Characterization of Singular Velocities in Quantum Turbulence

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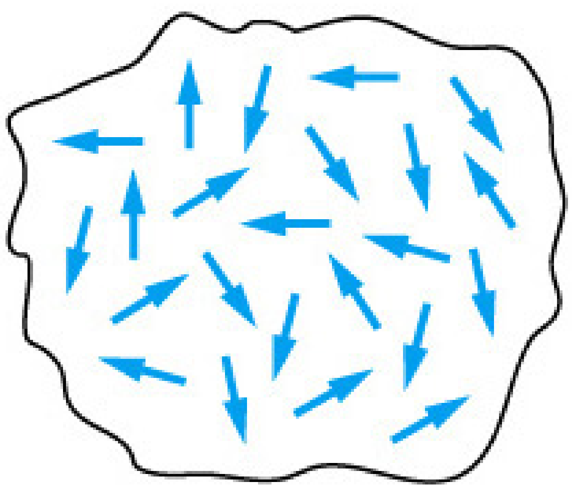
Order Parameters

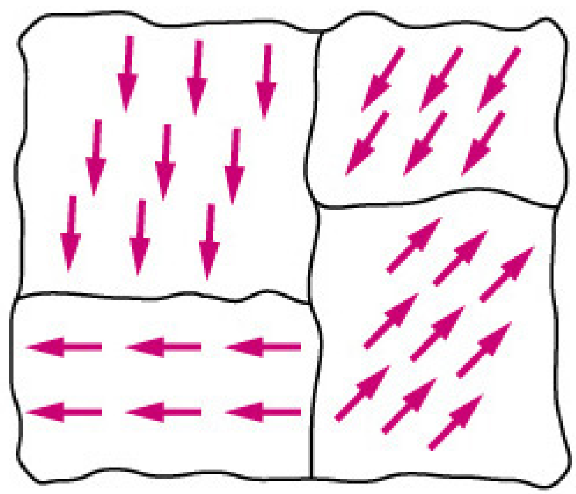
Symmetry breaking transitions require extra variables to describe state of system

- Example: Magnetization order parameter

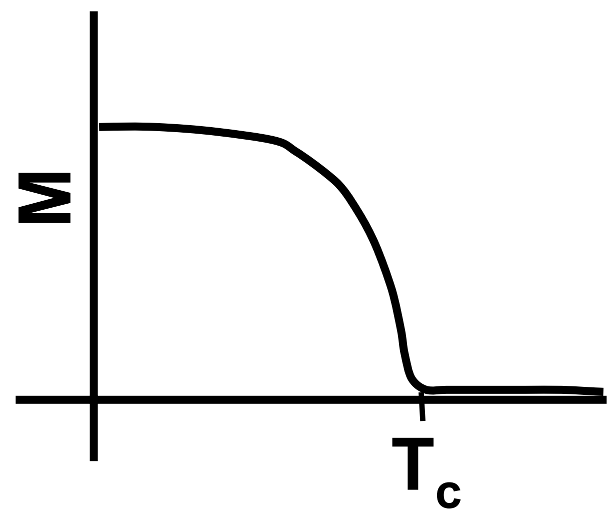
- Topological defects often form and separate areas of aligned magnetization



Above T_c



Below T_c

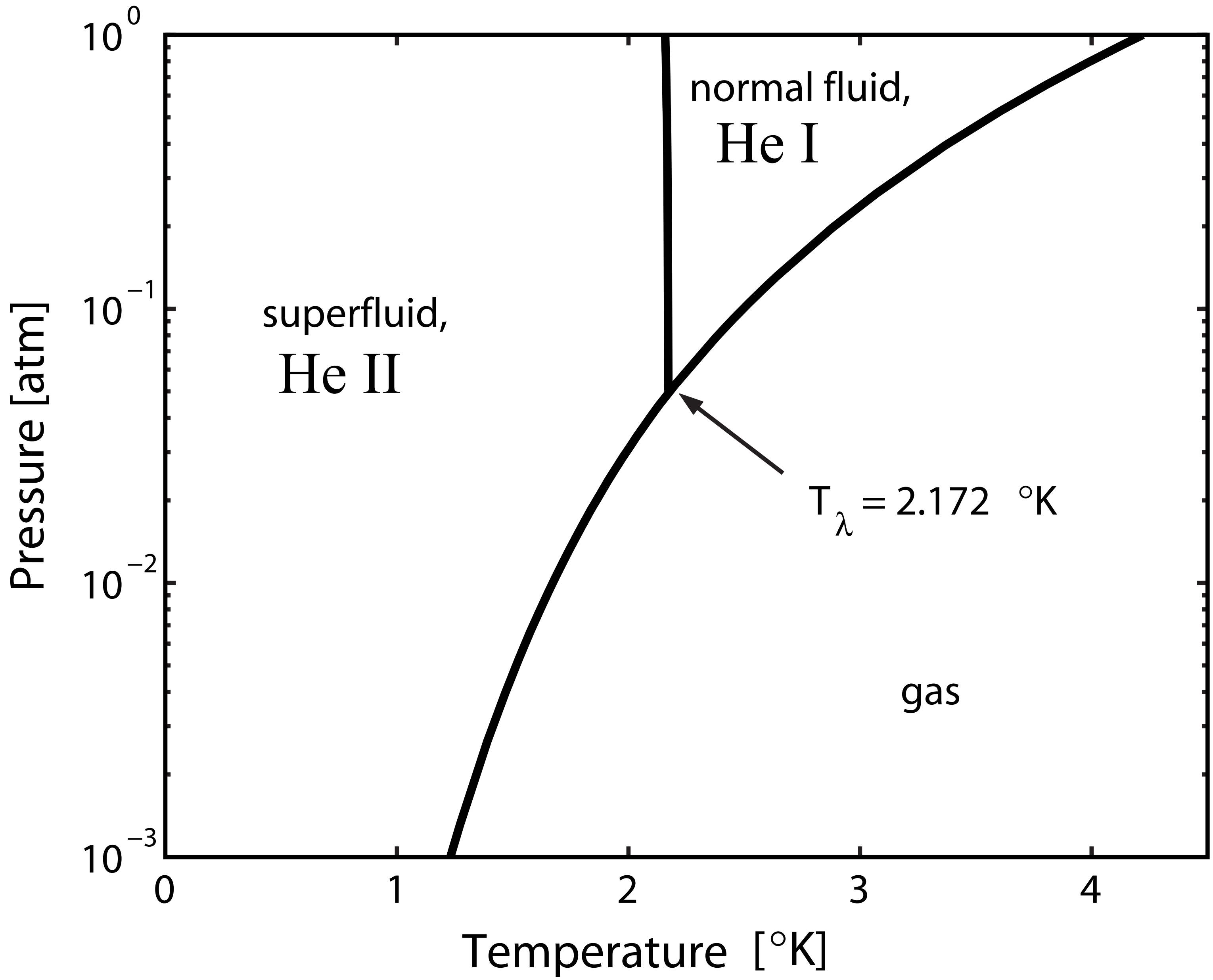


Dissipation vs. Topological Defects

- Dissipative processes relax systems toward equilibrium (i.e. diffusion)
- Topological defects cannot diffuse and frustrate a system’s ability to equilibrate
- Ex: magnetic domains can prevent a ferromagnet from reaching its minimum energy state

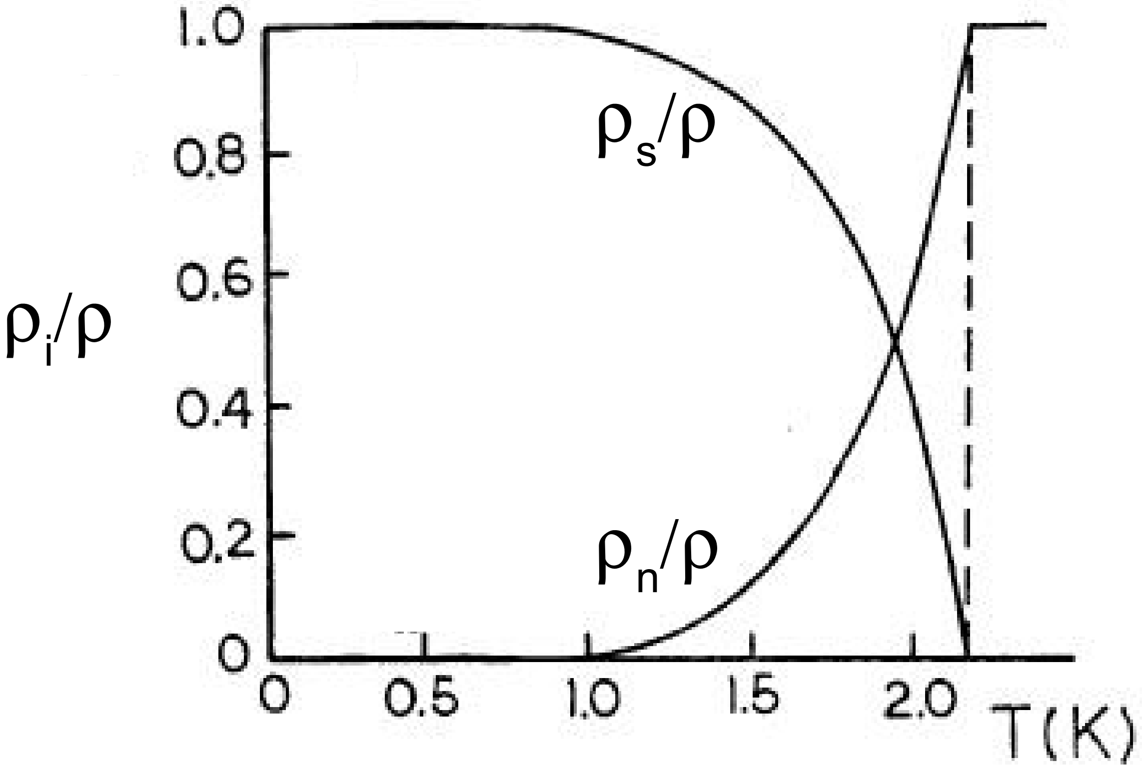
How do systems remove topological defects in order to reach equilibrium?

