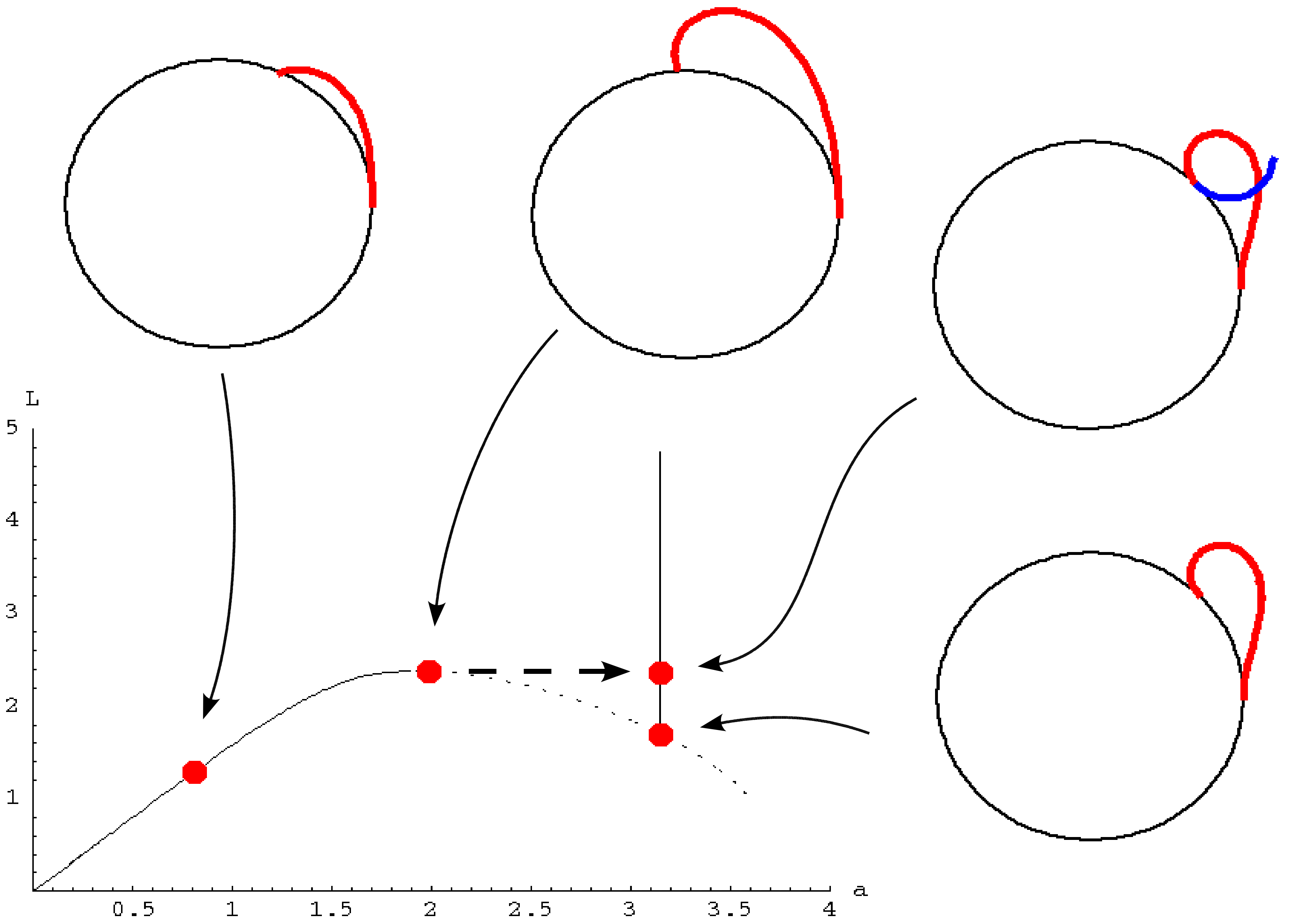
Numerical continuation of solutions : bifurcation diagram K = R / R0 = 3.5



Numerical continuation of solutions : bifurcation diagram K = R / R0 = 3.5



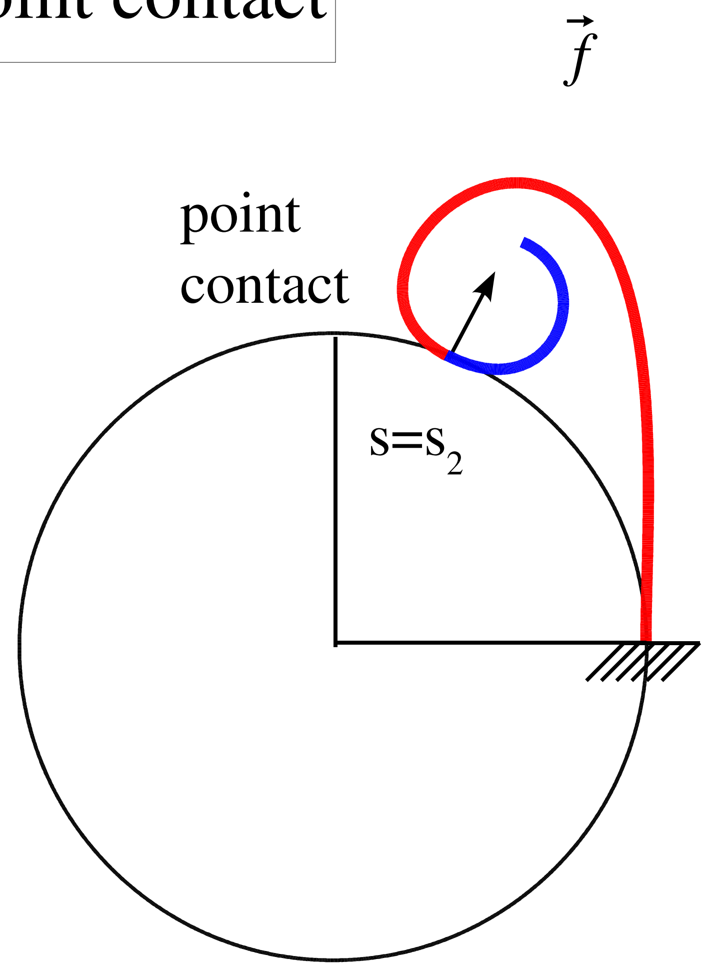
Numerical continuation of solutions : bifurcation diagram K = R / R0 = 3.5



