Drops sliding down an incline:

Singular ”corners”.

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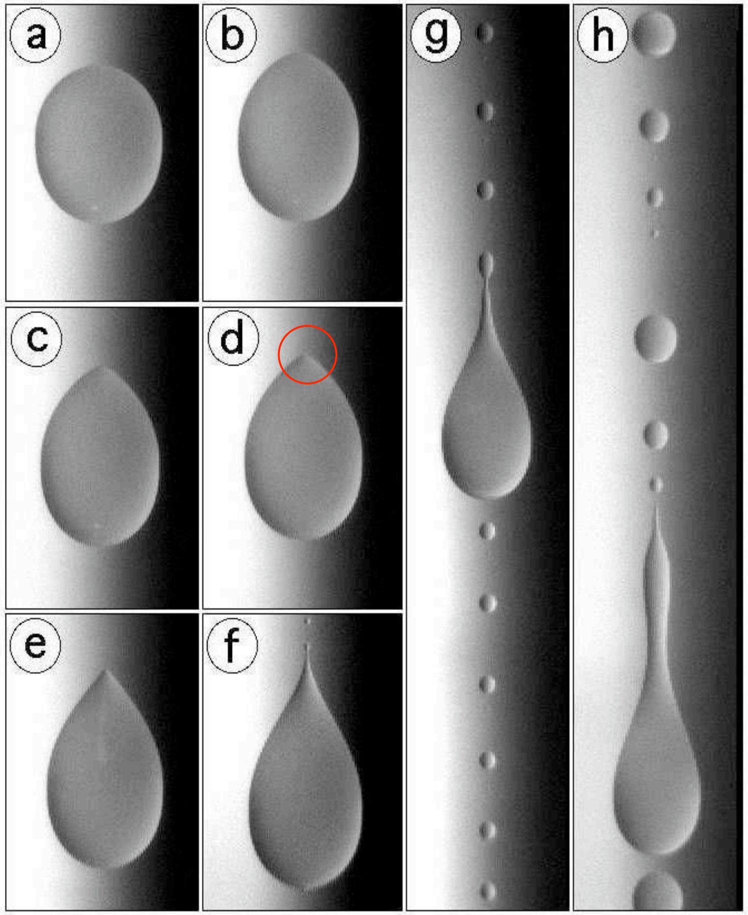
with:

-Jean-Marc Flesselles , Thomas Podgorski (initial experiments)

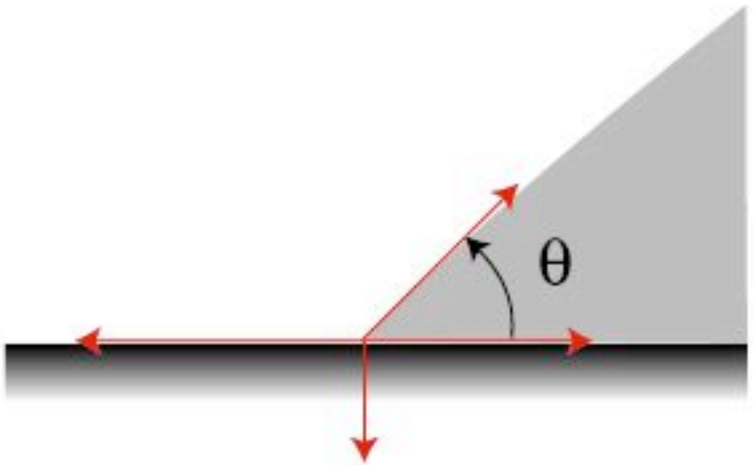
-Adrian Daerr, Nolwenn Le Grand
Bruno Andreotti, Emmanuelle Rio, Ivo Peters (3D visualization + PIV+optics)

- Jacco Snoeijer, Howard Stone, Jens Eggers (corner models)



