The flow in the glottis: self oscillation of a 2D elastic stenosis

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Introduction

The flow in the glottis is an example of fluid structure interaction. The wall has a stenosed shape and is elastic. This kind of problem is mainly solved by simplified methods for the fluid and the solid (Lous *et al.* (1998), Titze (1988)): 1-D approximation, Bernoulli law and flux conservation, finally the wall is simplified as a system of spring and masses. Here we use a more complex description though we do not solve complete Navier Stokes equations as Luo and Pedley (1998) do.

Method

The advantage of the first simplified 1D approach is that most of physical phenomena are incorporated in the model, though simplified (this is necessary to allow a real time computation). The second one (full NS 2D) is to slow, but takes into account all the phenomena. Here, we will use asymptotic expansion to simplify the flow, because we believe that this kind of simplification allows to scale some of the most salient features of the flow and leads to a numerical problem which maybe solved with a reasonable computing time.

We will use the oversimplified "one mass model" (Titze (1988)). This simple model consists in a system of two mass/ spring of opposite displacement. We suppose that the stenosis is of shape $y_w(x,t)=f(x)\ \square(t)$, where f(x) is a given fixed shape and $\square(t)$ is solution of a mass spring relation:

$$\frac{d^2}{dx^2}\square(t)+\square_0^2\square(t) = F(t)$$

The *ad hoc* coefficient $\square_0^2$ is the equivalent oscillating frequency of the tissues (dissipation is neglected). Here F(t) is proportional to the integral of the fluid pressure on $y_w(x,t)$.

We solve for the fluid the so called RNSP(x) equations (Reduced Navier Stokes with no transverse pressure variation). Those equations are a dimensionalised by the distance between the wall h_0 for y, and h_0Re for x:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,\ \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\square\frac{dp}{dx}+\frac{\partial^2 u}{\partial y^2},$$

those equations are valid for $Re=U_0h_0/\square>>1$. The fluid and the solid are coupled by the localisation of the wall: $u(x,y_w(x,t)=0$ and $v(x,y_w(x,t))=\frac{\partial y_w}{\partial t}$ and by the integral of the pressure over the bump.

Results :

If the system is solved with an imposed flux of velocity, self sustained growing oscillations appear. This is explained by a linearized "triple deck" theory. But, if the pressure drop over the whole domain ($\square$p) is imposed, then, there exist a threshold value $\square p_0$: if $\square p<\square p_0$, oscillations are damped, else, they are growing. On the figure we see the cycle of the force versus the minimum section of the glottis.

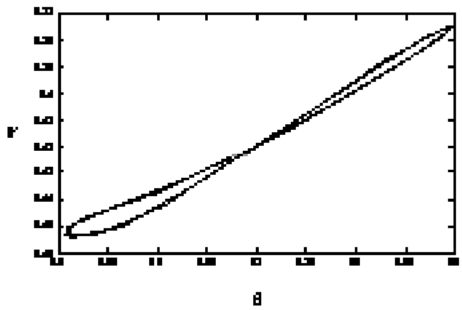

Figure : Force exerted by the air on the glottis as a function of the section S at the throat (the curve is parametrized by the time). Near the critical case $\square p=\square p_0$: during one half period the work furnished to the wall is positive, it is negative during the second half period.

Discussion and conclusion :

The chosen description is asymptotically coherent. The limits for the flow are the long bump approximation (linked to the choice of scales), and the fact that the two walls do not touch. Those hypothesis make it difficult to apply this work to real biomechanics cases, nevertheless those results confirm some aspects of the previous simple descriptions. Of interest is the observed fact that a *single (symmetrical) oscillator* is enough to obtain a self oscillatory system, which is a new result, in previous studies only "two mass models" where able to oscillate. A more detailed resolution for the solid is on progress.

References :

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