Conduction

équation de conservation de la masse :

$$\cancel{\frac{d\rho}{dt} + \rho \underline{\nabla} \cdot \underline{u} = 0.}$$

équation de conservation de la quantité de mouvement :

$$\cancel{\rho \frac{d\underline{u}}{dt} = \underline{\nabla} \cdot \underline{\underline{\sigma}} + \underline{f}.}$$

équation de l'énergie :

$$\rho \frac{de}{dt} = \cancel{\underline{\underline{\sigma}} : \underline{\underline{D}}} - \underline{\nabla} \cdot \underline{q} + r.$$

relations constitutives :

$$\cancel{\underline{\underline{\sigma}} = -p\underline{\underline{I}} + \lambda \underline{\nabla} \cdot \underline{u}\underline{\underline{I}} + 2\mu \underline{\underline{D}}}$$

$$\underline{q} = -k\underline{\nabla} T.$$

loi d'état :

$$\cancel{p(\rho, T)}$$

coefficients :

$$c_v(T), \quad c_p(T), \quad \lambda(T), \quad \mu(T), \quad k(T)...$$

conditions aux limites T_w **OU** q_w imposés, adhérence à la paroi.

Conduction

$$de = c\,dT$$

équation de conservation de la masse :

$$\cancel{\frac{d\rho}{dt} + \rho \underline{\nabla} \cdot \underline{u} = 0.}$$

équation de conservation de la quantité de mouvement :

$$\cancel{\rho \frac{d\underline{u}}{dt} = \underline{\nabla} \cdot \underline{\underline{\sigma}} + \underline{f}.}$$

équation de l'énergie :

$$\rho \frac{de}{dt} = \cancel{\underline{\underline{\sigma}} : \underline{\underline{D}}} - \underline{\nabla} \cdot \underline{q} + r.$$

$$\frac{\partial}{\partial t}$$

relations constitutives :

$$\cancel{\underline{\underline{\sigma}} = -p\underline{\underline{I}} + \lambda \underline{\nabla} \cdot \underline{u}\underline{\underline{I}} + 2\mu \underline{\underline{D}}}$$

$$\underline{q} = -k \underline{\nabla} T.$$

loi d'état :

$$\cancel{p(\rho, T)}$$

coefficients :

$$c_v(T), \quad c_p(T), \quad \lambda(T), \quad \mu(T), \quad k(T)...$$

conditions aux limites T_w **OU** q_w imposés, adhérence à la paroi.

Conduction

équation de conservation de la masse :

$$\cancel{\frac{d\rho}{dt} + \rho \underline{\nabla} \cdot \underline{u} = 0.}$$

équation de conservation de la quantité de mouvement :

$$\xcancel{\rho \frac{d\underline{u}}{dt} = \underline{\nabla} \cdot \underline{\underline{\sigma}} + \underline{f}.}$$

$$de = c\,dT$$

équation de l'énergie :

$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T$$

$$\frac{\partial}{\partial t}$$

relations constitutives :

$$\cancel{\underline{\underline{\sigma}} = -p\underline{\underline{I}} + \lambda \underline{\nabla} \cdot \underline{u}\underline{\underline{I}} + 2\mu \underline{\underline{D}}}$$

$$\underline{q} = -k \underline{\nabla} T.$$

loi d'état :

$$\cancel{p(\rho, T)}$$

coefficients :

$$c_v(T), \quad c_p(T), \quad \lambda(T), \quad \mu(T), \quad k(T)...$$

conditions aux limites T_w **OU** q_w imposés, adhérence à la paroi.

$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T$$

Conduction

$$\rho c\frac{\partial T}{\partial t} = k\nabla^2 T$$

conduction dans l'objet, l'extérieur est modélisé par un coefficient d'échange

$$(k[\frac{\partial T}{\partial n}]_w \underline{n} + h(T_w - T_f)))\underline{n} = 0.$$

Biot $\frac{hL}{k}$

Valeurs de k et de h

k		matériau	k en $Wm^{-1}K^{-1}$	$k/(\rho c_p)$ en $m^2 s^{-1}$
0.01		air	$2.5\ 10^{-2}$	$2\ 10^{-5}$
	gaz			
0.1		bois	0.13	$2.4\ 10^{-7}$
	liquides	glycérine	0.29	$0.98\ 10^{-7}$
		eau	0.60	$1.44\ 10^{-7}$
1				
		mercure	8.0	$4.2\ 10^{-6}$
		granit	2.51	$1.1\ 10^{-6}$
100	métaux	acier	46	$1.2\ 10^{-5}$
		alu	200	$0.86\ 10^{-4}$
		argent	418	$1.71\ 10^{-4}$

"gamme des valeurs" de h (unité $\mathrm{Wm^{-2}K^{-1}}$)

rayonnement (linéarisé a 300K)	1
convection libre (air)	5-25
convection libre (eau)	100-900
convection forcée (air)	10-500
convection forcée (eau)	100-15000
convection forcée (huile)	50-2000
conv. f. (métaux fondus)	6000-120000
eau bouillante	2500-25000
vapeur d'eau se condensant	50000-100000

Le nombre de Biot varie de 0 à l'infini

Biot $\dfrac{hL}{k}$

Dans la suite du cours, nous allons dans des cas particuliers simples estimer *h* à partir de nombres sans dimension caractéristiques du fluide et de l'écoulement

$$h = \frac{k}{L} Nu$$

Nombre de Nusselt $$Nu = 0.66 Pr^{1/3} R^{1/2}$$

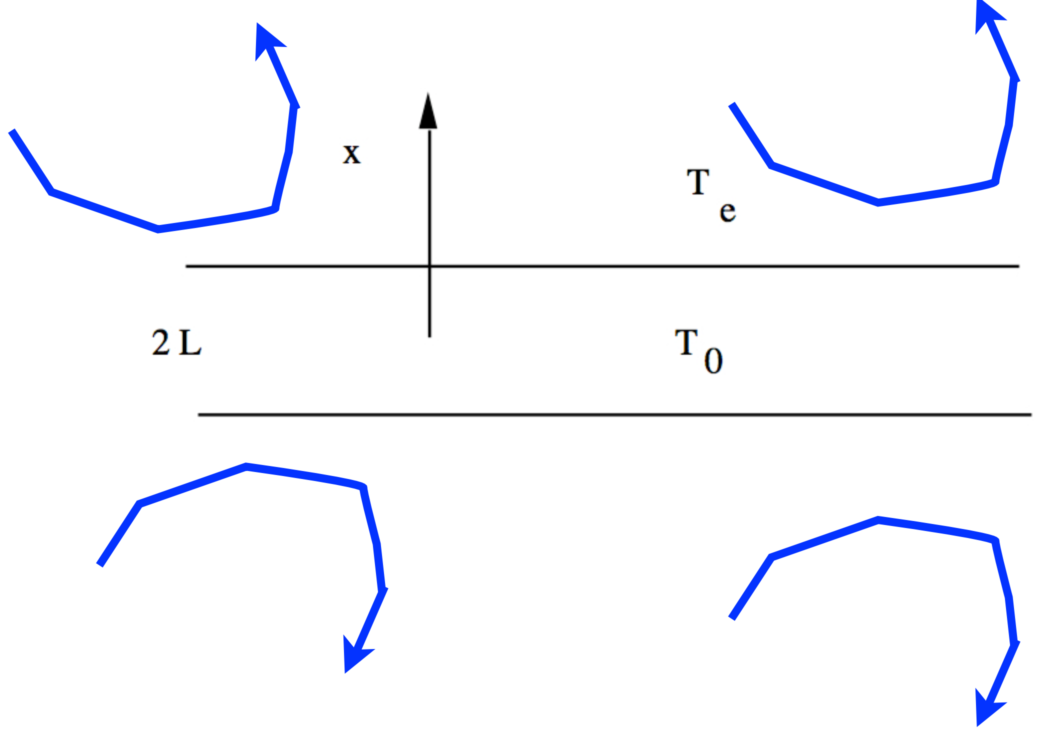
Nombre de Reynolds $$R = UL/\nu$$

Nombre de Prandtl $$Pr = \nu/(k/\rho c_p))$$

en PC:

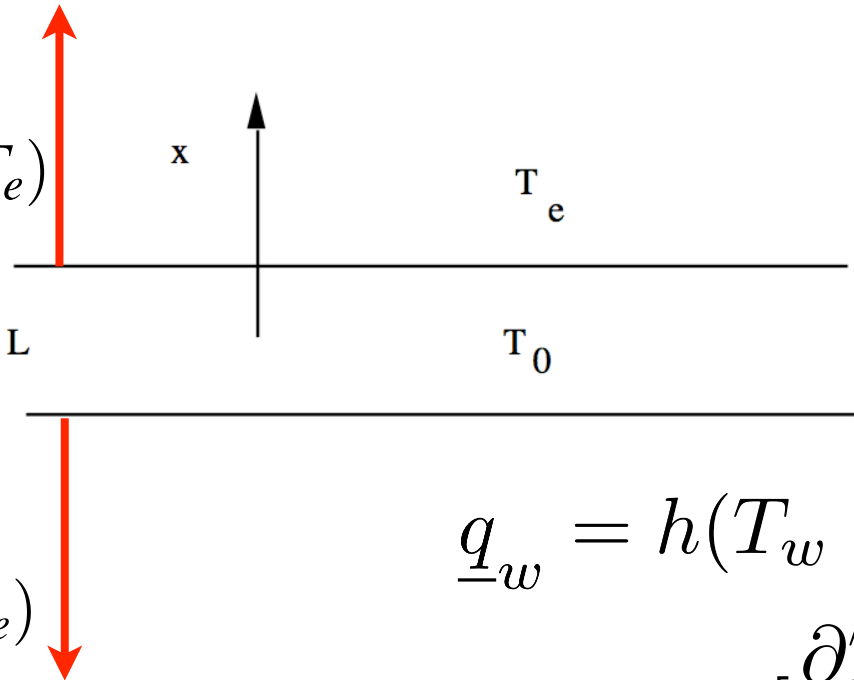
refroidissement d’une lamelle infinie

Echange



Echange

$$-k\frac{\partial T}{\partial x} = h(T - T_e)$$



$$k\frac{\partial T}{\partial x} = h(T - T_e)$$

$$\underline{q}_w = h(T_w - T_f)\underline{n}.$$

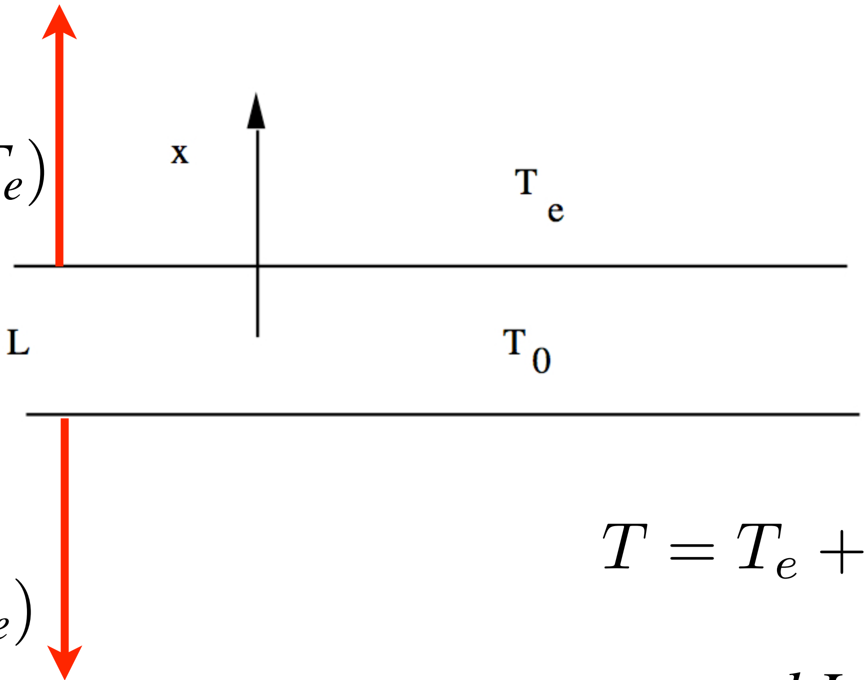
$$\underline{q}_w = -k[\frac{\partial T}{\partial n}]_w \underline{n},$$

$$k[\frac{\partial T}{\partial n}]_w \underline{n} + h(T_w - T_f)\ \underline{n} = 0.$$

$$Bi = \frac{hL}{k}$$

Echange

$$-k\frac{\partial T}{\partial x} = h(T - T_e)$$



$$k\frac{\partial T}{\partial x} = h(T - T_e)$$

$$T = T_e + (T_0 - T_e)\bar{T}$$

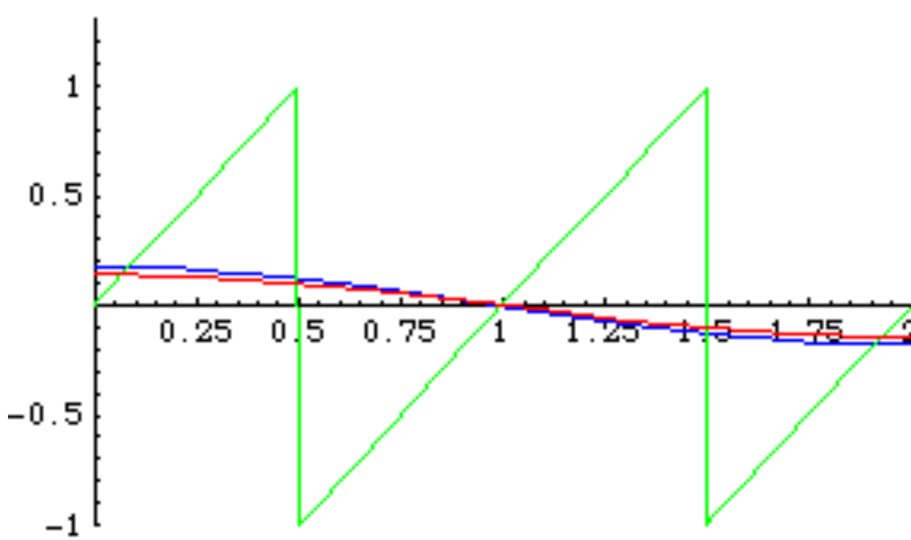
$$Bi = \frac{hL}{k}$$

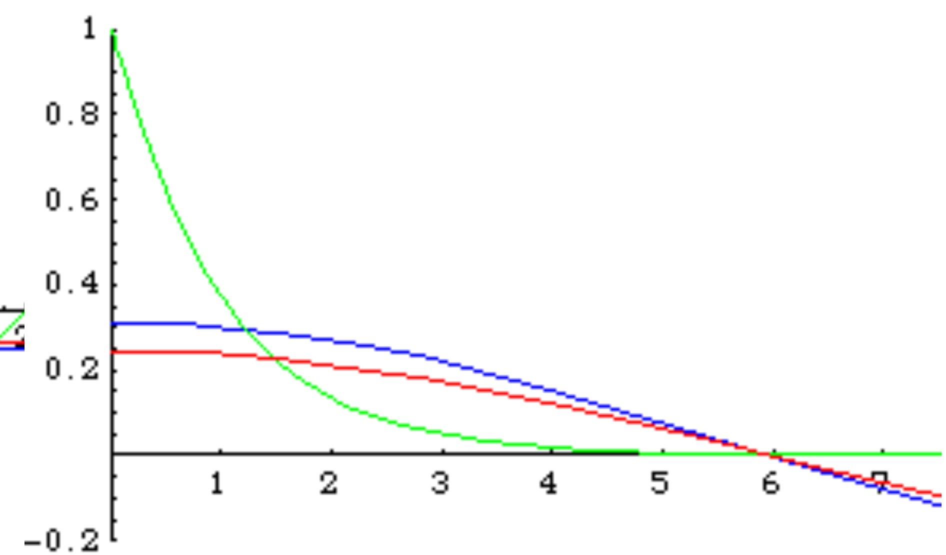
$$\frac{\partial \bar{T}}{\partial \bar{t}} = (\frac{\partial^2 \bar{T}}{\partial \bar{x}^2})$$

$\bar{T} = 1$ en $\bar{t} = 0$ et $-(\frac{\partial \bar{T}}{\partial \bar{x}}) = \pm Bi\bar{T}$ en $\bar{t}$>0 et $\bar{x} = \pm 1$.

Solution en Séries de Fourier

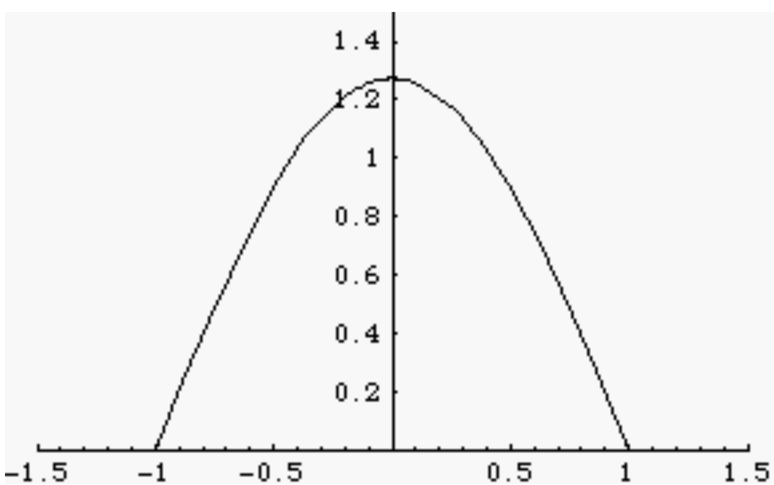
$$\Sigma_{i>0} A_i exp(-k_i^2 \bar{t}) cos(k_i \bar{x})$$





Solution en Séries de Fourier

$\Sigma_{i>0} A_i exp(-k_i^2 \bar{t}) cos(k_i \bar{x})$

TFI 

$= \sum_{i>0} A_i cos(k_i \bar{x}),$

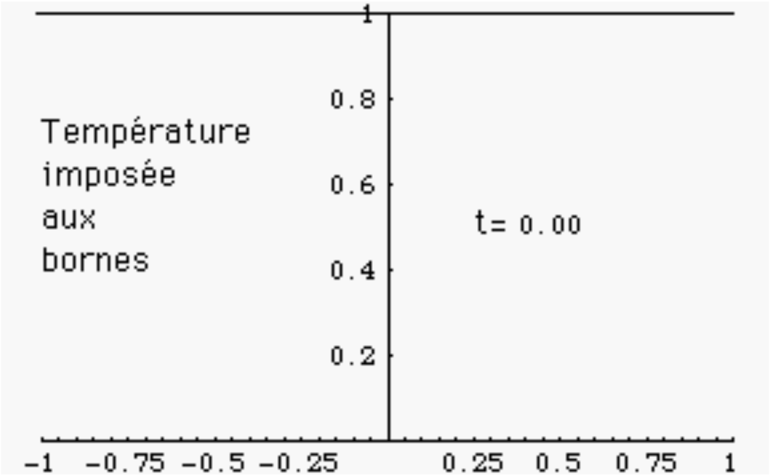
Solution en Séries de Fourier

$$\Sigma_{i>0} A_i exp(-k_i^2 \bar{t}) cos(k_i \bar{x})$$

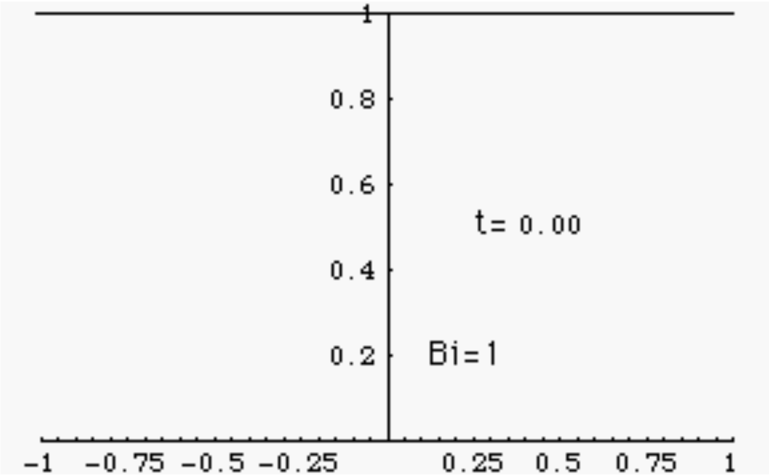
$$A_i = \frac{2 sin(k_i)}{k_i + sin(k_i) cos(k_i)}$$