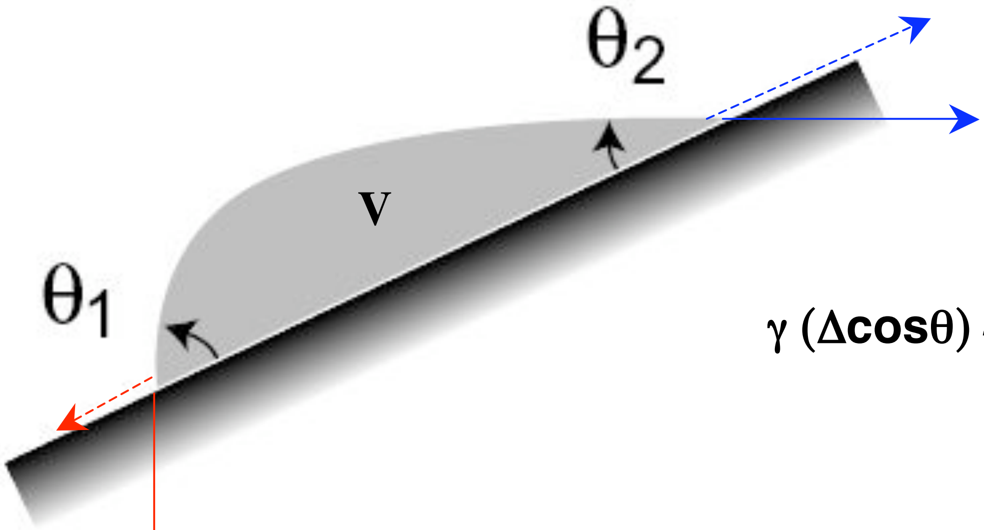
Partial wetting

$$\gamma_{sv} < \gamma_{sl} + \gamma$$



$$\gamma_{sv} = \gamma_{sl} + \gamma \cos\theta$$

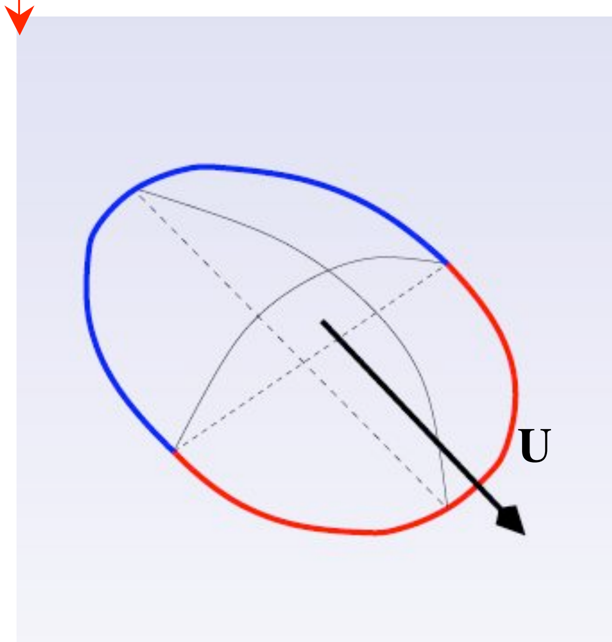
What happens on a tilted plate?



Hysteresis: drop can remain pinned...

$\gamma\,(\Delta\cos\theta) \sim \rho g\, V^{2/3}$

or begins to slide…



-viscous effects -> shape changes?
-wetting dynamics…
with 3D aspects -> ?
-Inclined, curved contact line possibly developing a singular point

3D Sliding drops:

-Bikerman, JCS 1950, Furmidge, JCS 1962 -> onset of motion
-Dussan (and Chow), JFM 1983, 1985 -> calculations for rounded drops yield condition
-Kim, Lee + Kang, JCIS 2002 -> sliding velocities of oval drops related to viscous dissipation

-Podgorski, Flesselles, LL, PRL 2001 -> Singularity at drop rear
-Le Grand, Daerr, LL, JFM 2005 -> 3D structure of interface
-Rio, Daerr, Andreotti, LL, PRL 2004 -> flow structure, contact angle distribution

