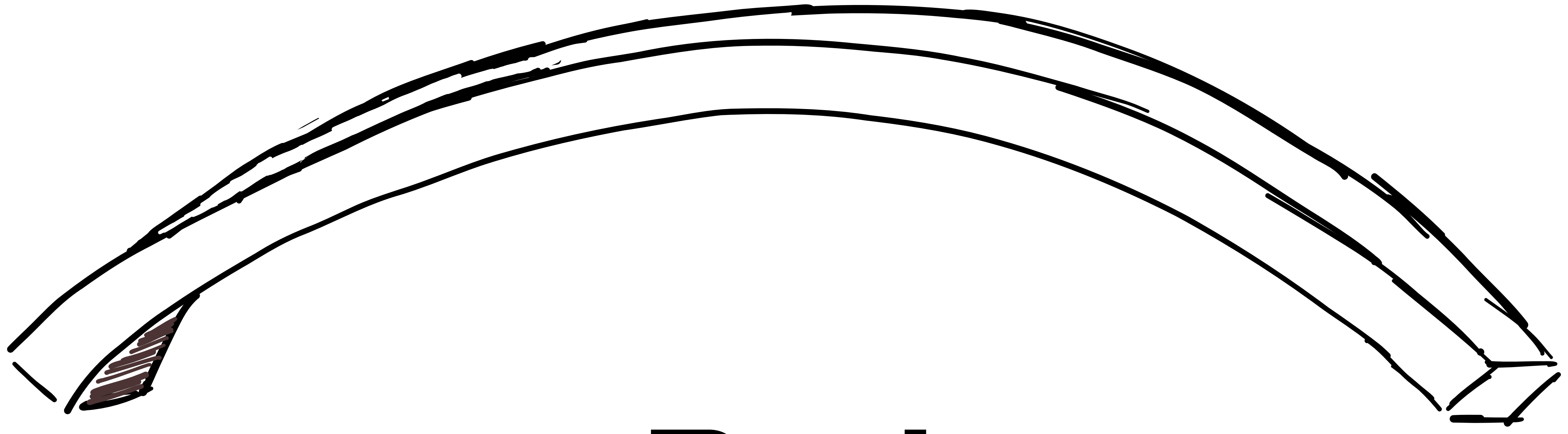
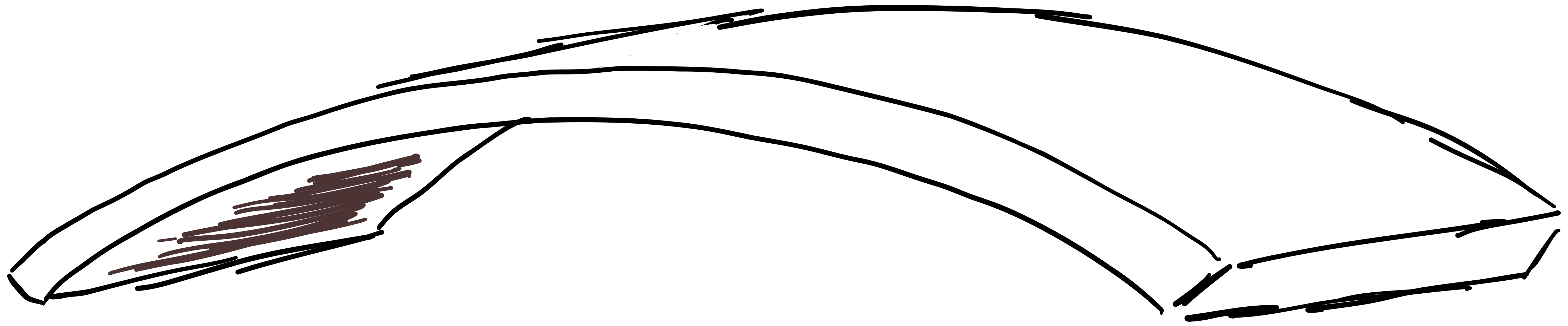
Benoit Roman's Question (2004)

$$M = B\,\kappa$$

torque **rigidity** **curvature**



Rod



Plate

$$B = EI$$

$$B = \frac{EI}{1-\nu^2}$$

Shield. Quart. J. Mech. & Applied Math. 1992

Ribbons are extensible

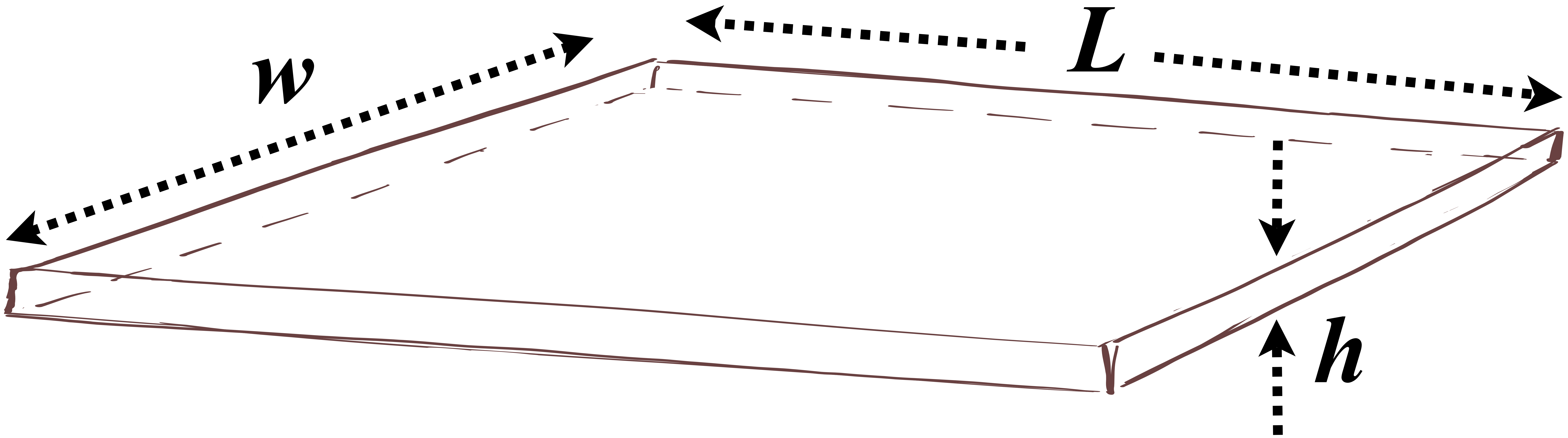
talking	Sebastien Neukirch (CNRS & Sorbonne University, FR)
theory	Basile Audoly (CNRS & Ecole Polytechnique, FR)
numerics	Florence Bertails (INRIA, FR) **Raphael Charrondiere (INRIA, FR) - PhD**

Acknowledgments:

Initial remark of *Paul Grandgeorge* (EPFL, CH & U. of Washington , USA)
Experiments of *Victor Romero* (INRIA, FR)
Help with FEniCS-Shell from *Corrado Maurini* (Sorbonne University, FR)
Abaqus black magic from *Arnaud Lazarus* (Sorbonne University, FR)

Rod vs Ribbon vs Plate

ODE ODE PDE



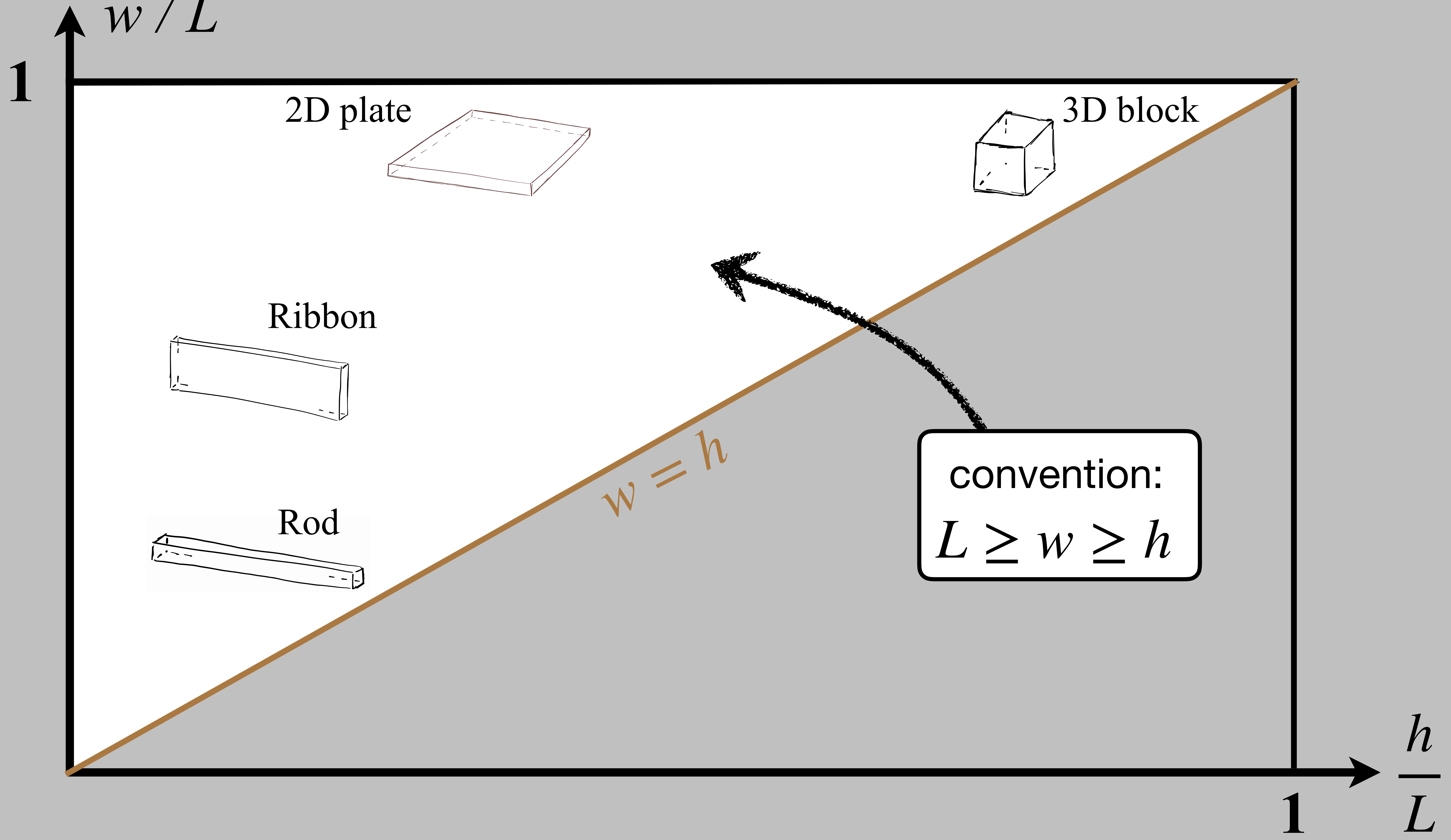
Rod $h \sim w \ll L$

Ribbon $h \ll w \ll L$

Plate $h \ll w \sim L$

convention:

$L \geq w \geq h$


w / L
1
2D plate
3D block
Ribbon
Rod
w = h
convention:
L ≥ w ≥ h
h / L
1

Ribbons at all length-scales

corse-grained **H bonds** **all atoms**

alpha-helix structure

0.54 nm

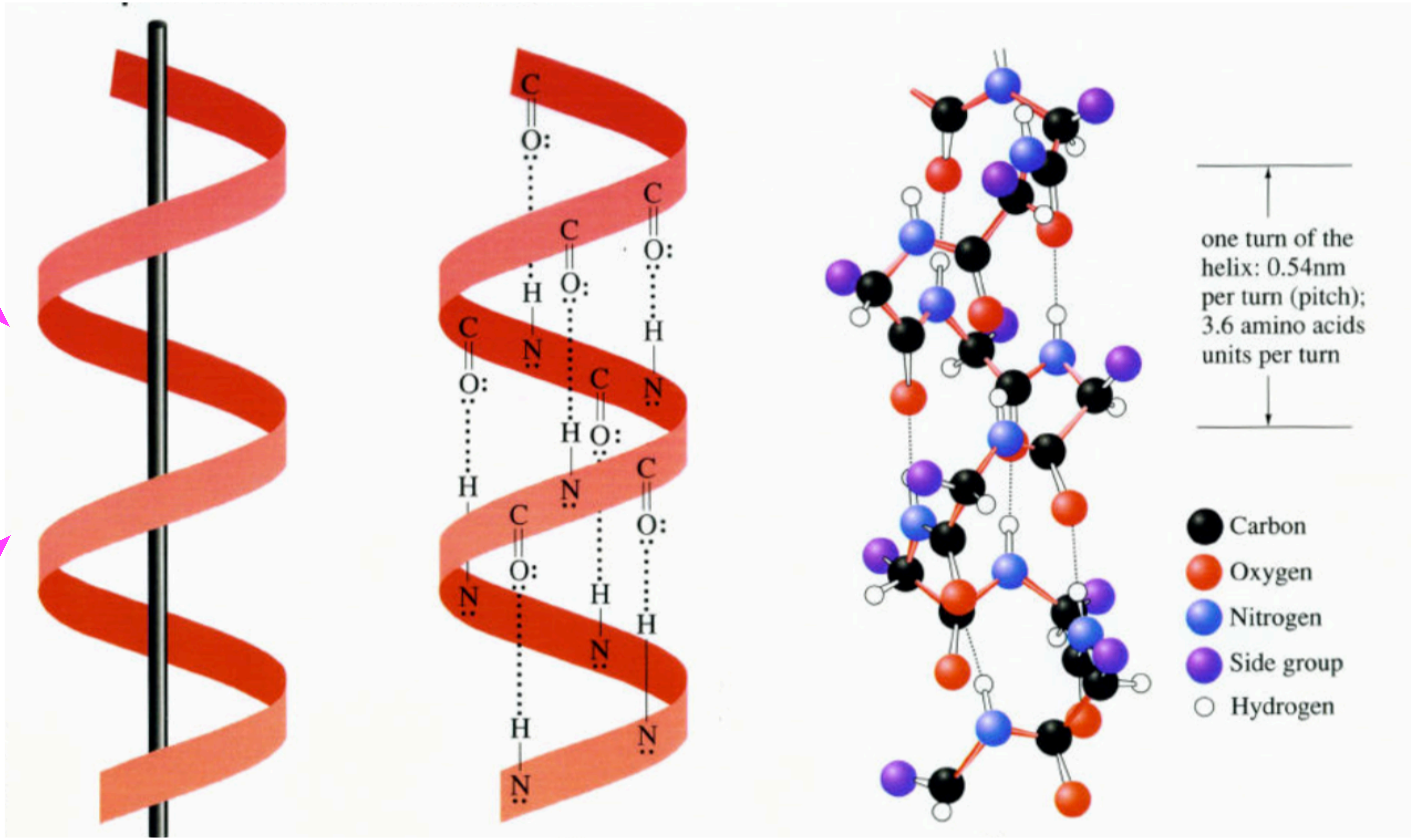
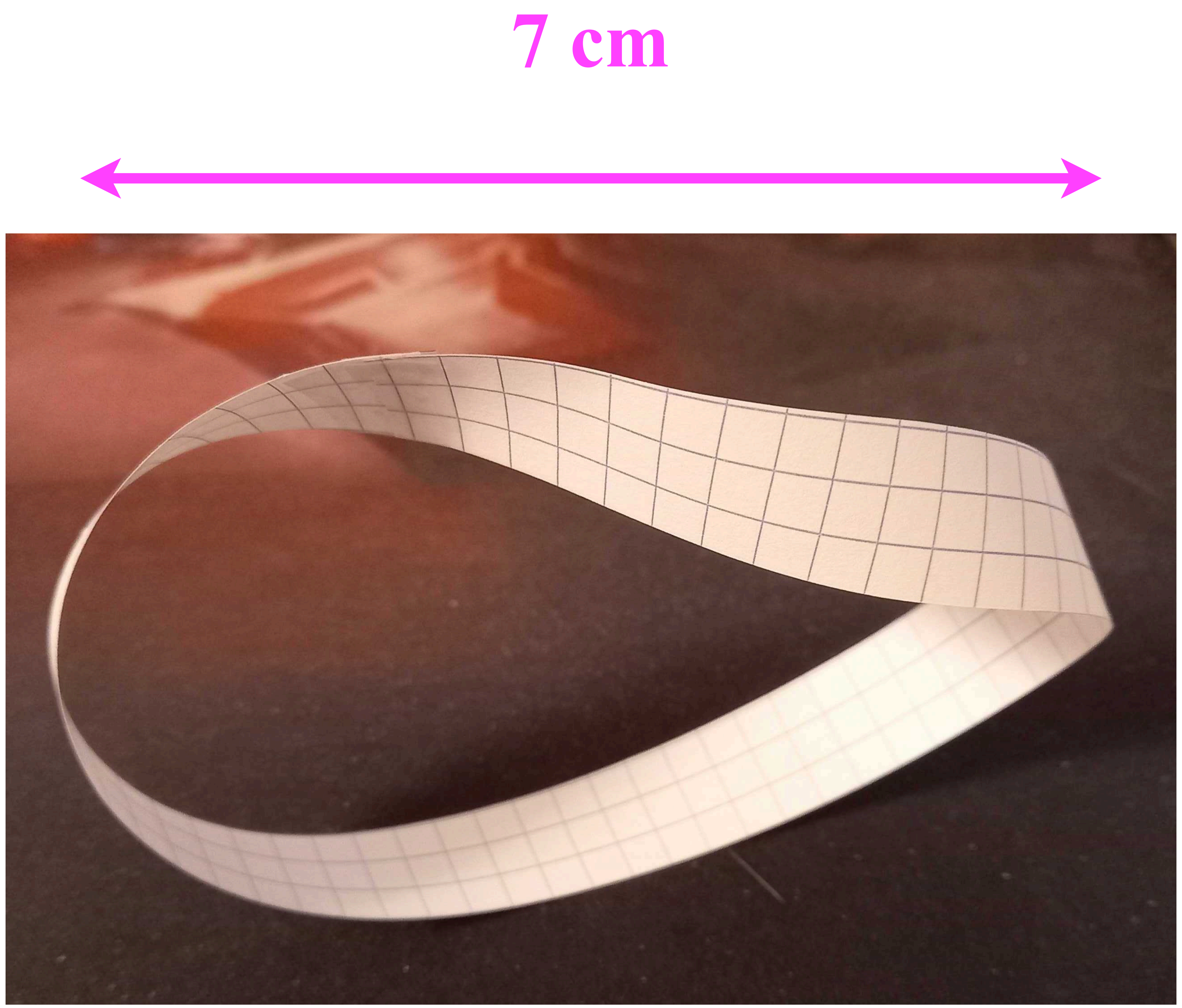