Configurations with point contact

$$at\ s = s_2 : \quad \vec{N}^+ - \vec{N}^- + \vec{f} = \vec{0}$$

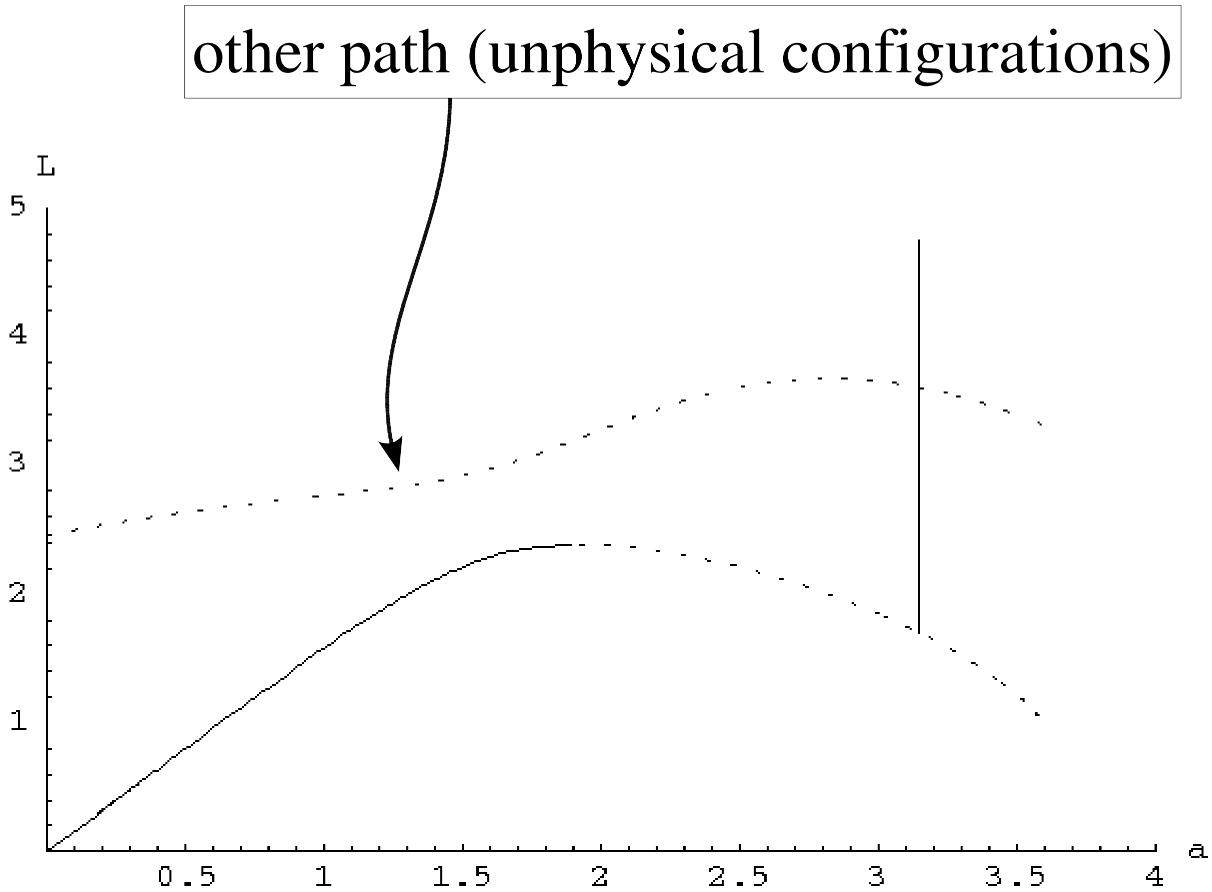
for $s > s_2$, unstressed shape



The free part can be lengthen arbitrarily

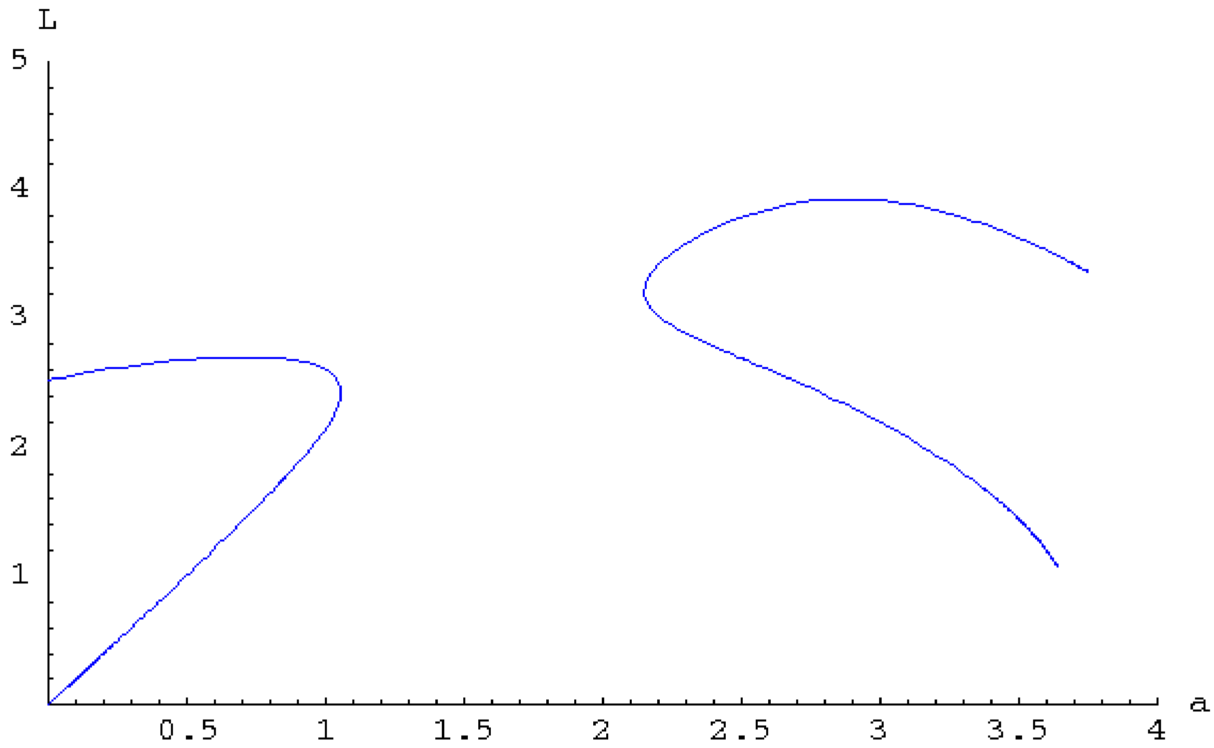
These configurations correspond to *non-climbing* cases