-Stone + LL, Europhys. Lett. 2004 -> similarity solution of hydrodynamics
-Snoeijer, Le Grand, Rio, LL, Phys. Fl. 2005 -> flow structure, rounded corners…
-Snoeijer, Le Grand, LL, Stone, Eggers, Phys. Fl. 2007 -> opening angle selection, pearling transition

Other model: Cummins, Ben Amar , Pomeau, Phys. Fl. 2003 -> model based on Laplace +directional Young condition

Numerical simulations:
-Schwartz et al., Physica D, 2005
--Thiele et al
--Gaskell et al. (Leeds)

Corners, Cusps, and Pearls in Running Drops

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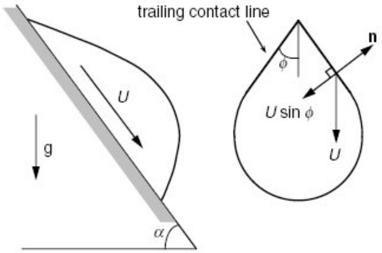
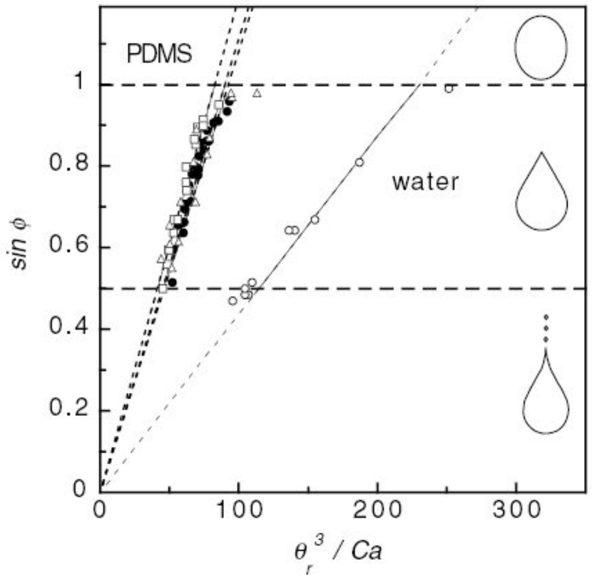
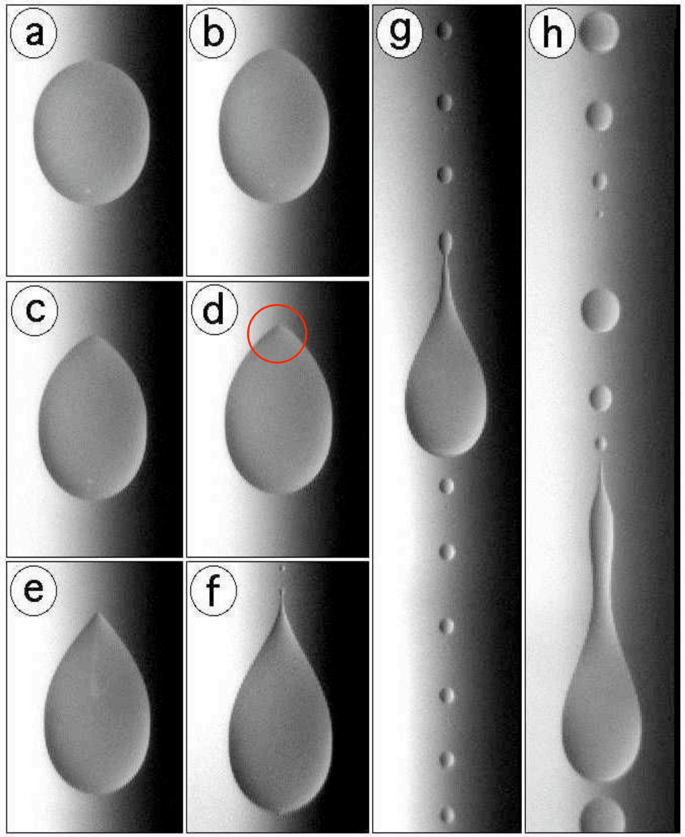
(Received 6 February 2001; published 27 June 2001)