Superfluid Helium



Two-Fluid Model

- Superfluid helium behaves as if it's a mixture of two interpenetrating fluids
- Normal and superfluid components have individual densities and velocity fields



Superfluid Order Parameter

- Order parameter for superfluid helium is a complex field,

$$\Psi(\mathbf{x}) = Ae^{i\phi}$$

A is amplitude, and ϕ is phase

- Superfluid velocity given by:

$$\mathbf{v}_s = \kappa \nabla \phi, \quad \kappa = \frac{h}{m}$$

$$h = \text{Planck's constant}$$

$$m = \text{mass of helium atom}$$

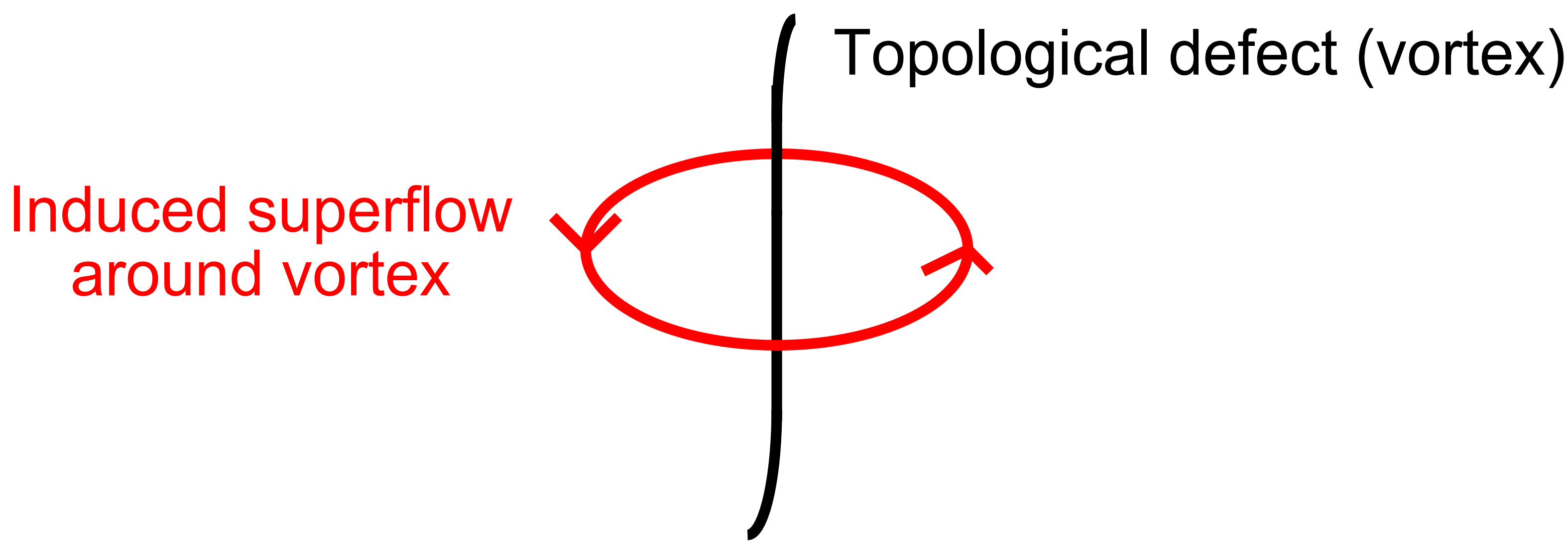
Superfluid Topological Defects

- By QM ϕ must be 2π periodic, which quantizes angular momentum and produces vortex-like velocity field:

$$\mathbf{v}_s = \kappa \nabla \phi = \frac{n\kappa}{2\pi s}$$

κ is quantum of circulation

- Topological defects are 1-dimensional, A=0

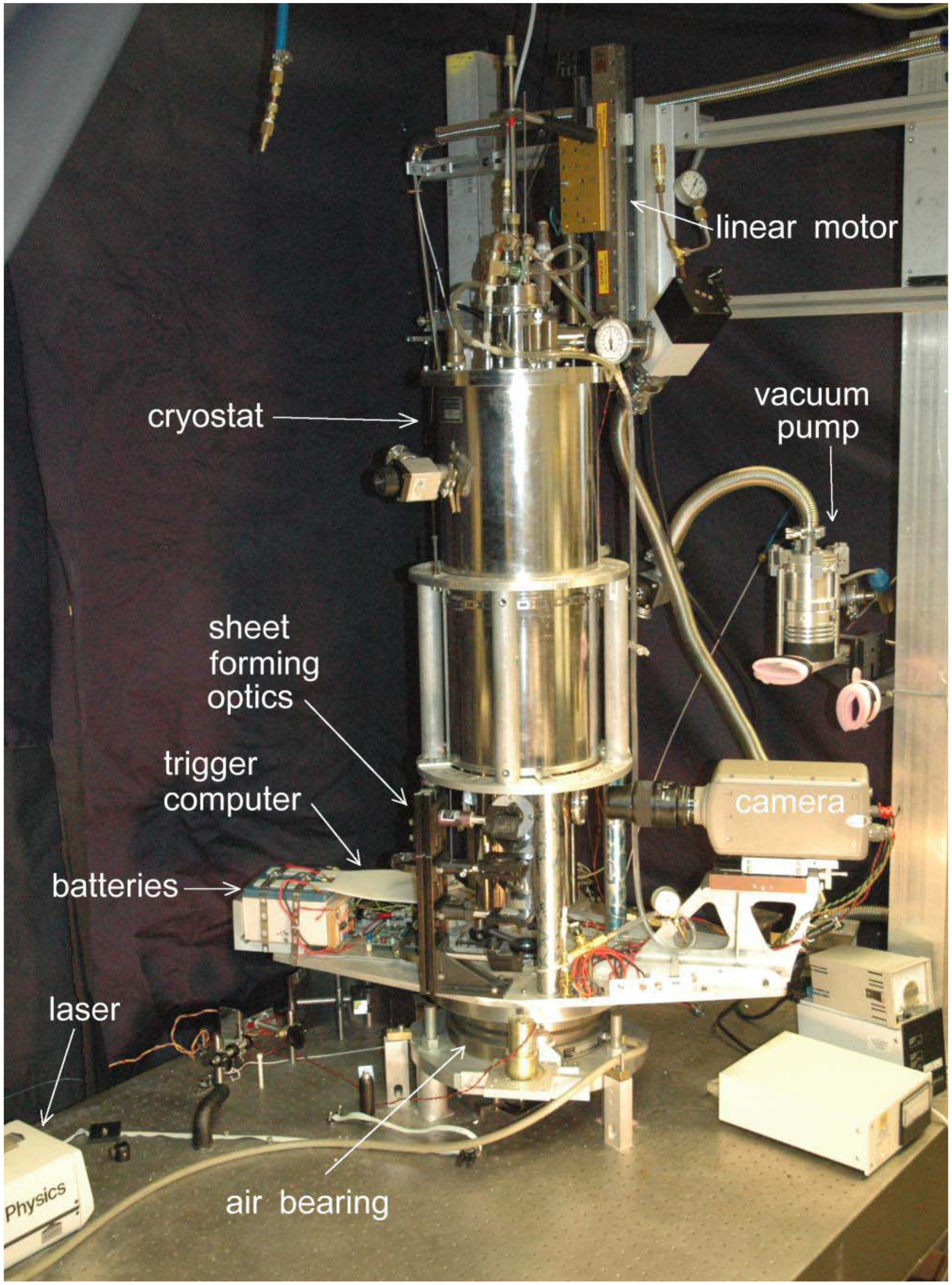


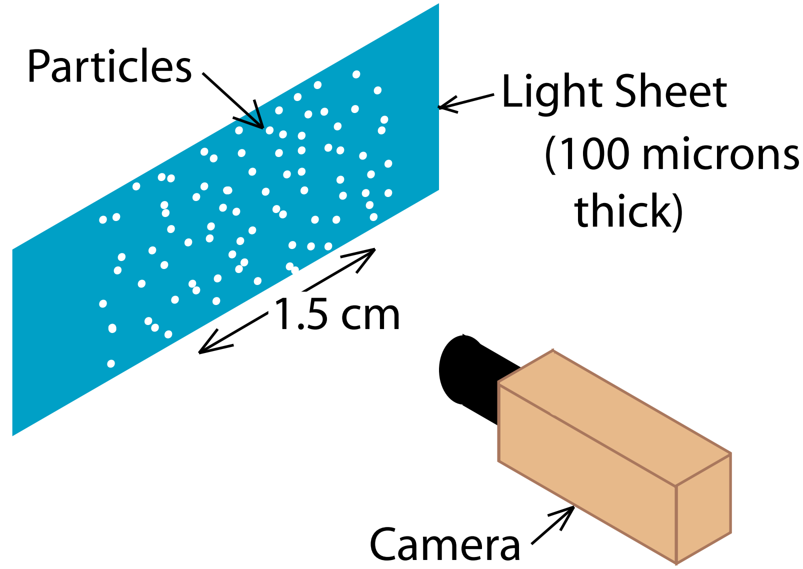
Main Points

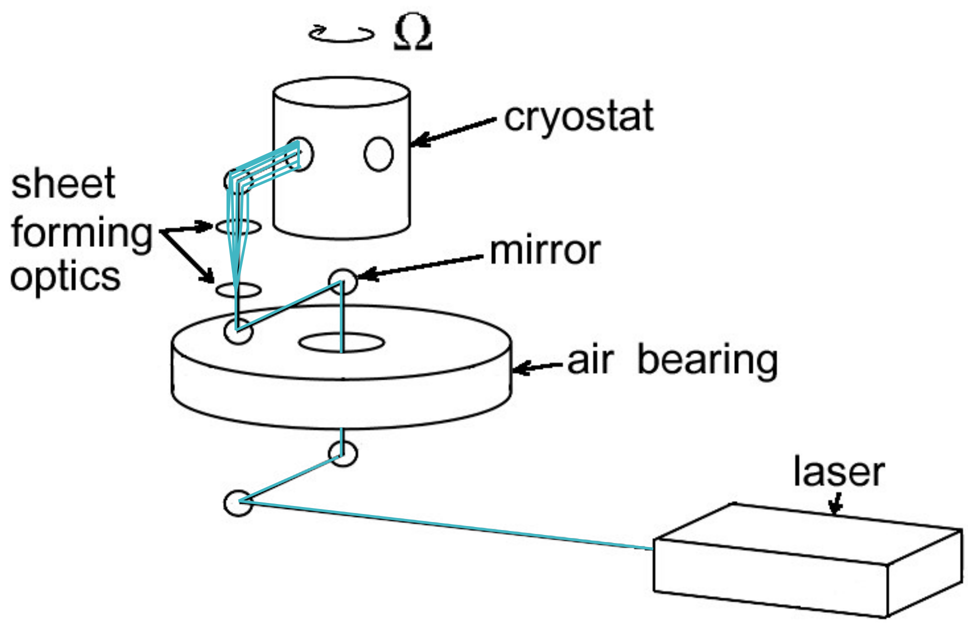
Reconnection plays significant role in decay of superfluid turbulence