Si $Bi = 0$, $k_i = (i-1)\pi$.
Si $Bi = \infty$, $k_i = (2i-1)\pi/2$.
Si $Bi = 1$, $k_1 = 0.863$, k_2=3.4256, k_3=6.4373, k_4=9.5293...

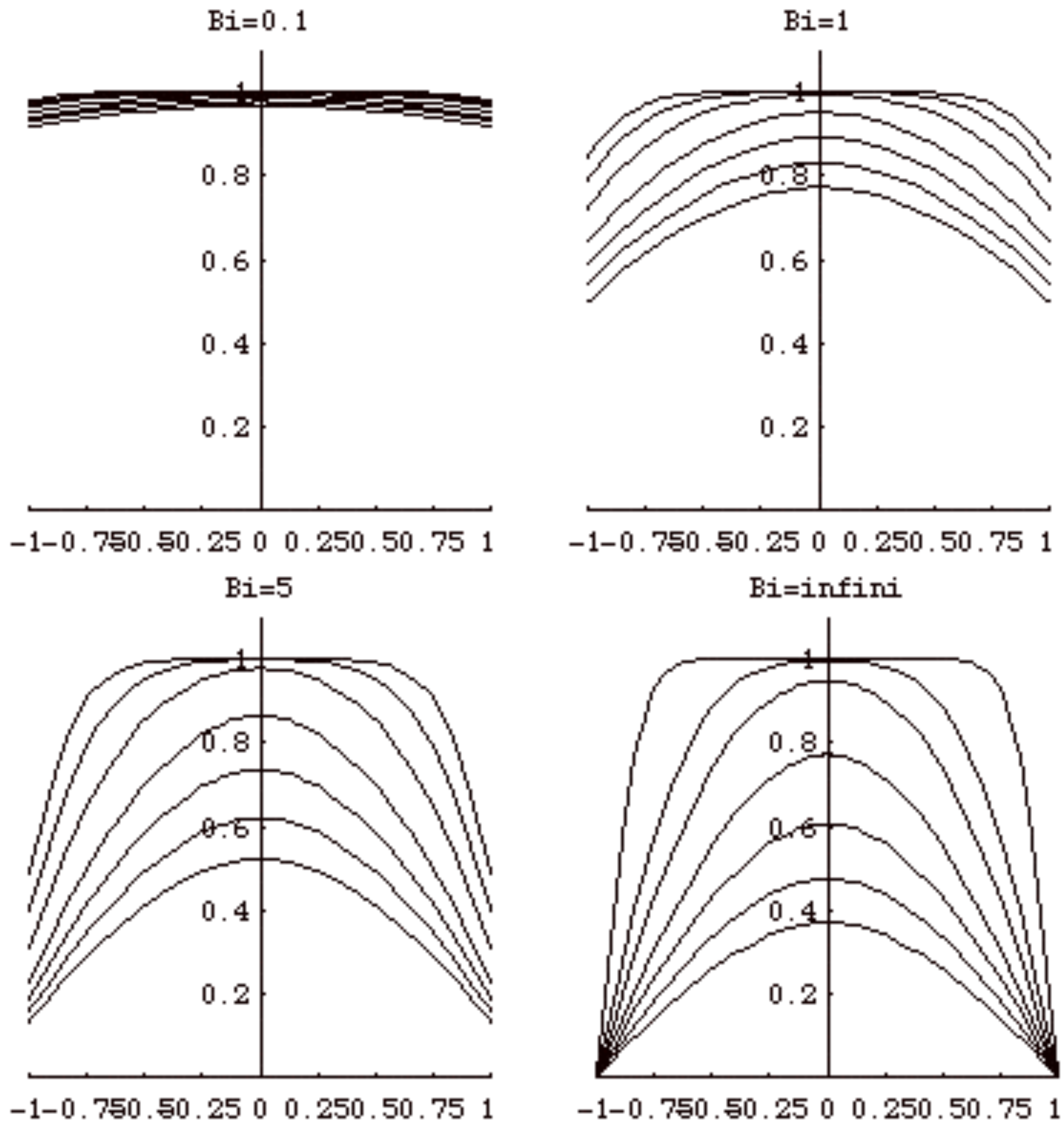
$$k tg(k) = Bi.$$



Bi infini



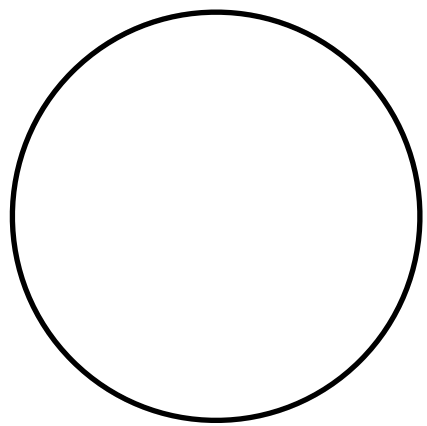
Bi=1



figures pour t=0.025 0.05 0.1 0.2 0.3 0.4 et 0.5 à différents nombres de Biot

En Cylindriques

$$\frac{\partial \bar{T}}{\partial \bar{t}} = \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\Big[\bar{r}\frac{\partial \bar{T}}{\partial \bar{r}}\Big]$$



En Cylindriques

$$\frac{\partial \bar{T}}{\partial \bar{t}} = \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} [\bar{r} \frac{\partial \bar{T}}{\partial \bar{r}}]$$

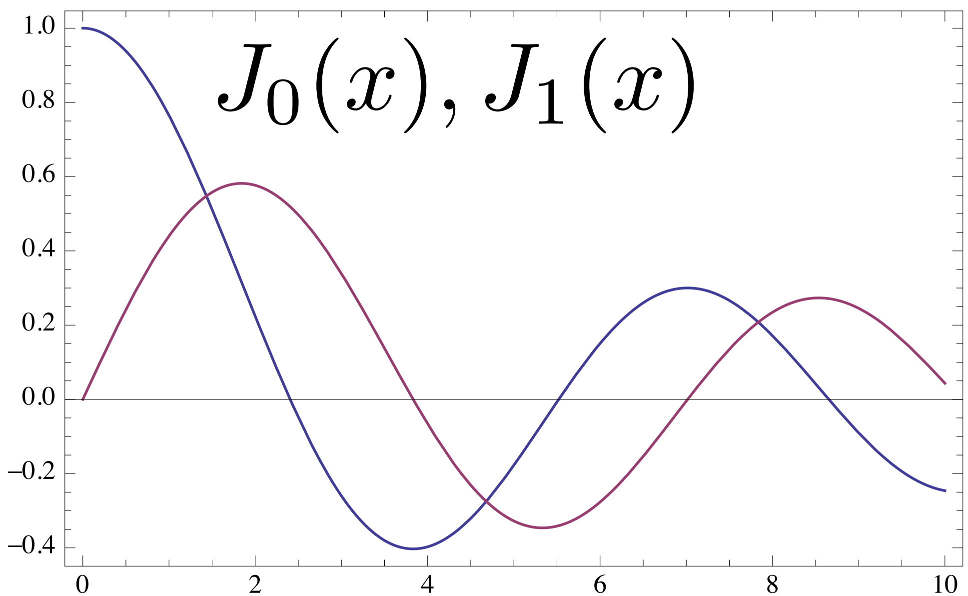
variables séparées $\bar{T} = f(\bar{r})g(\bar{r})$

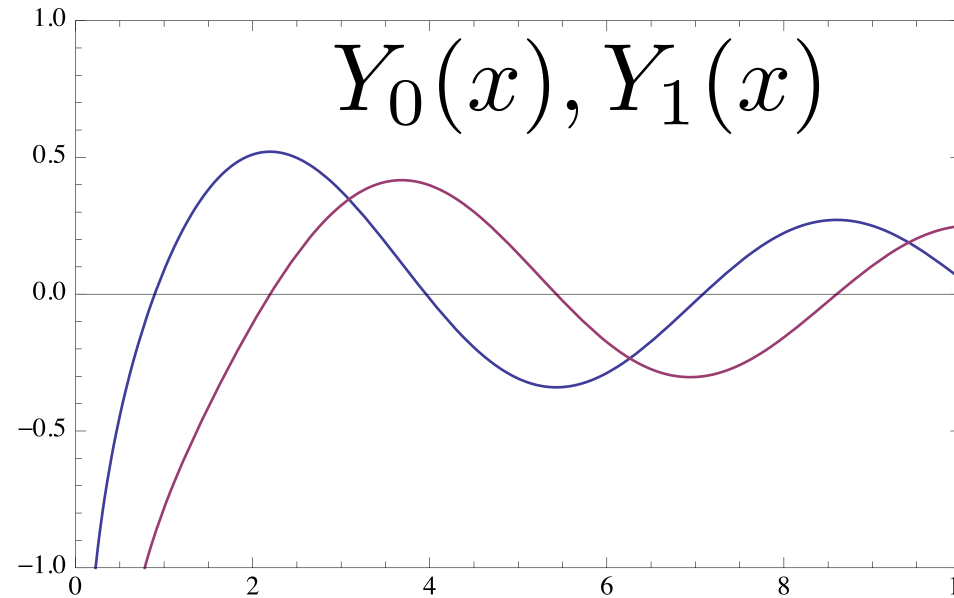
$$\frac{\bar{r}f'' + f'}{\bar{r}f} = \frac{g'}{g} = -\mu^2$$

exponentielle en temps et ...