Numerical continuation of solutions : bifurcation diagram K = R / R0 = 3.5



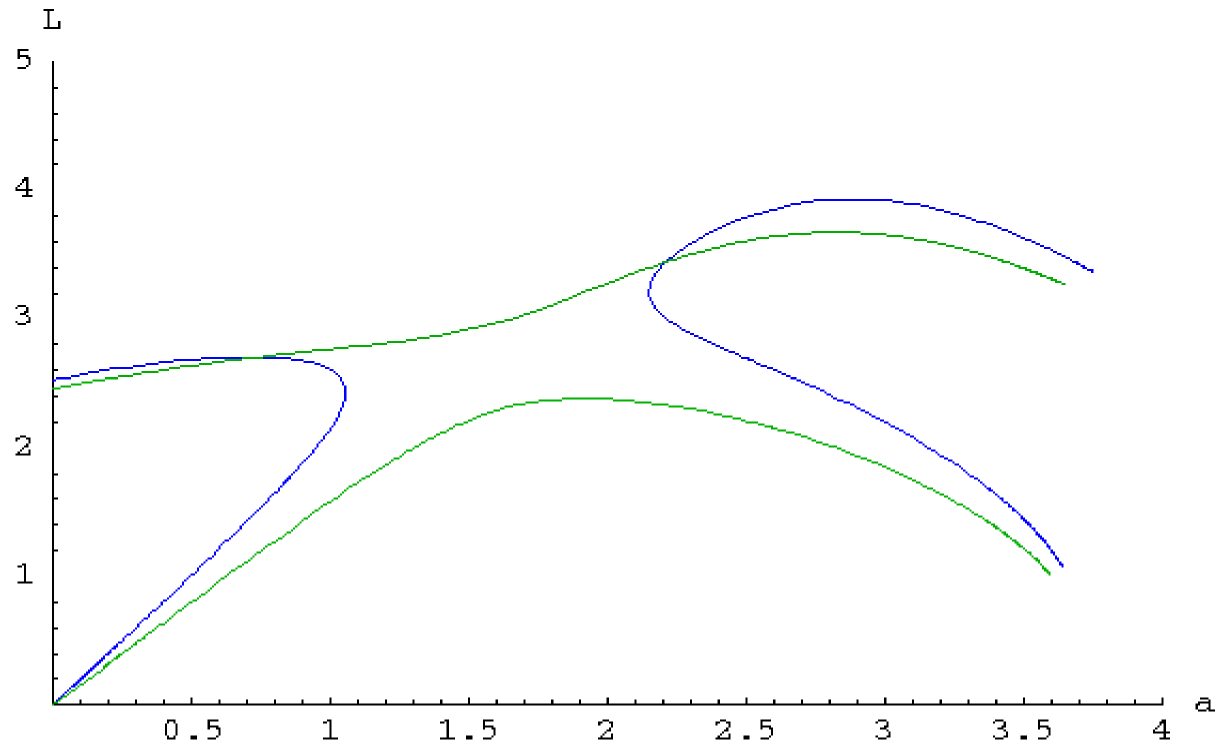
Numerical continuation of solutions : bifurcation diagrams when K varies

K=3



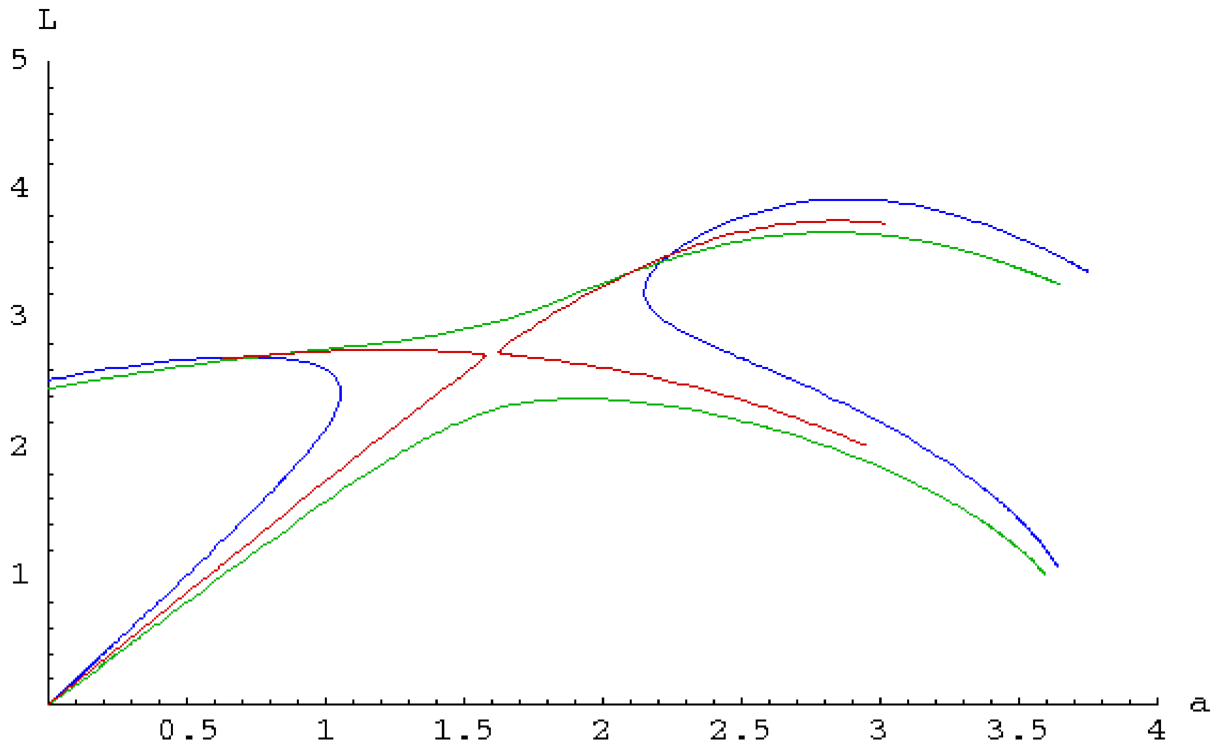
Numerical continuation of solutions : bifurcation diagrams when K varies