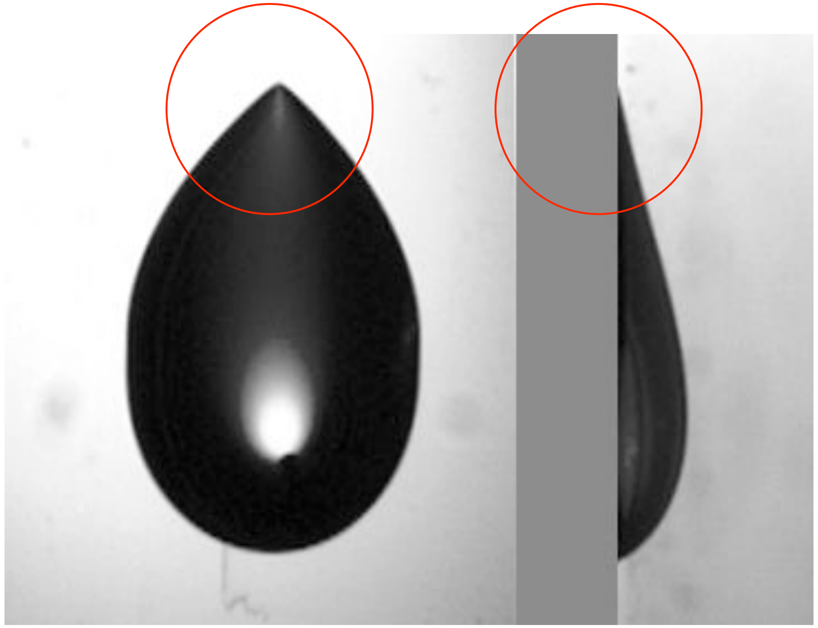
Ca=ηU/γ

sinφ~1/Ca

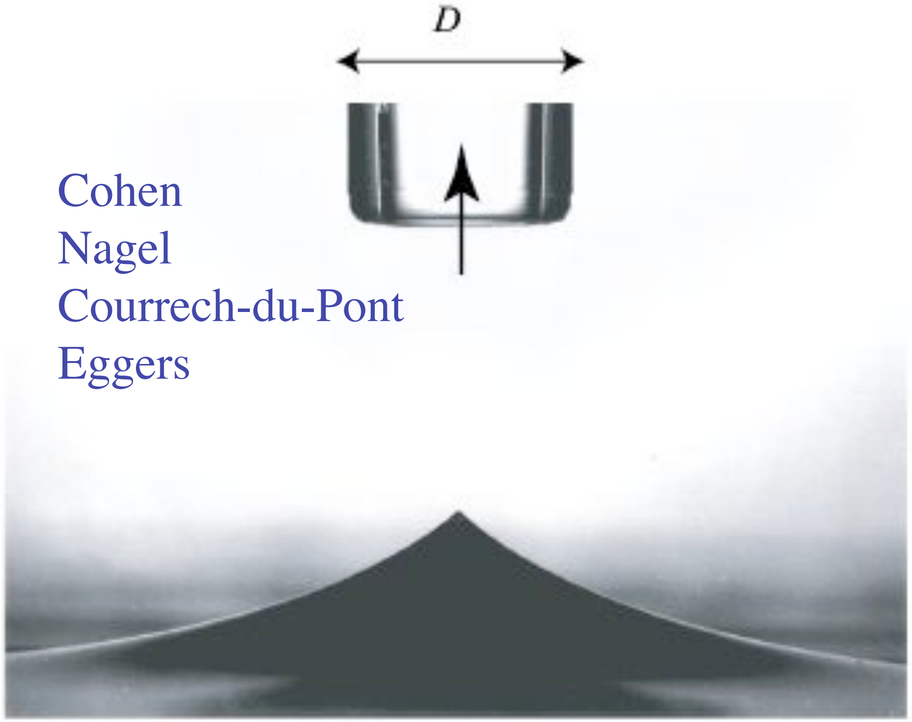




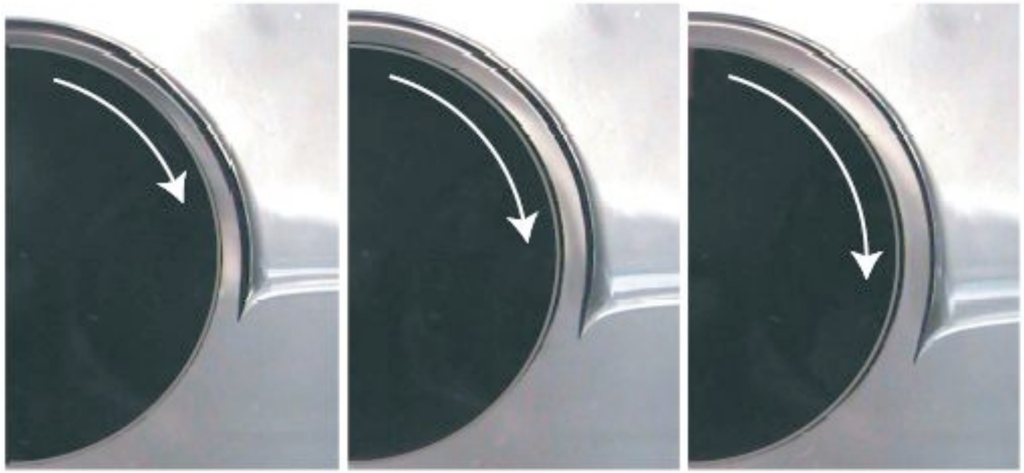
