Reduced Navier Stokes in axisymmetrical stenoses

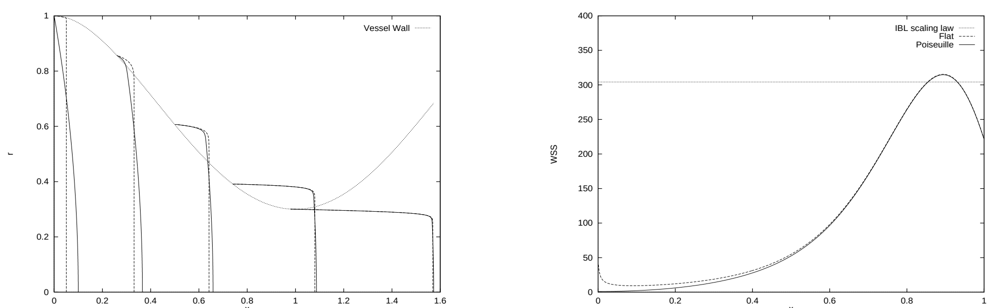
Pierre-Yves Lagrée and Sylvie Lorthois
L. M. M., Univ. PARIS VI, FRANCE ; I. M. F. T., TOULOUSE, FRANCE
Email: pyl@ccr.jussieu.fr, lorthois@imft.fr

Keywords - Reduced Navier-Stokes (RNS) - Stenosis - Biomechanics -

Abstract - The high values of the wall shear stresses (WSS) in advanced occlusive lesions are likely to play a role in the mechanism of thrombo-embolism and artherosclerotic plaque rupture. Navier-Stokes solvers are now very efficient in computing WSS. However, asymptotic methods provide a better understanding of the structure of the flow and of the relevant scalings. For example, the use of an interactive boundary-layer (IBL) method leads to a simple scaling law between maximal WSS, Reynolds number and geometrical parameters of the stenosis, for a steady, newtonian, axisymetrical flow [1]. This was not achieved previously using Navier-Stokes solvers [2]. However, due to the underlying assumption of existence of a potential core of perfect fluid, the IBL theory breaks down if the boundary-layer fills out the whole vessel's cross-section. Therefore, it is not suited to study transitions from boundary-layer/perfect fluid to fully viscous flow regions (entry flows) and vice versa. For this kind of flow, the relevant non-dimensional quantities are $x = \frac{x^\star}{r_0^\star Re}$, $r = \frac{r^\star}{r_0^\star}$, $u = \frac{u^\star}{\overline{u_0}^\star}$ and $v = \frac{v^\star Re}{\overline{u_0}^\star}$, where $x^\star$, $r^\star$, $u^\star$ and $v^\star$ are respectively the axial and radial coordinates and velocity components, and Re is the Reynolds number based on the entry radius ($r_0^\star$) and mean velocity ($\overline{u_0}^\star$). This holds if Re is large. At first order, Navier-Stokes equations are then written as :

$$\frac{\partial u}{\partial x} + \frac{1}{r}\frac{\partial (rv)}{\partial r} = 0, \quad u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = -\frac{\partial p}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right), \quad \frac{\partial p}{\partial r} = 0, \qquad (1)$$

This set of RNS equations is solved by a marching finite-differences scheme. The left figure displays the evolution of the velocity profile along the convergent part of a 70% stenosis, for two different imposed entry profiles: a flat profile (fully potential entry) and a Poiseuille profile (fully viscous entry). When the



Evolution of the velocity profile (left) and WSS distribution (right) along the convergent part of a 70% stenosis ($Re = 500$) ; solid line: Poiseuille entry profile ; broken line: flat entry profile.

entry flow is fully viscous, the strong acceleration causes the velocity profile to flatten. At the stenosis throat, the flow is thus independant of the entry. This is interesting because the *in vivo* entry profile is unknown but not parabolic as assumed in most studies [2]. In particular, the maximal WSS, independant of the entry, is in good agreement (7% discrepancy) with maximal WSS obtained by our IBL scaling law [1] (see right figure). In conclusion, the described set of RNS equations is "fully interactive" without any matching step and well suited for studying stenotic flow fields. Extension to unsteady and non-axisymetrical flows is currently in progress.

References

[1] S. Lorthois and P.-Y. Lagrée, *CRAS IIb* **328** (2000), 33–40.
[2] J.M. Siegel et al, *ASME J. Biomech. Egng.* **116** (1994), 446–451.