image:
Pr. C. Hrycyna
(Purdue Univ.)

Ribbons at all length-scales



Ribbons at all length-scales

Chebydesic Pavilion

1.5 m

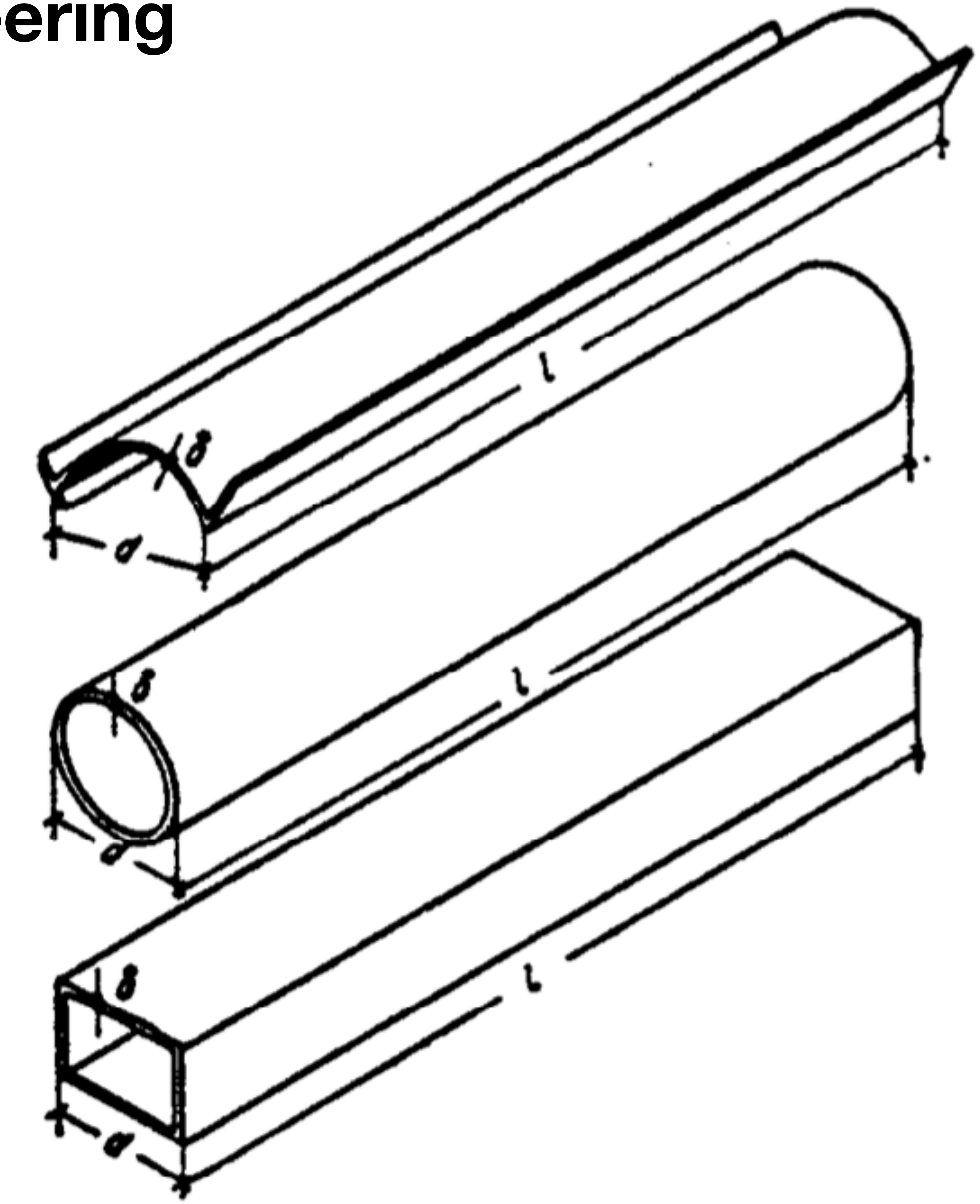
Pr. O Baverel
ENPC, FR
thinkshell.fr

Ribbons at all length-scales

Civil Engineering

V.Z. VLASOV

THIN-WALLED
ELASTIC
BEAMS



Ribbons at all length-scales

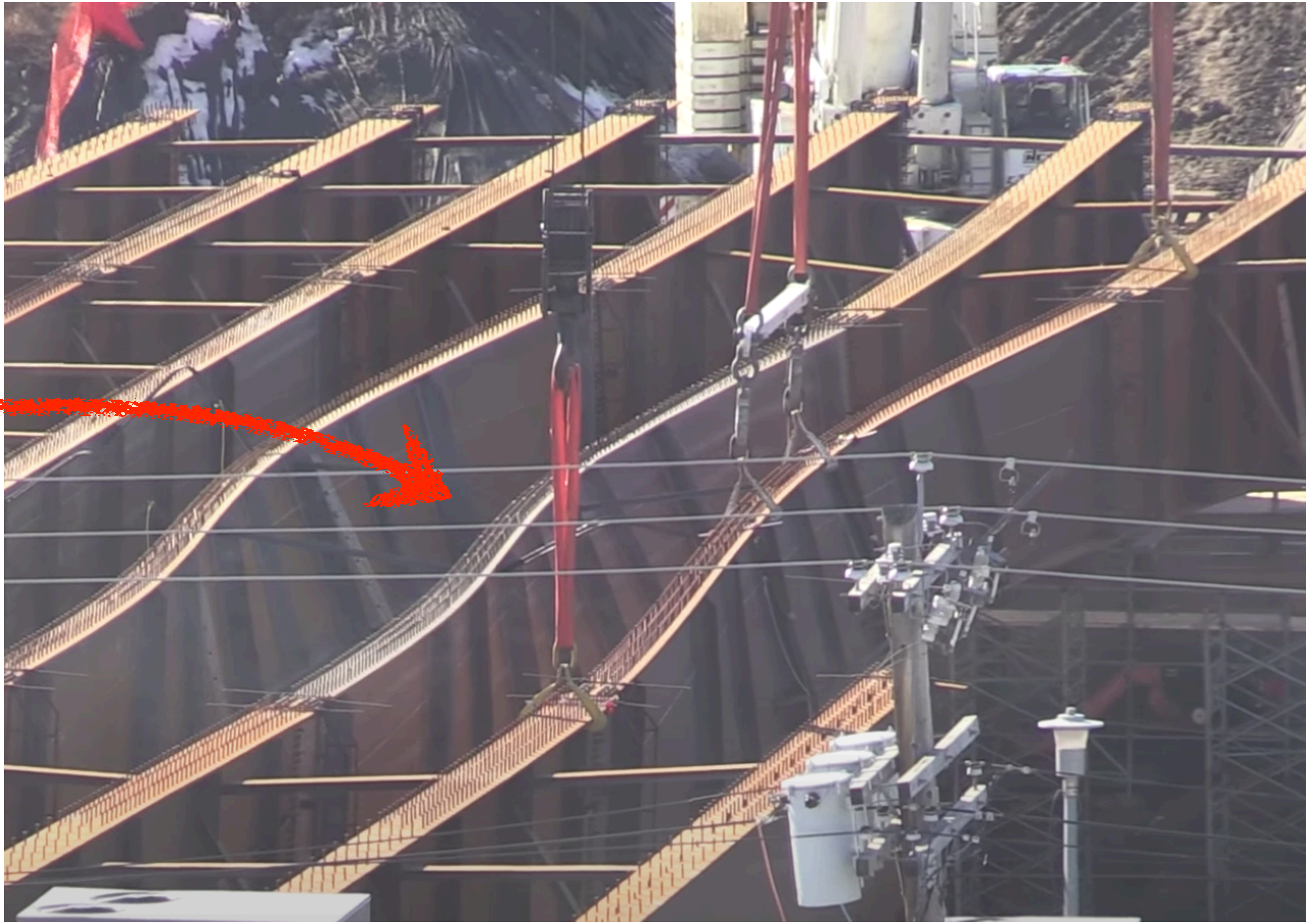
Edmonton bridge (Canada)

total
span
~ 100 m

image:
Bruce Edwards
Edmonton Journal

Ribbons at all length-scales

Edmonton bridge (Canada)