Silicon oil drops on fluoropolymers



Daerr, Le Grand, LL, 2005

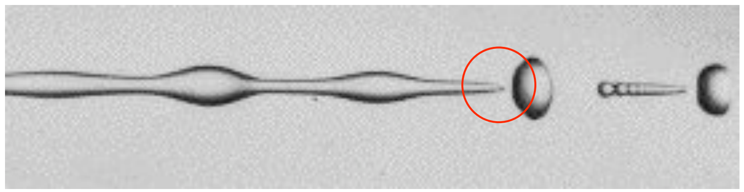


Cohen
Nagel
Courrech-du-Pont
Eggers

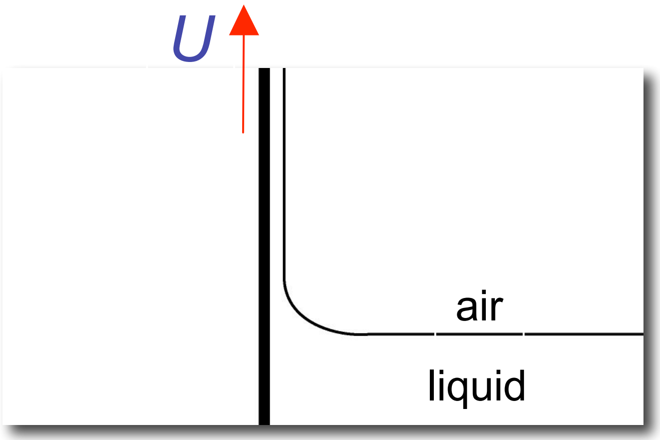


