The spreading of a granular column from a Bingham point of view

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Abstract.
The collapse and spreading of granular columns has been the subject of sustained interest in the last years from both mechanical and geophysical communities. Yet, in spite of this intensive research, the adequate rheology allowing for a reliable continuum modeling of the dynamics of granular column collapse is still open to discussion. Essentially, continuum models rely on shallow-water approximation for which dissipation and sedimentation processes are taken into account through the introduction of *ad hoc* laws. However, the rheological origin of the experimental scaling laws exhibited by the granular columns when spreading remains unclear. On these grounds, we adopt an alternative approach consisting of studying the collapse of columns of material obeying a Bingham rheology. Therefore we carried out series of numerical simulations using the Gerris Flow Solver solving the time dependent incompressible Navier-Stokes equation in two dimensions for the specified rheology. We first check that the mass exhibit similar scaling laws as those shown by granular columns. Then we investigate in which extent rheological parameters do reflect on these scaling laws. A comparative analysis of Bingham and granular flow characteristics ensues.

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THE GRANULAR COLLAPSE

Because it provides a simple, yet rich model for catastrophic natural flows, the collapse and spreading of granular material has been extensively studied in recent years, experimentally [3, 6, 4, 1, 7], and also numerically using discrete methods [11, 10] and theoretical modeling [5, 2]. As an example, different stages of a 2D numerical collapse with an aspect ratio $a = 9.1$ can be seen in Figure 1 (taken from [10]). Many mechanisms contribute to the global behavior, such as local deposition, interaction with the substrate, mass propagation in the flow induced by the vertical acceleration. However, the most interesting result lies in the experimental scaling laws exhibited by the deposit left once the flow resulting from the collapse has come to a rest. If H_0 and L_0 are respectively the initial height and initial radius of the column, and $a = L_0/H_0$ is the initial aspect ratio, then the final length of the deposit L obeys the following relation:

$$\frac{L-L_0}{L_0} = \begin{cases} \lambda_1 a & \text{si } a < a_0 \\ \lambda_2 a^{\alpha} & \text{si } a > a_0 \end{cases} \tag{1}$$

where a_0, λ_1 and λ_2 slightly vary with the grains properties (characteristic values for sand in 3D are $a_0 = 1.7$, $\lambda_1 = 1.24$ and $\lambda_2 = 1.6$, see [6], while [3] finds for glass beads $a_0 = 0.74$, $\lambda_1 = 1.35$ and $\lambda_2 = 2$), whereas α is dependent only on the geometry: $\alpha = 1/2$ in 3D and $\alpha = 2/3$ in 2D. Figure 2, taken from [3], shows an example of 3D experimental points for various flow conditions. This scaling law, which implies non-trivial dissipation mechanisms, is not yet elucidated. More specifically, it remains unclear how much it depends on the geometry on the one hand, and on the actual rheology of the flowing material on the other hand. Since the rheology of granular material is in itself a long lasting riddle, an alternative way of improving our

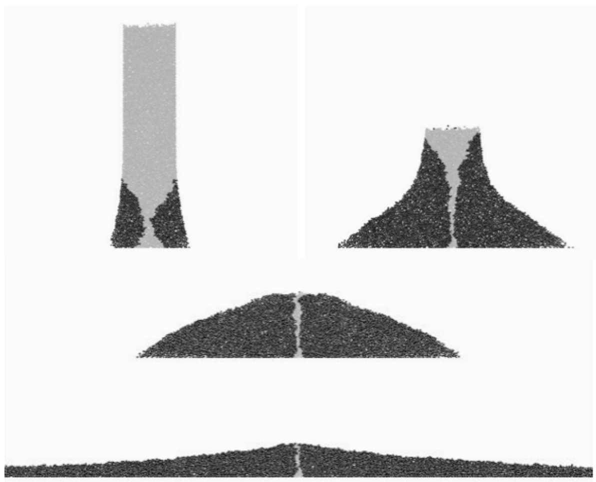

FIGURE 1. Four stages of the collapse of a 2D numerical column simulated by contact dynamics with an initial aspect ratio $a = 9.1$ [10]. Black identify grains whose cumulated displacement exceeds the mean grains diameter.

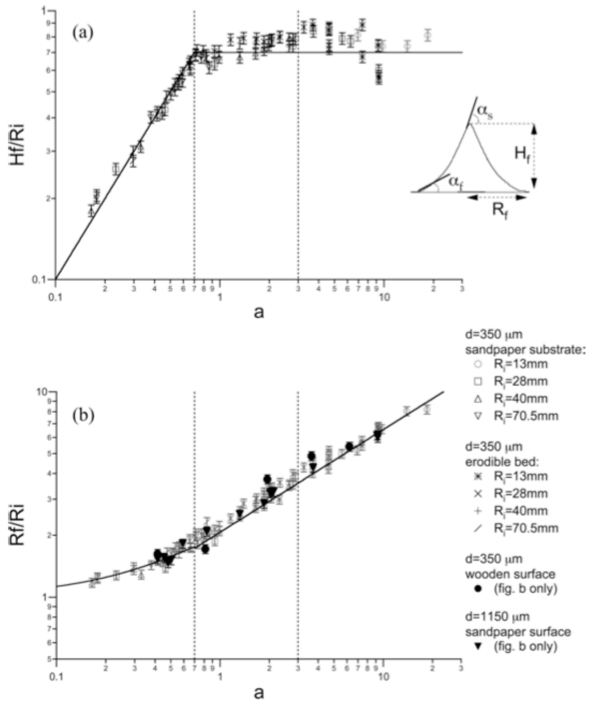

FIGURE 2. Normalized final height (top) and run-out distance (bottom) as a function of the aspect ratio a for different granular material. The experiments are 3D. Graph taken from [3].