3 buckled girders

March 2015

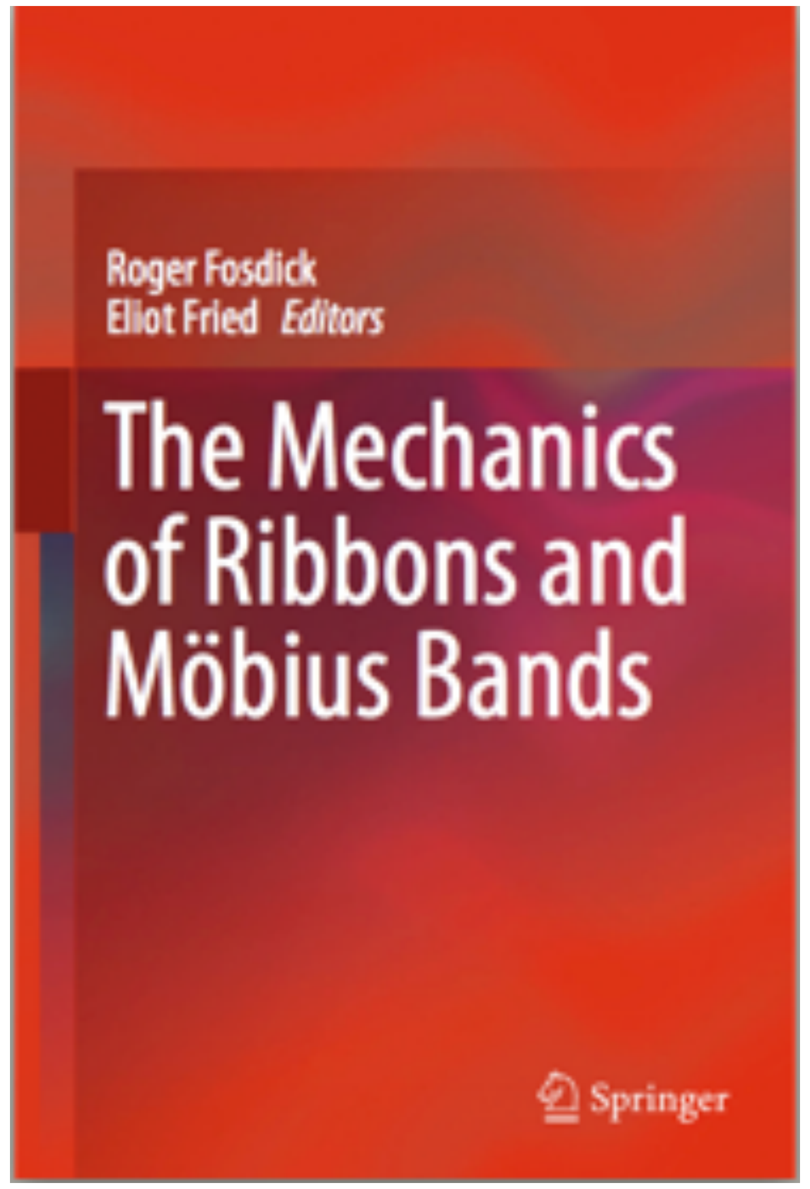
image:
Bruce Edwards
Edmonton Journal

Up to now: Inextensible Ribbon models

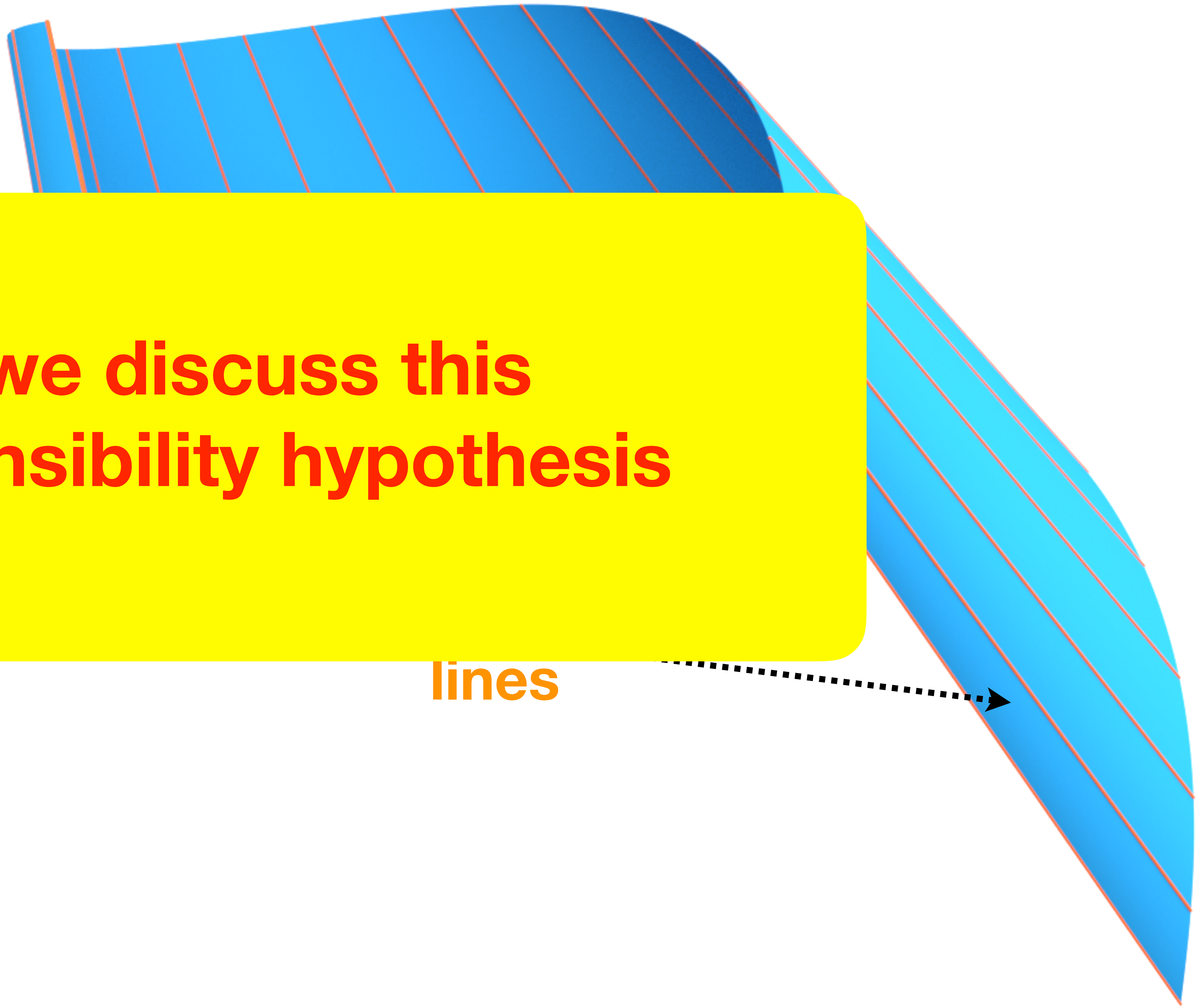
plate theory: extension is expensive

hypothesis: no extension at all
==> equilibrium: develo
==> presence of lines

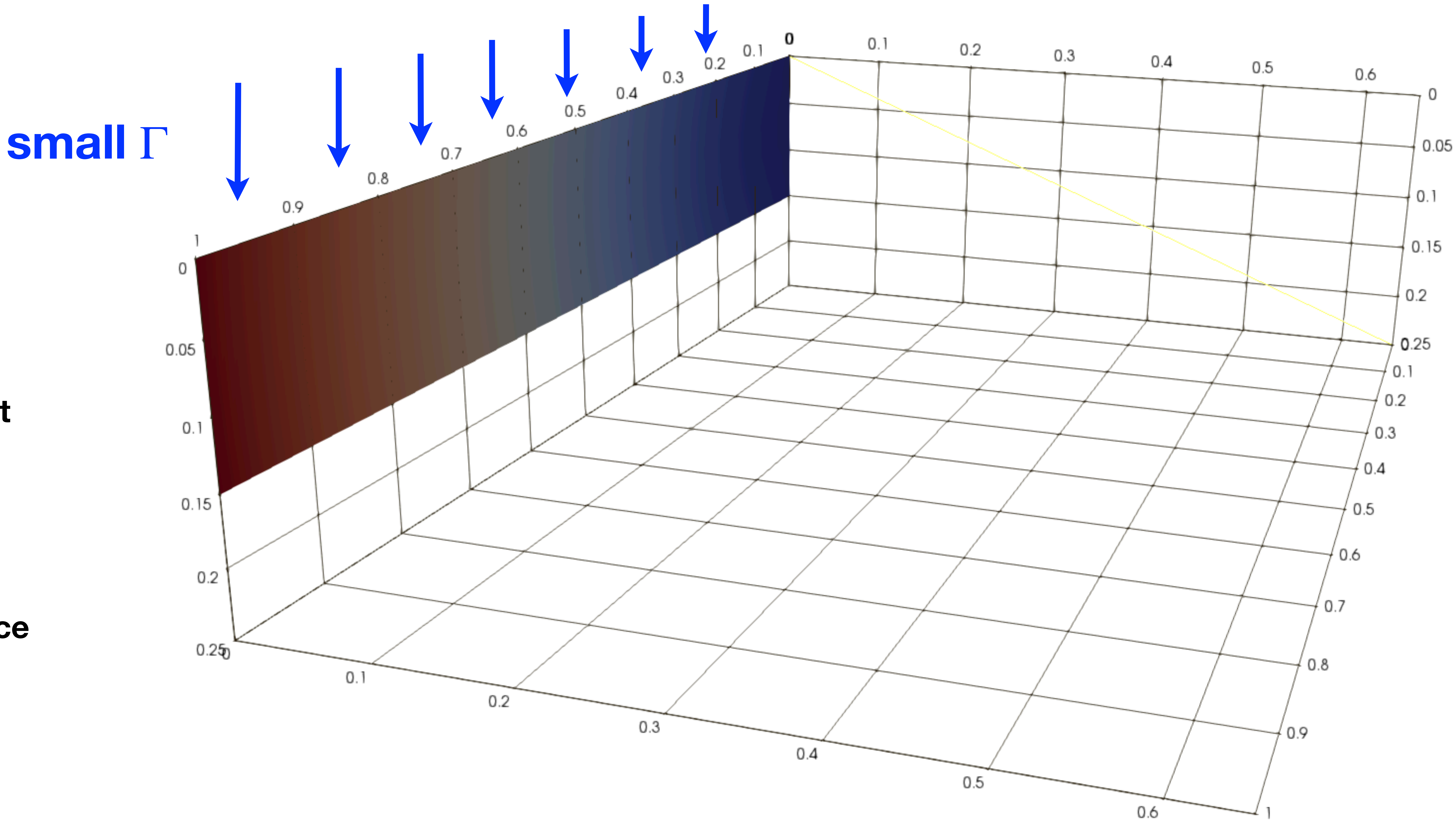
Here, we discuss this inextensibility hypothesis