Quéré, Lorenceau

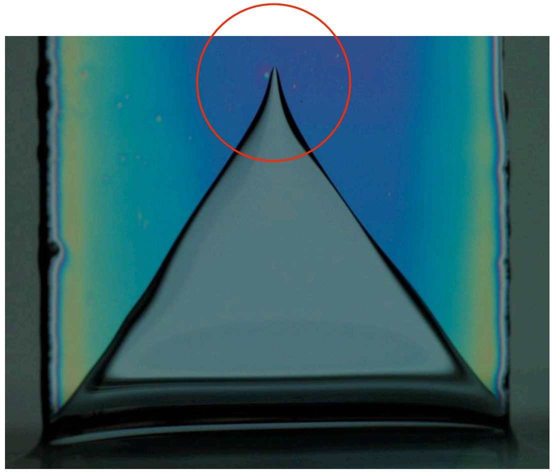
Singularities at interfaces

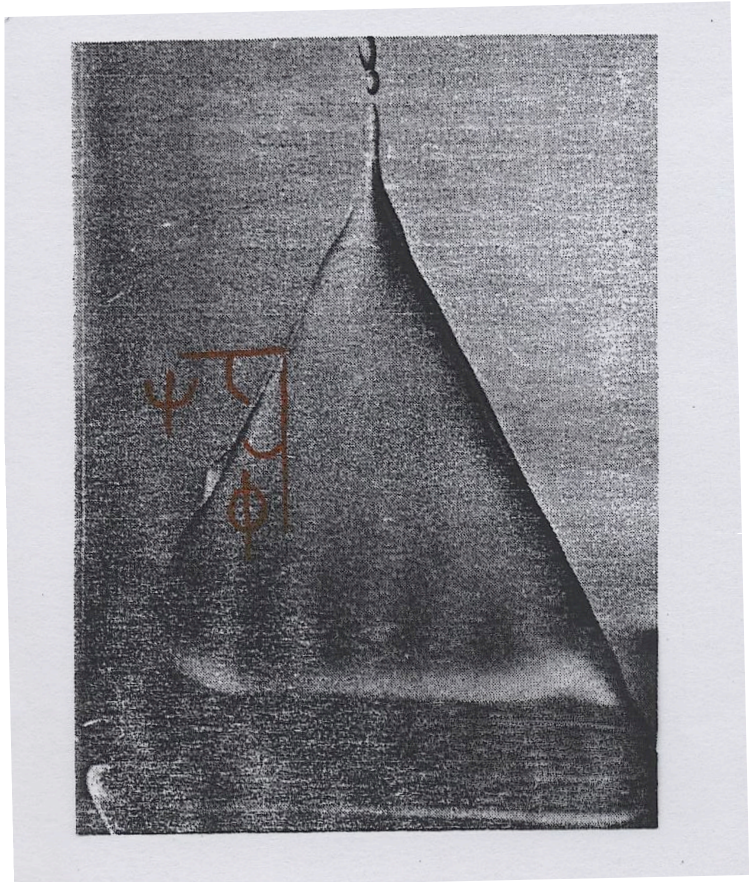


(Rutland and Jameson ’71)



