Erosion and sedimentation of a bump in a fluvial flow

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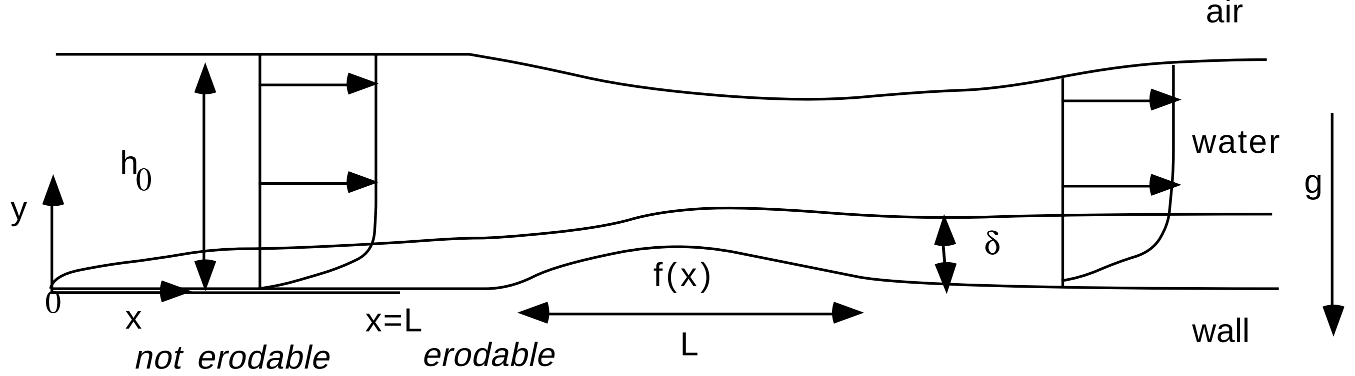


Asymptotic description:

With scales:

$x^* = Lx$, $y^* = LRe^{-1/2}y$, $u^* = U_0u$ ...

$\varepsilon = (L/h_0)Re^{-1/2}$ $\varepsilon<<1$,

$Re = U_0L/\nu$ $Re >>1$

$Fr^2=U_0^2/(gh_0)$ $Fr =O(1)<1$

we obtain the quasistatic Interacting Boundary Layer Problem:

$$\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0,$$

$$(u \frac{\partial}{\partial x}u + v \frac{\partial}{\partial y}u) = u_e\frac{du_e}{dx} + \frac{\partial^2}{\partial y^2}u.$$

$$\delta_1 = \int_0^\infty (1 - \frac{u}{u_e})dy$$

* no slip condition on y= f(x,t)

* $y\to\infty$, $u \to u_e =1 + \varepsilon \frac{\delta_1 + f(x)}{1-Fr^2}$

BL equations + matching condition with the perfect Fluid.

The movement of the dune (f(x,t)) is slow

There is a strong coupling (δ_1), boundary layer separation is possible

Turbulence may be added (mixing length theory)

Transport equation

Model of convective/ diffusive transport with a settling velocity $-V_f < 0$ (Boundary Layer scales):

$$(u \frac{\partial}{\partial x}c + (v-V_f) \frac{\partial}{\partial y}c) = \frac{1}{S} \frac{\partial^2}{\partial y^2}c.$$

No income, and flux condition at the wall depending on the value of the skin friction:

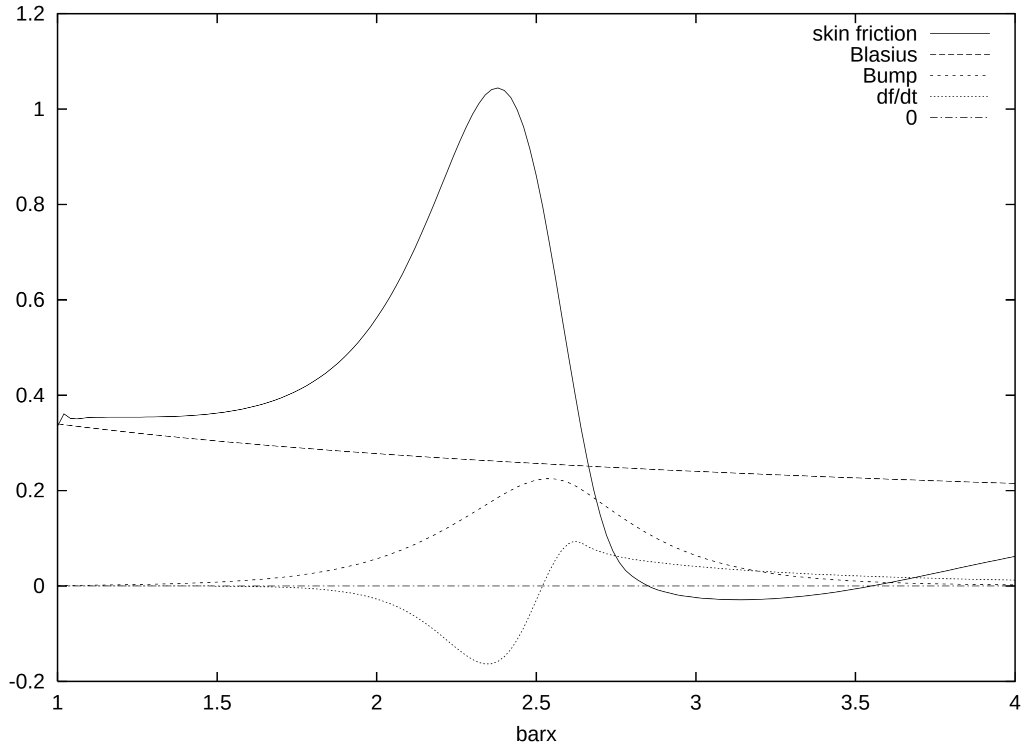
if $\frac{\partial}{\partial y}u(x,0) < \tau_w$ then $- \frac{\partial}{\partial y}c(x,0) = 0$

if $\frac{\partial}{\partial y}u(x,0) > \tau_w$ then $- \frac{\partial}{\partial y}c(x,0) = \beta (\frac{\partial}{\partial y}u(x,0) - \tau_w)^\gamma$