1929 M.
1962 W.
2015 Journal of Elasticity
special issue edited by
R. Fosdick & E. Fried



Lateral Torsional Buckling



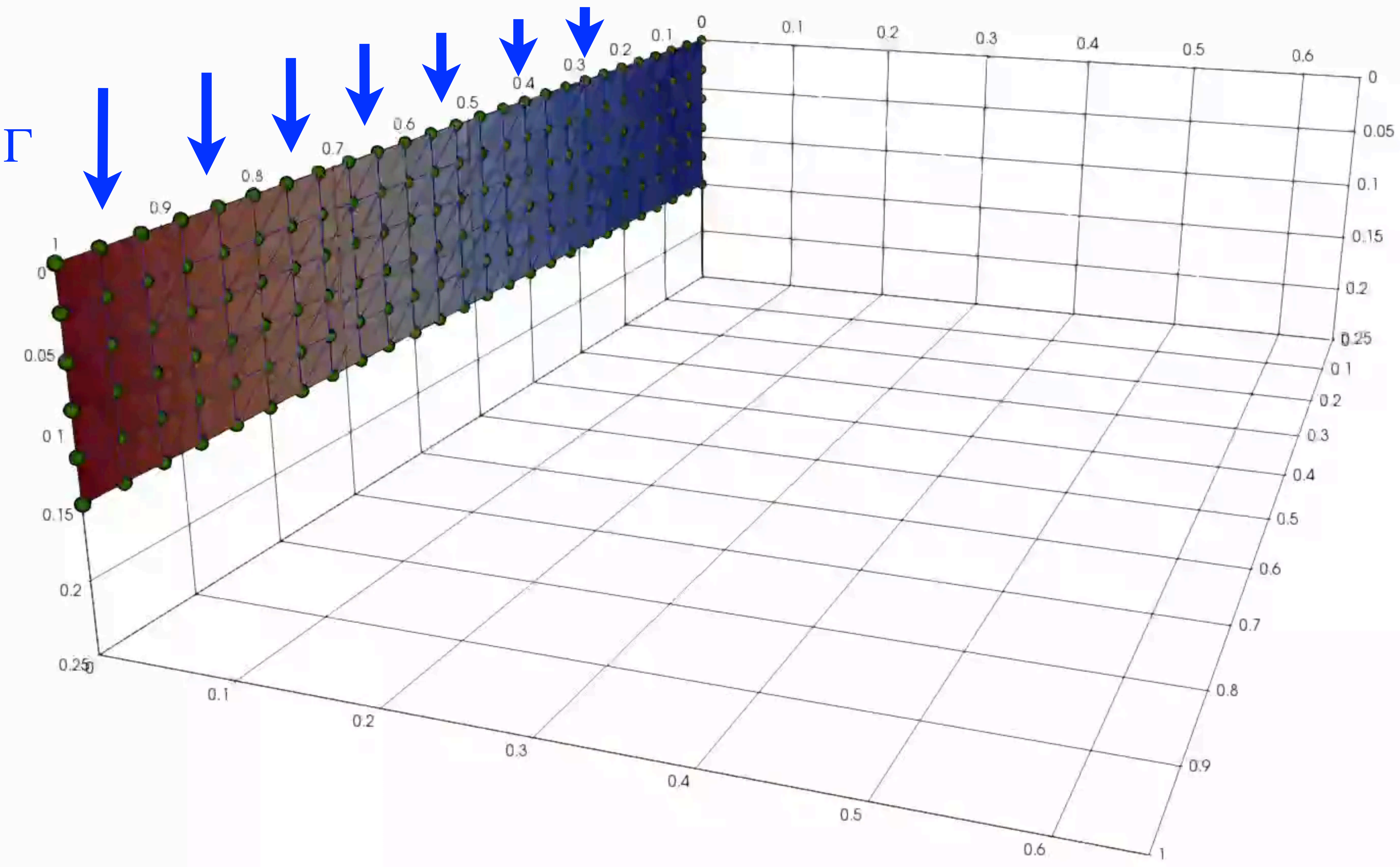
total weight

$$\Gamma = \frac{M g}{D w / L^2}$$

buckling force

Lateral Torsional Buckling

large Γ



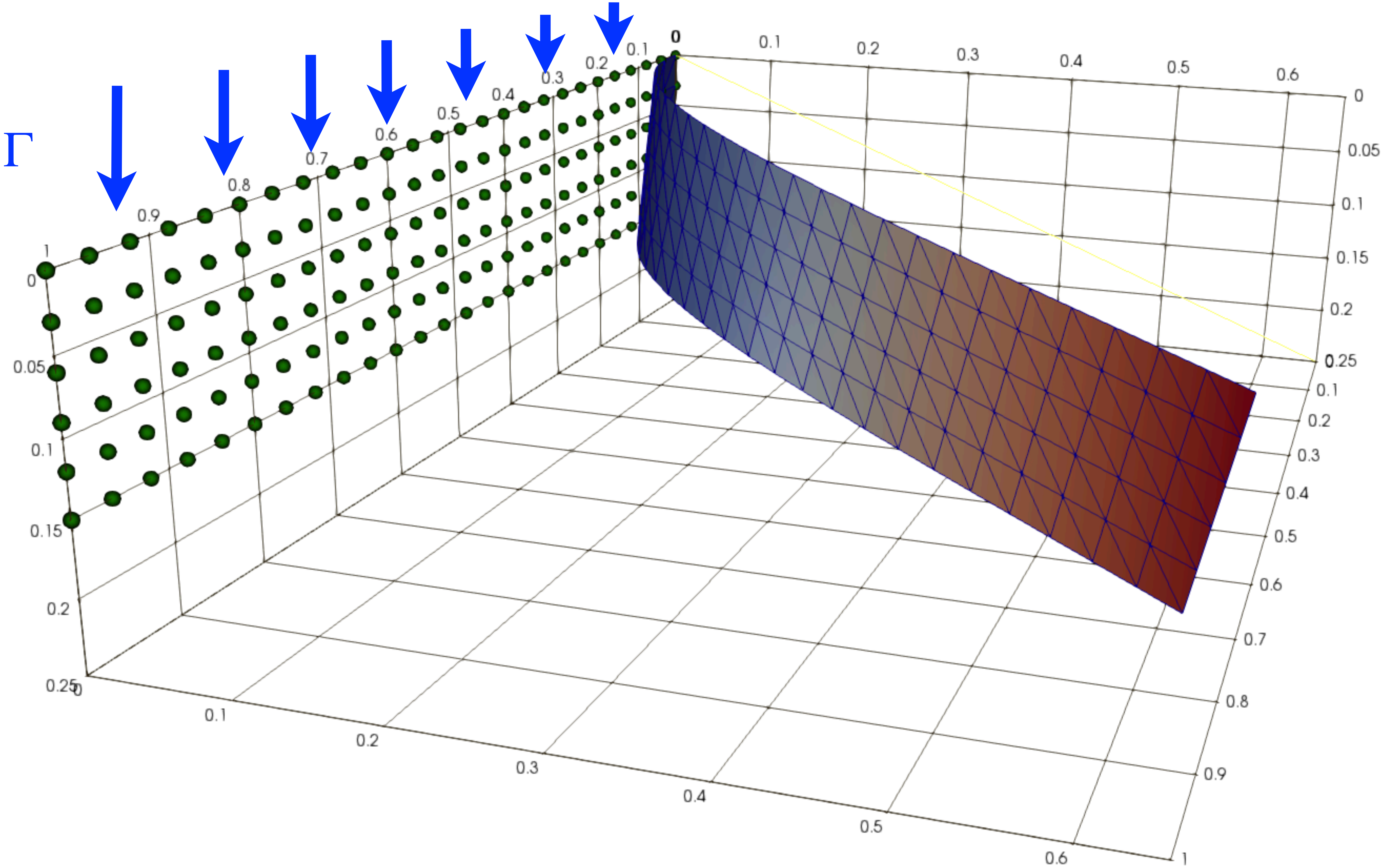
total weight

$$\Gamma = \frac{M\,g}{D\,w/L^2}$$

buckling force

Lateral Torsional Buckling

large Γ



total weight

$$\Gamma = \frac{M\,g}{D\,w/L^2}$$

buckling force

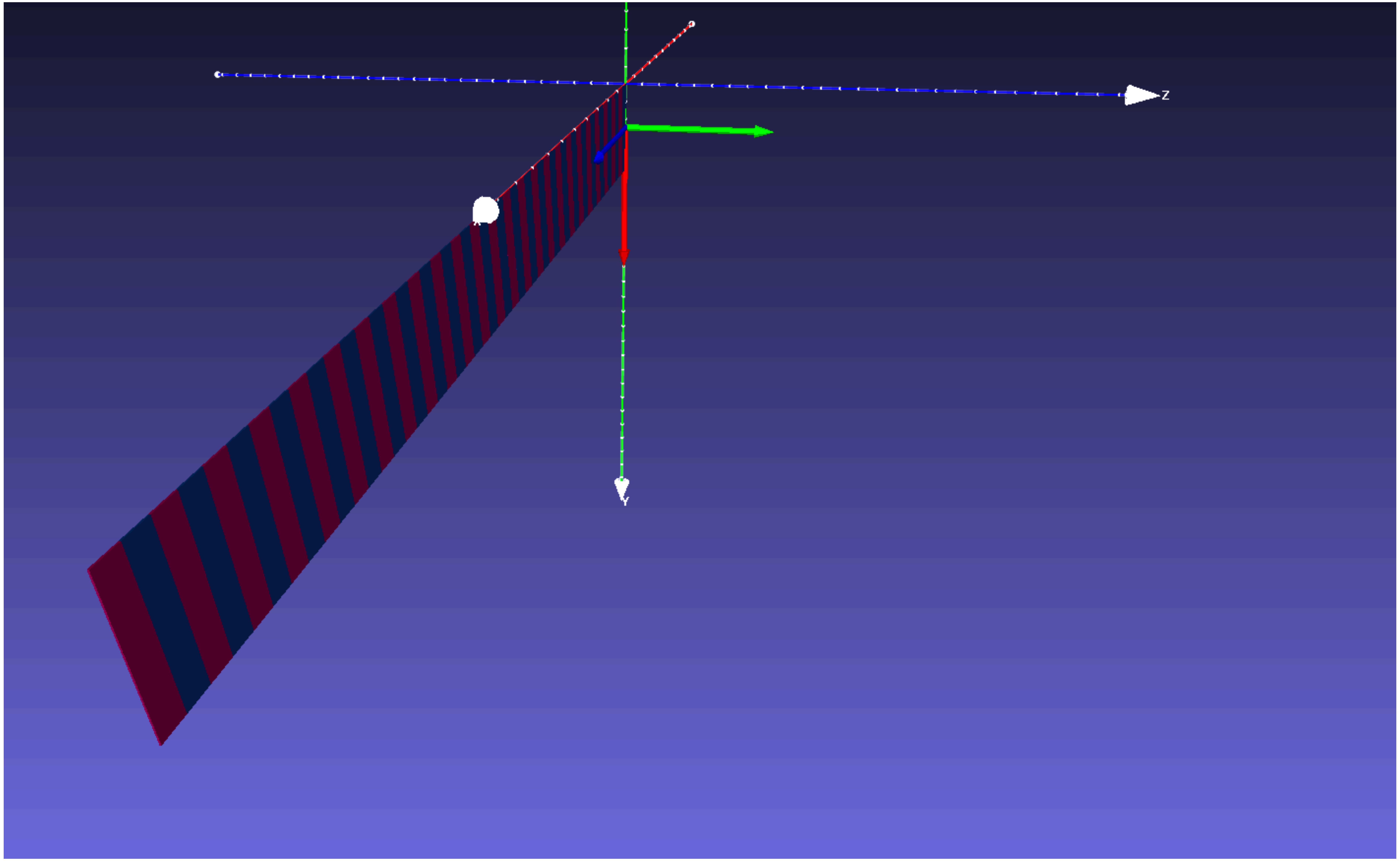
Lateral Torsional Buckling

$w/L = 0.1$

$h/L = 0.001$

$\nu = 0.35$

$\Gamma_{\textbf{buck}} = 15$



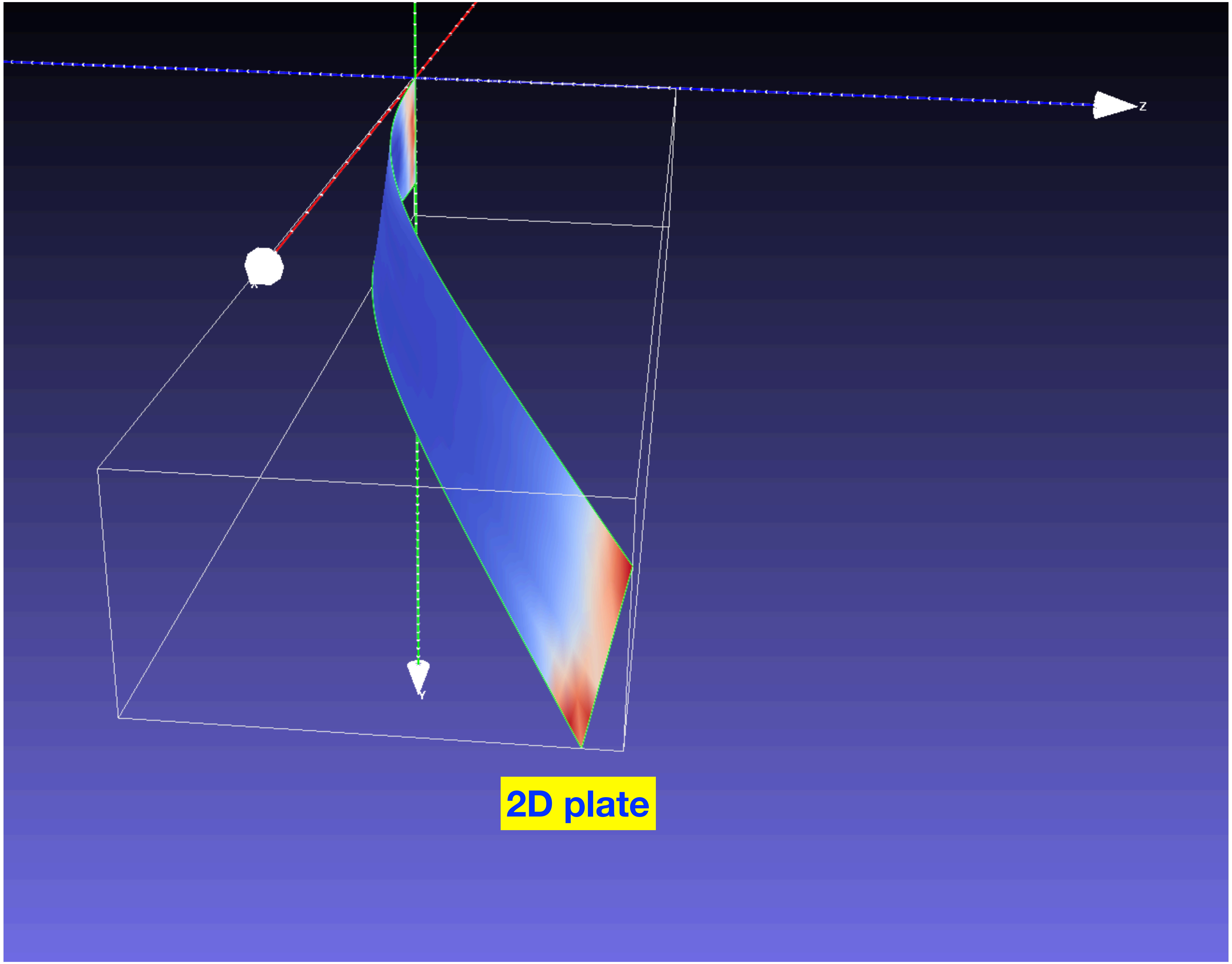
Lateral Torsional Buckling

$w/L = 0.1$

$h/L = 0.001$

$\nu = 0.35$

$$\Gamma = \frac{M\,g}{D\,w/L^2} = 20$$



$\eta(0) = 0$

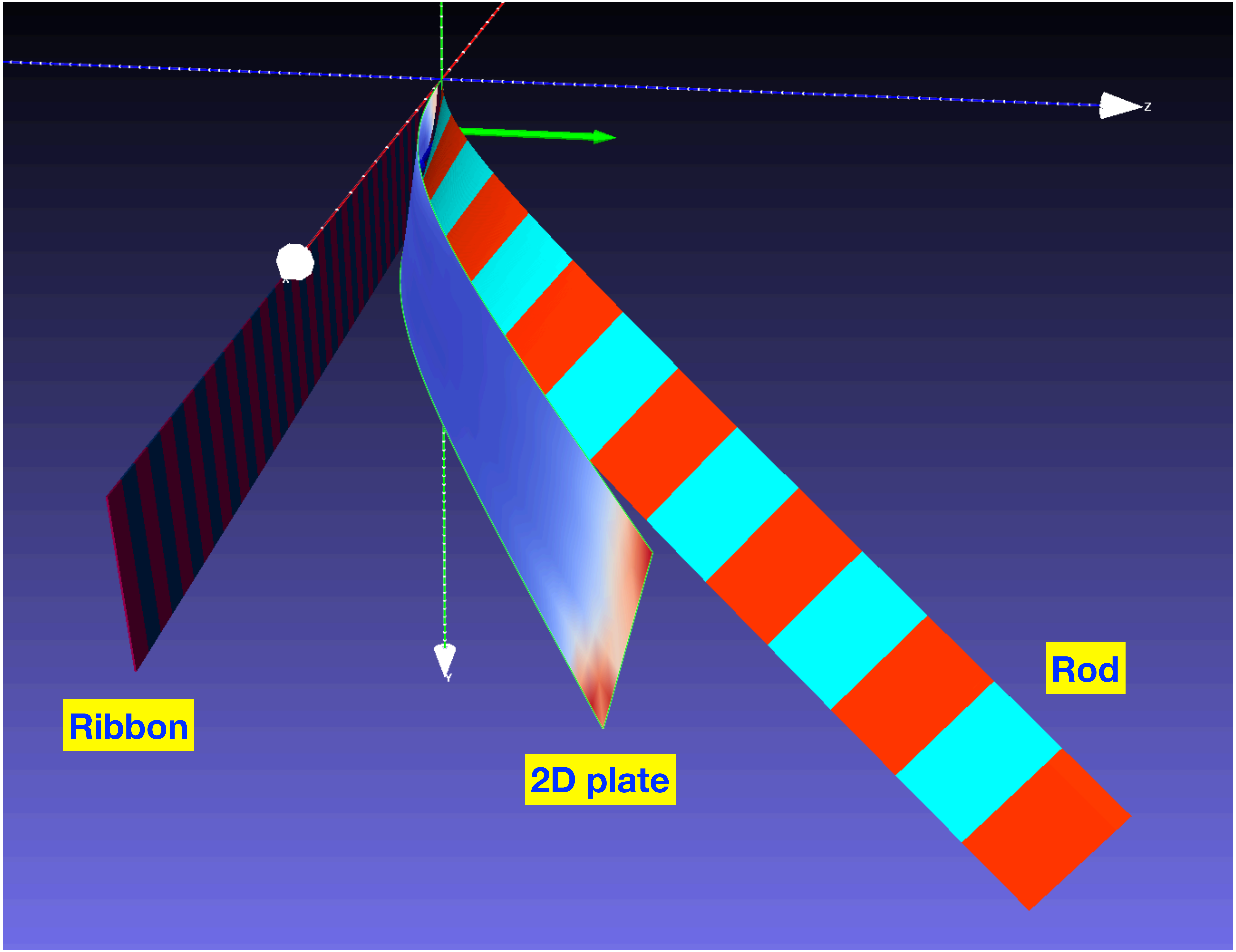
Lateral Torsional Buckling

$w/L = 0.1$

$h/L = 0.001$

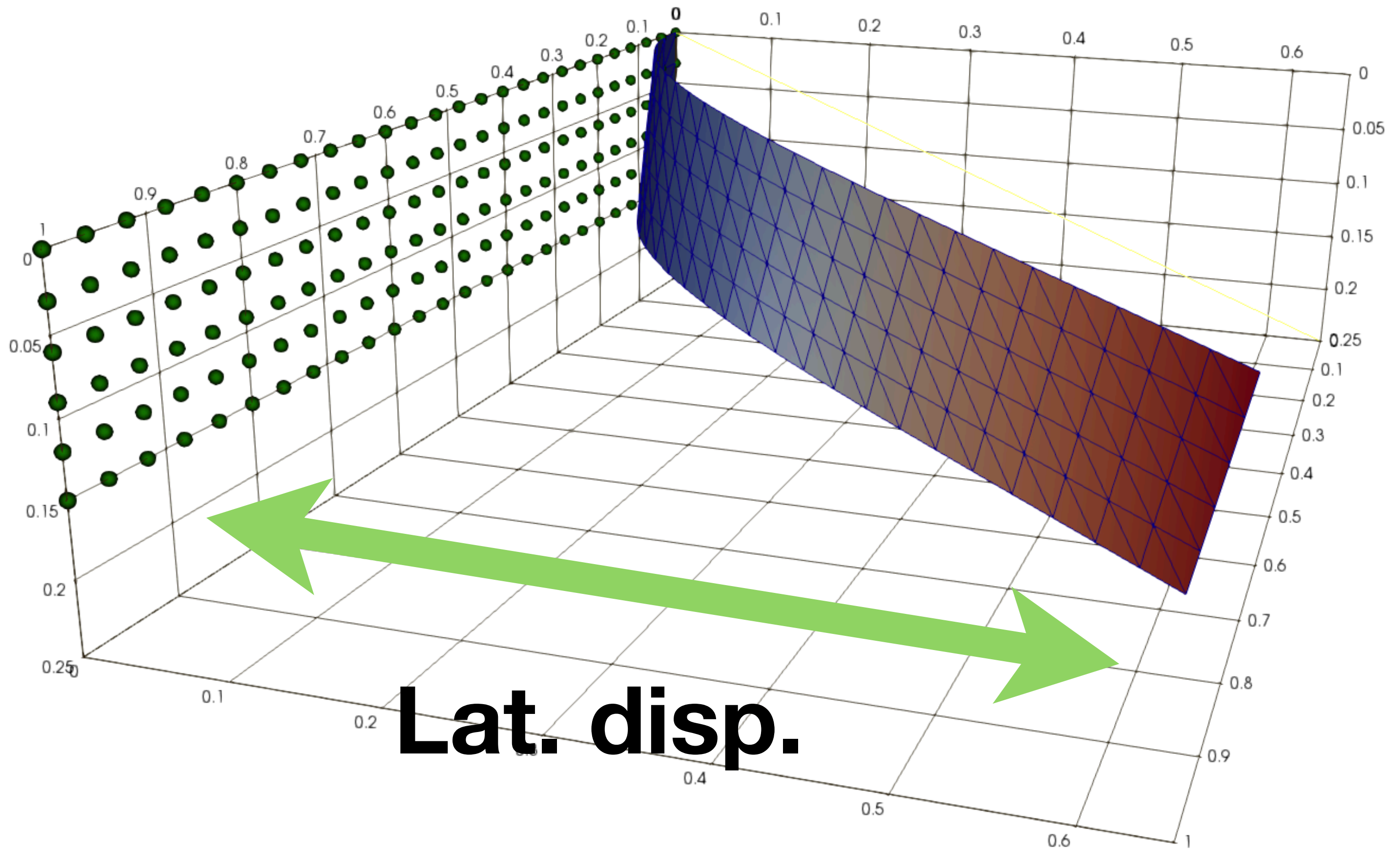
$\nu = 0.35$

$$\Gamma = \frac{M\,g}{D\,w/L^2} = 20$$



Lateral Torsional Buckling

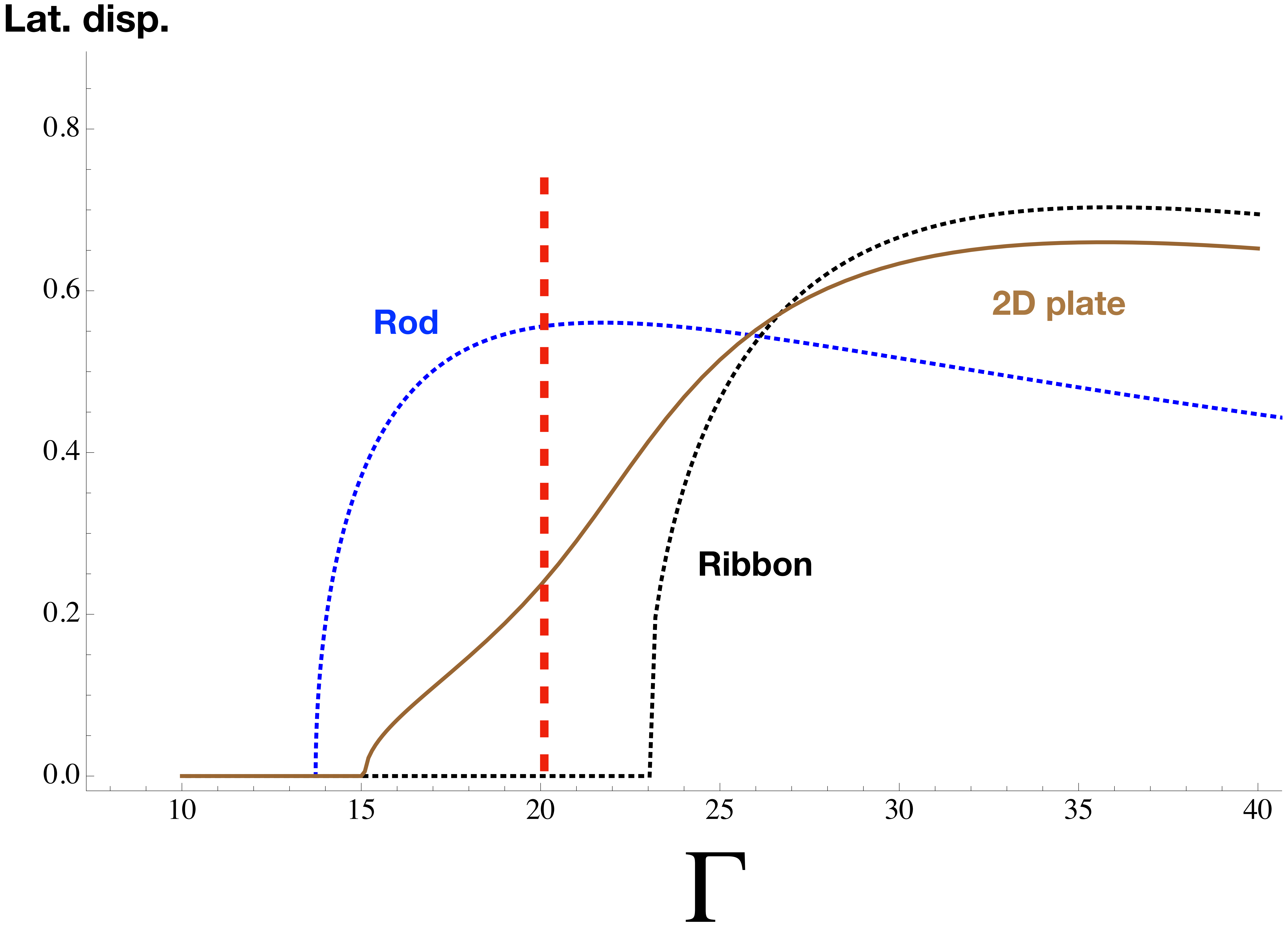
$\eta(0) = 0$



$w/L = 0.1$

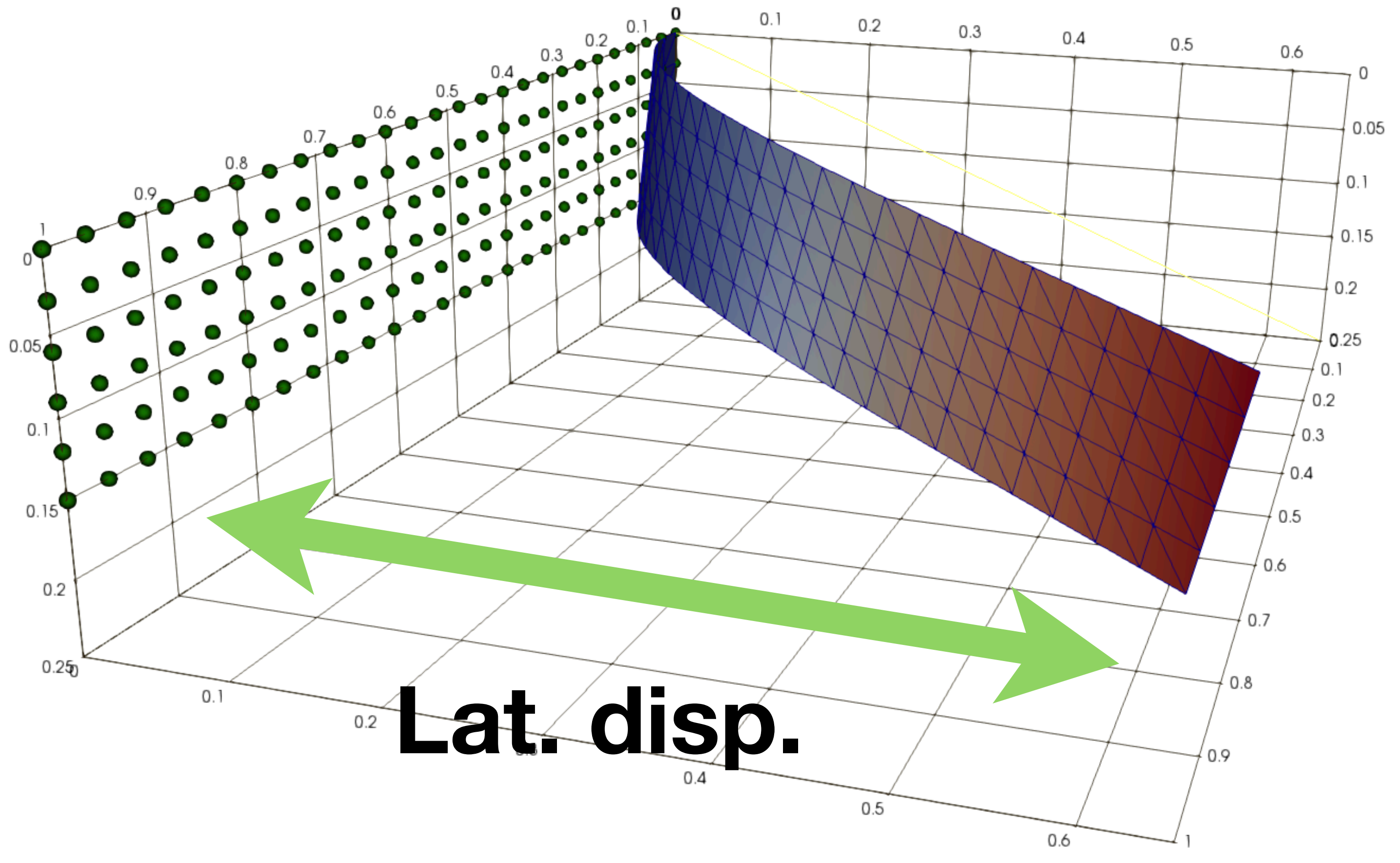
$h/L = 0.001$

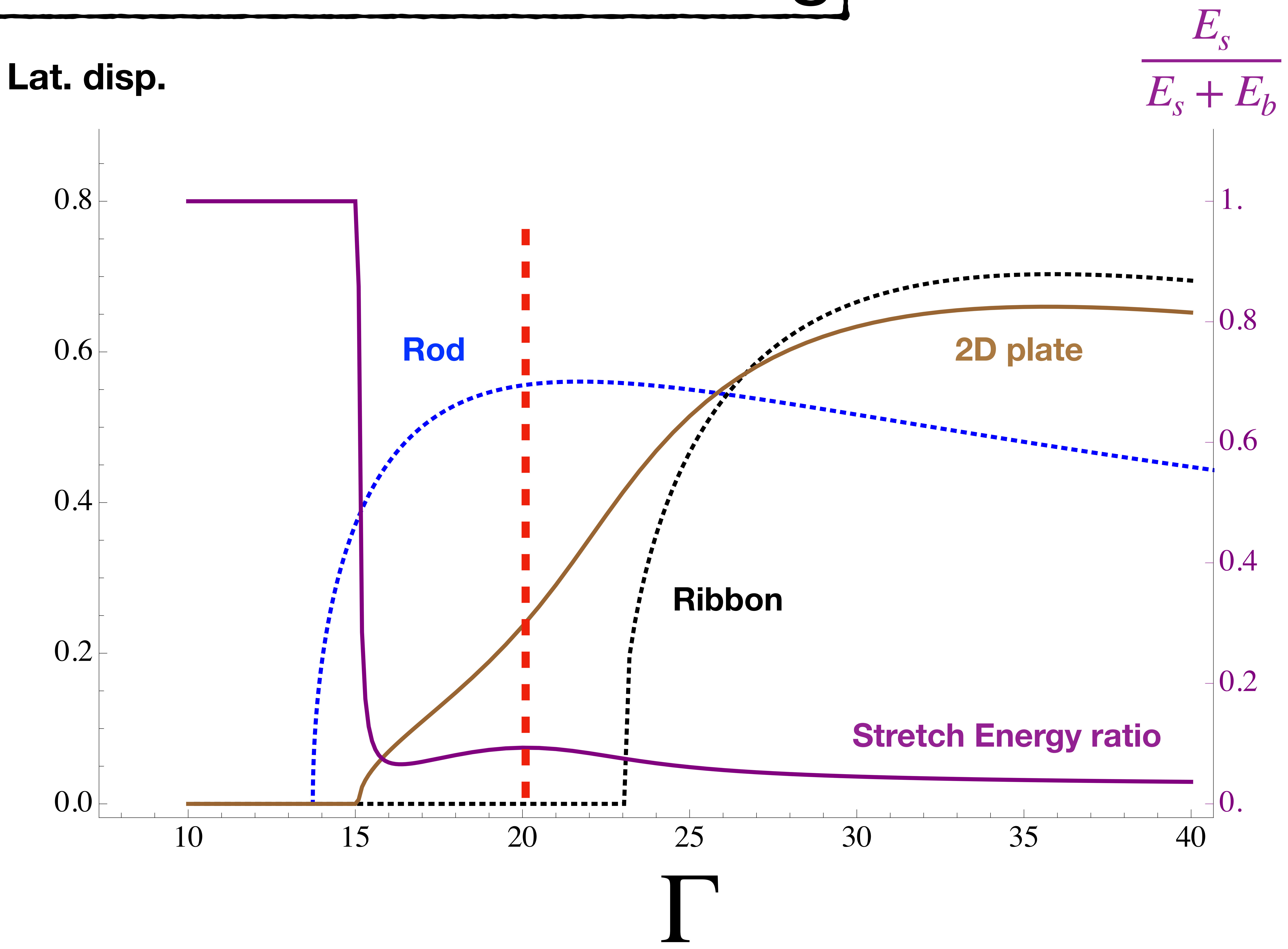
$\nu = 0.35$



Lateral Torsional Buckling

$\eta(0) = 0$





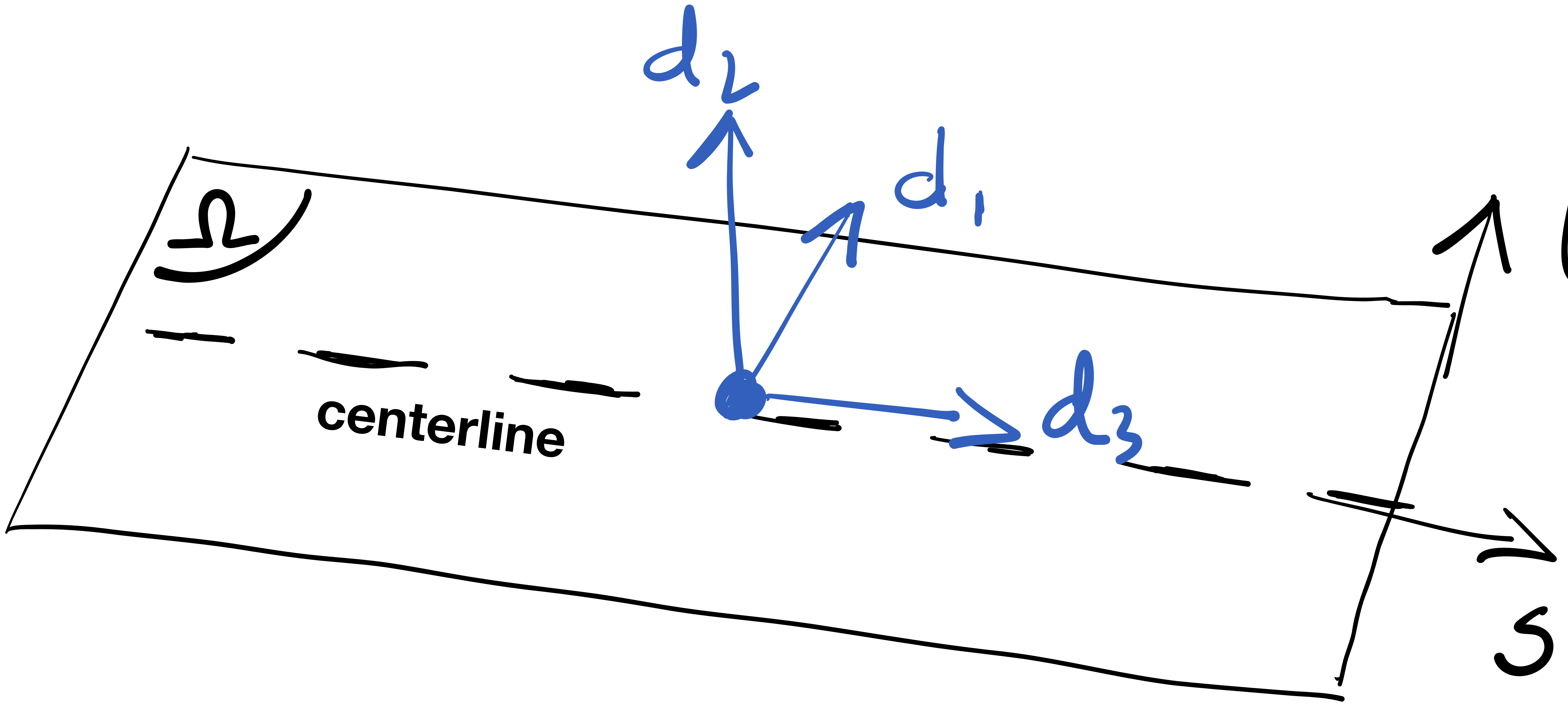
$w/L = 0.1$

$h/L = 0.001$

$\nu = 0.35$

The Ribext model

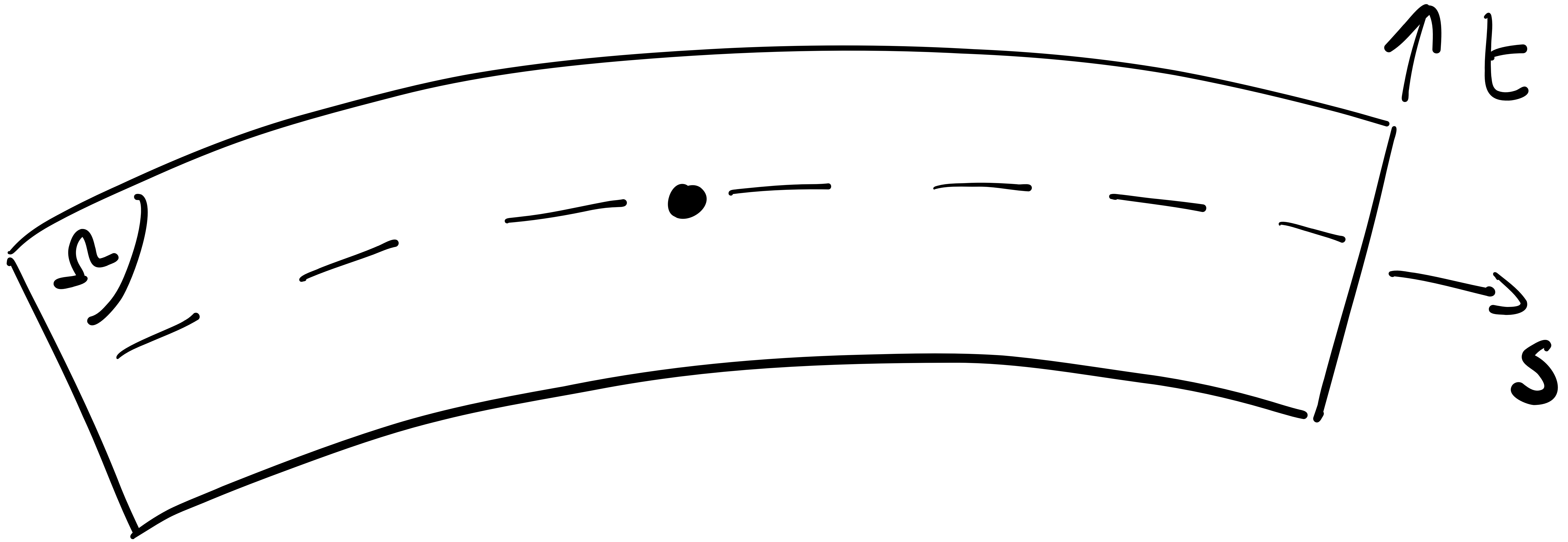
Ribbon **with** **extension**



start with plate model

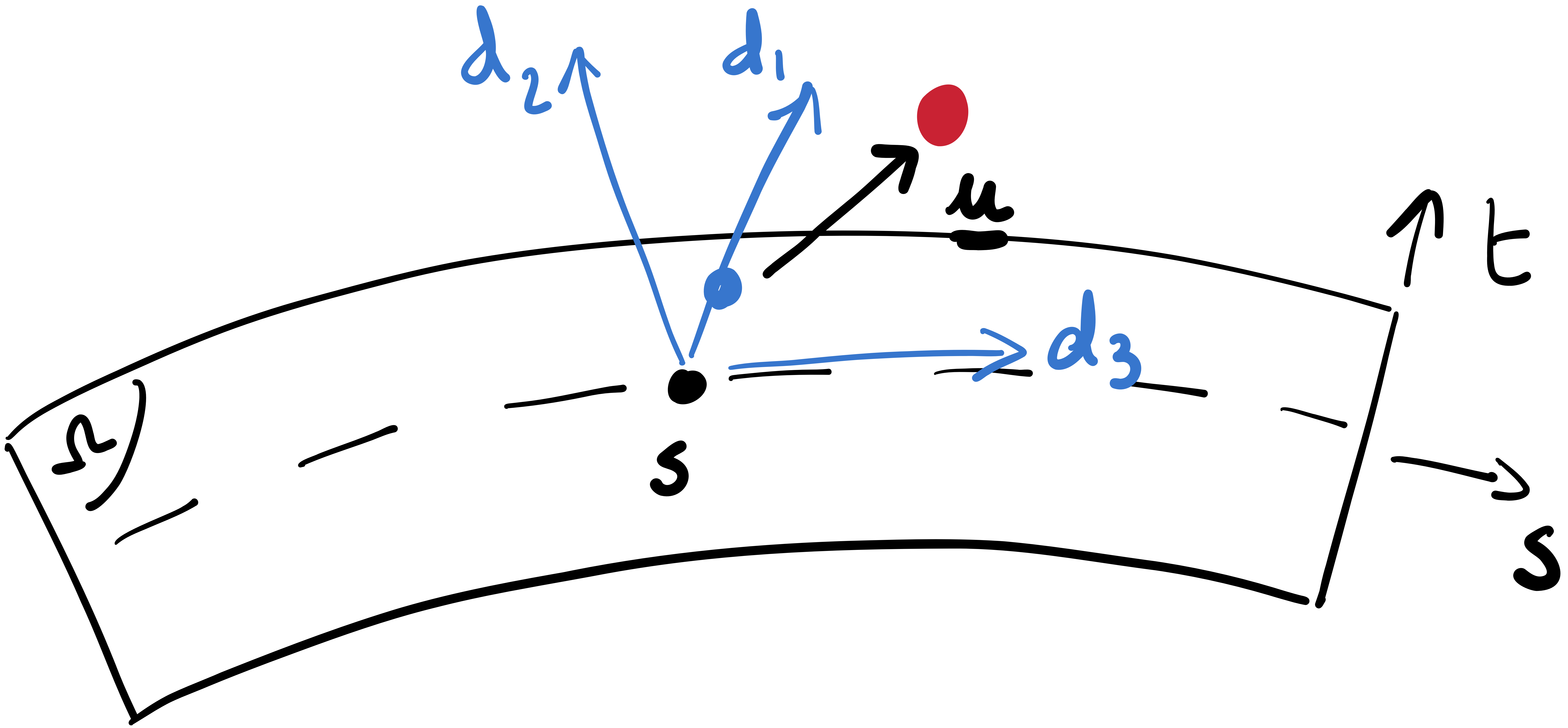