fonction de Bessel en r

$$x^2 y'' + x y' + (x^2 - \alpha^2) y = 0$$





zéros de J_0 $\quad J_0(\mu) = 0 \; \mu_1 = 2.405 \; \mu_2 = 5.52 \; \mu_3 = 8.65 \; \mu_4 = 11.79$

Forme générale $\quad \bar{T} = \Sigma_i A_i J_0(\mu_i \bar{r})) exp(-\mu_i^2 \bar{t})$

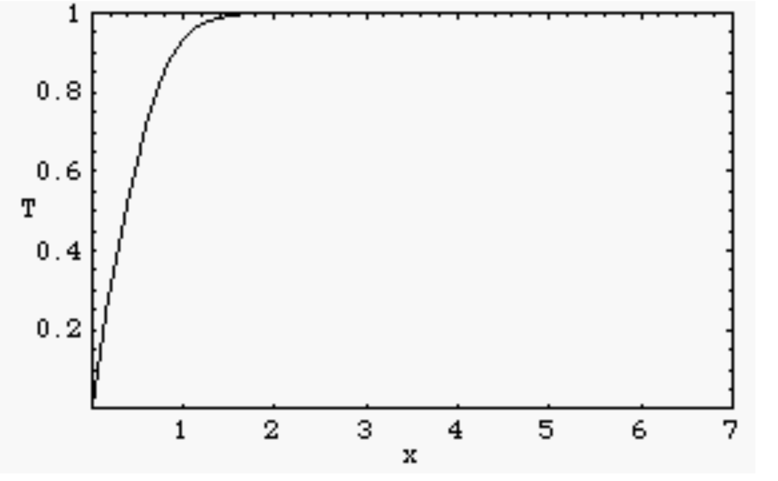
formules de produit scalaire
et normalisation

$$\int_0^1 x J_n^2(\mu x) dx = \frac{1}{2}[J_n'^2(\mu) + (1 - \frac{n^2}{\mu^2}) J_n^2(\mu)]$$

$$\int_0^1 x J_n(\alpha x) J_n(\beta x) dx = \frac{1}{\beta^2 - \alpha^2}[\alpha J_n(\beta) J_n'(\alpha) - \beta J_n(\alpha) J_n'(\beta)]$$

Milieu semi infini

$$\frac{\partial \bar{T}}{\partial \bar{t}} = (\frac{\partial^2 \bar{T}}{\partial \bar{x}^2})$$

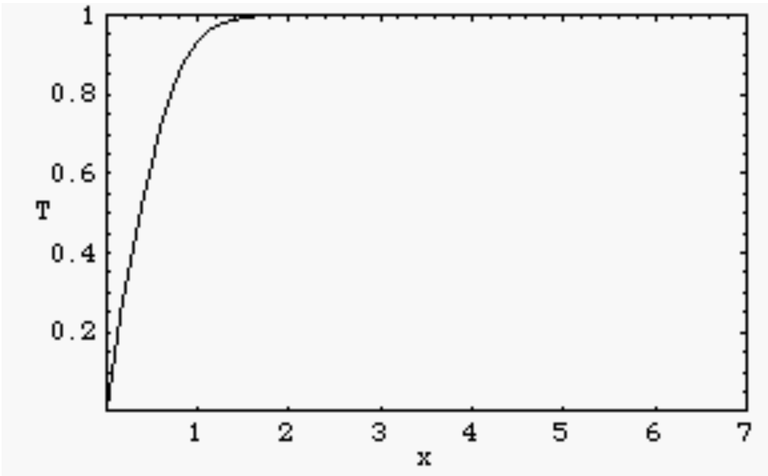


$$\bar{T}(\bar{x}, \bar{t} = 0) = 1$$

$$\bar{T}(\bar{x} = 0, \bar{t}) = 0$$

$$\bar{T}(\bar{x} = \infty, \bar{t}) = 1$$

Milieu semi infini



variable de similitude

$$\eta = \bar{x}/\sqrt{\bar{t}}$$

On remarque que $\frac{\partial \bar{T}}{\partial \bar{t}} = \frac{\partial \bar{T}}{\partial \eta}\frac{\partial \eta}{\partial \bar{t}}$, donc:
Lorsque l'on dérive $\frac{\partial \bar{T}}{\partial \bar{t}} = \theta'(\eta)\frac{\partial \eta}{\partial \bar{t}}$, or $\frac{\partial \eta}{\partial \bar{t}} = \frac{-\bar{x}}{2\bar{t}^{3/2}} = -\frac{\eta}{2\bar{t}}$
Lorsque l'on dérive $\frac{\partial \bar{T}}{\partial \bar{x}} = \theta'(\eta)\frac{\partial \eta}{\partial \bar{x}}$, or $\frac{\partial \eta}{\partial \bar{x}} = \frac{-1}{\bar{t}^{1/2}}$

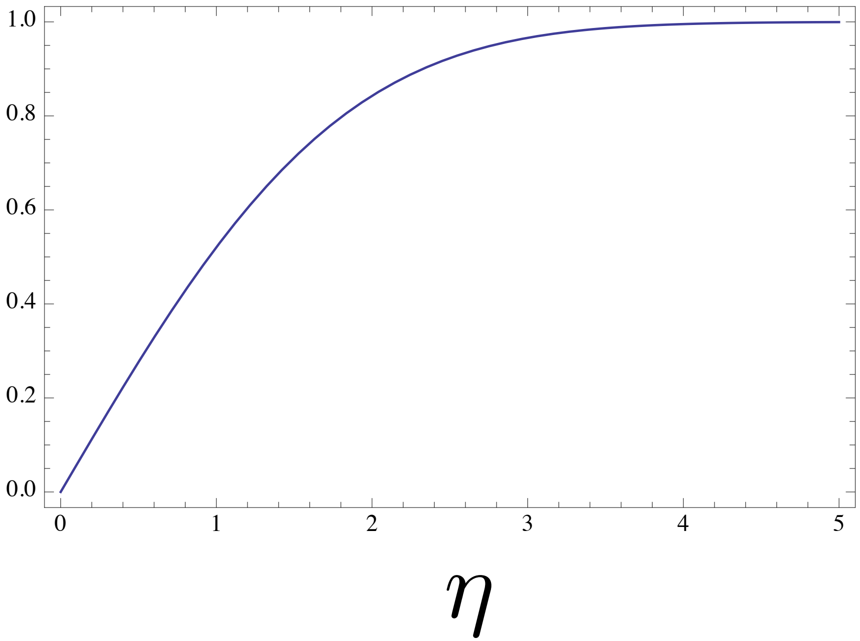
donc $\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} = \theta''(\eta)\bar{t}^{-1}/2$, et $\frac{\partial \bar{T}}{\partial \bar{t}} = -\eta\theta'(\eta)\bar{t}^{-1}$ on constate donc que:

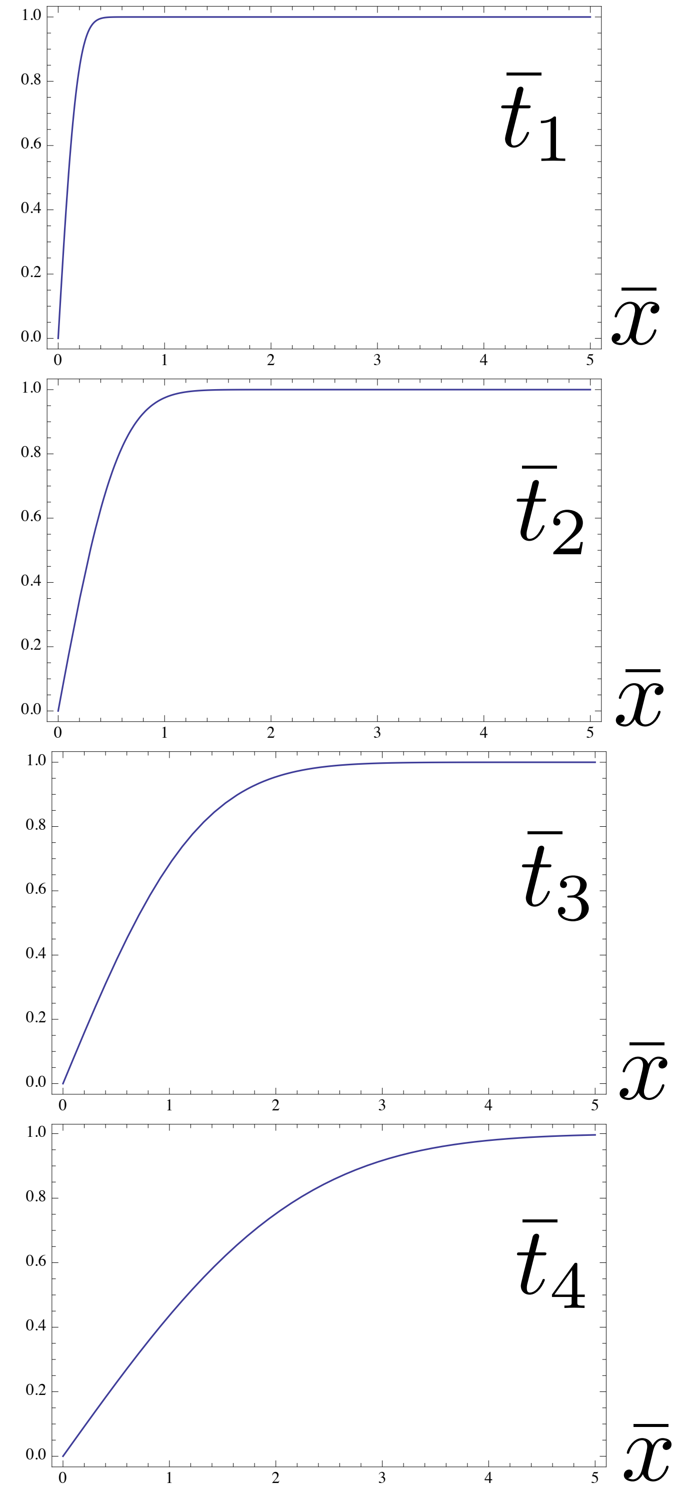
$$\theta''(\eta) = -\frac{\eta}{2}\theta'(\eta).$$