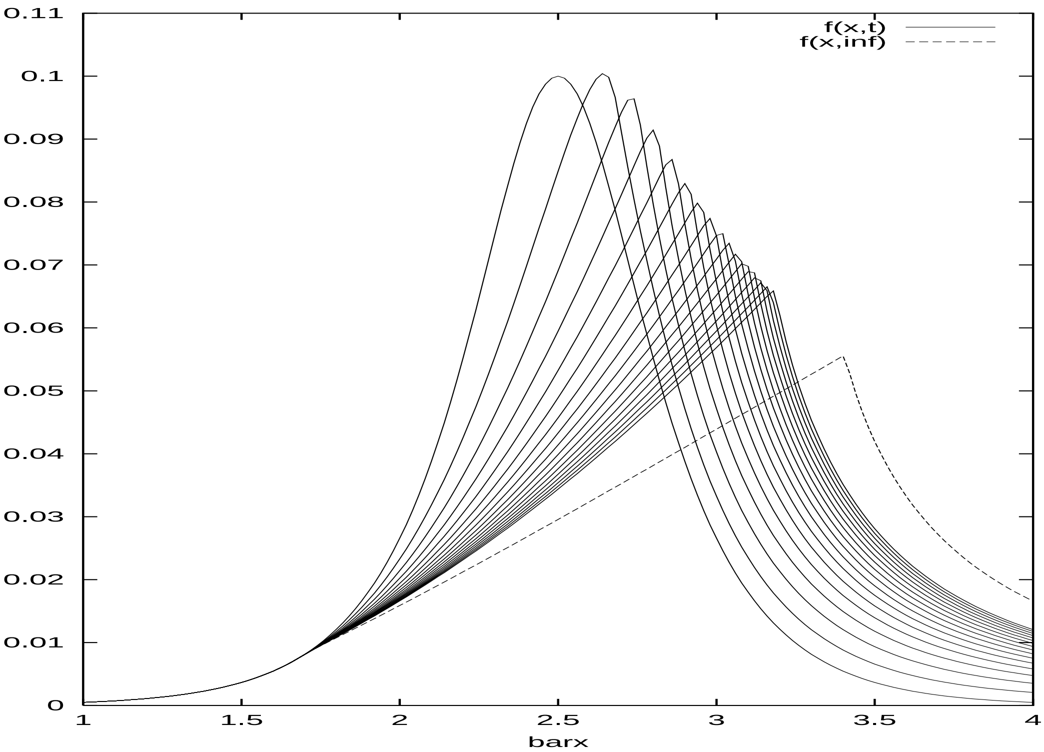
$\gamma=3/2$, $\beta=O(1)$

Dune evolution at slow time scale:

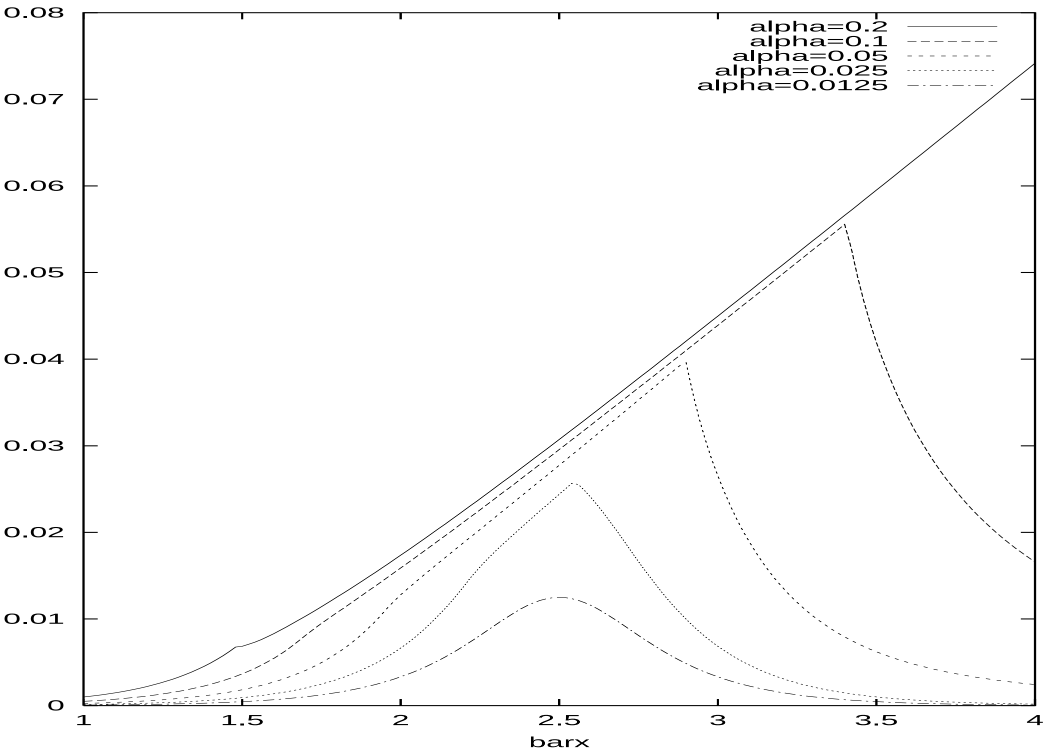
$$\frac{\partial f(x,t)}{\partial t} = S^{-1} \frac{\partial}{\partial y}c(x,0) + V_f\, c(x,0).$$



At initial time, the initial bump: f(x,t)=0) and the associated computed skin friction at the wall