understanding of the column collapse dynamics is to check whether any prescribed rheology allows us for the recovering of the scaling law.
In this contribution, we present preliminary numerical results on the collapse of a Bingham fluid, for which both the initial geometry of the column and the rheological parameters were varied. Therefore, the Gerris Flow Solver was applied.

THE BINGHAM COLLAPSE

The Bingham rheology

Bingham fluids are characterized by their ability to resist shear at low stresses, while they flow like newtonian viscous fluids at higher stresses. This implies the existence of a yield value for the shear stress τ_y, which separates the rigid and the viscous response of the Bigham fluid. This behavior can be written as follows:

$$\frac{\partial u}{\partial z} = \begin{cases} 0 & , \tau < \tau_y \\ (\tau - \tau_0)/\mu & , \tau \geq \tau_y \end{cases} \tag{2}$$

where μ is the viscosity.
Just as granular material resist flowing when the shear stress is below the so-called internal angle of friction, and hence exhibit solid-like behavior, Bingham fluid have the ability to remain solid, and most importantly, to stop flowing when the stress becomes small. This analogy makes Bingham fluid a suitable candidate for testing the column collapse scaling laws on more general grounds than the granular media experiments.

The numerical method

The Bingham column collapse was performed in 2D using the Gerris Flow Solver which solves the time dependent incompressible Navier-Stokes equation in two dimensions for the rheology specified in equation (2). More details about the numerical method will be found in [8]. A condition of non-slipping at the horizontal plane is implemented. The non-dimensional equation is solved in the domain of size H_m; in the following, we consider the normalized viscosity $\mu/(\rho(gH_m)^{3/2})$, the normalized yield stress $\tau_y/(\rho g H_m)$, and the normalized time $t/(H_m/g)^{1/2}$. Four sets of simulations were performed for two values of the viscosity $\mu = 1.10^{-3}$ and $\mu = 5.10^{-3}$, and three values of the yield stress $\tau_y = 1.10^{-2}$, $\tau_y = 2.10^{-2}$ and $\tau_y = 4.10^{-2}$. As an example, Fig 3 shows the collapse of a column with an aspect ratio $a = 6$ at different times.

COMPARING THE SCALING LAWS

The run-out

The normalized run-out distance $(L - L_0)/L_0$ is plotted against a in Fig 4 for the four sets of simulations performed. We observe that the general trend shown by the points differs from experimental results shown in Fig 2, specifically for small aspect ratios (namely $a < 1$). However, for larger aspect ratios, a power-law approximation seems acceptable, in agreement with the experimental scaling law observed for granular material (see equation (1)). The corresponding exponents are reported in Fig 5: the values obtained are much higher than the value $2/3$ observed in the scaling (1). This discrepancy can be related to the difference of energy dissipation mechanisms at the base of the collapsing column: while the agitation and many collision existing at the base of the granular column is very efficient and induce smaller exponent, the smoother deformation characterizing the Bingham collapse implies less dissipation and leads to larger exponent. Consistently,

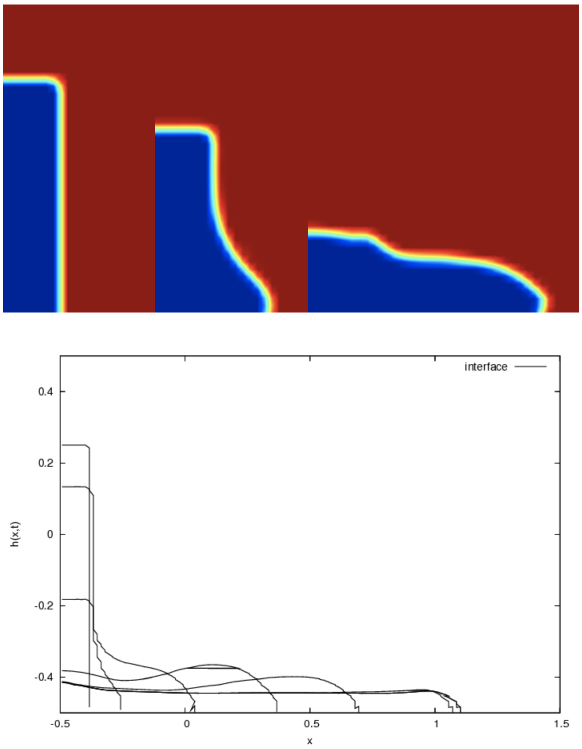

FIGURE 3. Example of the collapse of a column with $a = 6$, $\mu = 1.10^{-3}$ and $\tau_y = 1.10^{-2}$. In the bottom graph, the profiles coincides with times $t = 0$, $t = 0.5$, $t = 1$, $t = 1.5$, $t = 2$, 3 and $t = 4$, where time is normalized by $\sqrt{H_m/g}$