$\eta = \frac{\bar{x}}{\sqrt{\bar{t}}}$ et $\bar{T} = \theta(\eta)$ avec $\theta(\eta) = \frac{2}{\sqrt{\pi}}\int_0^{\eta/2} exp(-\xi^2)d\xi$

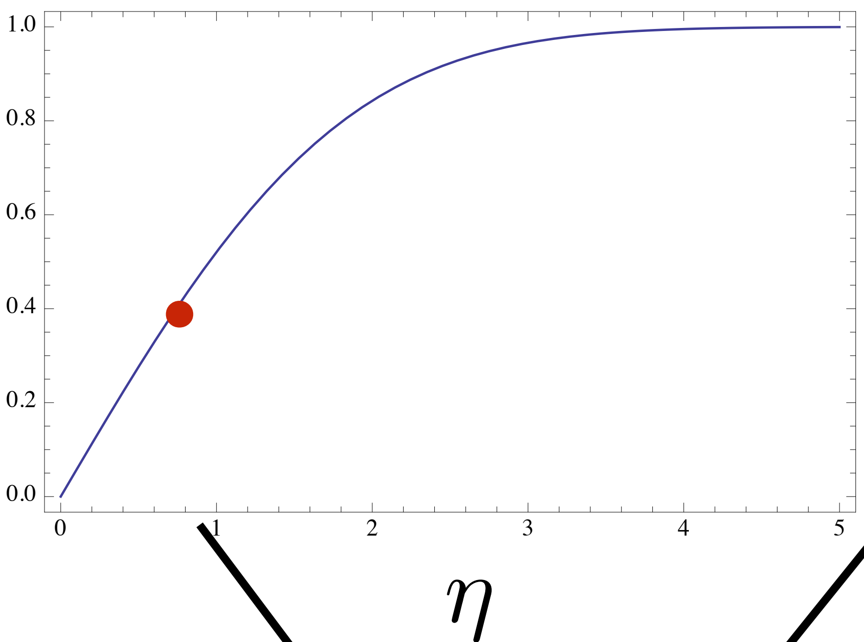
$$\theta = erf(\eta/2)$$

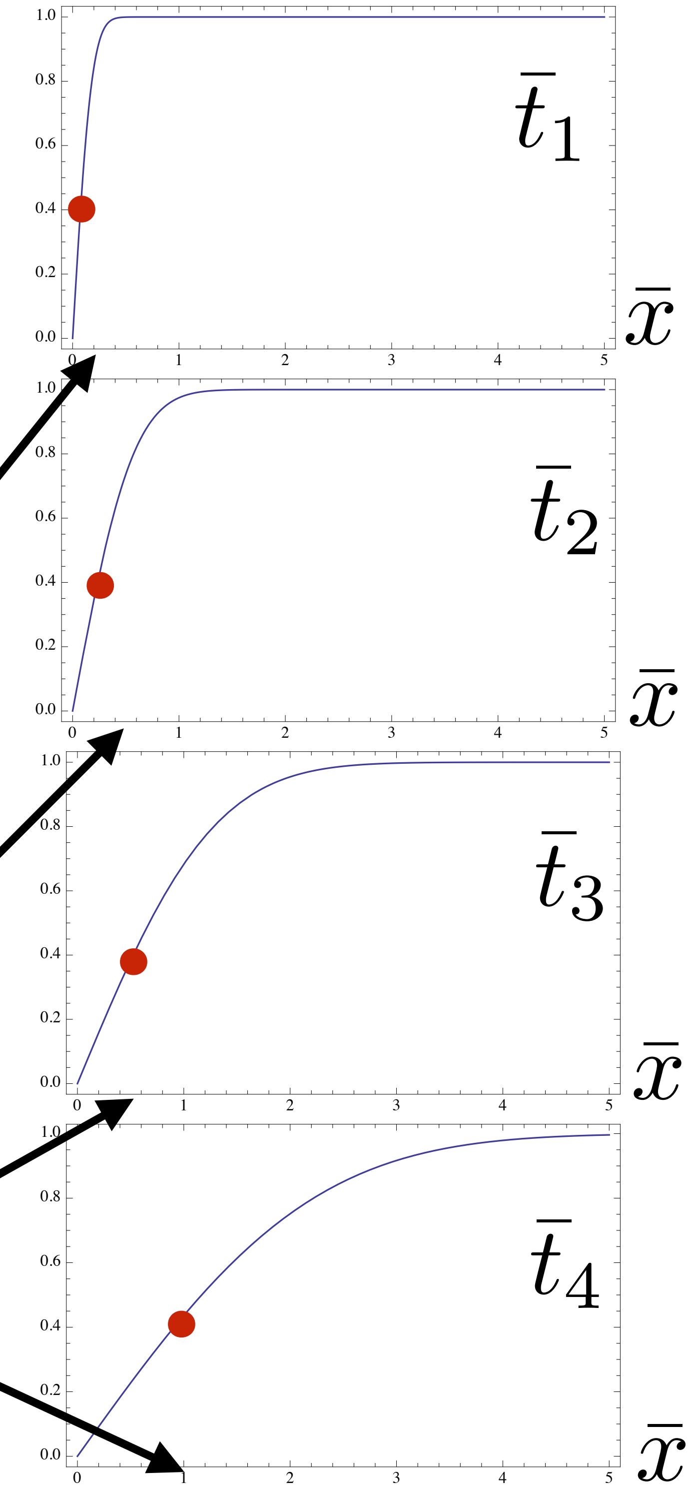
Milieu semi infini





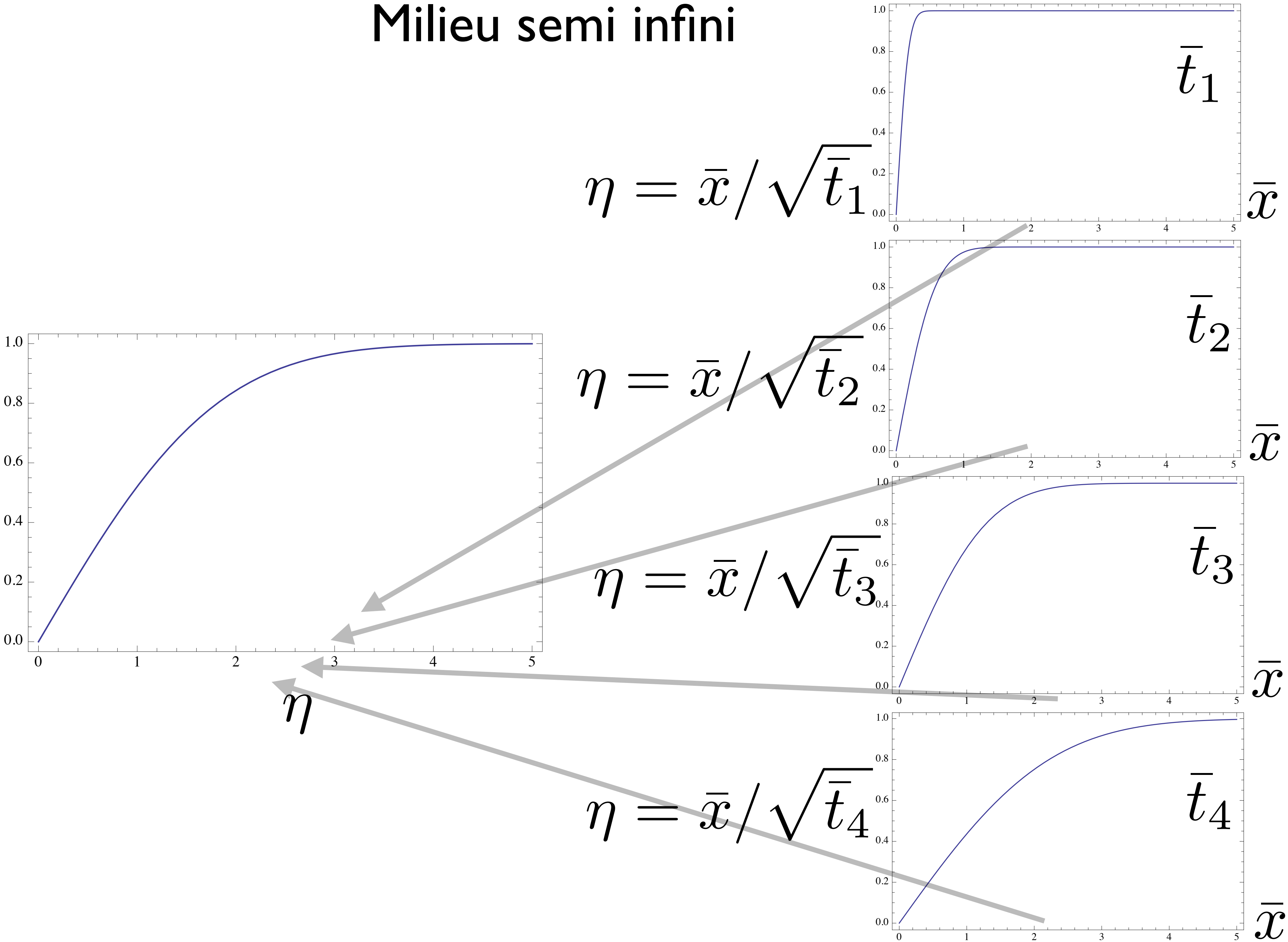
Milieu semi infini





$$\eta_0 = \frac{\bar{x}_1}{\sqrt{\bar{t}_1}} = \frac{\bar{x}_2}{\sqrt{\bar{t}_2}} = \frac{\bar{x}_3}{\sqrt{\bar{t}_3}} = \frac{\bar{x}_4}{\sqrt{\bar{t}_4}}$$