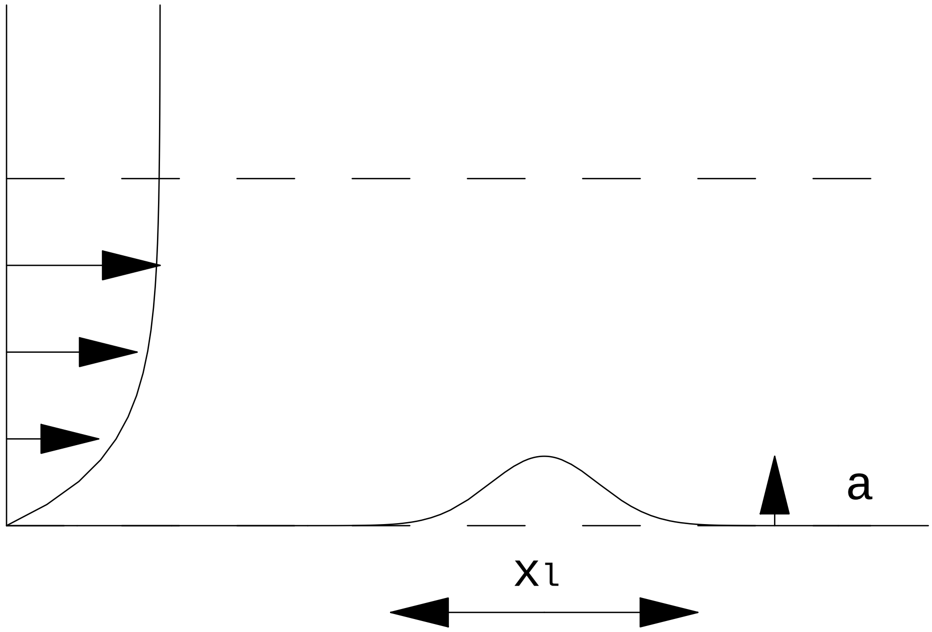
The dune shape as a function of time t=0, 1, 2, 3...,16 ,∞



Final dune shapes for different starting values of α

Triple Deck description (Gajjar Smith 83)

With *ad hoc* scales (small bump)



$$\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0,$$

$$(u\frac{\partial}{\partial x}u + v\frac{\partial}{\partial y}u) = -\frac{dp}{dx} + \frac{\partial^2}{\partial y^2}u.$$

$$y\to\infty \quad u \to y + A$$

and

$p = A$ fluvial flow $(Fr<1)$

$p = - A$ supercritical flow $(Fr>1)$

$p = \frac{1}{\pi}\int \frac{A'}{(x-\xi)}d\xi$ infinite depth

$A = 0$, very small bump which does not perturb the perfect fluid

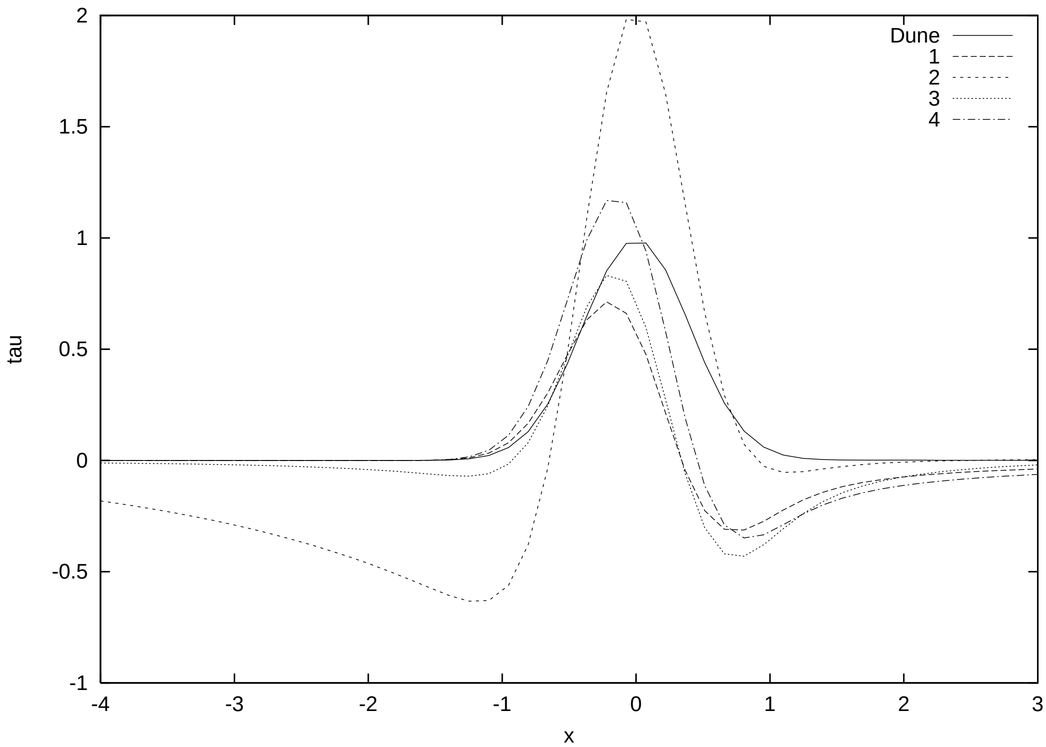
The linearized Fourier solution for a small bump:

Perfect fluid response $\beta p = A$, with $\beta = 1, -1, 1/|k|, 0$
and $\beta^* = (-i\,k)^{1/3} /(3\,Ai'(0))$,
perturbation of pressure is:

$$TF[p] = \frac{TF[f]}{\beta^* - \beta}$$

perturbation of skin friction is:

$$TF[\tau] = (-i\,k)^{2/3}\, \frac{Ai(0)}{Ai'(0)}\, TF[p]$$



evolution of the skin friction for a given bump in various flow régime:
1 subcritical flow ($\beta = 1$)
2 supercritical flow ($\beta = -1$)
3 infinite depth ($\beta = 1/|k|$)
4 no perturbation in the perfect fluid ($\beta = 0$)

We are here at the limit τ_w (τ_w=1 with the chosen scales)

The mass transport equation:

$$\frac{\partial}{\partial x}\left(\int_0^\infty cu\,dy\right) = -\,c(x,0)(V_f) - S^{-1}\frac{\partial}{\partial y}c(x,0) \qquad \text{with} \qquad q = \left(\int_0^\infty cu\,dy\right)$$

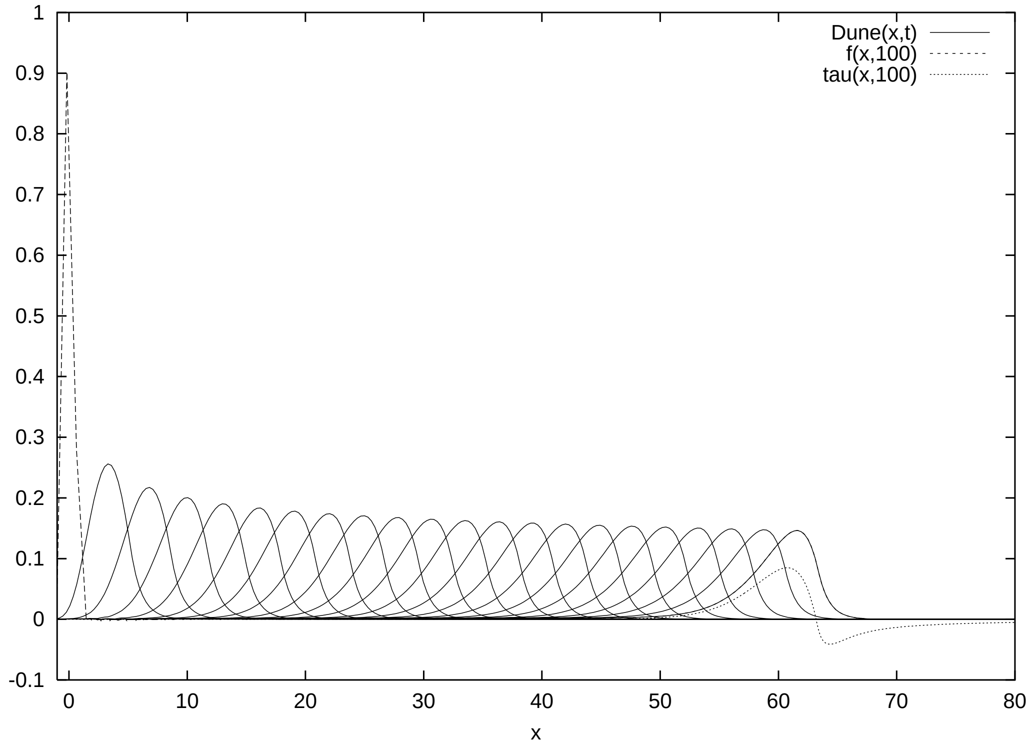
$$A_r = -\,S^{-1}\frac{\partial}{\partial y}c(x,0) \quad \text{is} \quad \frac{\partial}{\partial y}u(x,0)-1 \qquad \text{if} \quad \frac{\partial}{\partial y}u(x,0)>1$$