K=3 K=3.5



Numerical continuation of solutions : bifurcation diagrams when K varies

K=3 K=3.309 K=3.5



Numerical continuation of solutions : bifurcation diagrams when K varies

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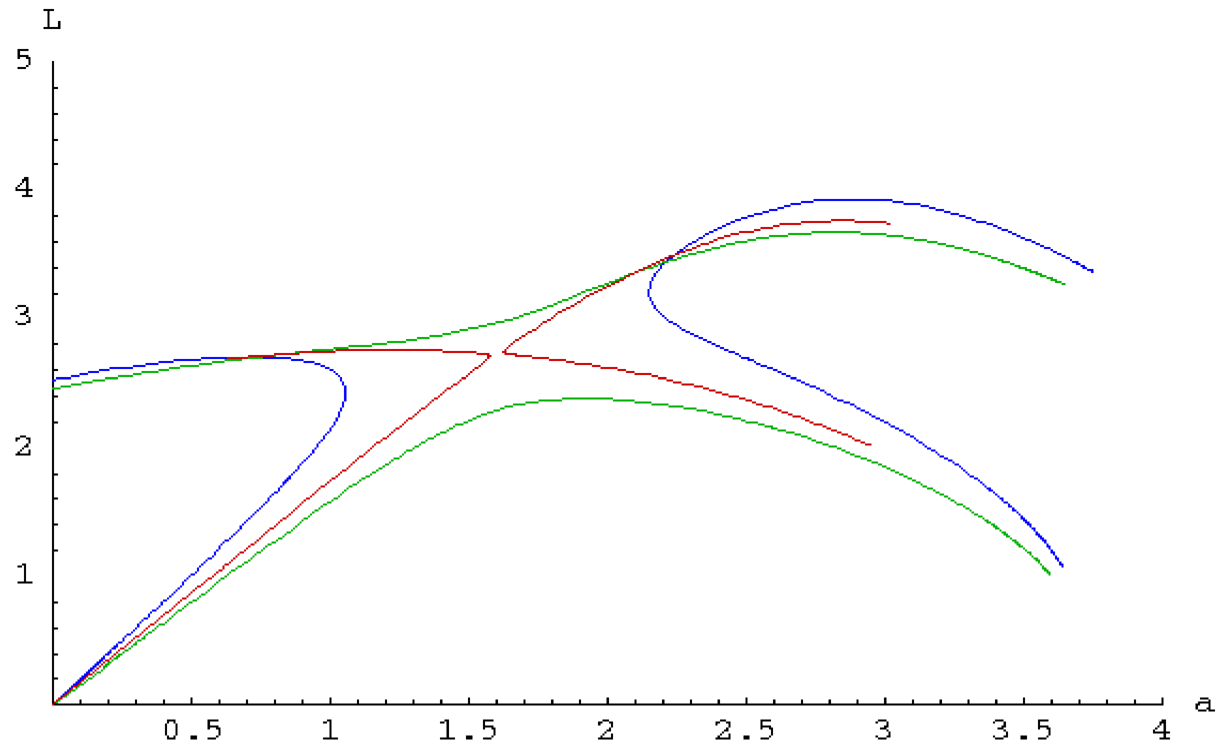
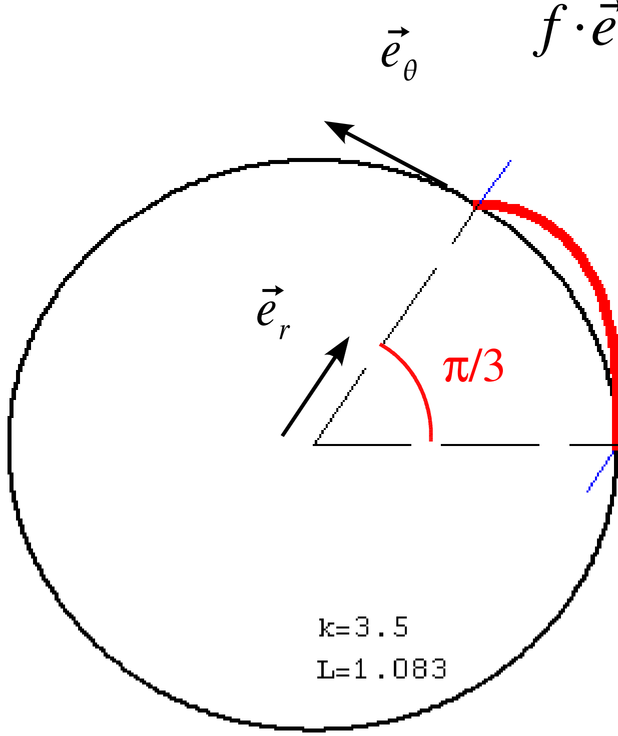