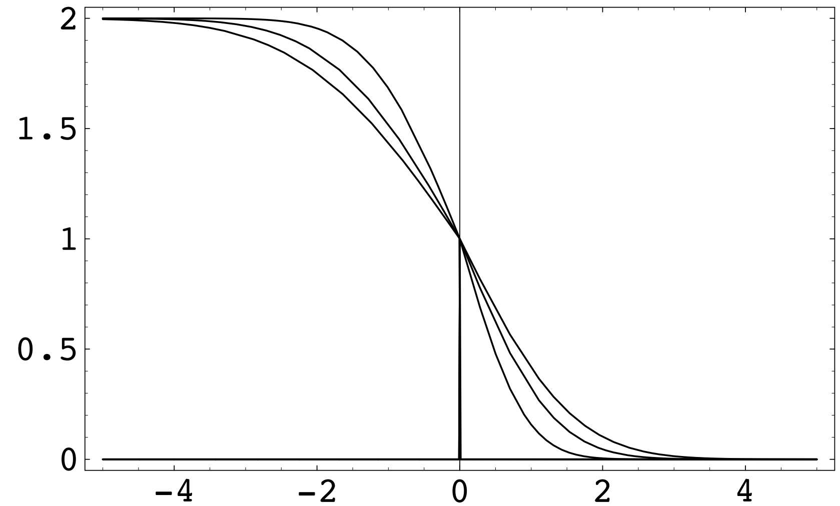
Milieu semi infini



Cas avec deux milieux

T_1

$$\frac{\partial T}{\partial t} = a_1 \frac{\partial^2 T}{\partial x^2}$$



$$\frac{\partial T}{\partial t} = a_2 \frac{\partial^2 T}{\partial x^2}$$

T_2

$$T|_1 = T|_2$$

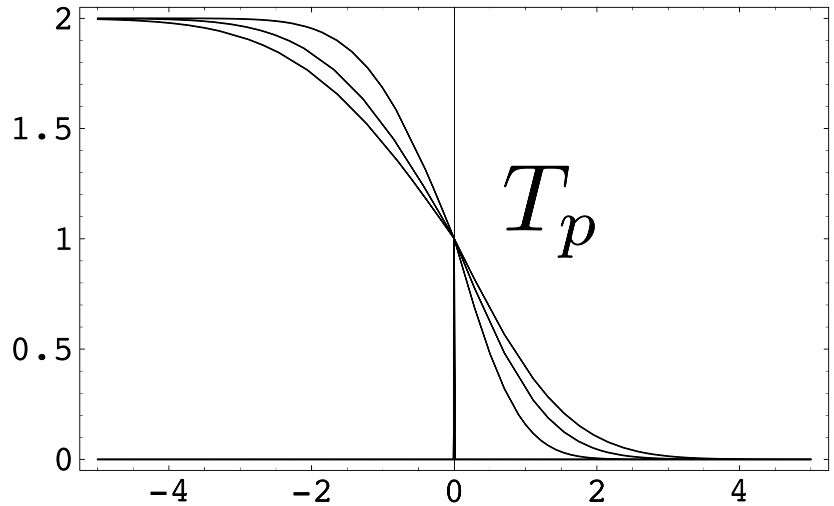
$$k_1 \frac{\partial T}{\partial x}|_1 = k_2 \frac{\partial T}{\partial x}|$$

égalité des températures et des flux à l'interface

Cas avec deux milieux

$$T_1$$

$$\frac{\partial T}{\partial t} = a_1 \frac{\partial^2 T}{\partial x^2}$$



$$\frac{\partial T}{\partial t} = a_2 \frac{\partial^2 T}{\partial x^2}$$

$$T_2$$

$$T|_1 = T|_2$$

$$k_1 \frac{\partial T}{\partial x}|_1 = k_2 \frac{\partial T}{\partial x}|$$

Pour $x > 0$ on a donc $T - T_p = (T_2 - T_p) erf(x/\sqrt{4a_2 t})$
Pour $x < 0$ on a donc $T - T_p = (T_1 - T_p) erf(-x/\sqrt{4a_1 t})$
L'égalité des flux en $x = 0$

$$\sqrt{k_1\rho_1 c_1}(\frac{(T_1 - T_p)}{\sqrt{\pi t}}) = \sqrt{k_2\rho_2 c_2}(\frac{(T_2 - T_p)}{\sqrt{\pi t}})$$

donc

$$T_p = \frac{b_1 T_1 + b_2 T_2}{b_1 + b_2}$$

avec $b_1 = \sqrt{k_1 \rho c_1}$ et $b_2 = \sqrt{k_2 \rho c_2}$ l'effusivité du matériau.

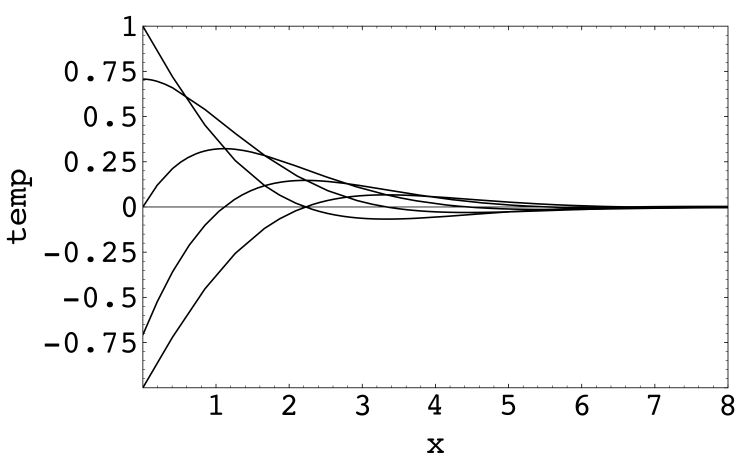
sinusoidal forcé en 0

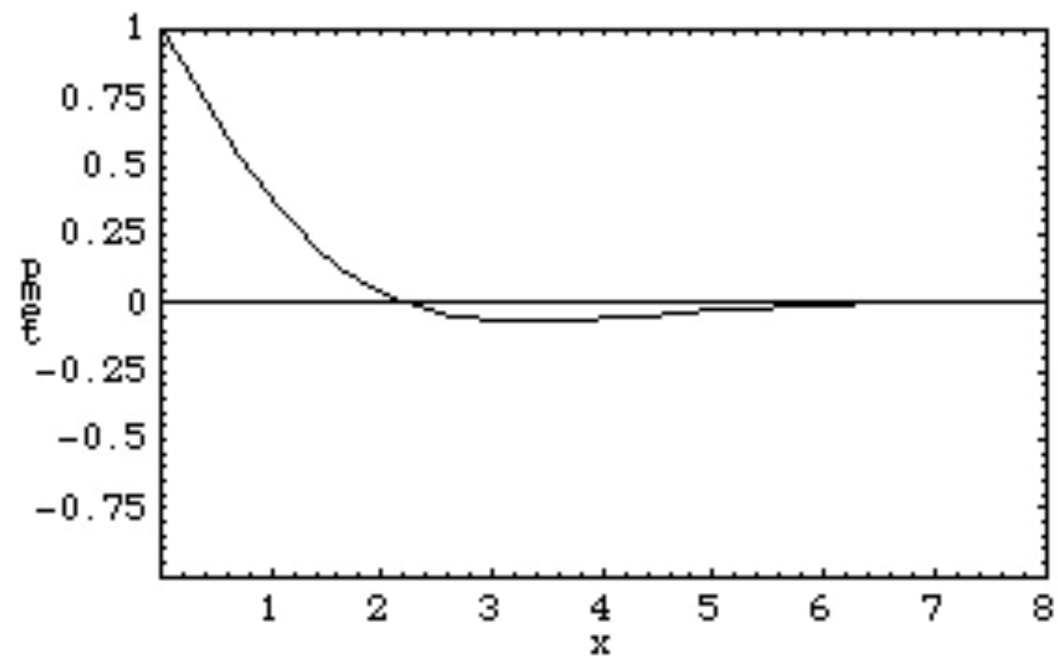
$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2}$$

solution forcée $T = \Delta T e^{i(\omega t - kx)} + T_0$

avec $-k^2 = i\omega/a.$ or $(-i)^{1/2} = \frac{(1-i)}{\sqrt{2}}$:

$$T = T_0 + \Delta T e^{-x\sqrt{\frac{\omega}{2a}}} \cos\left(\omega t - x\sqrt{\frac{\omega}{2a}}\right)$$