$$A_r = -\,S^{-1}\frac{\partial}{\partial y}c(x,0) \quad \text{is} \quad 0 \qquad \text{if} \quad \frac{\partial}{\partial y}u(x,0)<1$$

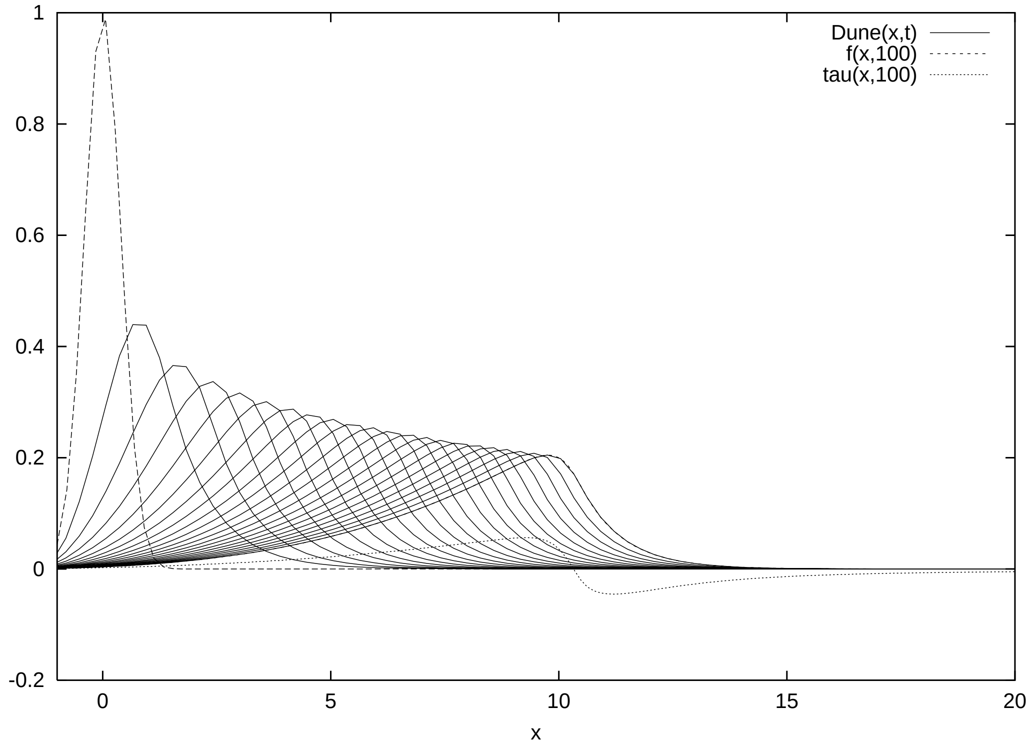
The final problem for the sediments is:

$$\frac{\partial}{\partial x} q = -\,q/\lambda + A_r \quad \text{and} \quad \frac{\partial}{\partial t} f = -\frac{\partial}{\partial x} q$$