Conclusion : in the 2D case, $K_{max} \simeq 3.31$

Friction

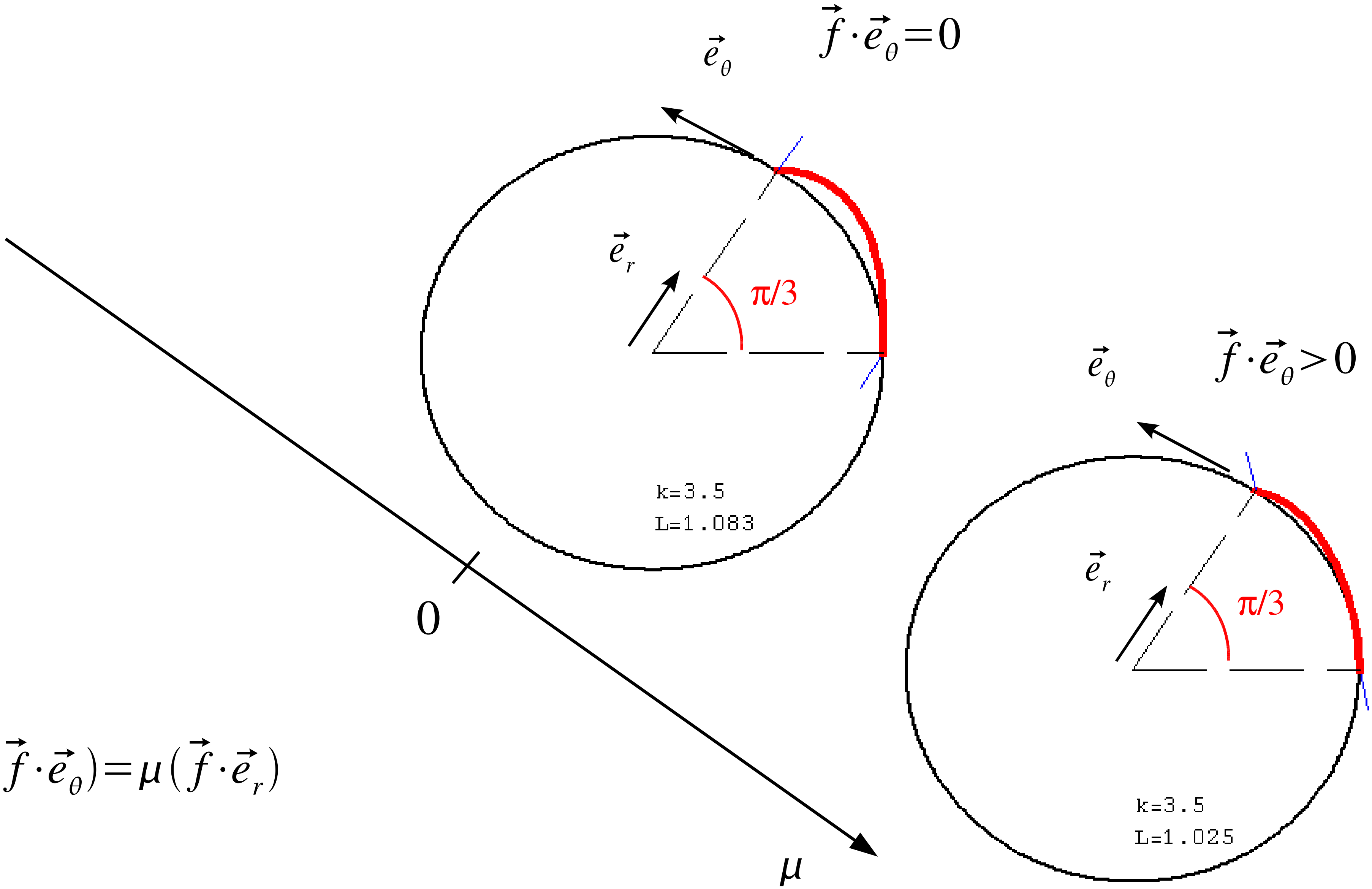
$$\vec{f}\cdot\vec{e}_\theta=0$$



$$\mu=0$$

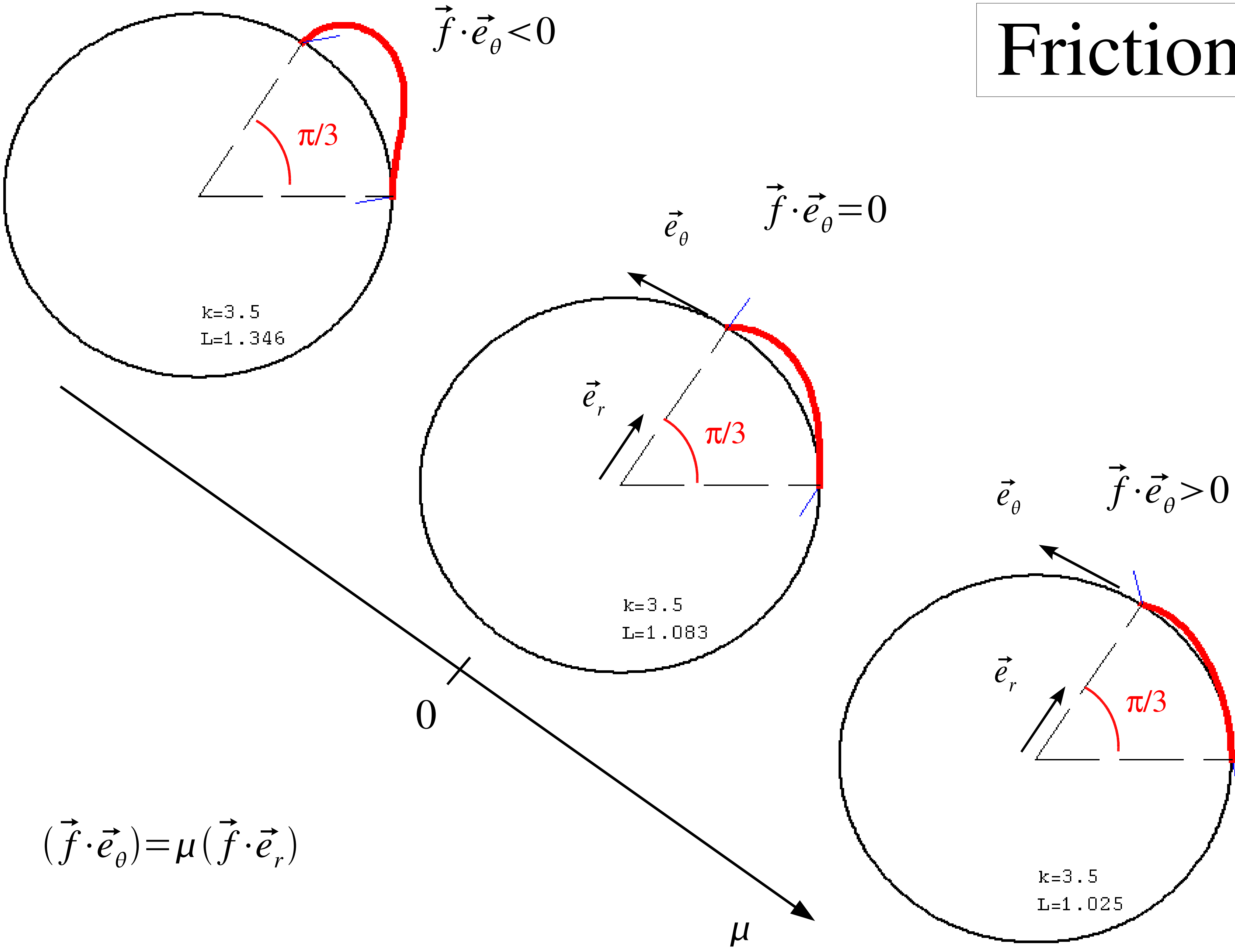
$$(\vec{f}\cdot\vec{e}_\theta)=\mu(\vec{f}\cdot\vec{e}_r)$$