Reconnection produces non-classical velocity and energy statistics

Apparatus







Laser Beam routing

Previous work on visualization

Experimental

K.L. Chopra and J.B. Brown, *Phys. Rev.* **108**, 157 (1957)

D.Y. Chung and P.R. Critchlow, *Phys. Rev. Lett.* **14**, 892 (1965)

T.A. Kitchens, W.A. Steyert and R.D. Taylor, *Phys. Rev. Lett.* **14**, 942 (1965)

M. Murakami and N. Ichikawa, *Cryogenics* **29**, 438 (1989)

R.E. Boltnev, G. Frossati E.B. Gordon, I.N. Krushinskaya, E.A. Popov and A. Usenko, *J. Low Temp. Phys.* **127**, 245 (2002)

D. Celik and S.W. Van Sciver, *Expt. Thermal and Fluid Sci.* **26**, 971 (2002)

T. Zhang, D. Celik and S.W. Van Sciver, *J. Low Temp. Phys.* **134**, 985 (2004)

Yarmchuk, E.J., Gordon, M.J.V. and Packard, R.E. (1979), *Phys. Rev. Lett.* **43**, 214-217.

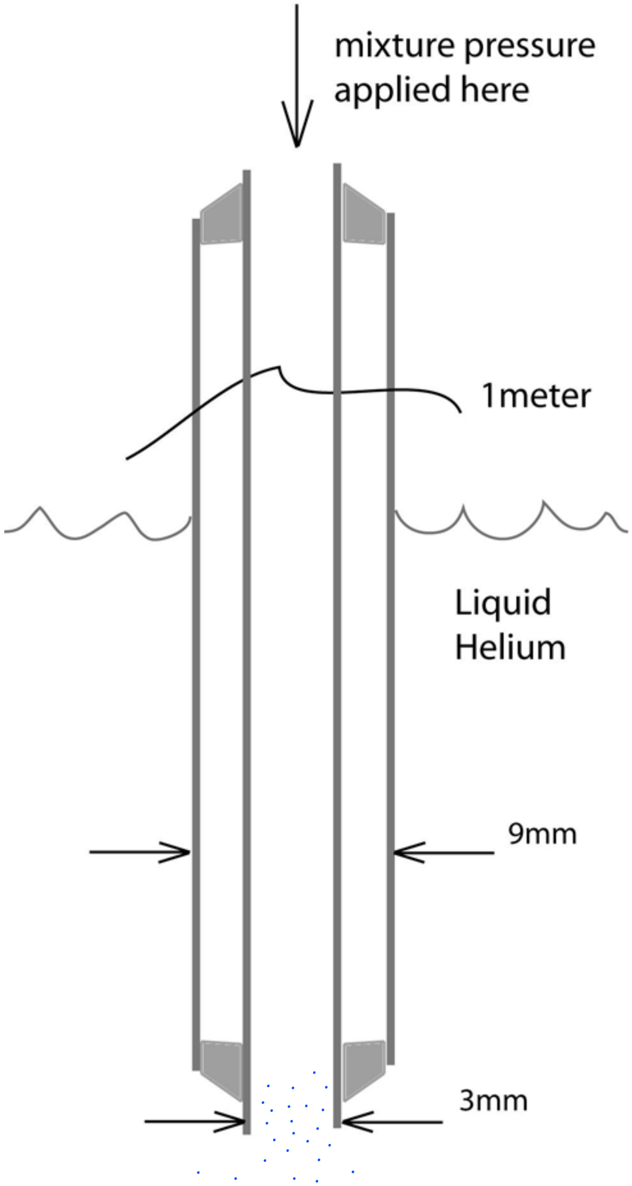
Theoretical

P.E. Parks and R.J. Donnelly, *Phys. Rev. Lett.* **16**, 45 (1966)

D.R. Poole, C.F. Barenghi *et al., Phys. Rev. B* **71**, 064514 (2005)

Particle Production

$1\ H_2 : \chi\ He,\ \chi >> 1$



In the normal fluid, $T > T_\lambda$:

8
mm

Visualizing Superfluid Vortices in He II

- Below T_λ hydrogen particles collect onto filaments

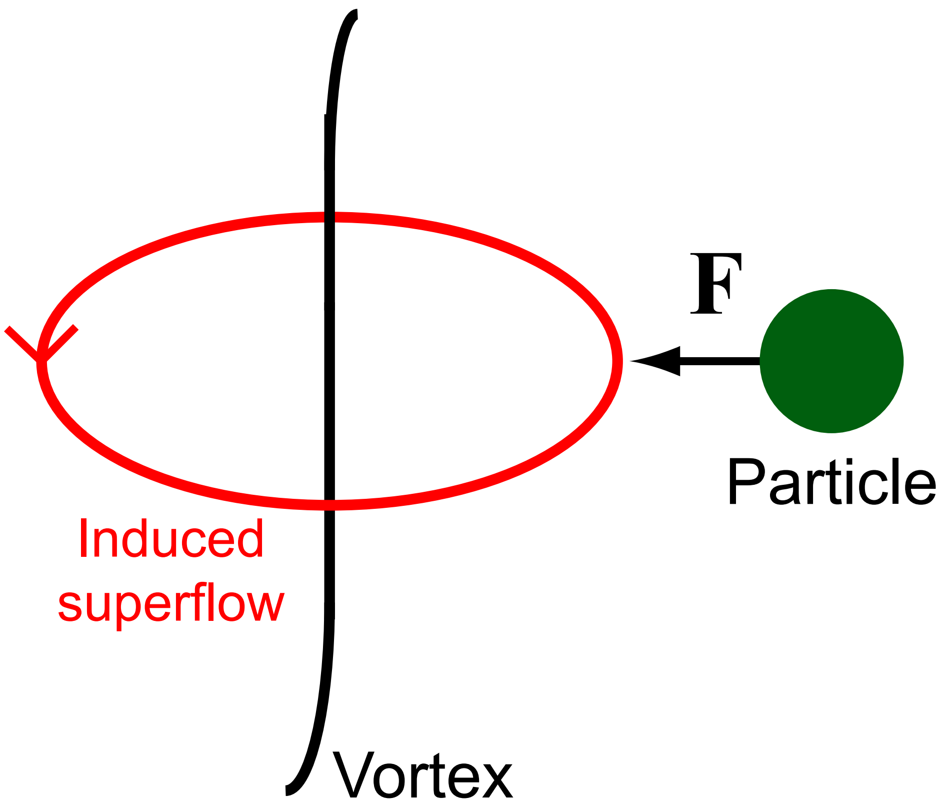
- Previous work has shown these filaments are particles trapped on the superfluid vortices (Bewley, et al., Nature 2006)

8 mm

Movie in real time
Begins 180 s after transition
T_λ-T~50 mK

Particle Trapping Mechanism

- Pressure gradient acts to balance centrifugal force of circulating superfluid around vortex
- Hydrogen particles do not circulate and only feel pressure gradient, which traps them along the cores of the defects



$$P = \frac{\rho_s \kappa^2}{8\pi^2 s^2}, \quad \kappa = \frac{h}{m}$$

$$\nabla P = -\frac{\rho_s \kappa^2}{4\pi^2 s^3}\hat{s}$$

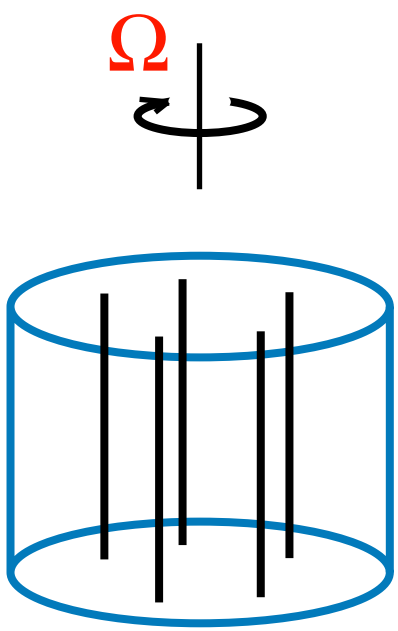
$$\mathbf{F} = \mathbf{F}(T) = \oint_{part.surface} P$$

Parks, P.E. and Donnelly, R.J. (1966), *Phys. Rev. Lett*. **16**, 45–48.

