MASTER SDI SPECIALTY M2FA

Hydrodynamics

Test

December 2, 2011

2 hours – all documentation is authorized. Internet access unauthorized.

1 We consider viscous incompressible flow. Recall the definition of the stream function for axisymmetric flow in spherical coordinates.

2 We consider a bubble of radius a rising in incompressible Stokes flow. The viscosities and densities are μ_i, ρ_i with $i = 1$ for the bubble and $i = 2$ for the fluid outside. There is no flow at infinity and no surface tension. Write the conditions on the fluid velocity at the bubble interface.

3 Recall the general solution for the pressure and vorticity in the flow around a sphere.

4 Using the same arguments as in the course, give the forms of pressure and vorticity inside the sphere.

5 Explain how the coefficiens for the vorticity and pressure are connected.

6 Find the expression for the stream function inside the sphere, including some unknown coefficients.

superscript

7 The matching conditions at a fluid interface without surface tension are $\mathbf{u}^{(1)} = \mathbf{u}^{(2)}$ where the superscript indicates on which side of the interface the quantity is estimated, and the stress balance condition $\sigma_{ij}^{(1)} n_j = \sigma_{ij}^{(2)} n_j$ where σ is the total stress tensor in the fluid (including the pressure) and n_i is the unit normal. Find expressions connecting the unknown coefficients to those for the stream function outside the sphere using the velocity condition.

8 What other condition on the velocity can one write ?

9 Write two more conditions from the stress balance condition.

M2, Fluid mechanics 2011/2012
Friday, December 2nd, 2011

Multiscale Hydrodynamic Phenomena

Heat transfer in natural convection, influence of a large Grashof number

We consider the heat transfer from a vertical plate to a viscous incompressible dilatable fluid in the Boussinesq approximation. The set of equations are, incompressibility, momentum and heat equation, notice the dilatable density $\rho = \rho_0(1 - \alpha(T - T_0))$ which produces an Archimedes force. The dynamic viscosity μ and the heat Fourier coefficient k are supposed constant.

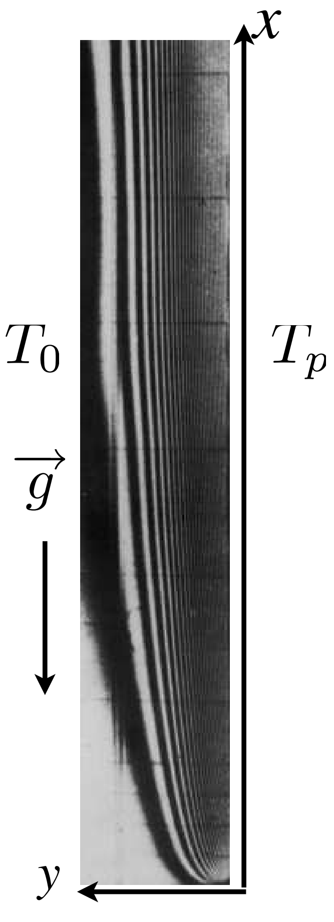

FIG. 1 – Interferogram showing iso density lines (photo Van Dyke 82).

$$\begin{cases} \vec{\nabla} \cdot \vec{u} = 0 \\ \rho_0(\frac{\partial}{\partial t}\vec{u} + \vec{u} \cdot \vec{\nabla}\vec{u}) = \\ -\vec{\nabla} p - \rho_0 g(1 - \alpha(T - T_0))\vec{e}_x + \mu \vec{\nabla}^2 \vec{u}, \\ \rho_0 c_p(\frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla})T = k\vec{\nabla}^2 T, \end{cases}$$

We will consider a steady 2D flow. The x axis is vertical y is horizontal, the gravity is $\vec{g} = -g\vec{e}_x$. The flat plate is in $x > 0$ and has no thickness. By thermal diffusion, the heated semi infinite flat plate at temperature T_p increases the temperature in the air near the plate. This heated layer has a smaller density than the surrounding air, so that a buoyancy force appears $-\rho_0 g(1-\alpha(T-T_0))\vec{e}_x$. The flow is generated by this buoyancy force, and viscosity slows down the flow. The flow convects then the temperature.

A steady configuration exists, this is the "Natural Convection", see figure 1.

The equations are the same than for the Rayleigh Bénard instability, but the configuration is different. The relevant number is no more the Rayleigh number but the Grashof number (and the Prandtl number is $Pr = \mu c_p/k$) so that

$$G = \frac{g\alpha(T_p - T_0)L^3}{\nu^2}$$

and of course the inverse of this number is small...