$$V = \int_s \int_t \frac{1}{2} M_{\alpha\beta}\,\kappa_{\alpha\beta} + \frac{1}{2} N_{\alpha\beta}\,\epsilon_{\alpha\beta}\,d\Omega$$

The Ribext model: kinematics



deformation of the centerline: $\kappa_1(s) \quad \kappa_2(s) \quad \kappa_3(s)$

The Ribext model: kinematics



deformation of the section

$$\underline{u}(s,t) = u(s,t)\, d_1 + v(s,t)\, d_2 + w(s,t)\, d_3$$

The Ribext model: two-scale expansion

$$V = \int_s \int_t \frac{1}{2} M_{\alpha\beta}\, \kappa_{\alpha\beta} + \frac{1}{2} N_{\alpha\beta}\, \epsilon_{\alpha\beta}\, d\Omega \;=\; \int_s \int_t W_{plate}\, d\Omega$$

with $M_{\alpha\beta} = D\left[(1-\nu)\,\kappa_{\alpha\beta} + \nu\,\kappa_{\gamma\gamma}\,\delta_{\alpha\beta}\right]$ **and** $N_{\alpha\beta} = K\left[(1-\nu)\,\epsilon_{\alpha\beta} + \nu\,\epsilon_{\gamma\gamma}\,\delta_{\alpha\beta}\right]$

$$\kappa_{\alpha\beta} = \kappa_{\alpha\beta}(\kappa_{123}(s), u\, v\, w(s,t)) \simeq \varphi_{\kappa_1 \kappa_2 \kappa_3}(u(t)\, v(t)\, w(t))$$

two-scale expansion

slow variable s

rapid variable t

$$\epsilon_{\alpha\beta} = \epsilon_{\alpha\beta}(\kappa_{123}(s), u\, v\, w(s,t)) \simeq \phi_{\kappa_1 \kappa_2 \kappa_3}(u(t)\, v(t)\, w(t))$$

$\kappa_{123}(\cancel{s})$ **are 'parameters'**

The Ribext model: two-scale expansion

plate model

$$V = \int_s \int_t \frac{1}{2} M_{\alpha\beta}\,\kappa_{\alpha\beta} + \frac{1}{2} N_{\alpha\beta}\,\epsilon_{\alpha\beta}\,d\Omega \; = \int_s \int_t W_{plate}\,d\Omega$$