Bi < 0.1 Système Mince/ Corps thermiquement mince

- "lumped system analysis"
- par définition les systèmes minces : faible nombre de Biot
- température uniforme dans le corps

$$\frac{\partial}{\partial t}\int_{\Omega} \rho e dv = \int_{\partial\Omega} (-\overrightarrow{q}) \cdot \overrightarrow{dS}$$

$$\overrightarrow{q} = h(T - T_{ext})\overrightarrow{n}$$

$$\rho V c_p \frac{\partial T}{\partial t} = -hS(T - T_{ext})$$

Bi < 0.1 Système Mince/ Corps thermiquement mince

- "lumped system analysis"
- par définition les systèmes minces : faible nombre de Biot
- température uniforme dans le corps

$$\rho V c_p \frac{\partial T}{\partial t} = -hS(T - T_{ext})$$

"Loi de Newton"

Bi petit

$$\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} = \frac{\partial \bar{T}}{\partial \bar{t}}$$

$$\int \frac{\partial^2 \bar{T}}{\partial \bar{x}^2} dx = \int \frac{\partial \bar{T}}{\partial \bar{t}} dx$$

moyenne de la température

$$-2Bi(\bar{T}) \sim \frac{\partial}{\partial \bar{t}} \int \bar{T} d\bar{x}$$

$$-Bi\bar{T}_m = \frac{\partial}{\partial \bar{t}} \bar{T}_m$$

temps lent $Bi\bar{t} = \tilde{t}$

Bi petit

moyenne de la température

Systèmes Minces Bi<0.1

$$-Bi\bar{T}_m = \frac{\partial}{\partial \bar{t}}\bar{T}_m$$

temps lent $Bi\bar{t} = \tilde{t}$

Bi>0.1

$$\bar{T} = A_1 exp(-k_1^2\bar{t})cos(k_1\bar{x}) + ...,$$

$\bar{t}$>0.2)

$$\text{pour} Bi = \infty \text{ on a } \bar{T} = \frac{4e^{-\frac{\pi^2\bar{t}}{4}}cos\left(\frac{\pi\bar{x}}{2}\right)}{\pi}$$

$$\text{pour} Bi = \infty \text{ on a } \bar{T} = 1.27324e^{-(1.5708)^2\bar{t}}cos(1.5708\bar{x})$$

$$\text{pour} Bi = 5 \text{ on a } \bar{T} = 1.2402e^{-(1.3138)^2\bar{t}}cos(1.3138\bar{x})$$

$$\text{pour} Bi = 1 \text{ on a } \bar{T} = 1.1192e^{-(0.8603)^2\bar{t}}cos(0.8603\bar{x})$$

$$\text{pour} Bi = .1 \text{ on a } \bar{T} = 1.0160e^{-(0.3111)^2\bar{t}}cos(0.3111\bar{x})$$

Pour $Bi < .1$

$$\bar{T} = e^{-Bi\bar{t}}$$

- Bi<<1 “lumped system analysis” T(t) température uniforme dans le corps
- Bi>>1 température imposée aux bords
- Bi quelconque : résolution numérique (système épais)

Bi quelconque

Résolution numérique

$$\frac{\partial T}{\partial t} = \frac{\partial^2}{\partial x^2} T$$

discrétisation

$$\frac{T(t+\Delta t) - T}{\Delta t} = \frac{(T(t, x+\Delta x) - 2T(t,x) + T(t, x-\Delta x))}{\Delta x^2}$$

Explicite

$$\frac{T(t+\Delta t) - T}{\Delta t} = \frac{(T(t+\Delta t, x+\Delta x) - 2T(t+\Delta t,x) + T(t+\Delta t, x-\Delta x))}{\Delta x^2}$$

Implicite

http://www.lmm.jussieu.fr/~lagree/SOURCES/Appliquette-JavaChal/guiChalomega_v_implicite/index.html

guiChalOmegaImp.class

discrétisation explicite

$$T(t+\Delta t,x) = T(t,x) + \Delta t \frac{T(t,x+\Delta x) - 2T(t,x) + T(t,x-\Delta x)}{\Delta x^2}$$

réécrit

$$T(t+\Delta t,x) = (1 - \frac{2\Delta t}{\Delta x^2})T(t,x) + \Delta t \frac{T(t,x+\Delta x) + T(t,x-\Delta x)}{\Delta x^2}$$