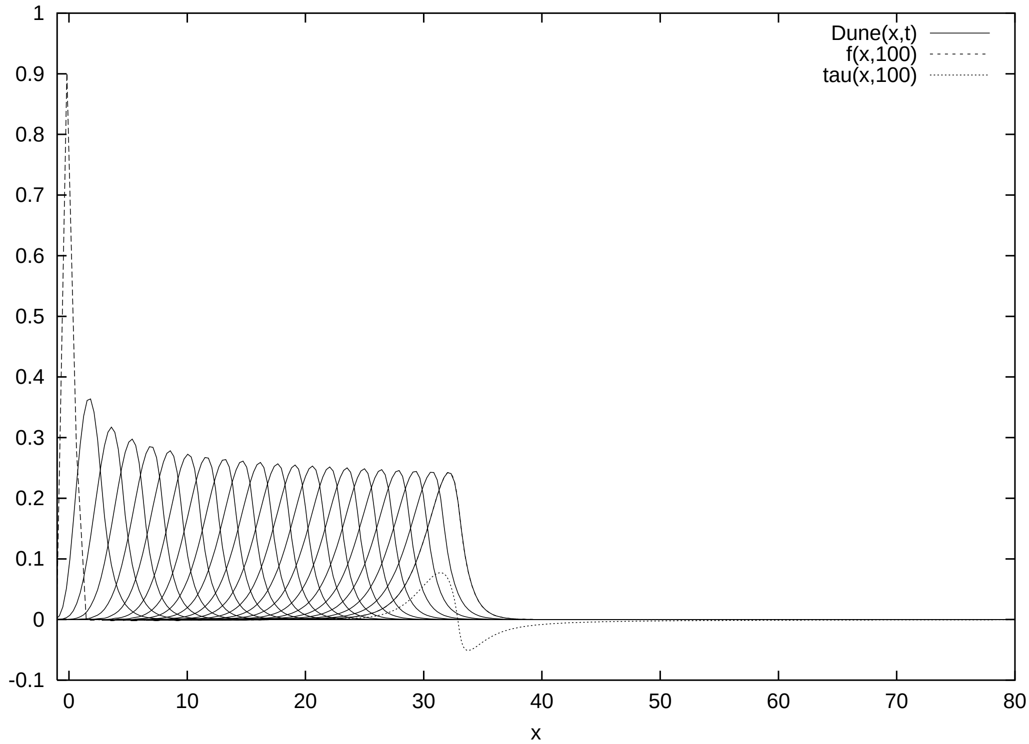
$$t=0,\ f(x,0) = \exp(-\pi x^2),\ q(x,0)=0.$$



no perturbation in the perfect fluid ($\beta = 0$)