Friction



$$(\vec{f}\cdot\vec{e}_\theta) = \mu(\vec{f}\cdot\vec{e}_r)$$

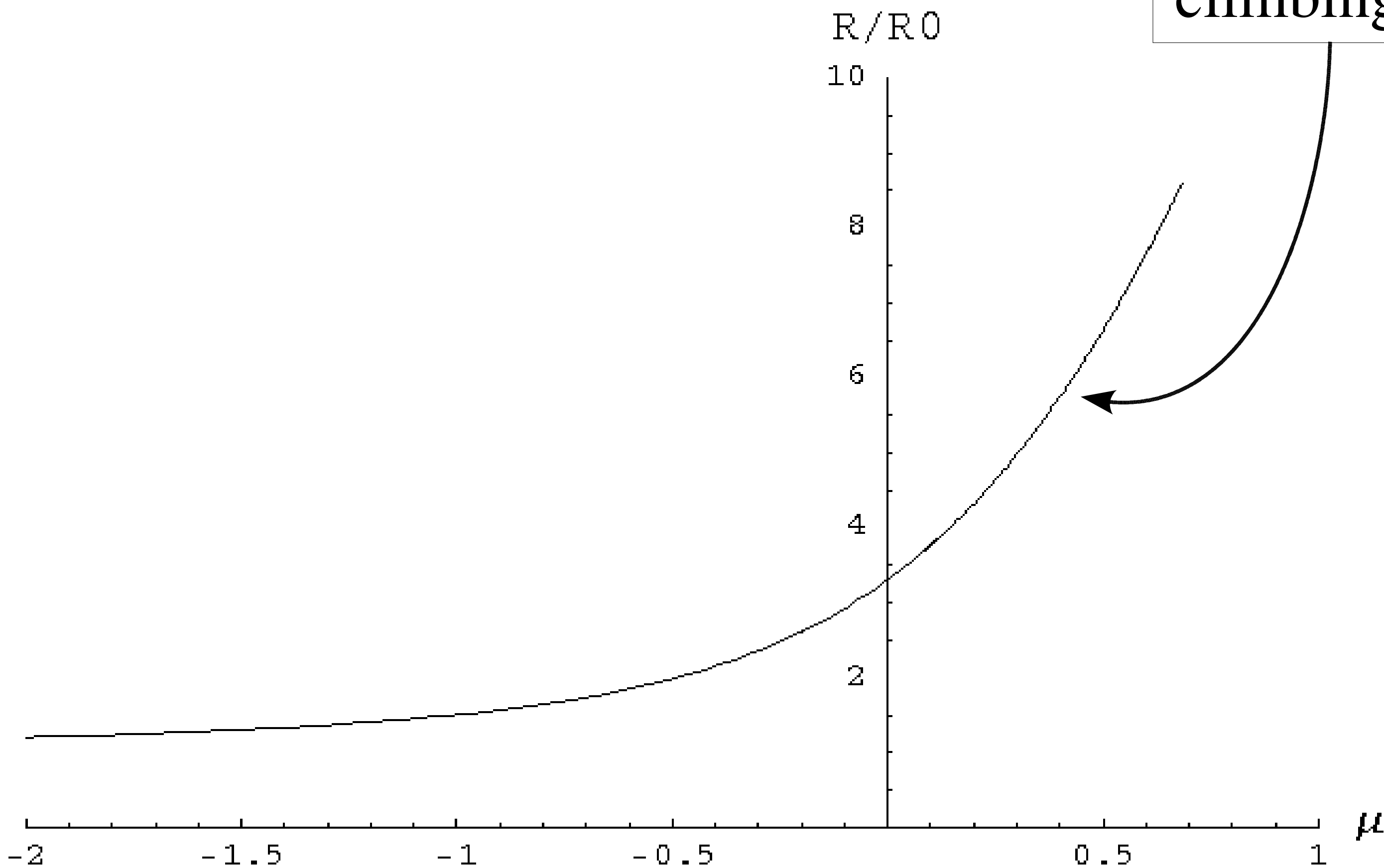
Friction



$$(\vec{f}\cdot\vec{e}_\theta)=\mu(\vec{f}\cdot\vec{e}_r)$$

Friction

friction 'helps' climbing habit



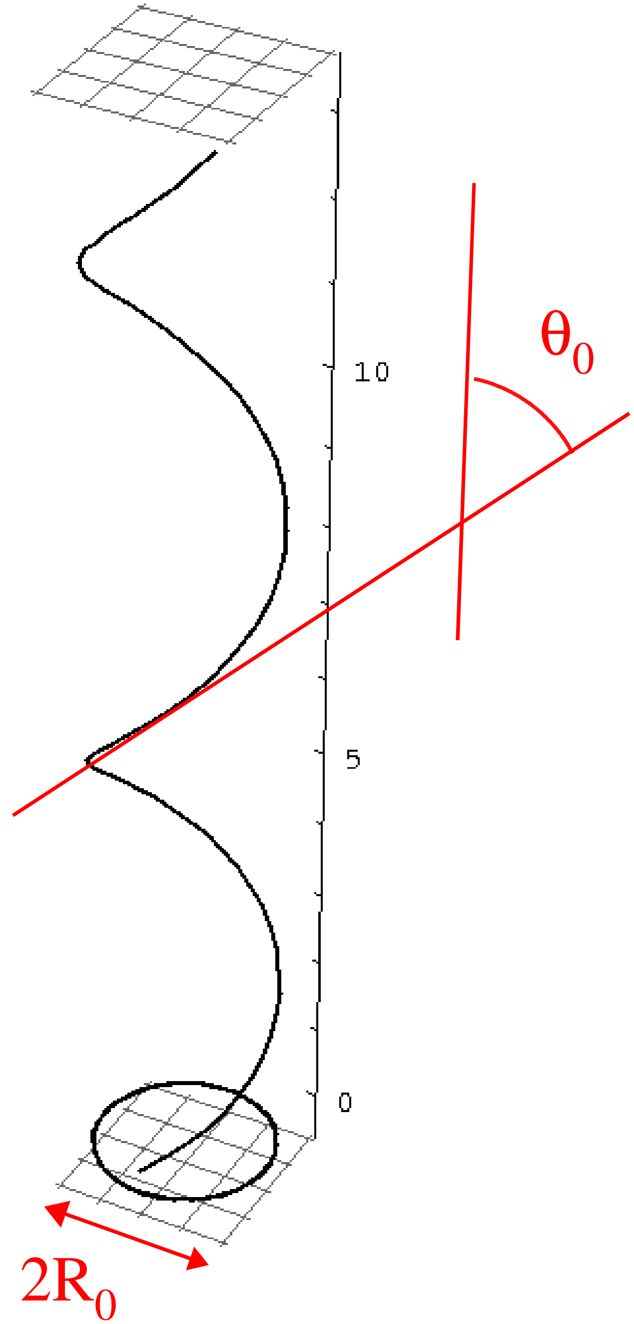
The 3D case

R : cylindrical support radius

R_0 : natural (intrinsic) radius of curvature

θ_0 : natural (intrinsic) helical angle