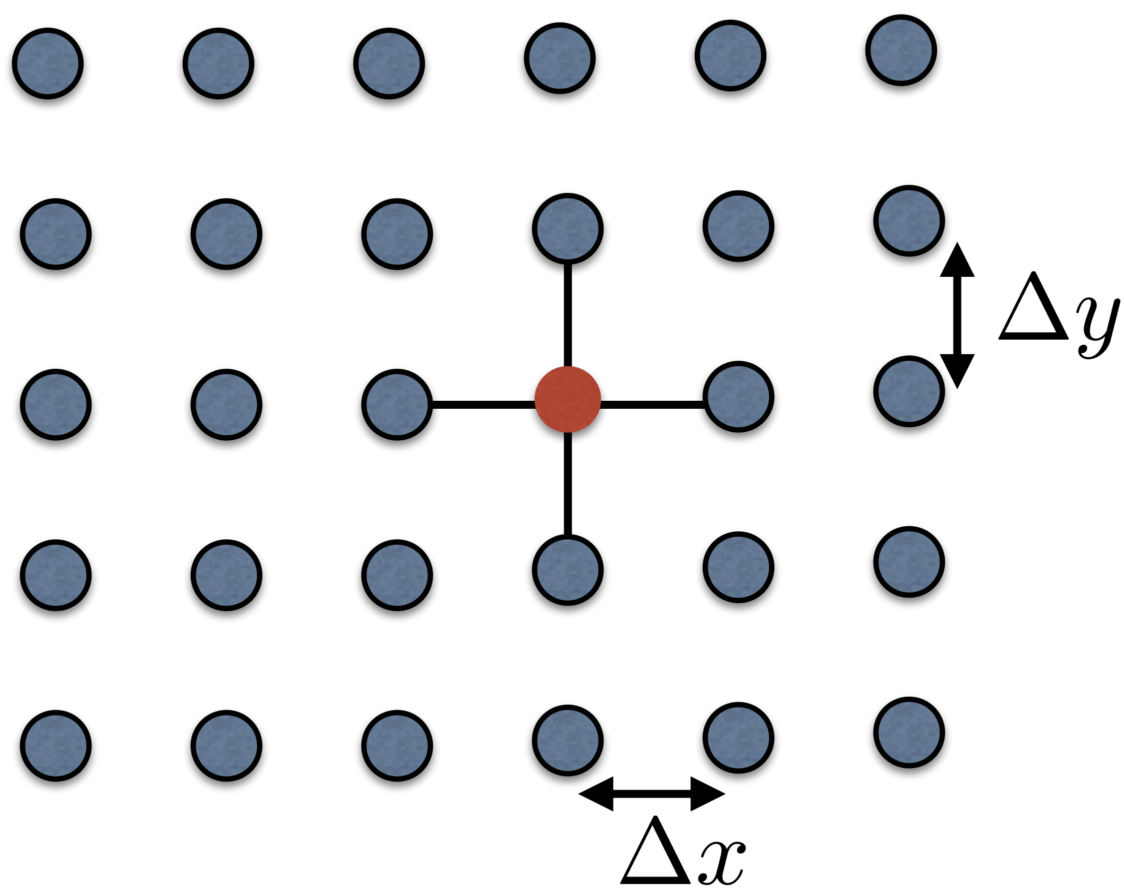
si $\Delta t = \dfrac{\Delta x^2}{2}$

$$\boxed{T(t+\Delta t,x) = \frac{T(t,x+\Delta x) + T(t,x-\Delta x)}{2}}$$

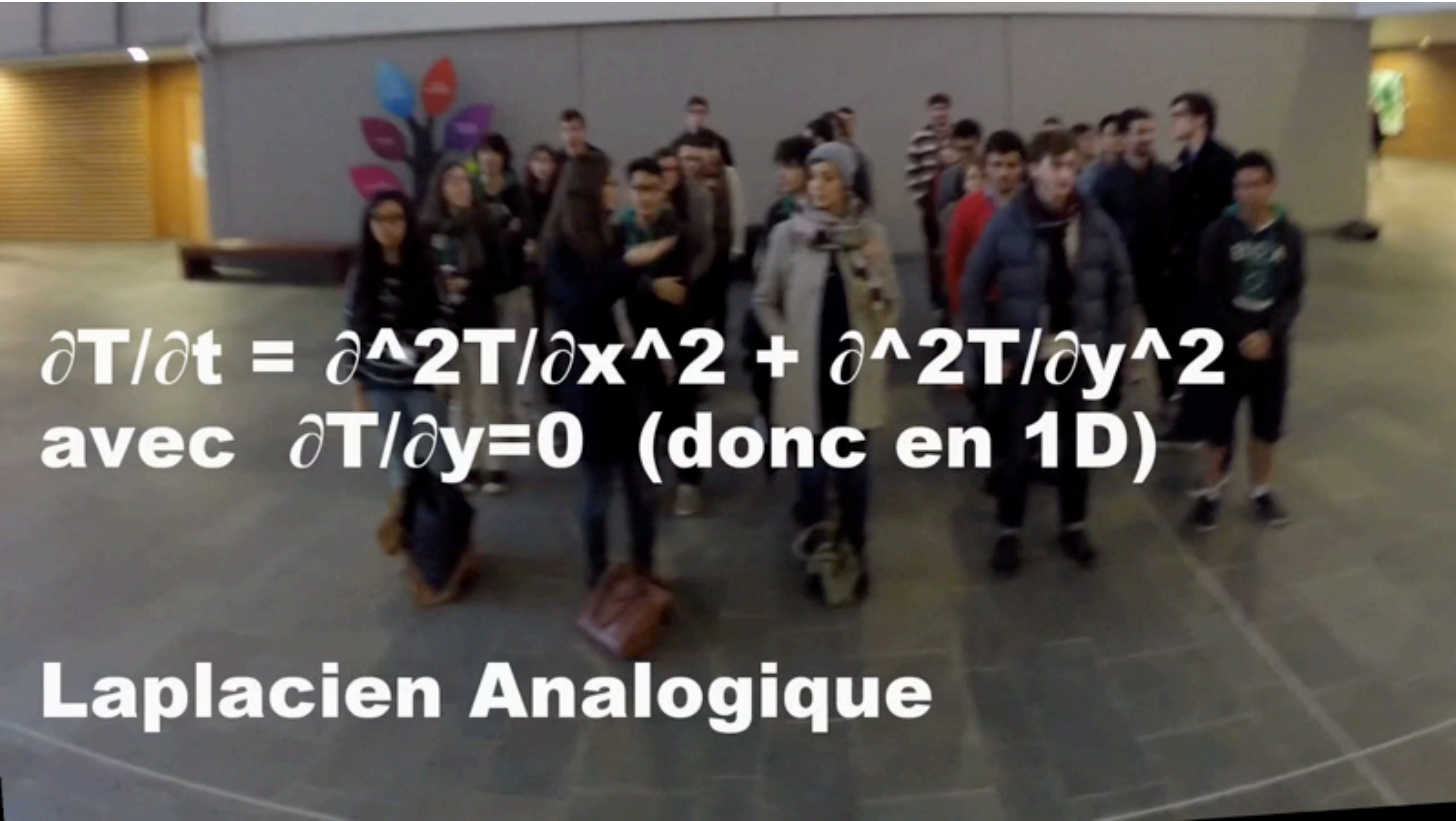
la nouvelle estimation est la moyenne…

idem en 2D

$$T(t+\Delta t, x, y) = \frac{T(t, x+\Delta x, y) + T(t, x-\Delta x, y) + T(t, x, y+\Delta y) + T(t, x, y-\Delta y)}{4}$$



Solveur Analogique ;-)




Résolution de l'équation
de la chaleur 2D
analogique
∂T/∂t=∂^2T/∂x^2+∂^2T/∂y^2

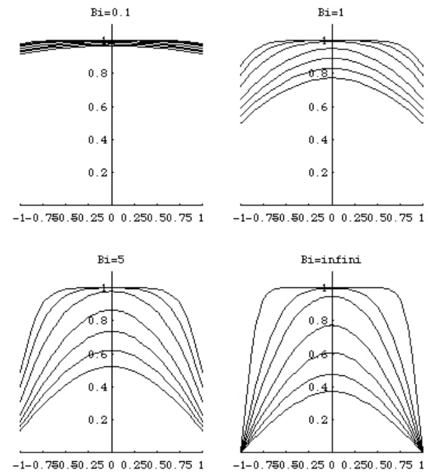


FIG. 16 – $\bar{T}(\bar{x},\bar{t})$ pour $\bar{t}$ =0.025 0.05 0.1 0.2 0.3 0.4 et 0.5

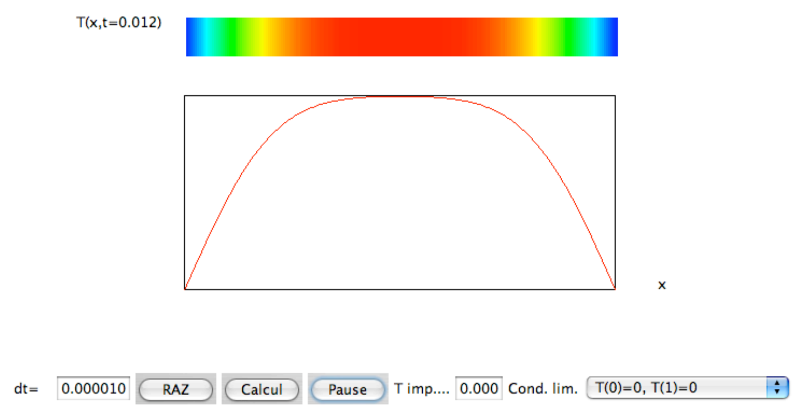
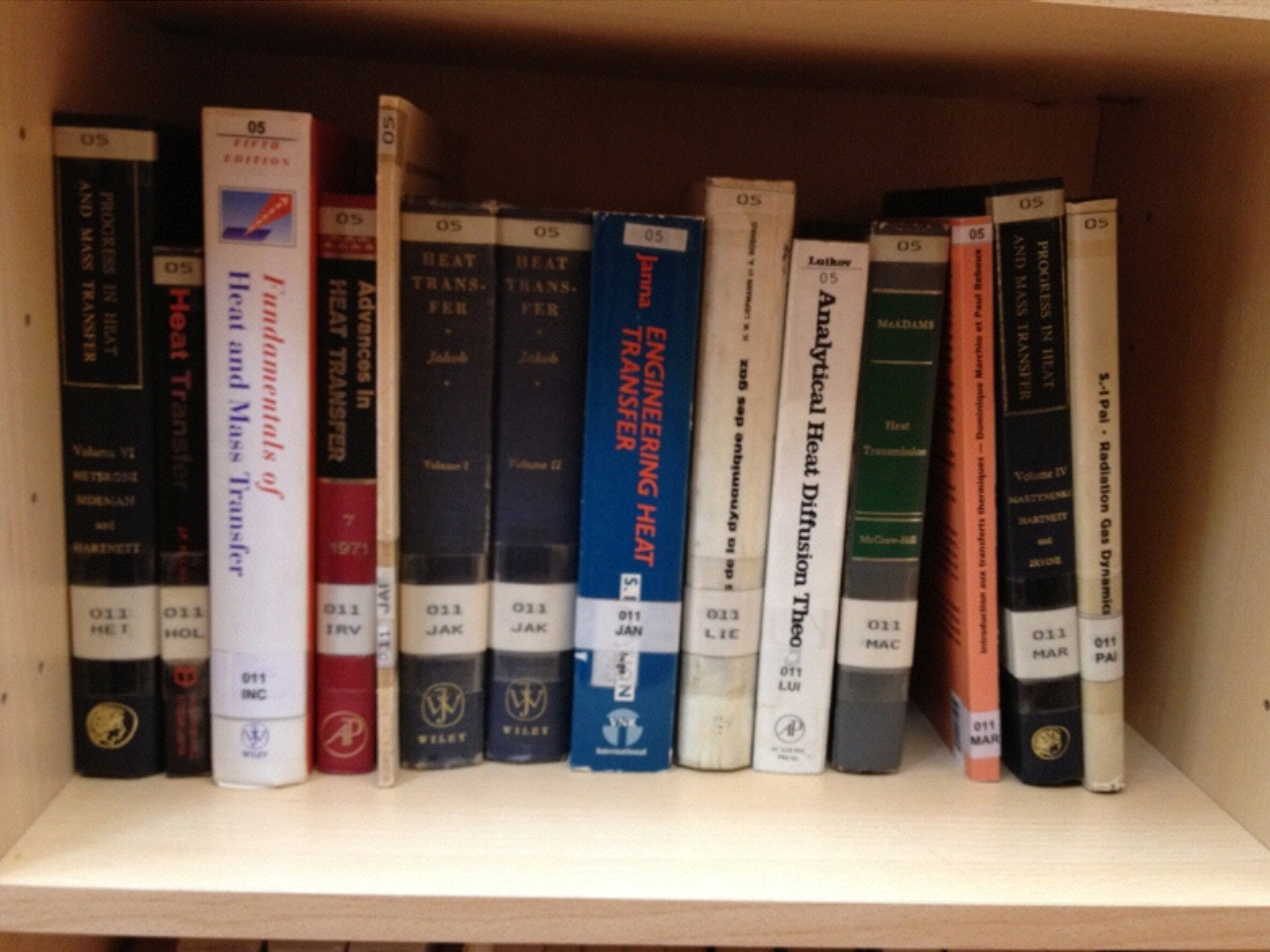


FIG. 17 – Résolution en direct par différences finies de l'équation de la chaleur.



05
PROGRESS IN HEAT AND MASS TRANSFER
011 HET
05
Heat Transfer
011 HOL
05
FIFTH EDITION
Fundamentals of Heat and Mass Transfer
011 INC
WILEY
05
Advances in HEAT TRANSFER
7
1971
011 IRV
05
011 JAI
05
HEAT TRANS-FER
Jakob
Volume I
011 JAK
WILEY
05
HEAT TRANS-FER
Jakob
Volume II
011 JAK
WILEY
05
Janna
ENGINEERING HEAT TRANSFER
011 JAN
05
011 LIE
Luikov
05
Analytical Heat Diffusion Theo
011 LUI
05
McADAMS
Heat Transmission
McGraw-Hill
011 MAC
05
011 MAR
05
PROGRESS IN HEAT AND MASS TRANSFER
Volume IV
011 MAR
05
S.-I Pai · Radiation Gas Dynamics
011 PAI


013 PRU
011 REB
05
ROLLE
THERMODYNAMICS AND HEAT POWER
Sixth Edition
011 ROL
Introduction to gas dynamics
011 ROT
jf sacadura
thermiques
trans
011 SAC
011 SCH
Conduction Heat Transfer
Schneider
011 SCH
THERMAL RADIAT
HEAT TRANSFER
011 SIE
011 SOE
OULING OF HEAT
RANSFER EQUIPMENT
SOMERSCALES
011 SOM
D. B. Spalding
Arnold
011 SPA
RADIATION HEAT TRANSFER
011 SPA
Taine/J.-P. Petit
Transferts thermiques : Applications
011 TAI
TRANSFERTS THERMIQUES
Introduction aux sciences des transferts
011 TAI