subcritical flow ($\beta = 1$) $Ar=(\tau-1)^{3/2}$



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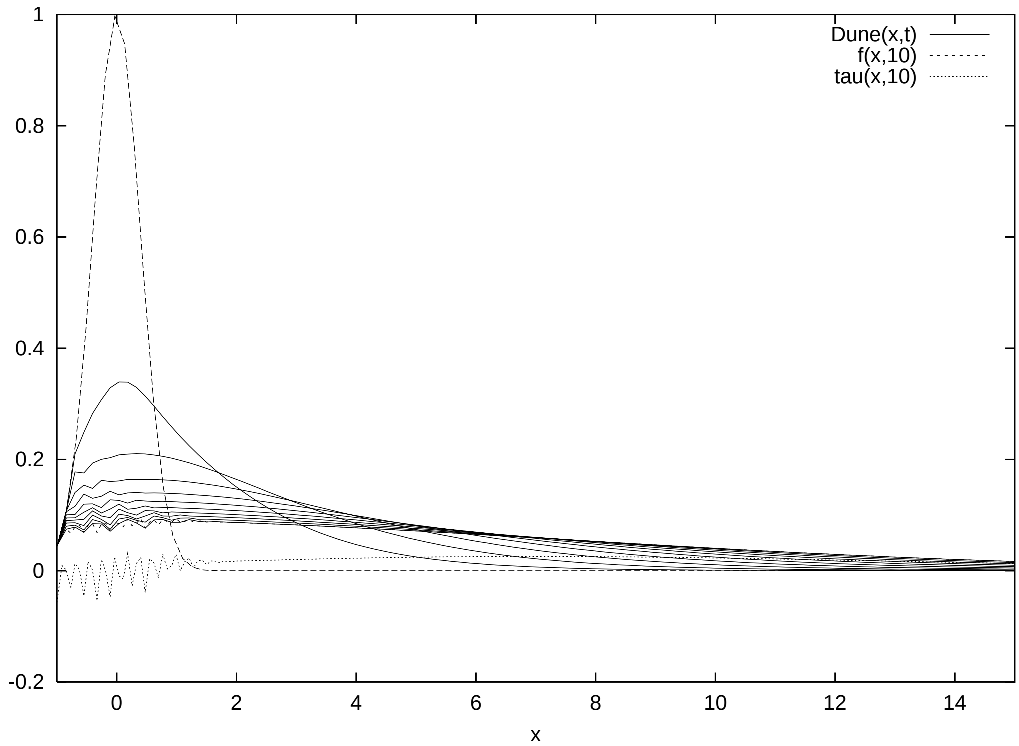
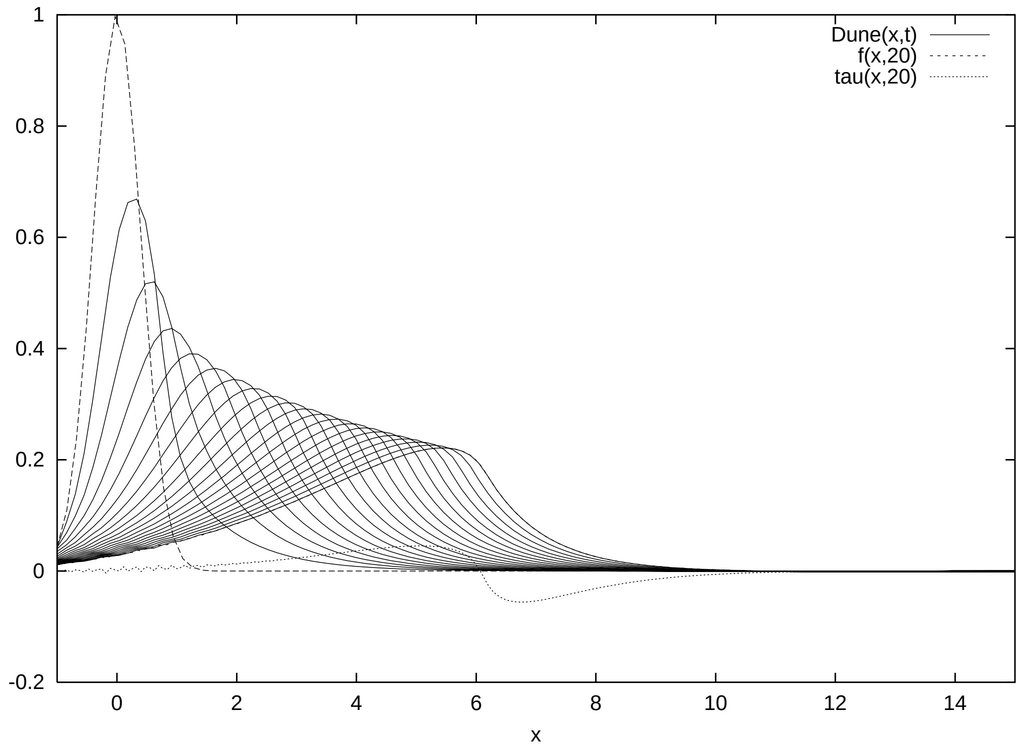


supercritical flow (β = -1)



infinite depth (β =1/|k|)

Conclusion

The advantage of this model is that a lot of hydrodynamical mecanisms have been put without usual integral simplifications.

Of course, the first hypotheses to introduce in the model would be a turbulent stress viscosity and diffusivity and for the river bed

It would be interesting to introduce the slope limitation.

The triple deck description allows the movement of the bump...

Ref:

K. Kroy G. Sauerman & J. Hermann "A minimal model for sand dunes", *subm.*

P.-Y. Lagrée "Erosion and sedimentation of a bump in fluvial flow", *to appear in CRAS.*