nearly helical solutions : climbing angle θ



-climbing angle $\theta = \theta(R_0, \theta_0, R)$

-contact pressure $P = P(R_0, \theta_0, R)$

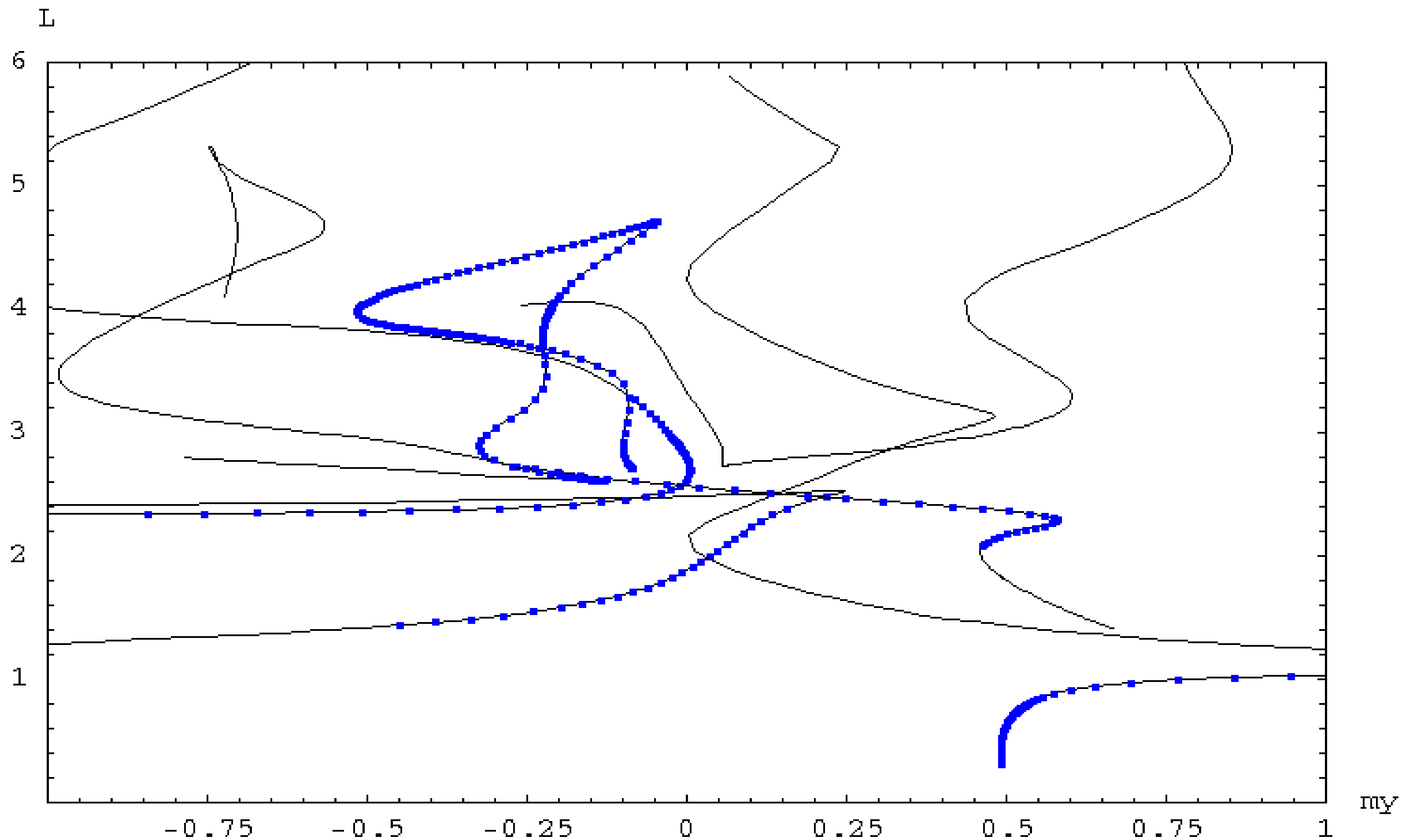
-limit $K_{max} = \dfrac{R}{R_0} = K_{max}(\theta_0)$

The 3D case : a bifurcation diagram

$$\theta_0 = 1.4 < \frac{\pi}{2}$$

$$\frac{R}{R_0} = 3$$

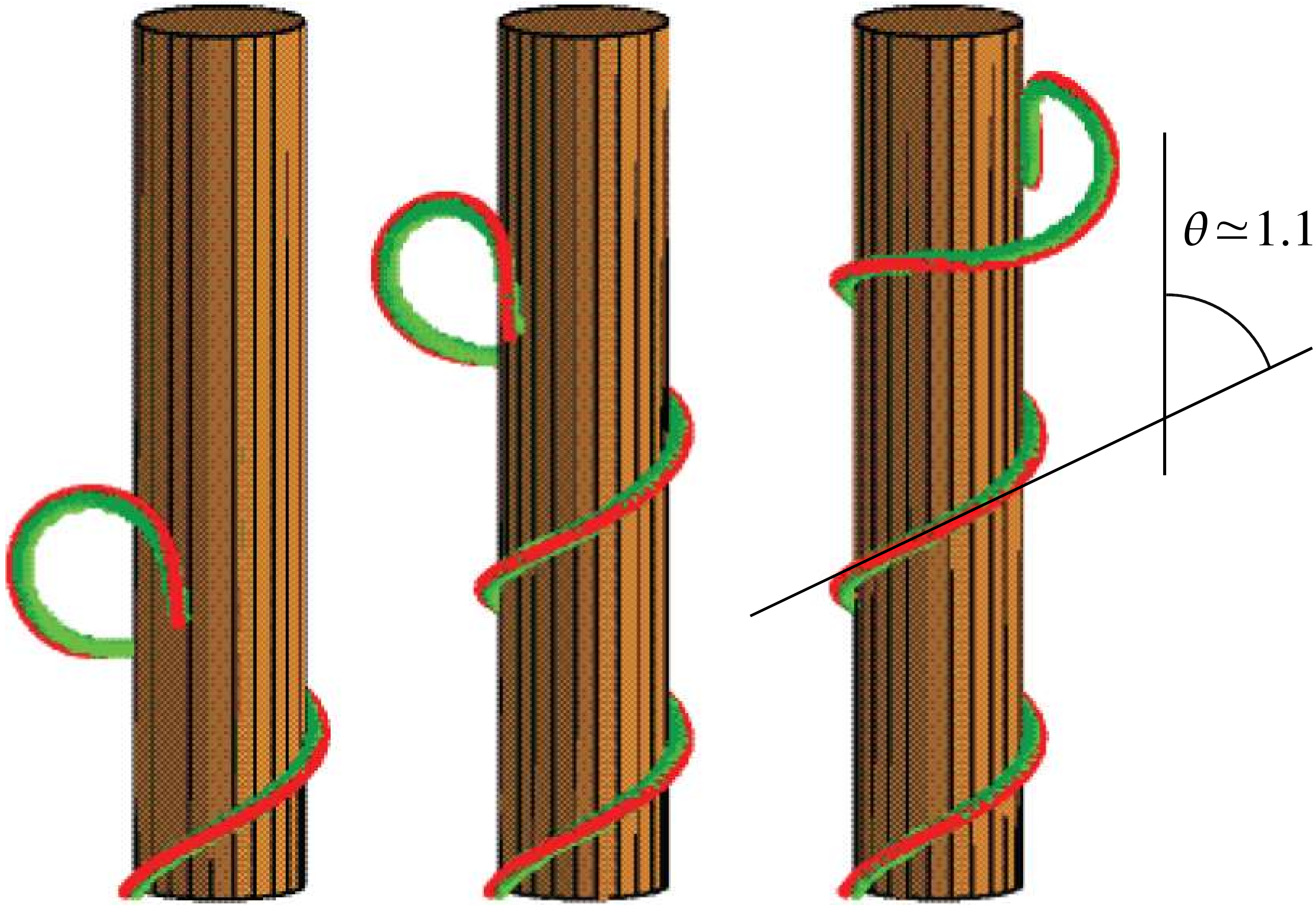
no “climbing” configurations
=> K_{max} decreases in 3D



