UPMC PARIS UNIVERSITAS

M2, Fluid mechanics 2015/2016
Friday, December 4th, 2015

Multiscale Hydrodynamic Phenomena

Part I. : 90 minutes, NO documents

1. Quick Questions In few words :
1.1 What is "dominant balance" ?
1.2 What is the dimension of the viscosity ?
1.3 Write Prandtl equations
1.4 What is the usual scale for pressure in incompressible NS equation ?
1.5 What is the usual scale for pressure in incompressible NS equation at small Reynolds ?
1.6-7-8 ∂'Alembert, Laplace, Heat : give the equation and any simple solution of it.
1.9 What is the Bürgers equation ? Which balance is it ?
1.10 What is the KDV equation ? Which balance is it ?

2. Exercice
Let us look at the following ordinary differential equation : $(E_\varepsilon) \quad \dfrac{d^2y}{dt^2} + 2\varepsilon\dfrac{dy}{dt} + y = 0,$ valid for any $t > 0$ with boundary conditions $y(0) = 0$ and $y'(0) = 1$. Of course ε is a given small parameter.
We want to solve this problem with Multiple Scales.
2.1 Expand up to order ε : $y = y_0(t) + \varepsilon y_1(t)$, show that there is a problem for long times.
2.2 Introduce two time scales, $t_0 = t$ and $t_1 = \varepsilon t$
2.3 Compute $\partial/\partial t$ and $\partial^2/\partial t^2$
2.4 Solve the problem.
2.5 Suggest the plot of the solution.
2.6 What is the exact solution for any ε, compare.

3. Exercice
Consider the following equation (of course ε is a given small parameter)

$$(E_\varepsilon) \quad \varepsilon\frac{\partial u}{\partial x} = -u. \text{ with }, \quad u(0) = 1.$$

We want to solve this problem with the Matched Asymptotic Expansion method.
3.1 Why is this problem singular ?
3.2 What is the outer problem and what is the possible general form of the outer solution ?
3.3 What is the inner problem of (E_ε) and what is the inner solution ?
3.4 Solve the problem at first order (up to power ε^0).
3.5 Suggest the plot of the inner and outer solution.
3.6 What is the exact solution for any ε.
3.7 Solve (E_ε) with WKB expansion $exp((S_0(x) + ...)/\delta)$.

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Friday, December 5th, 2014

Multiscale Hydrodynamic Phenomena

Part II. : 1h 15 min all documents.

triple Deck

1. This is a part of Landau & Lifschitz Fluid Mechanics Vol 6 1987 (but it was in first edition, 1959) Unfortunately a coffee drop fall on the book, a coffee stain hides the end of the solution.

L. D. LANDAU and E. M. LIFSHITZ

§41 *Stability of flow in the laminar boundary layer* 167

PROBLEM

Determine the order of magnitude of the least possible increase Δp in the pressure which can occur (in the main stream) over a distance Δx and cause separation.

SOLUTION. Let y be a distance from the surface of the body at which, firstly, Bernoulli's equation can be applied and, secondly, the squared velocity $v^2(y)$ in the boundary layer is less than the change $|\Delta U^2|$ in the squared velocity outside that layer. For $v(y)$ we can write, in order of magnitude, $v(y) \cong y\, dv/dy$ [illegible] here $\delta \sim \sqrt{(\nu l/U)}$ is the thickness of the boundary layer and l the dimension of the body. Equa[illegible] of magnitude, the two terms on the right-hand side of equation (40.6), we find

$$(1/\rho)\Delta p/\Delta x \sim \nu v(y)/y^2 \sim \nu U/\delta y.$$

From the condition $v^2 = |\Delta U^2| = (2/\rho)\Delta p$ we have $U^2y^2/\delta^2 \sim$ [illegible]. Eliminatin[illegible], we finally [illegible]

$$\Delta p \sim \rho U^2(\Delta x/l)^{[illegible]}.$$

1.1 Discuss the sentence "Bernoulli's equation can be applied", conclude that this is an overstatement and that a balance is involved, which one ?
1.2 What is the Blasius boundary layer thickness, in terms of characteristic quantities ?
1.3 Explain why the velocity in the boundary layer is linear near the wall, (the longitudinal velocity is written here $v(y)$)
1.4 Find the velocity in terms of the distance from the wall and the Boundary Layer thickness (the result is hidden by the large coffee stain)
1.5 What is equation (40.6) to your opinion ?
1.6 Explain the line with $(1/\rho)\Delta p/\Delta x) \sim \nu v(y)/y^2$...
1.7 What is the final response to the problem ?

2. IBL
2.1 Comment (8) Meyer page 643
2.2 Comment (9) Meyer page 644

3. Scaling of triple deck.
3.1 Meyer starts by the upper deck and says that the scales in x and y are the same (11) page 647. Explain why.
3.2 Where does equation (2.4a) Stewartson page 314 comes from ?
3.3 Where does equation (2.4b) Stewartson page 314 comes from ?
What is the argument to go from (30) and (31) in Meyer 652 ?
3.4 Check that (27) of Meyer is the same than the scales given by Stewartson page 314 at the end of the paragraph.
3.5 What is the scale of the transverse velocity in the lower deck ?
3.6 Check that all these scales give by substitution the equations (28a)-(28c) of Meyer page 650.
3.7 Check that it is consistent with question 1.7
3.8 As defined by Meyer page 651 section 7, what is the scale of angle of perturbations ?
3.9 Check that equations (2.2) Stewartson page 313 is the solution in the main deck.
3.10 Write the final system with all the boundary conditions.
3.11 Bottom of page 327, top 328 Stewartson suggests an unsteady triple deck. Find by dominant balance the scale of the time.