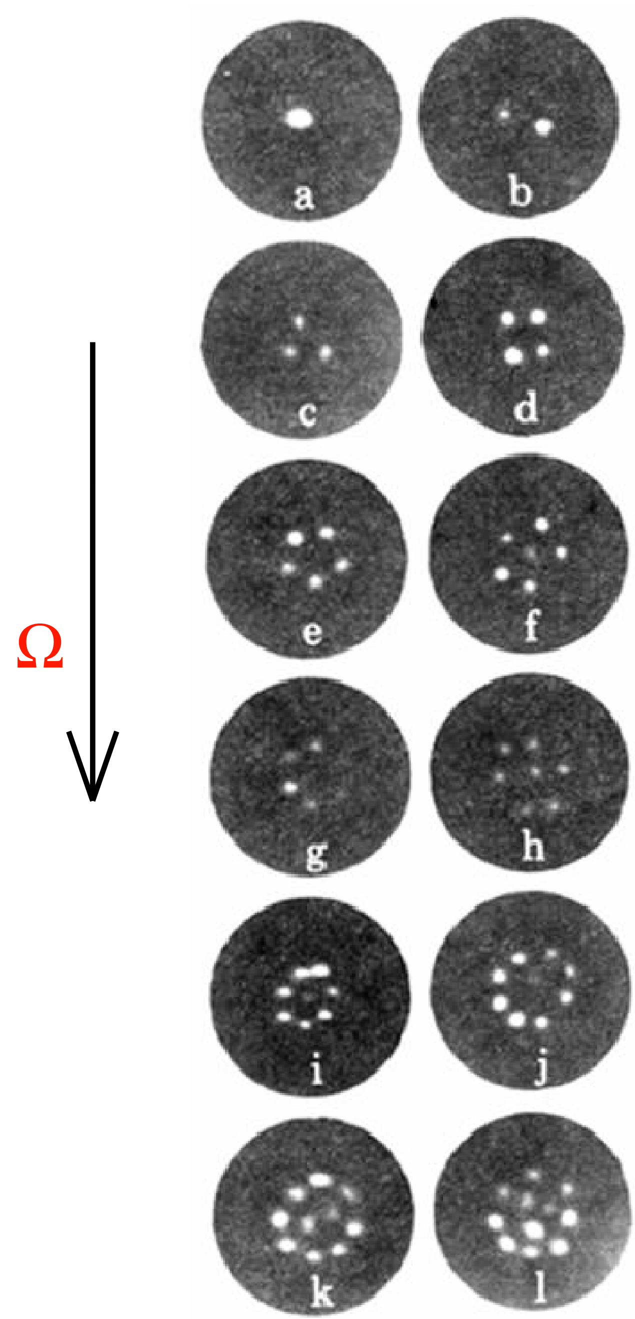
Previous Observation of Quantized Vortices

Rotating superfluid forms triangle lattice with line density given by Feynam's rule:

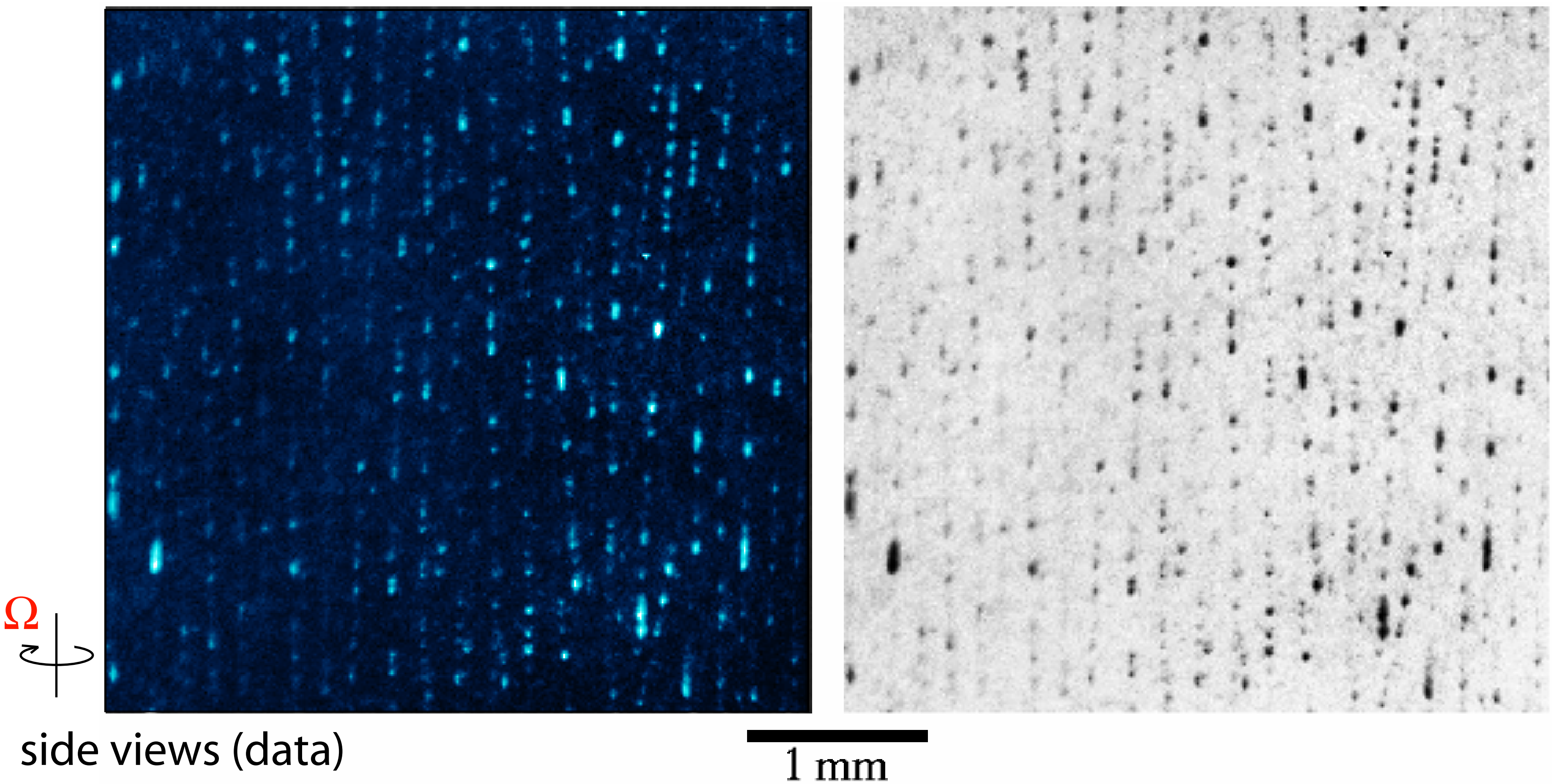
$$n \approx 2000\Omega \ \text{lines/cm}^2$$



E.J. Yarmchuk, M.J.V. Gordon and R.E. Packard, *Phys. Rev. Lett*. **43**, 214 (1979)

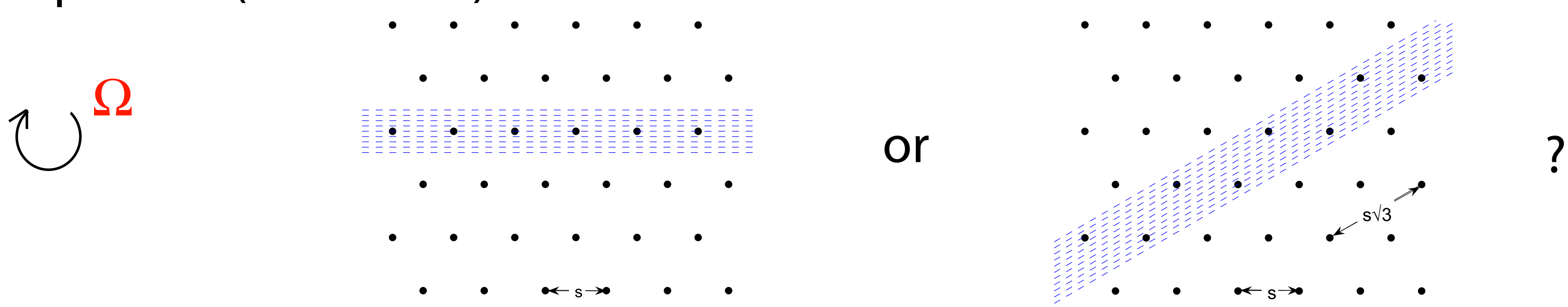


Rotating Superfluid Lattice



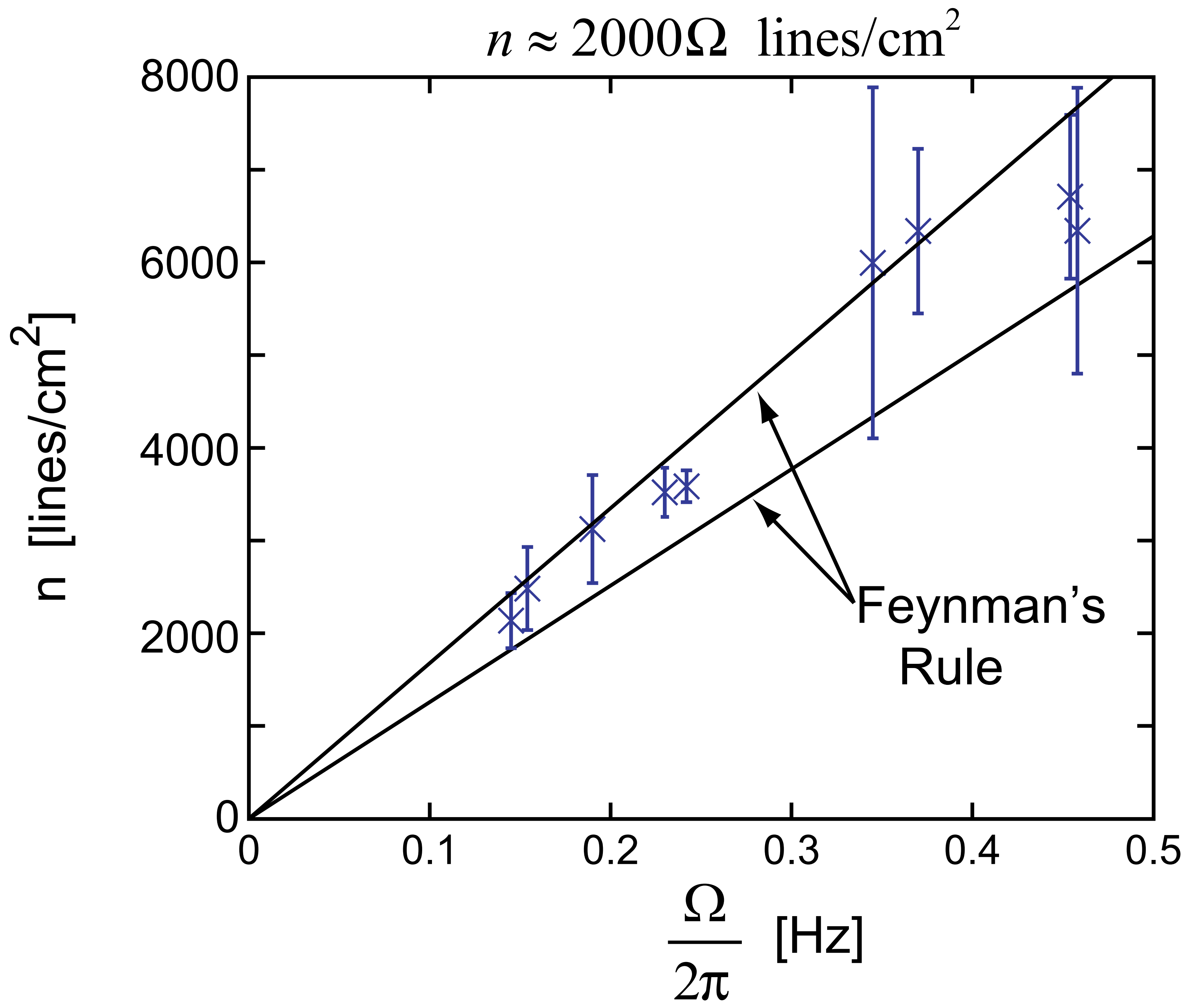
side views (data)

top views (schematic)