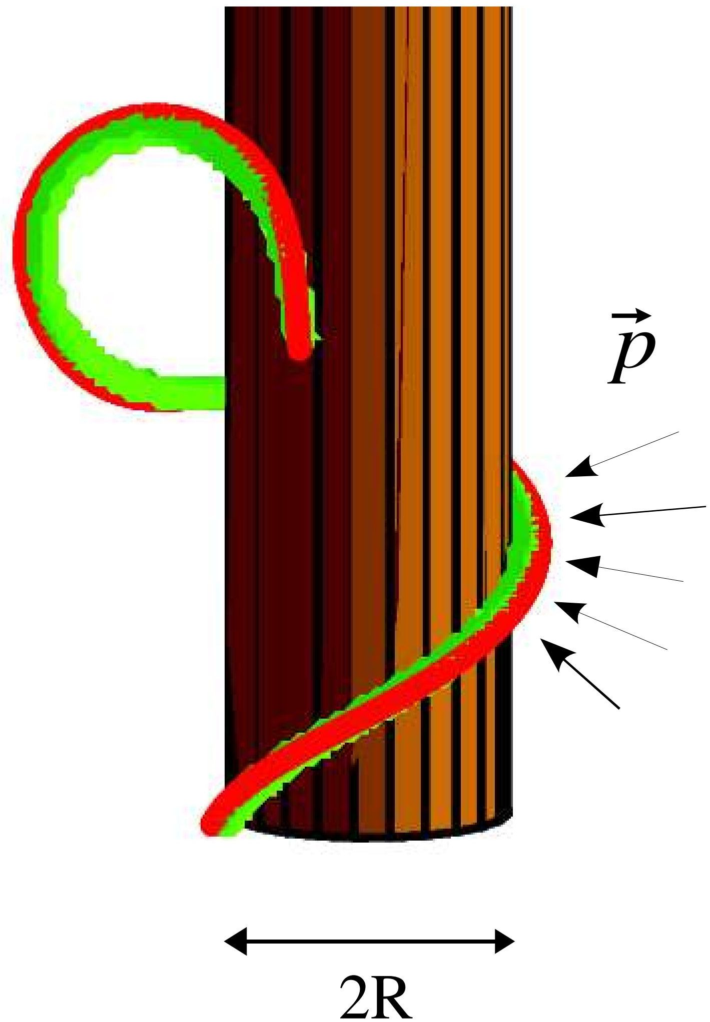
Shapes in 3D

$\theta_0 = 0.8$

$\frac{R}{R_0} = 2$



Contact pressure

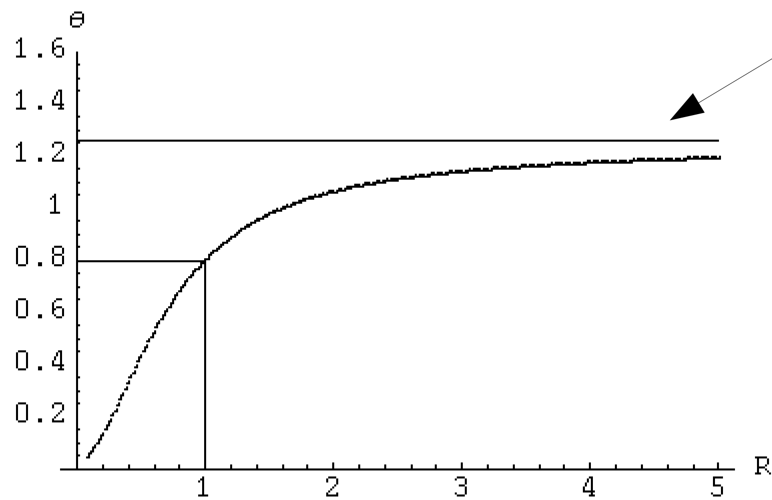


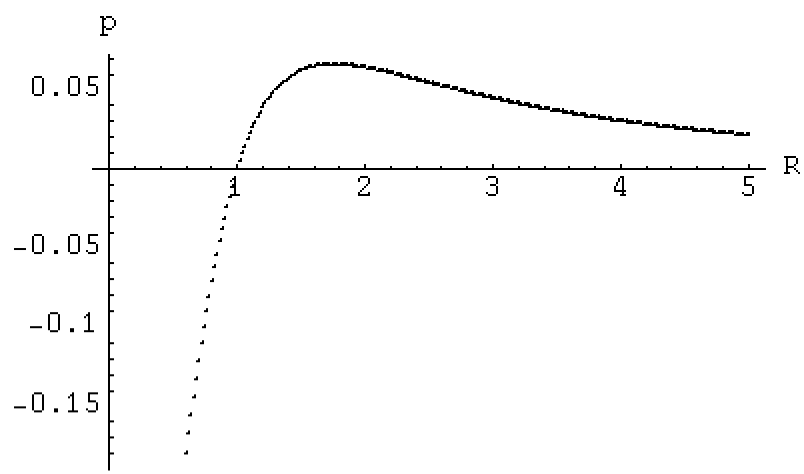
Shape and Pressure vs cylindrical radius

$\theta_0 = 0.8$

$R_0 = 1$

$$\tilde{\theta} = \frac{\theta_0}{2} + \frac{\pi}{2}$$





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