$$W_{plate} \simeq W_{\kappa_{123}}(u\,v\,w(t))$$

$$V \simeq \int_s \underbrace{\left[\int_t W_{\kappa_{123}}(u\,v\,w(t))\,dt \right]}_{F_{\kappa_{123}}[u(t)\,v(t)\,w(t)]} \; ds$$

End Result

Euler-Lagrange equations

$$u_{sol}(t)\,v_{sol}(t)\,w_{sol}(t)$$

$$\Rightarrow F_{\kappa_{123}}[u_{sol}\,v_{sol}\,w_{sol}] = F_{sol}(\kappa_1\,\kappa_2\,\kappa_3)$$

The Ribext model: two-scale expansion

2D plate model

$$V_{2D} = \int_s \int_t \frac{1}{2} M_{\alpha\beta}\, \kappa_{\alpha\beta} + \frac{1}{2} N_{\alpha\beta}\, \epsilon_{\alpha\beta}\, d\Omega$$

1D Ribbon model

$$V_{1D} = \int_s F(\kappa_1\, \kappa_2\, \kappa_3)\, ds$$

$$F(\kappa_1\, \kappa_2\, \kappa_3) = \frac{1}{2} E I_1 \left[\kappa_1^2 + \frac{2}{1+\nu} \kappa_3^2 + \frac{\left(\nu\kappa_1^2 + \kappa_3^2\right)^2}{(1-\nu^2)\,\kappa_1^2} \psi\left(\mu[\kappa_1]\right) \right] + \cancel{\frac{1}{2} E \frac{h w^3}{12} \kappa_2}$$

$$I_1 = \frac{h^3\, w}{12}$$

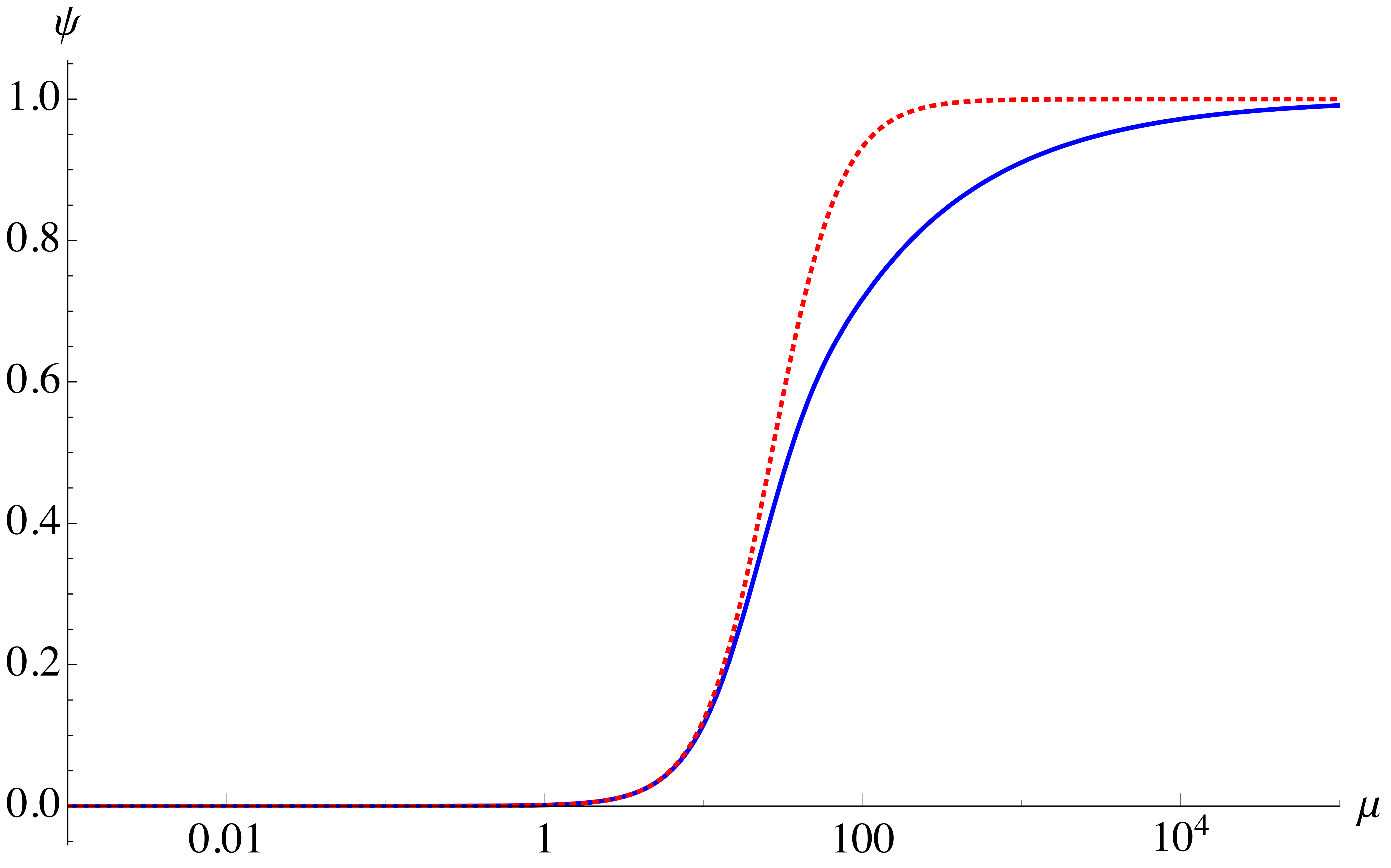
with $\mu[\kappa_1] = \sqrt{12\,(1-\nu^2)}\ \ \frac{w^2}{Lh}\ \ \kappa_1\, L$

$$\kappa_2 = 0$$

relaxation of stiff mode

The Ribext model: limits

$$F(\kappa_1\,\kappa_3) = \frac{1}{2}EI_1\left[\kappa_1^2 + \frac{2}{1+\nu}\kappa_3^2 + \frac{\left(\nu\kappa_1^2+\kappa_3^2\right)^2}{(1-\nu^2)\,\kappa_1^2}\psi\left(\mu[\kappa_1]\right)\right]$$



$$\psi(\mu) = 1 - \frac{2}{\sqrt{\mu/2}}\,\frac{\cosh\sqrt{\mu/2}-\cos\sqrt{\mu/2}}{\sinh\sqrt{\mu/2}+\sin\sqrt{\mu/2}}$$