Experiments performed by Greg Bewley

Lattice Density

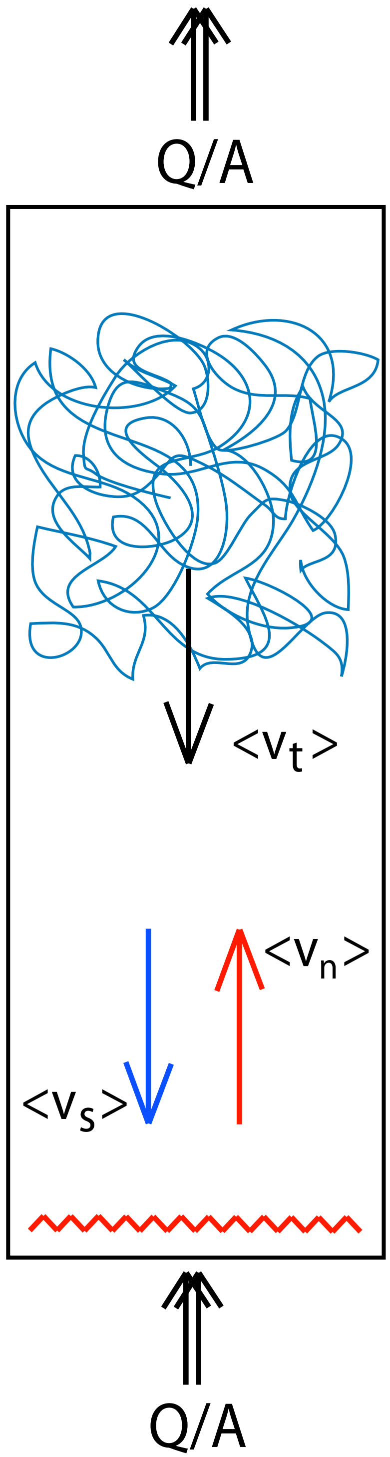


Particle Tracking

- Particle-tracking allows us to analyze the particle dynamics without assuming smooth velocity fields (as in PIV)

Particle-tracking software from Eric Weeks and John Crocker

Thermal Counterflow



- Drive the system from equilibrium by applying a heat flux Q/A to the bottom of the channel

$$\frac{dL}{dt} = \alpha \left| \mathbf{v}_{ns} \right| L^{3/2} - \beta \kappa L^2$$

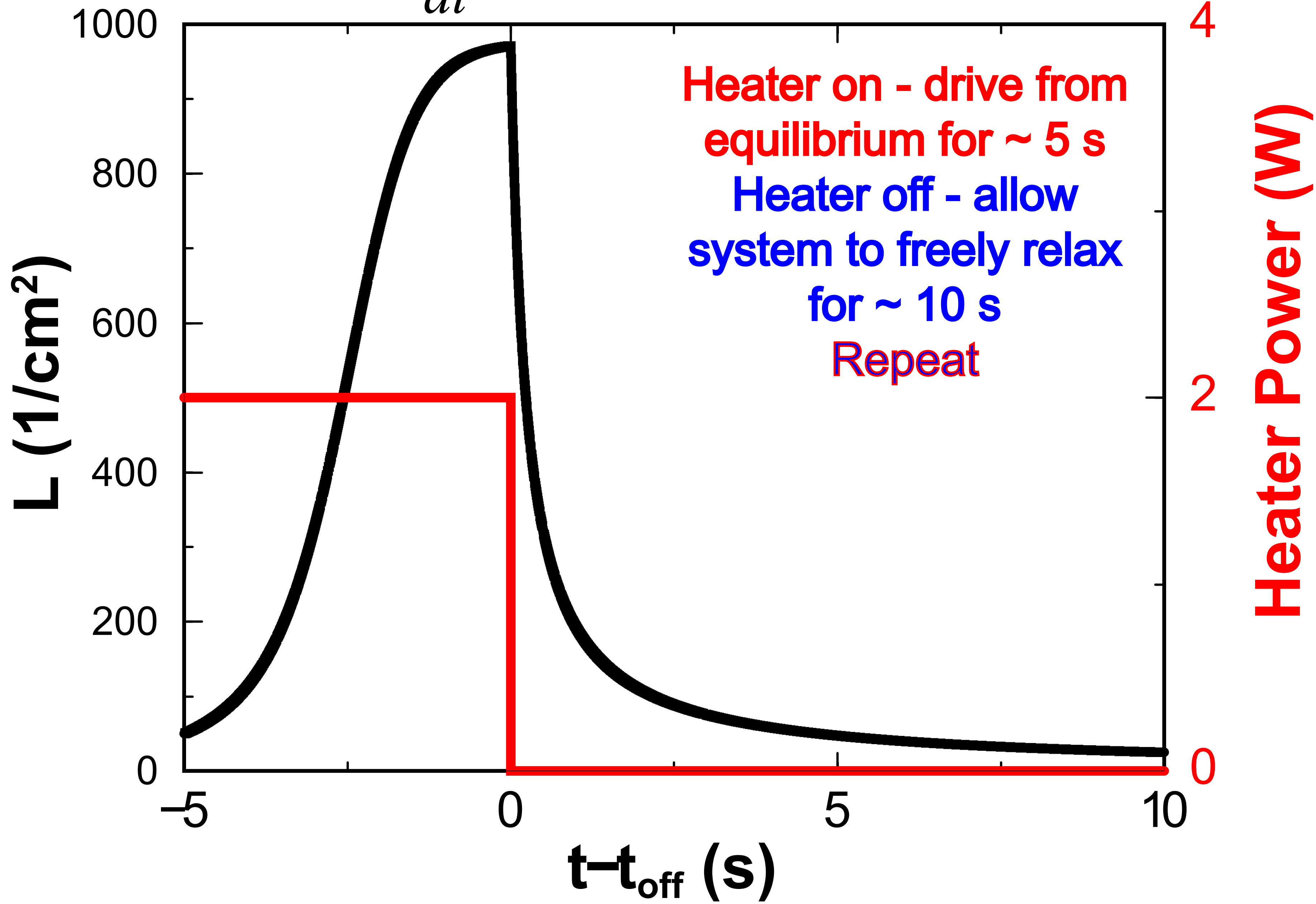
$$L = \frac{\text{vortex line length}}{\text{volume}}, \ \mathbf{v}_{ns} = \mathbf{v}_n - \mathbf{v}_s$$

The spatially averaged velocites are of:
$\langle v_n \rangle$ - the viscous component,
$\langle v_s \rangle$ - the superfluid,
$\langle v_t \rangle$ - the quantized vortex tangle

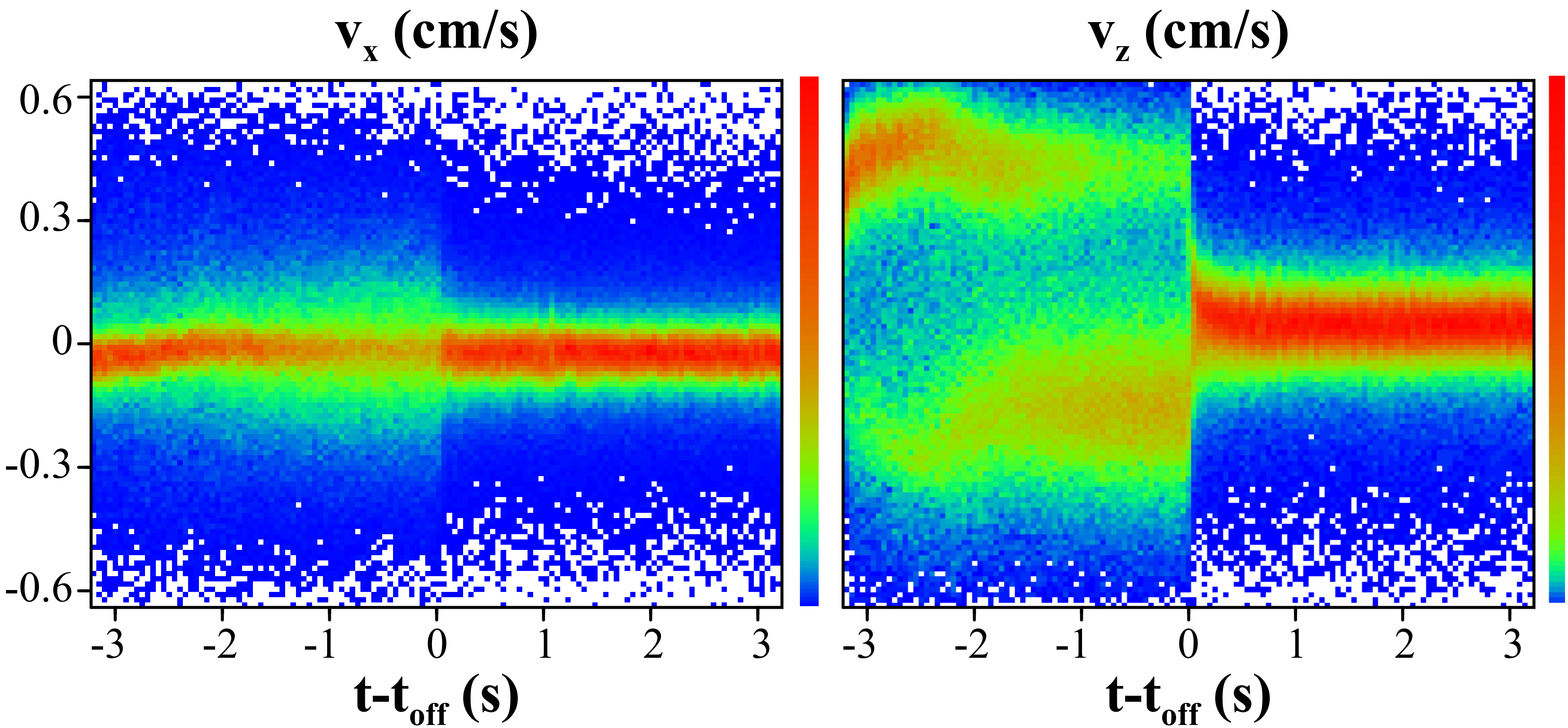
WF Vinen: Proc. R. Soc. London Ser. A **242**, 493 (1957)

Pulsed Counterflow Experiments

$$\frac{dL}{dt} = \alpha \left| \mathbf{v}_{ns} \right| L^{3/2} - \beta \kappa L^2$$



Pulsed Counterflow Experiments



What is the source of high velocity trajectories when heater is off?