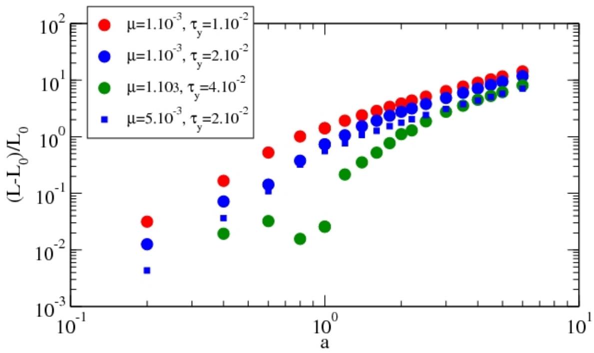

FIGURE 4. Normalized run-out distance $(L - L_0)/L_0$ as a function of the aspect ratio a for 4 series of simulations with varying viscosity μ and varying yield stress τ_y.

increasing the value of the yield stress τ_y leads to rapidly increasing exponents. The influence of the value of the viscosity μ, which we expect to be dominating in the lateral spreading, is comparatively very weak. This suggests that the origin of the exponents essentially lies in the early stages of the collapse.

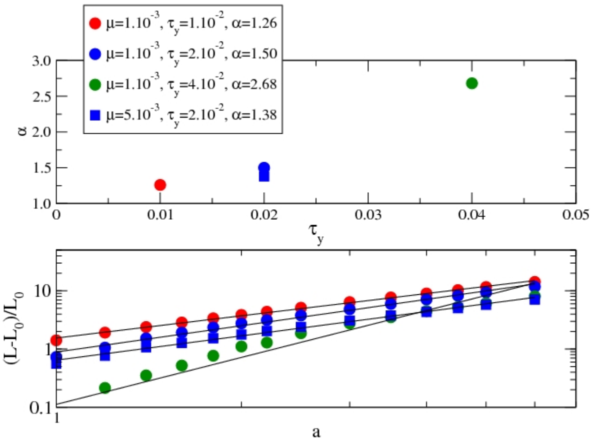

FIGURE 5. Normalized run-out distance $(L - L_0)/L_0$ as a function of the aspect ratio a for $a > 1$ (bottom). The straight lines show the best power-law fits, while the corresponding exponents are plotted as a function of τ_y in the top graph.

The final height

The normalized final height of the deposit H/L_0 is plotted against the aspect ratio a in Fig 6. The general behavior is similar to the experimental granular collapse reported in Fig 2. For very small aspect ratio, only the edge of the squat column flows in response to gravity while the center remains solid: this is due to the existence of the yield stress which plays for Bingham fluids a role similar to the role played by the angle of friction in granular material. Hence we observed that $H/L_0 = a$. For larger aspect ratios, H/L_0 becomes independent of a, and is essentially dominated by the value of both μ and τ_y. For larger value of τ_y, a peaked behavior emerges around intermediate values of a: we probably see here "erosion effect", namely the material falling with increasing momentum destroys the large solid core preserved by a stress under the yield τ_y for intermediate values of a.

CONCLUSION

Although a power-law scaling seems a reasonable approximation of the behavior of the run-out for Bingham column collapse, important features distinguish it from the behavior of its granular counterpart. In first place, the value of the exponent of the power law are much higher, but most importantly, they seem to be very sensitive to the rheological parameters. We can thus suspect that the Bingham model does not capture the energy dissipation occurring in granular collapse, which obey complex mechanisms not yet fully explained. However, it allows one to discriminate between the effects of geometry

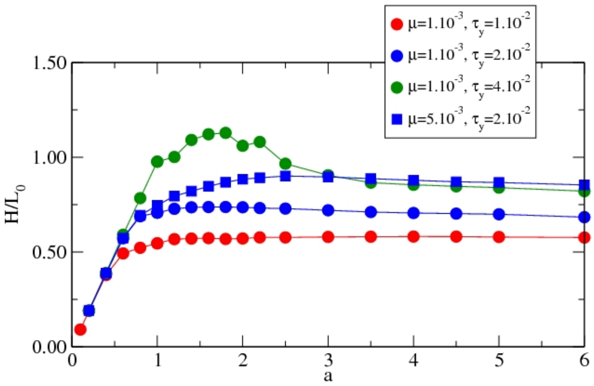

FIGURE 6. Normalized final height of the deposit H/L_0 as a function of the aspect ratio a

and rheology, and determinate whether the scaling law results at first order from the early stage of the collapse or from the lateral spreading. More simulations will be carried out in this perspective, and more comparison between the time evolution of the flow, and the shape of the deposit, will be done. In addition, implementing a slipping or mixed condition at the horizontal plane should bring us closer to the granular collapse phenomenology.

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