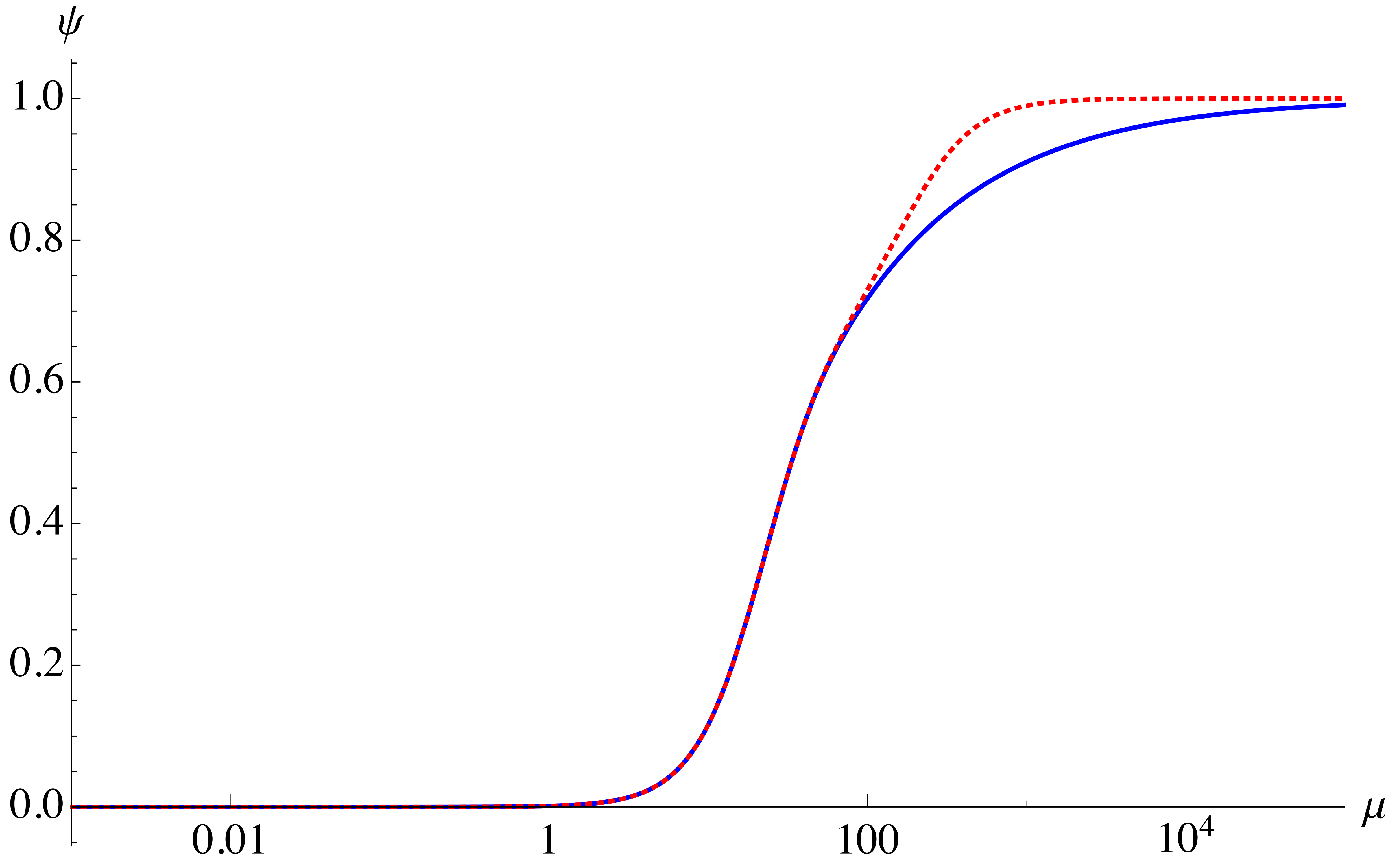
$$\psi(\mu) = \frac{\mu^2}{720+\mu^2}$$ Padé approximant

with $\mu[\kappa_1] = \sqrt{12\,(1-\nu^2)}\;\boxed{\frac{w^2}{Lh}}\;\kappa_1\,L$

Shield number

The Ribext model: limits

$$F(\kappa_1\,\kappa_3) = \frac{1}{2}EI_1\left[\kappa_1^2 + \frac{2}{1+\nu}\kappa_3^2 + \frac{\left(\nu\kappa_1^2+\kappa_3^2\right)^2}{(1-\nu^2)\,\kappa_1^2}\psi\left(\mu[\kappa_1]\right)\right]$$



$$\psi(\mu) = 1 - \frac{2}{\sqrt{\mu/2}}\,\frac{\cosh\sqrt{\mu/2}-\cos\sqrt{\mu/2}}{\sinh\sqrt{\mu/2}+\sin\sqrt{\mu/2}}$$

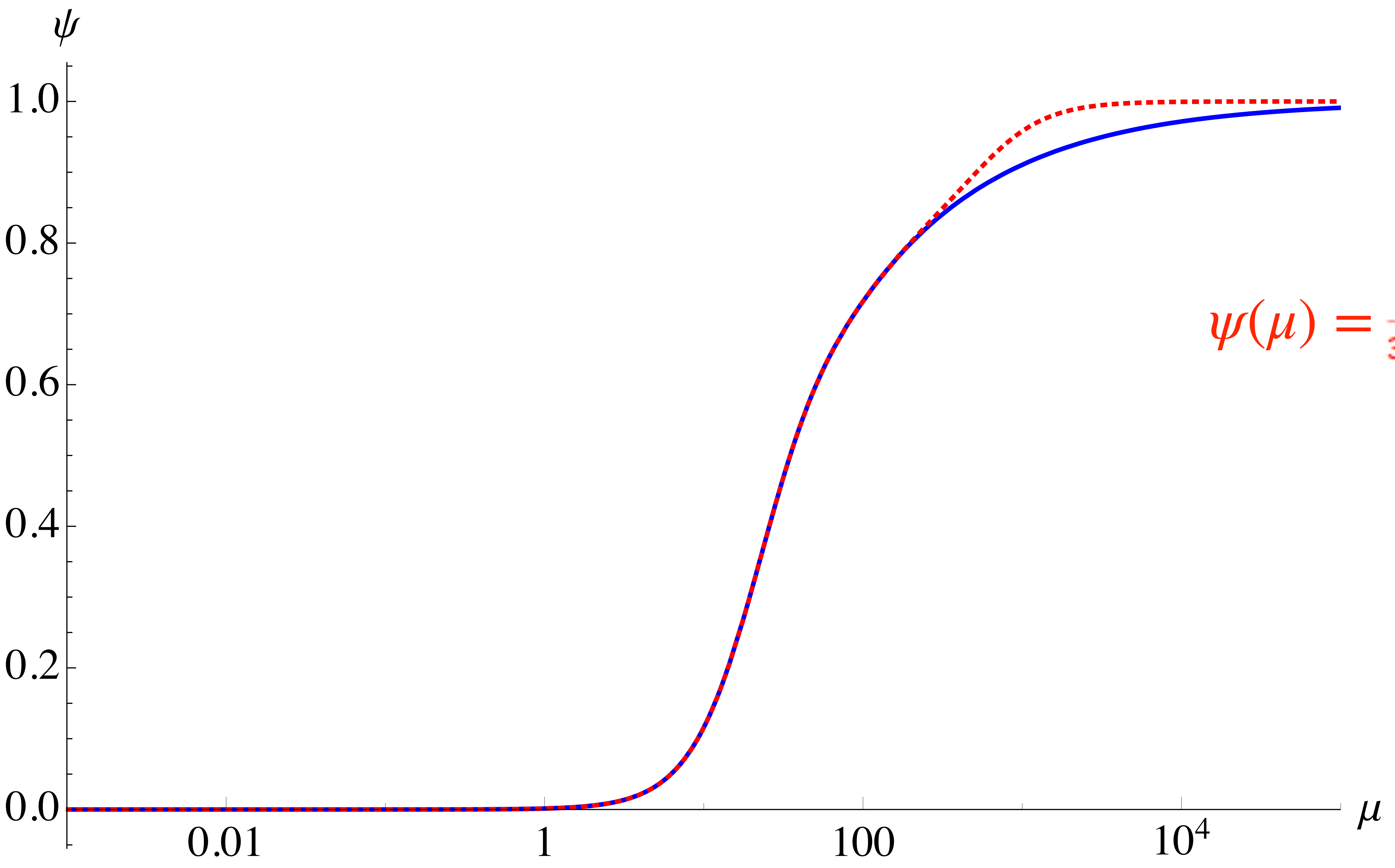
$$\psi(\mu) = \frac{\mu^2\left(432432+19\,\mu^2\right)}{311351040+631440\,\mu^2+19\,\mu^4}$$ Padé

with $\mu[\kappa_1] = \sqrt{12\,(1-\nu^2)}\;\boxed{\frac{w^2}{Lh}}\;\kappa_1\,L$

Shield number

The Ribext model: limits

$$F(\kappa_1\,\kappa_3) = \frac{1}{2} E I_1 \left[\kappa_1^2 + \frac{2}{1+\nu}\kappa_3^2 + \frac{\left(\nu\kappa_1^2 + \kappa_3^2\right)^2}{(1-\nu^2)\,\kappa_1^2}\,\psi\left(\mu[\kappa_1]\right) \right]$$



$$\psi(\mu) = 1 - \frac{2}{\sqrt{\mu/2}}\,\frac{\cosh\sqrt{\mu/2} - \cos\sqrt{\mu/2}}{\sinh\sqrt{\mu/2} + \sin\sqrt{\mu/2}}$$

$$\psi(\mu) = \frac{\mu^2\left(5\,483\,650\,369\,317\,120 + 465\,906\,128\,688\,\mu^2 + 1\,822\,309\,\mu^4\right)}{3\,948\,228\,265\,908\,326\,400 + 8\,169\,238\,654\,536\,960\,\mu^2 + 563\,633\,048\,160\,\mu^4 + 1\,822\,309\,\mu^6}$$

with $\mu[\kappa_1] = \sqrt{12\,(1-\nu^2)}\ \boxed{\dfrac{w^2}{Lh}}\ \kappa_1 L$

Shield number

The Ribext model: limits

$$F(\kappa_1\,\kappa_3) = \frac{1}{2}EI_1\left[\kappa_1^2 + \frac{2}{1+\nu}\kappa_3^2 + \frac{\left(\nu\kappa_1^2 + \kappa_3^2\right)^2}{(1-\nu^2)\,\kappa_1^2}\psi\left(\mu[\kappa_1]\right)\right]$$

$\mu \ll 1 \qquad \psi \to 0$ Rod $\qquad F(\kappa_1\,\kappa_3) = \frac{1}{2}EI_1\,\kappa_1^2 + \frac{1}{2}\frac{2EI_1}{1+\nu}\kappa_3^2$ Kirchhoff

$\mu \gg 1 \qquad \psi \to 1$ Ribbon $\qquad F(\kappa_1\,\kappa_3) = \frac{1}{2}\frac{EI_1}{1-\nu^2}\frac{\left(\kappa_1^2 + \kappa_3^2\right)^2}{\kappa_1^2}$ Sadowsky

with $\mu[\kappa_1] = \sqrt{12\,(1-\nu^2)}\;\boxed{\frac{w^2}{Lh}}\;\kappa_1\,L$

Shield number

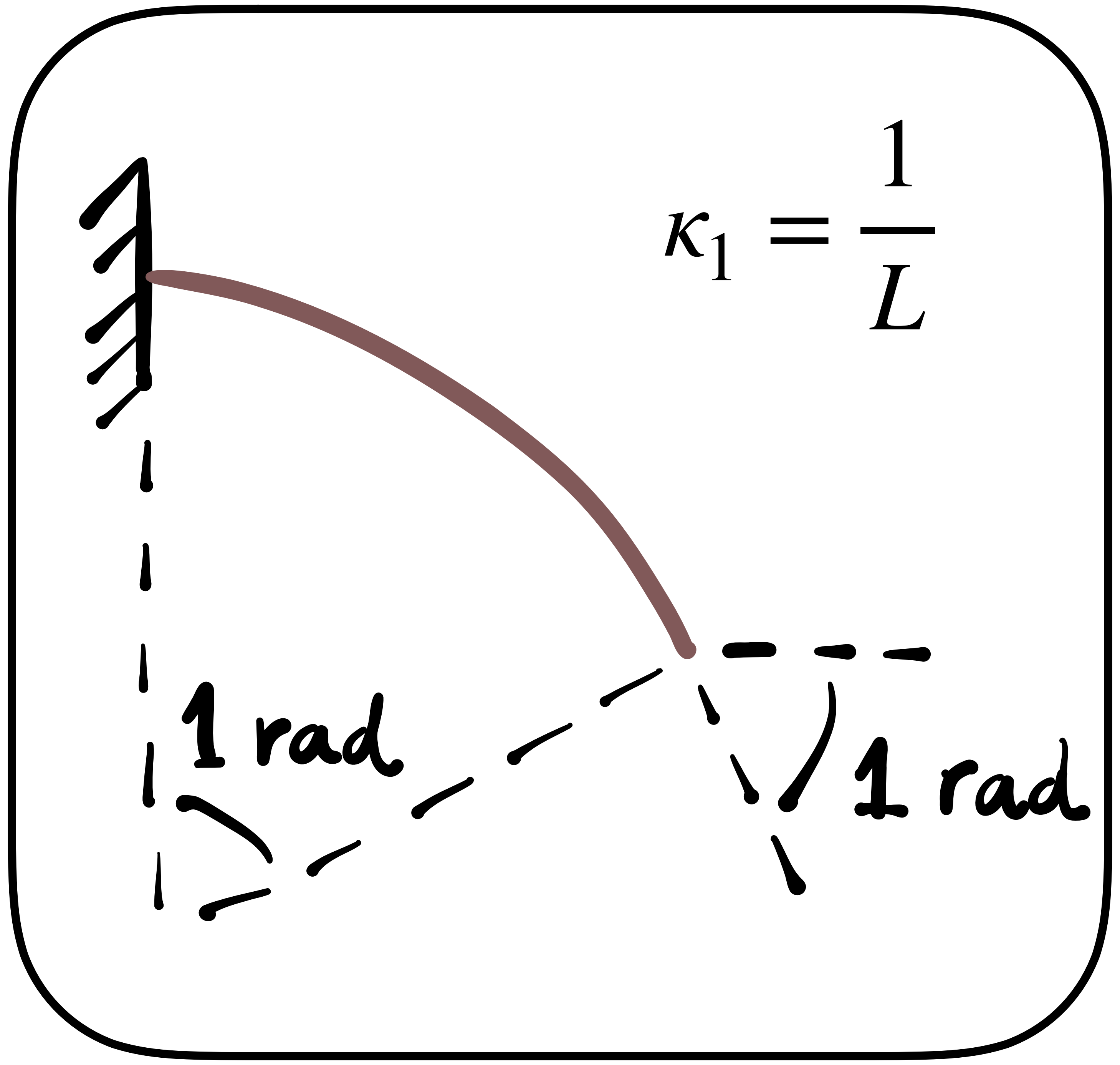
depends on the equilibrium solution !!