Superfluild Vortex Reconnection

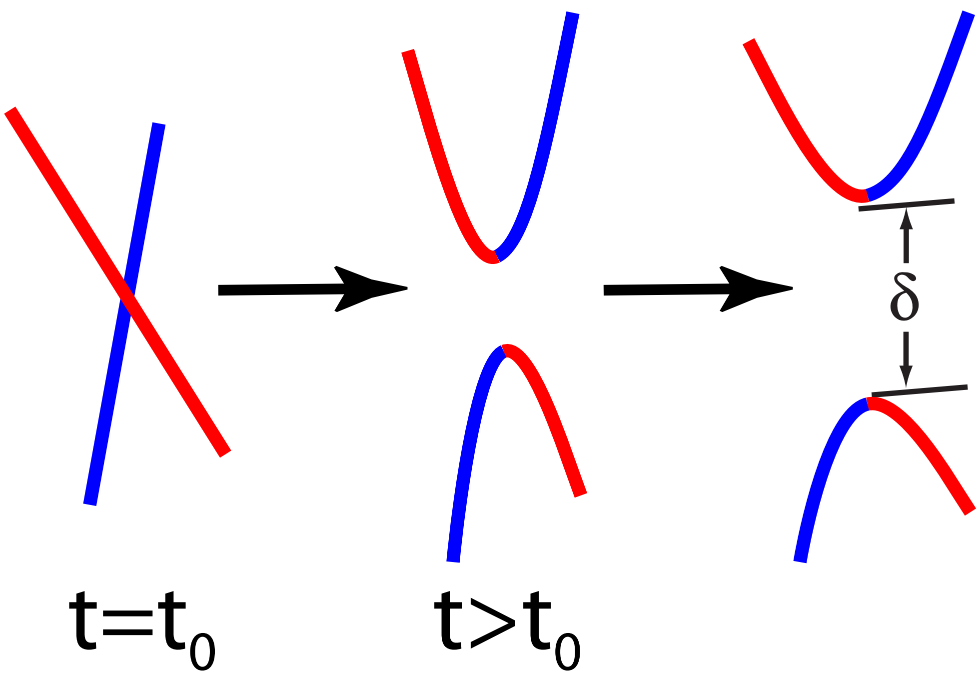
Schwarz, PRB 1985 (LV)
de Waele and Aarts, PRL 1994 (LV)
Koplik and Levine, PRL 1993 (NLSE)

Nazarenko and West 2003 (NLSE)
Tsubota and Maekawa, JPSJ 1992 (LV)

Previous theoretical studies predict that when two vortices cross they reconnect

Source of dissipative term in Vinen equation

$$\frac{dL}{dt} = -\beta\kappa L^2$$

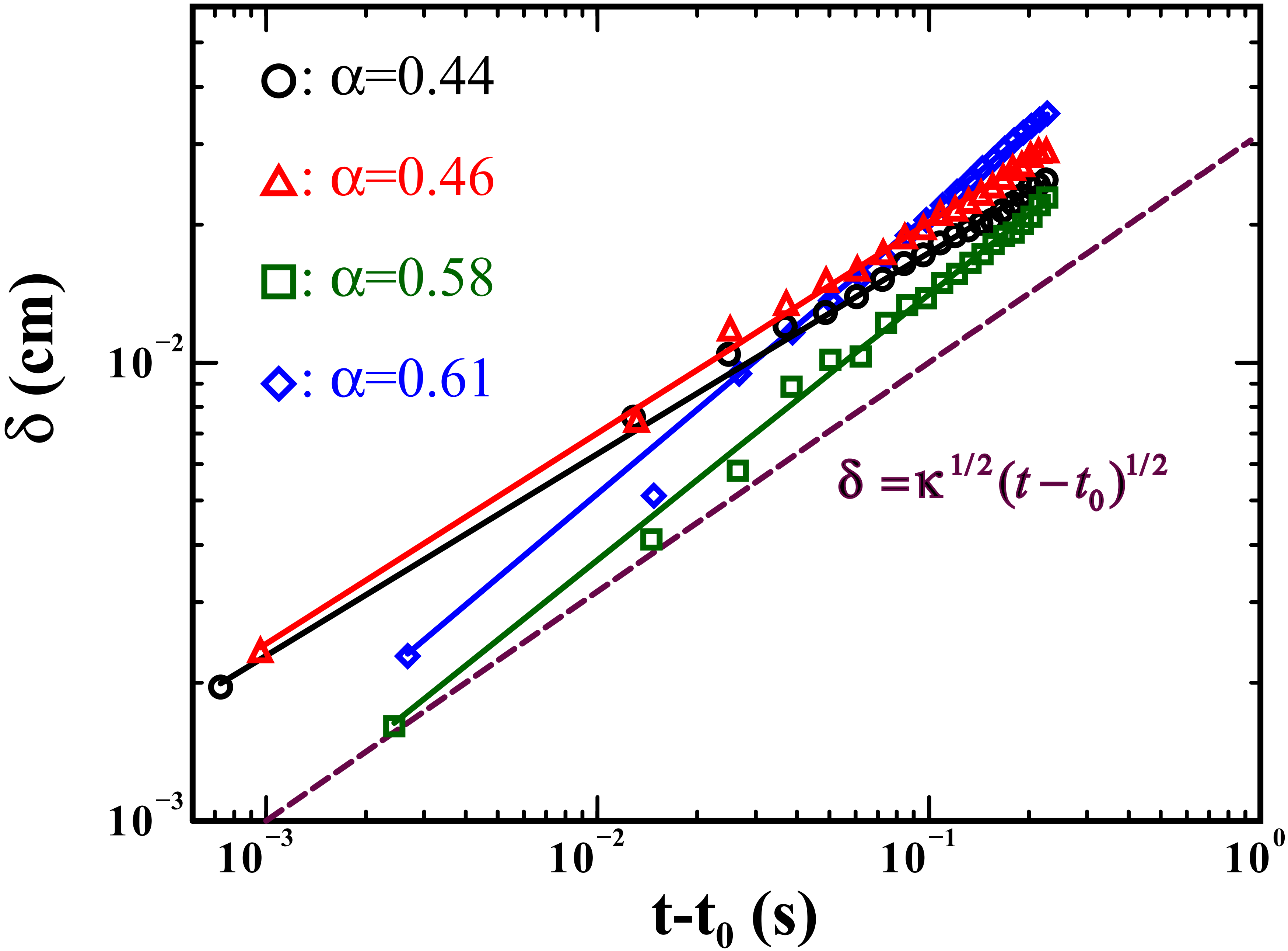


$$\delta \sim \kappa^{1/2}(t_0 - t)^{1/2}$$

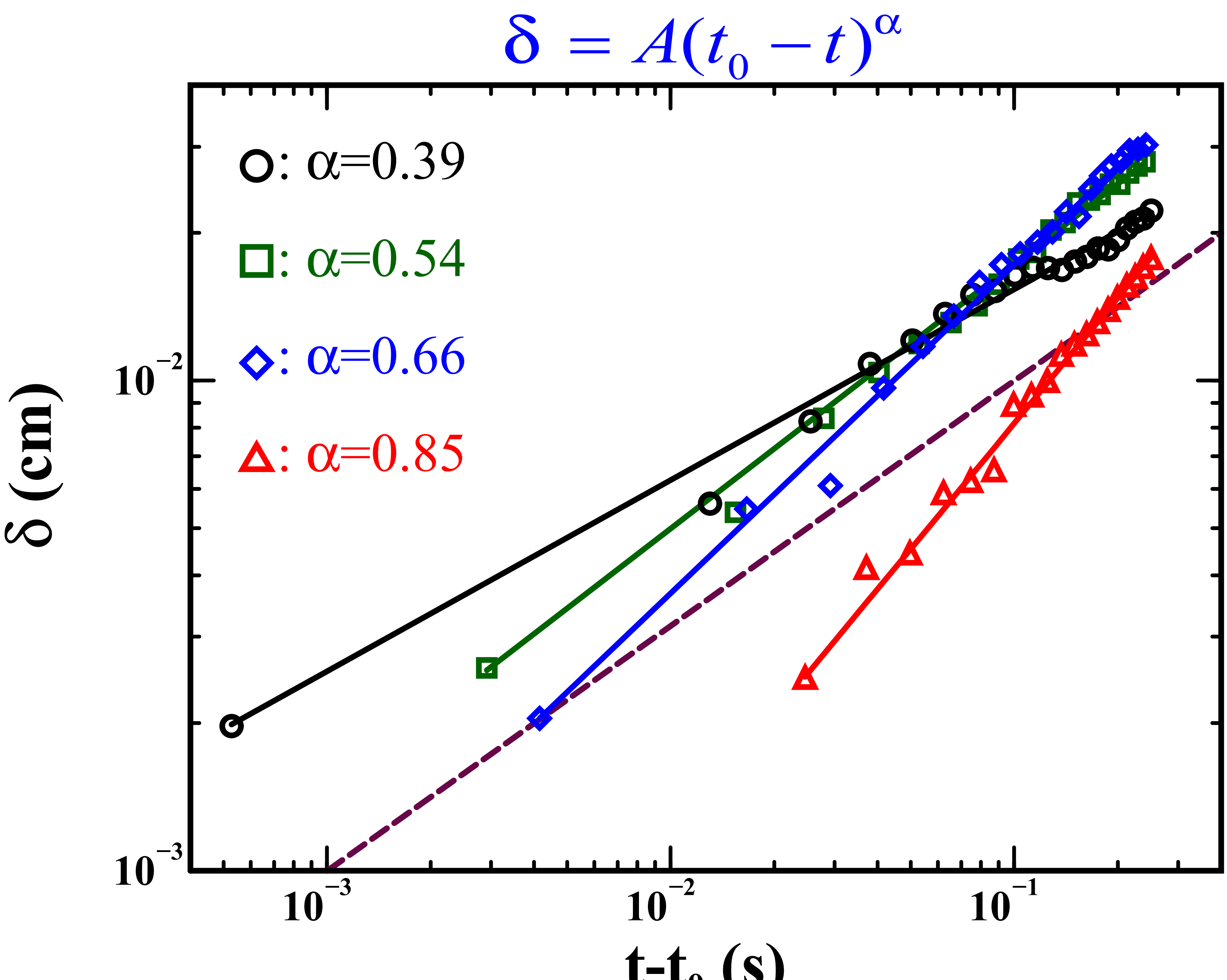
$$\delta \sim \kappa^{1/2}(t - t_0)^{1/2}$$