1.1 Define a new pressure P as $p = -\rho_0 g x + P$ and write the boundary conditions for P and the new momentum equation with P.
1.2 Using a standard approach, considering the flow at position L, where the induced flow is of velocity U_0, we want obtain a non dimensional problem. Why do we take first $u = U_0\bar{u}$, $v = U_0\bar{v}$, $x = L\bar{x}$, and $y = L\bar{y}$? Explain why $T = T_0 + (T_p - T_0)\bar{T}$ is a good idea. Define the scale for the excess of pressure P.
1.3 Which idea guides us to link directly $\rho U_0^2/L$ to ρ , g, α, and $(T_p - T_0)$?
1.4 Having Write the non dimensional steady 2D system with a Grashof number. What is the link between the Rayleigh and Grashof numbers ?
1.5 We have obtained a system without dimension, with the G number. Conclude that if $1/G$ is vanishingly small the problem is singular.
1.6 We have to introduce a thin boundary layer of thickness δ, why ? We keep the same $\bar{x}$, why ?
1.7 What is the relation between δ/L and G ?
1.8 Write all the equations without dimension in the new stretched coordinates.
1.9 Write all the boundary conditions.
1.10 This system may be solved with selfsimilar variables $\xi = \bar{x}, \quad \eta = \tilde{y}/\bar{x}^{1/4}$, the stream function ($u = \partial_y\psi$ and $v = -\partial_x\psi$) prove that they are :

$$\psi = \bar{x}^{3/4} f(\eta) \text{ and } \tilde{T} = g(\eta),$$

1.11 Show that the final system is

$$4f''' + 3ff'' - 2f'^2 + 4g = 0; \quad 4g'' + 3Prfg' = 0;$$

what are the boundary conditions ?
1.12 On figure 2 left is the computation with the Navier Stokes solver `gerris` of u/U_0 and $T - T_0/(T_p - T)$ at 4 different locations in x/L as a function of y/L. Identify every curve. Comments ? On figure 2 right the self similar solution (lines) and the same computed fields, what are the axes ? The ordinates ? Comments ?

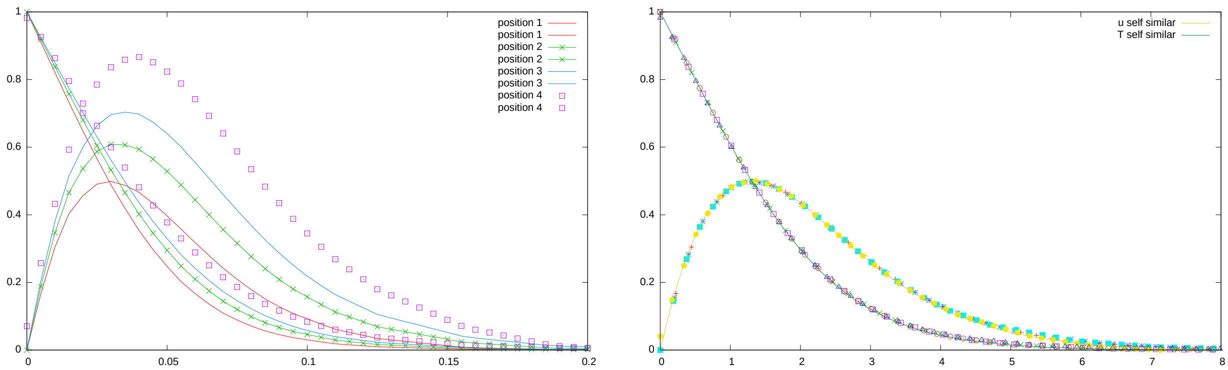

Fig. 2 – Numerical simulations with `gerris` of the system.

Exercice

Let us look at the following ordinary differential equation :

$$(E_\varepsilon) \qquad \varepsilon \frac{d^2y}{dx^2} - y = -1,$$

valid for $0 \leq x \leq 1$, with boundary conditions $y(0) = 0$ and $y(1) = 1$. Of course ε is a given small parameter.

We want to solve this problem with the Matched Asymptotic Expansion method.

1) Why is this problem singular ?

2) What is the outer problem obtained from (E_ε) and what is the possible general form of the outer solution ?

3) Discuss the position of the boundary layer ($x = 0$ or $x = 1$) find the new local scale $\delta(\varepsilon)$ at the singular point.

4) What is the inner problem of (E_ε) and what is the inner solution ?

5) Solve the problem at first order (up to power ε^0).

6) Suggest the plot of the inner and outer solution.

7) What is the exact solution for any ε.

8) Construct the composite expansion and draw it for a small ε, compare with the exact solution.