The Ribext model: limits

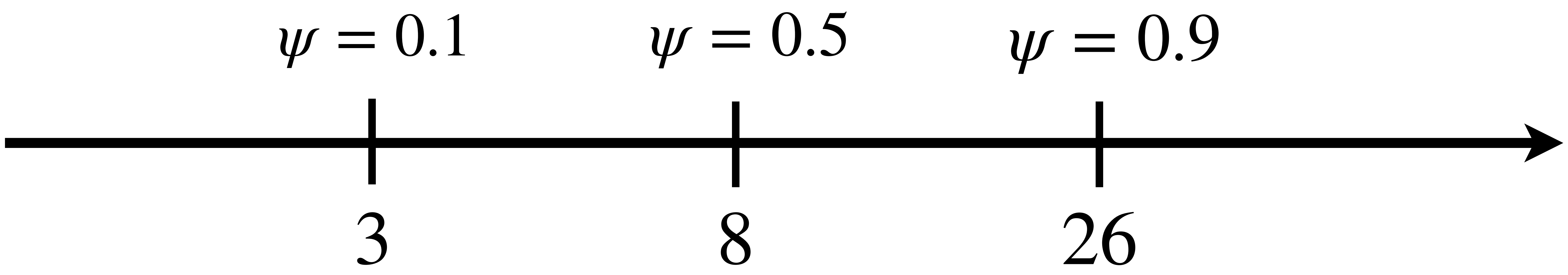
Shield number Sh

$$\mu[\kappa_1] = \sqrt{12\,(1-\nu^2)}\ \boxed{\frac{w^2}{Lh}}\ \kappa_1 L$$

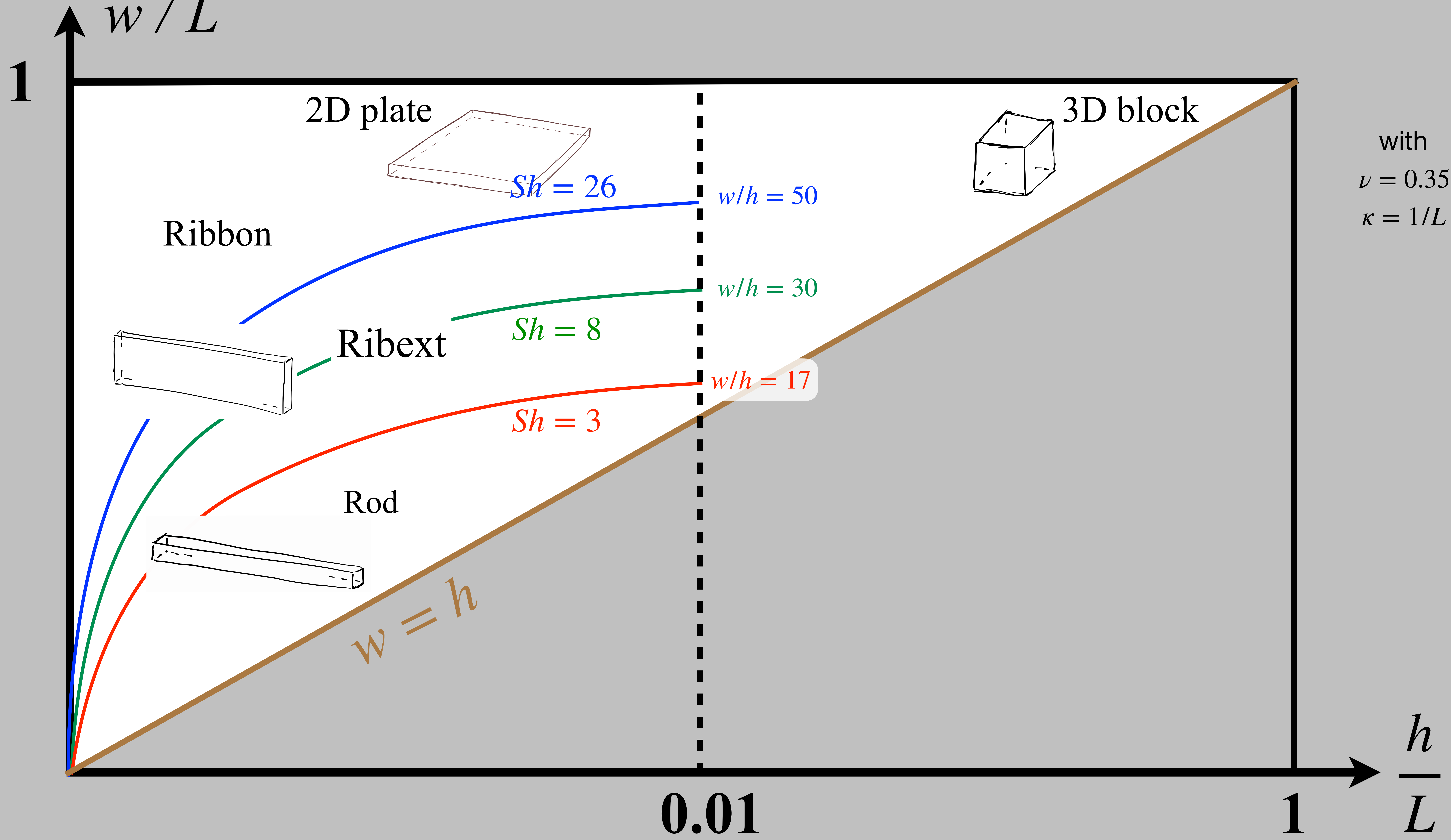
$\nu = 0.35$
$\kappa_1 = 1/L$ $\Rightarrow$ $\mu \simeq 3\,Sh$



rod **ribext** **ribbon**



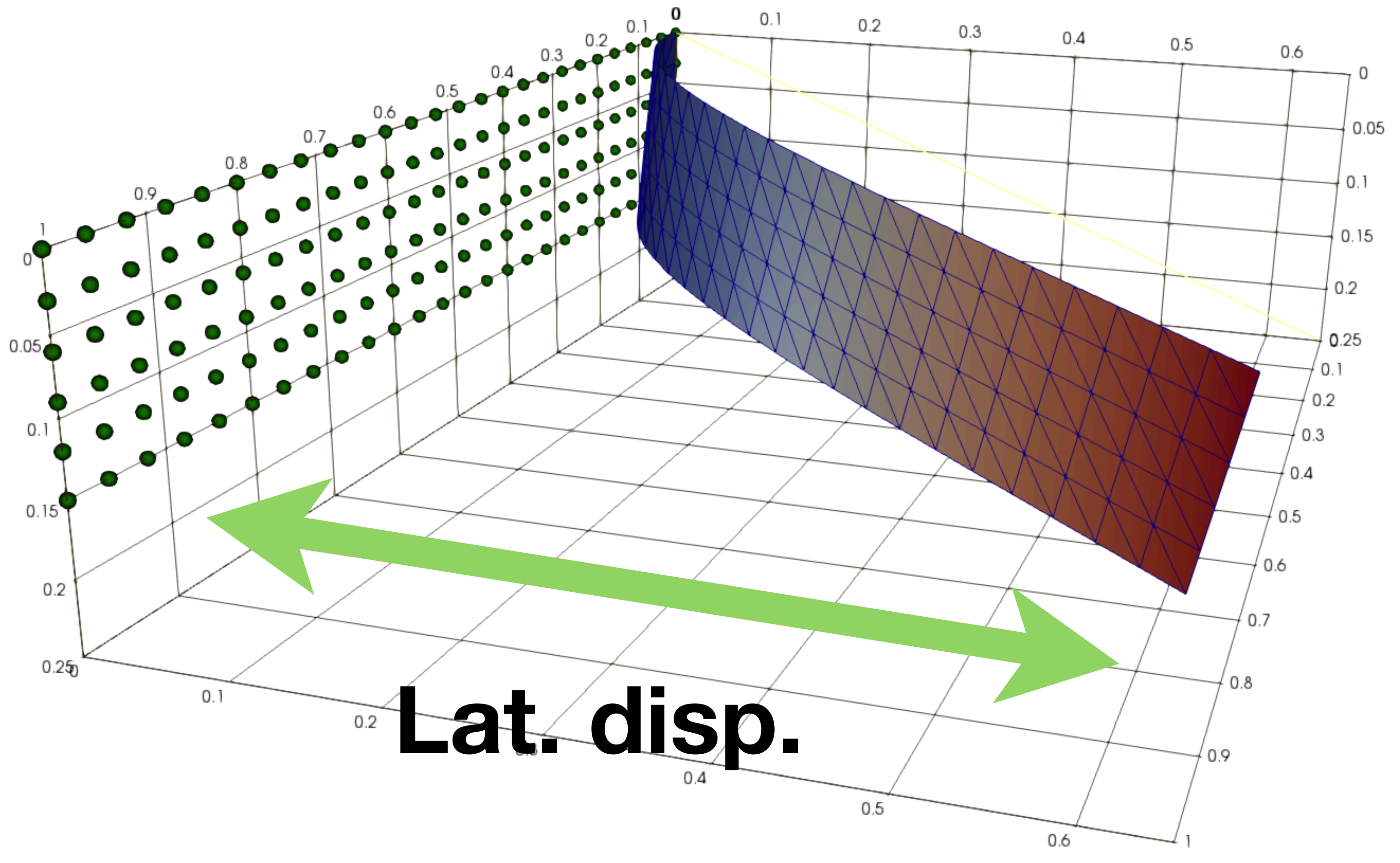
$$Sh = \frac{w^2}{L\,h}$$



with
$\nu = 0.35$
$\kappa = 1/L$

Lateral Torsional Buckling

$\eta(0) = 0$

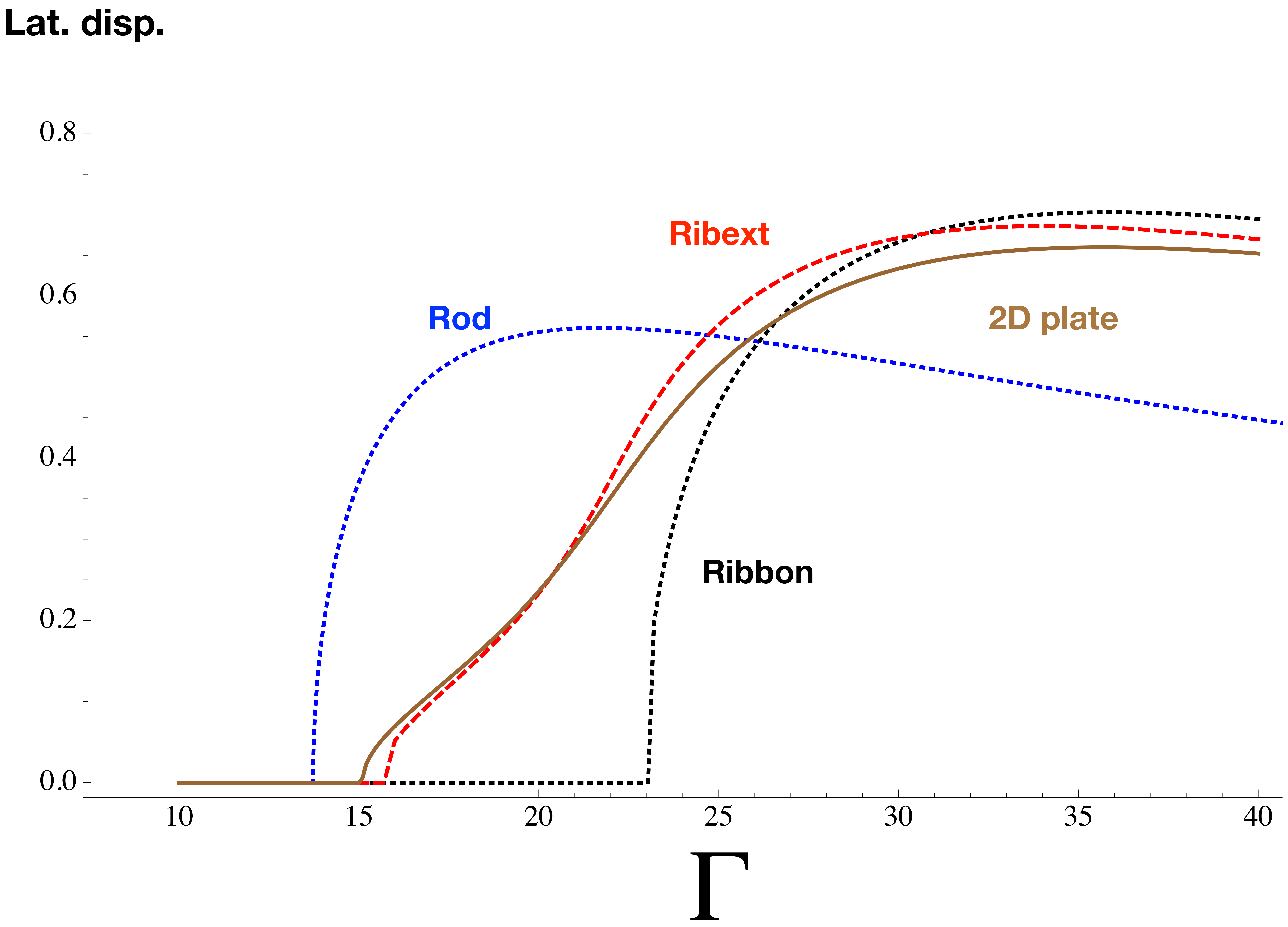


$w/L = 0.1$

$h/L = 0.001$

$\nu = 0.35$

$Sh = 10$



$\eta(0) = 0$

Lateral Torsional Buckling

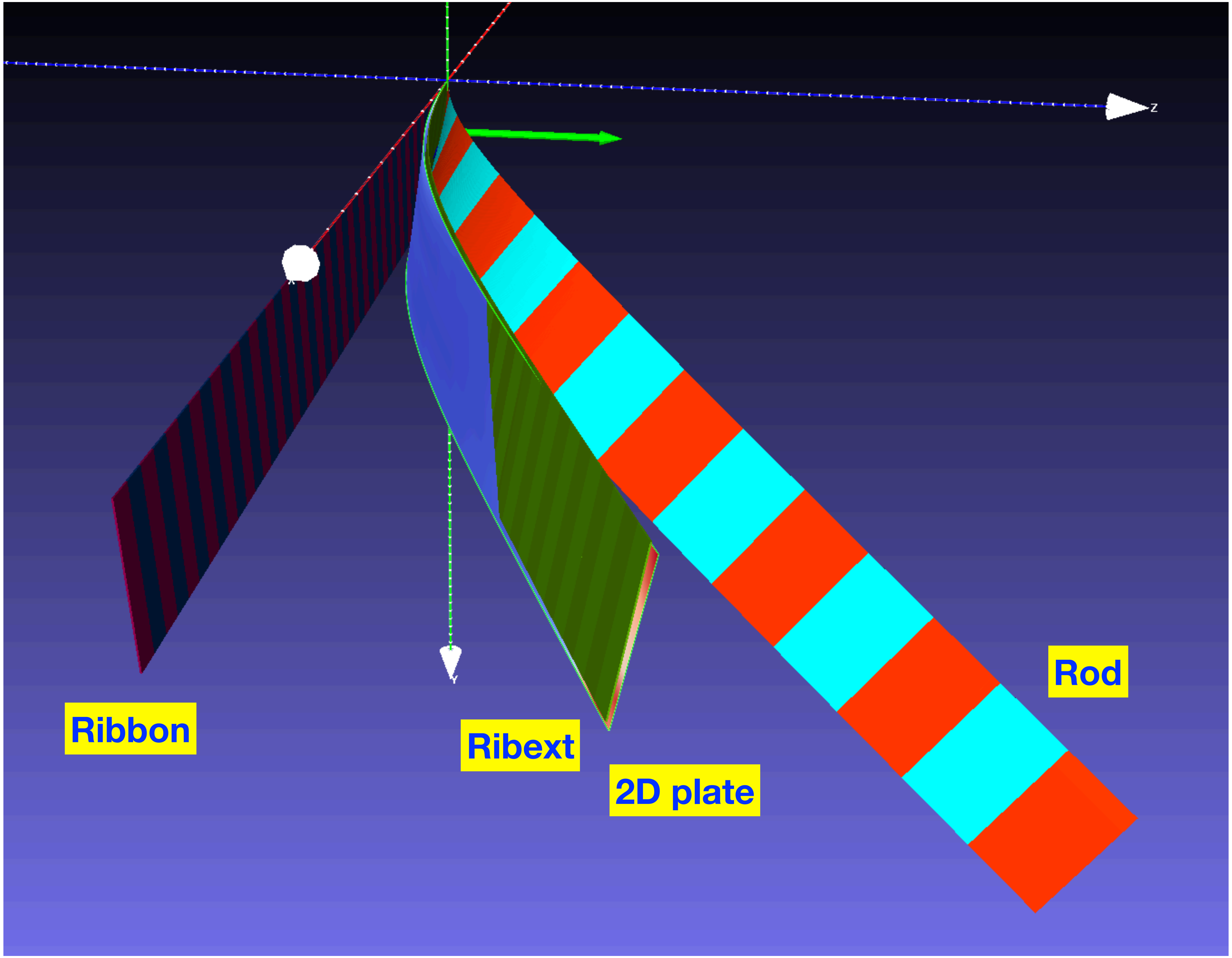
$w/L = 0.1$

$h/L = 0.001$

$\nu = 0.35$

$$\Gamma = \frac{M g}{D w / L^2} = 20$$

$Sh = 10$



Conclusion

- **Ribext, a model between rod and ribbon**

- **Even a plate may behave as a rod or as a ribbon,**
  it depends on the *actual* curvature

Thank you