Visualizing Vortex Reconnection

$$\delta = A(t - t_0)^{\alpha}$$



Visualizing Time-Reversed Dynamics



Statistical Measure

Event defind as:

$$\frac{\delta_{mn}(t \pm 0.25\ s)}{\delta_{mn}(t)} > 4$$

+ forward event

- reversed event

Goodness-of-fit requirement

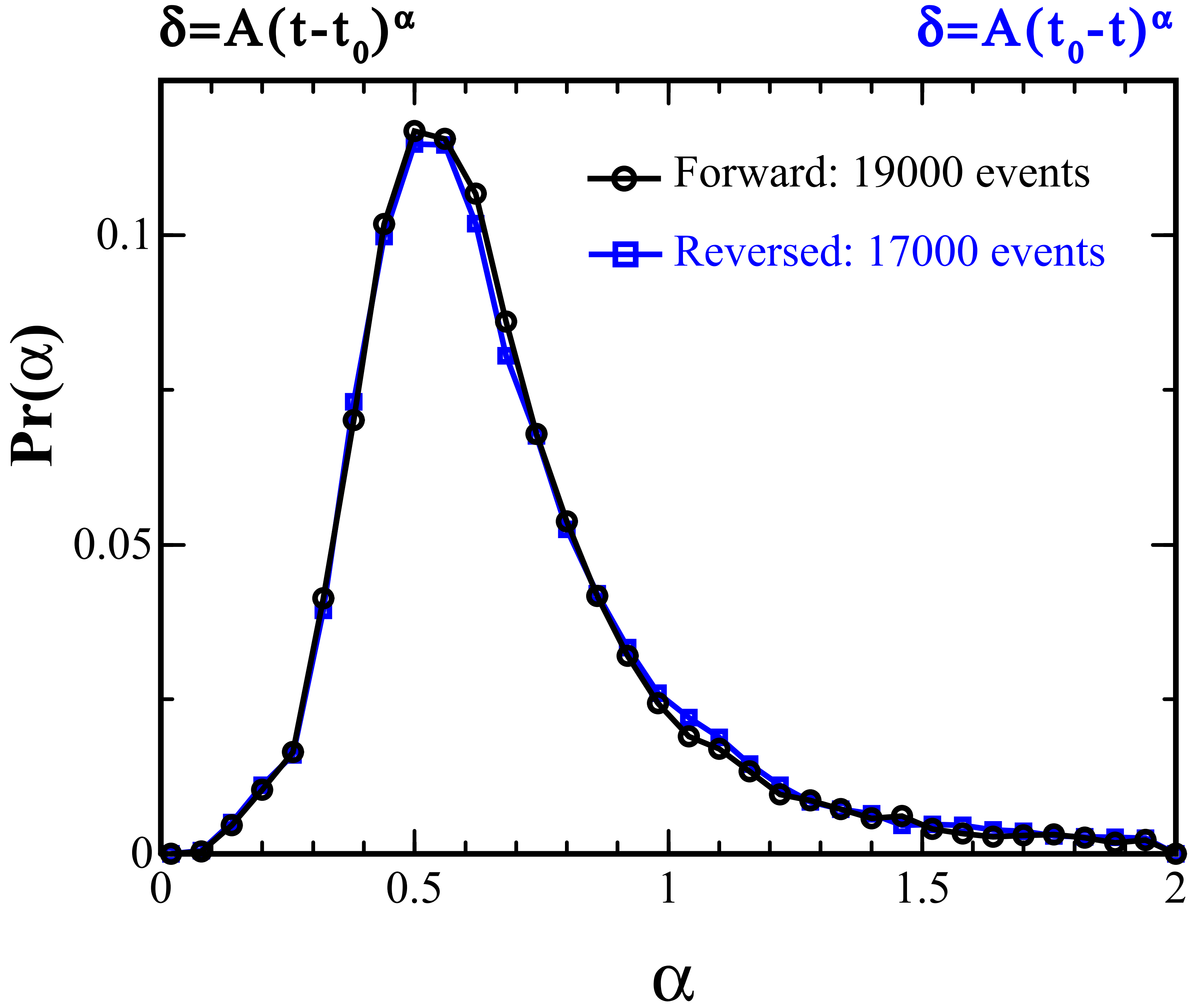
$$\chi^2 = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\delta_{fit_i} - \delta_i}{\sigma}\right)^2 < 4$$

σ=4 μm (0.25 pixels)

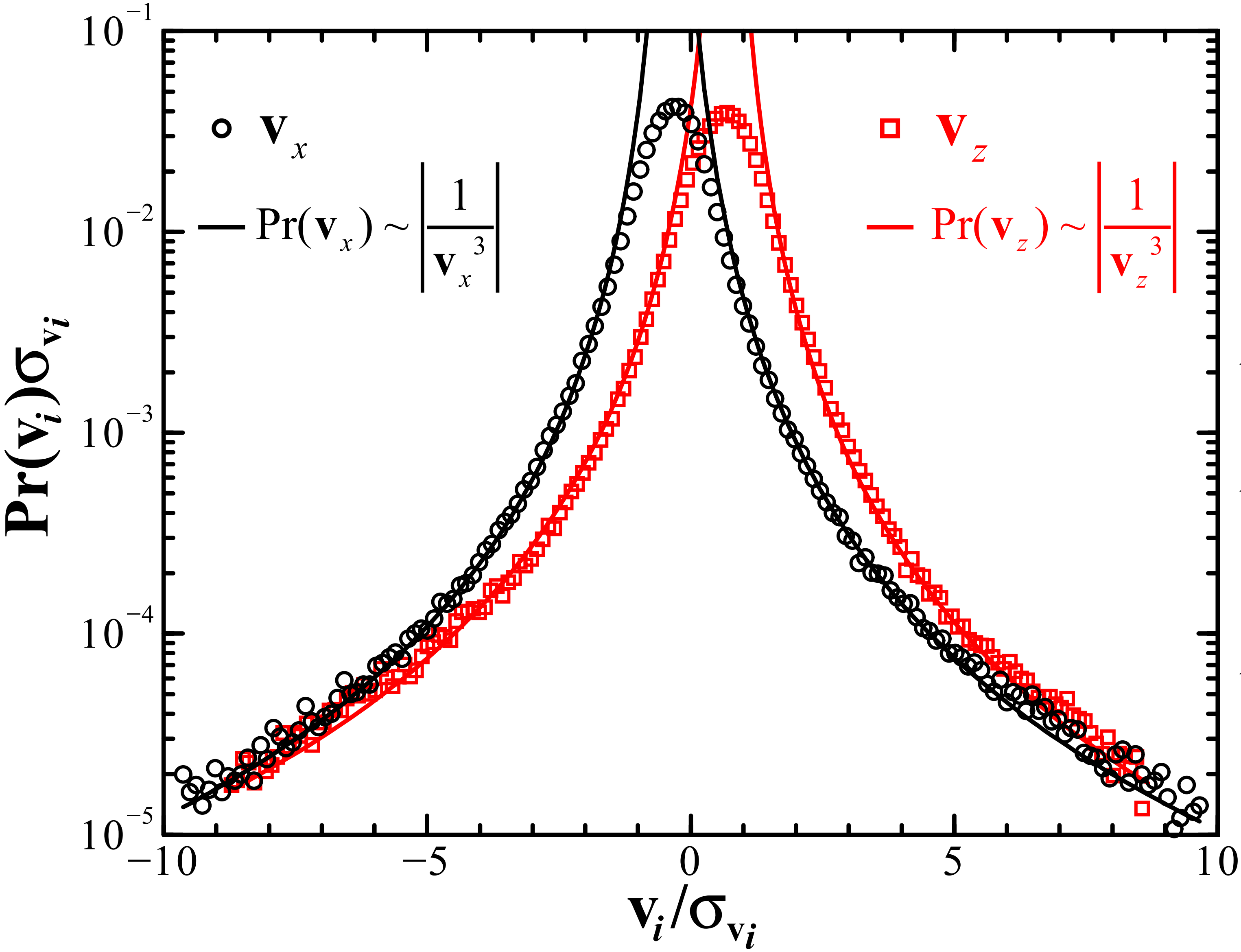
$$\delta_{fit} = A(t - t_0)^{\alpha}$$

$$\delta_{fit} = A(t_0 - t)^{\alpha}$$

Scaling of Reconnection



Velocity Statistics



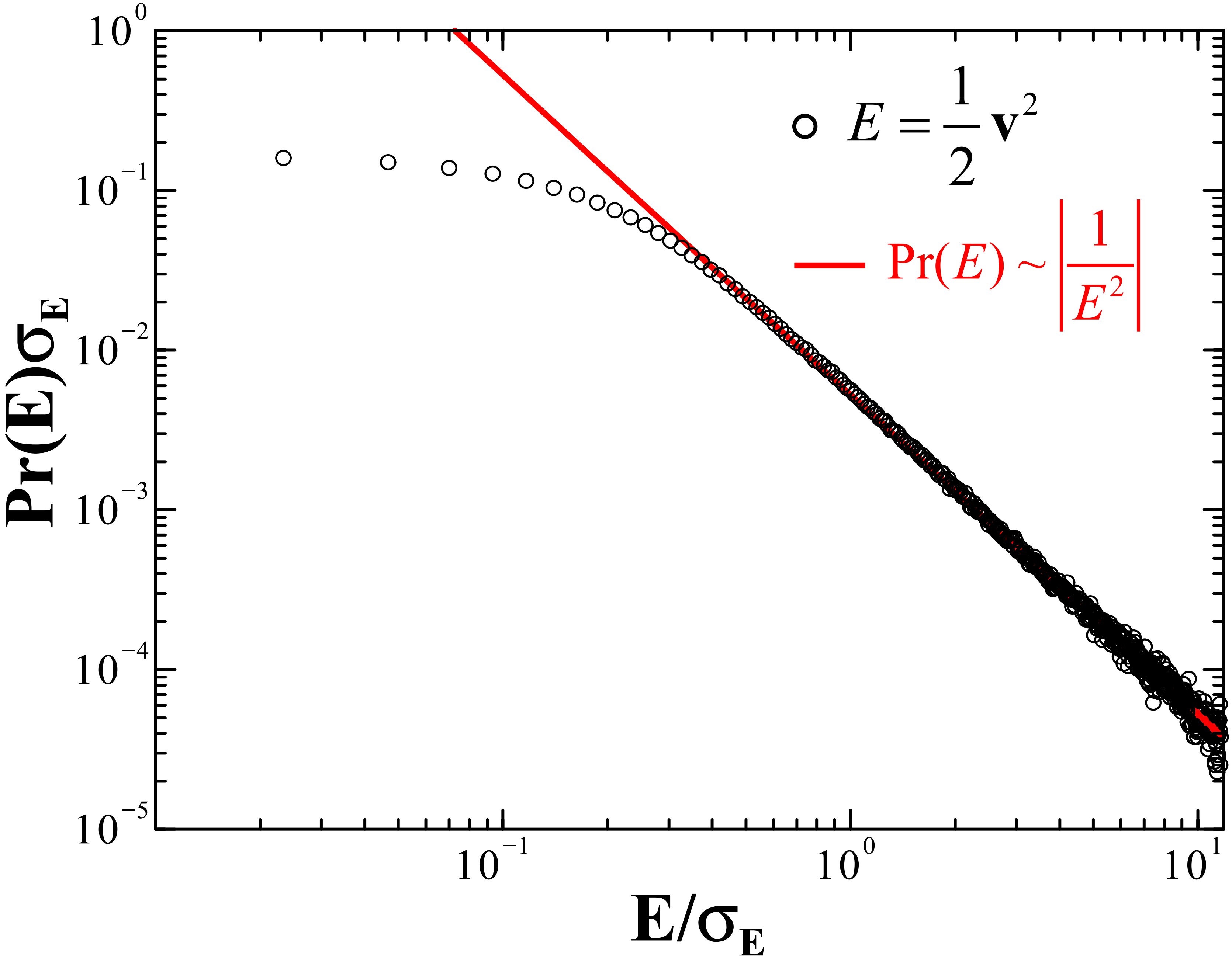
$$\mathbf{v}(t) \sim \kappa^{1/2}\,|t - t_0|^{-1/2}$$

$$P(\mathbf{v})d\mathbf{v} = P(t)dt$$

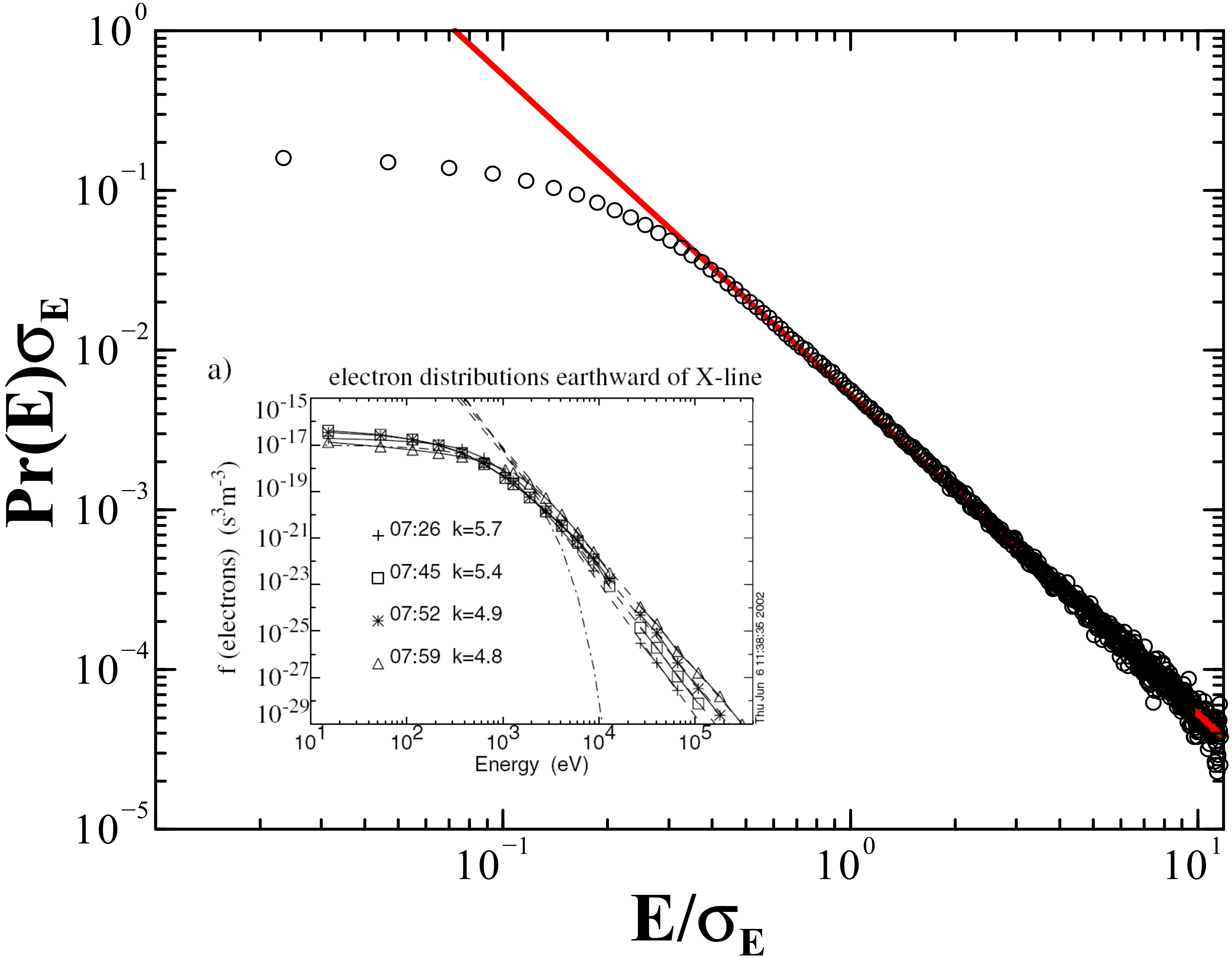
$$P(\mathbf{v}) \sim \left|\frac{dt}{d\mathbf{v}}\right|$$

$$P(\mathbf{v}) \sim \left|\frac{1}{\mathbf{v}^3}\right|$$

Energy Statistics



Energy Statistics



M. Oieroset et. al, *Phys. Rev. Lett.* **89**, 195001 (2002)

Conclusions

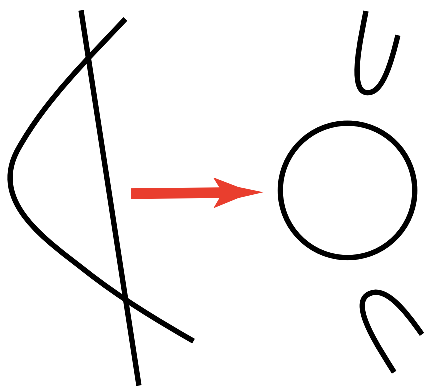
Reconnection is key to removing topological defects

$\Pr(\alpha)$ is distributed about $\frac{1}{2}$

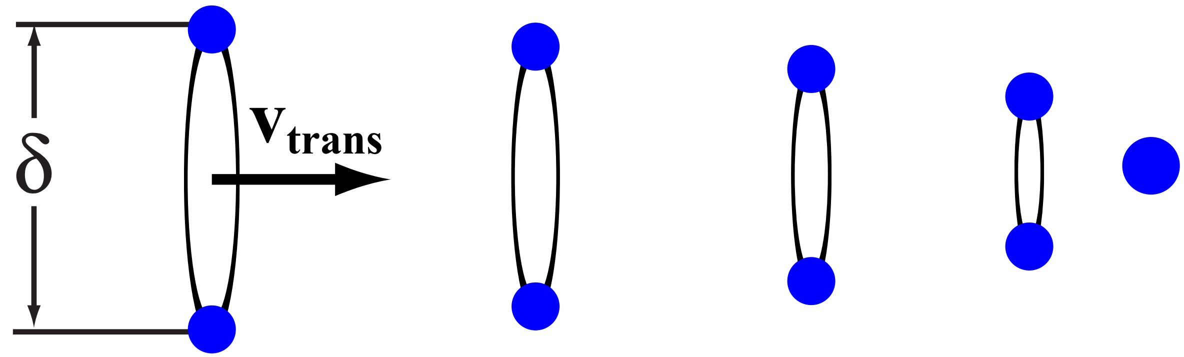
Power-law scaling of reconnection produces power-law velocity and energy statistics

Thanks to: Makoto Tsubota, Nigel Goldenfeld, Marc Swisdak, James Drake, and Michael Fisher for useful discussions

Quantized Vortex Rings



Reconnection can produce vortex rings



Pair of particles on collapsing rings look like reconnection backwards in time with additional transverse velocity