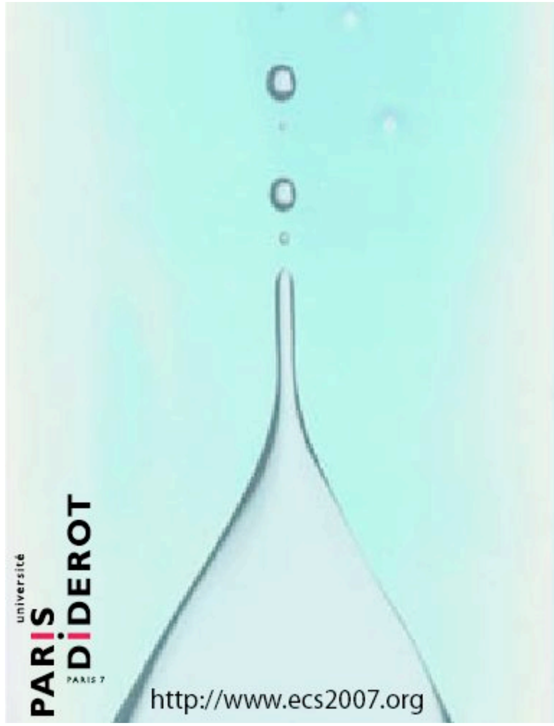
Analogy with Landau-Levich

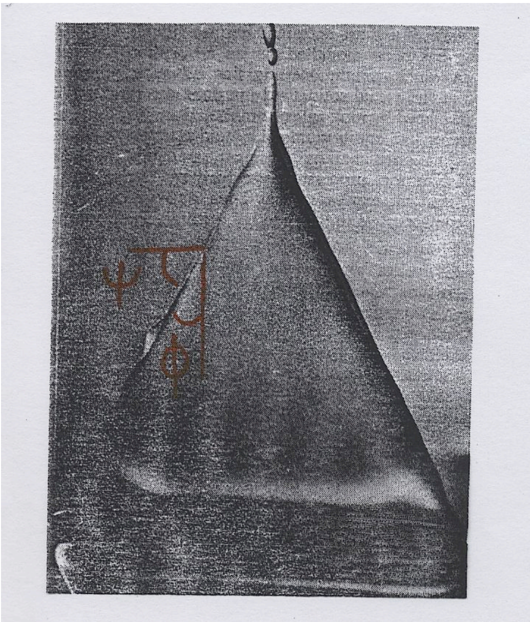


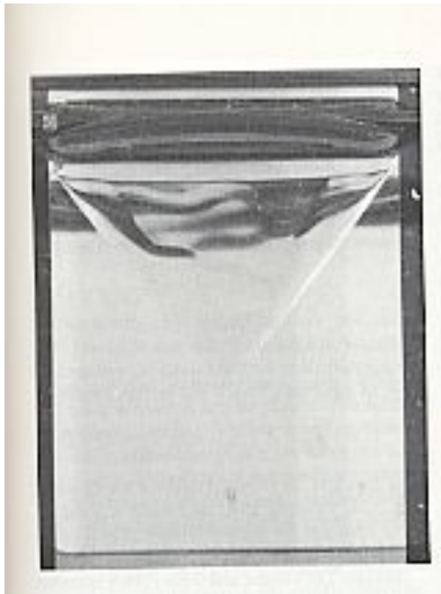


Blake, Ruschak
(Nature 1979)

Snoeijer, Delon,
Andreotti, Fermigier
PRL 2006

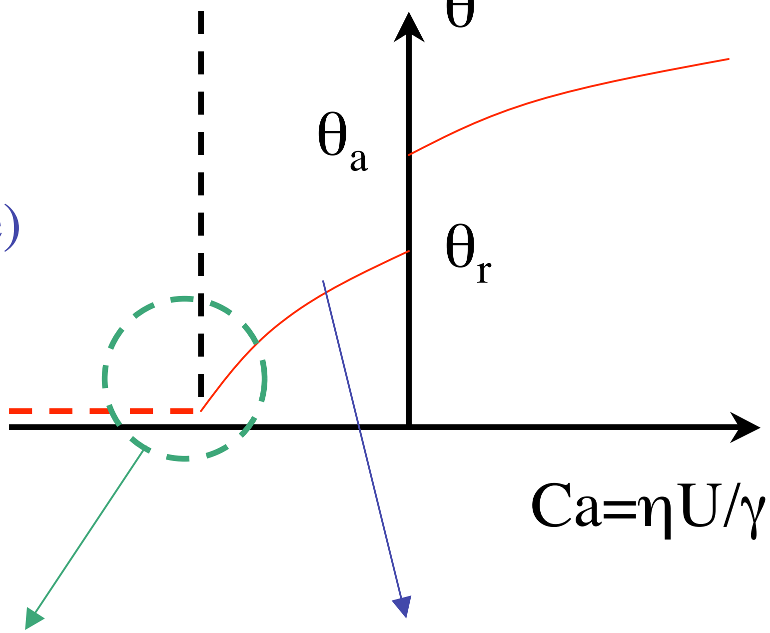


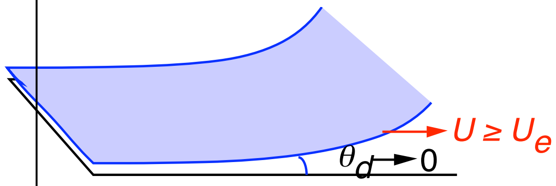


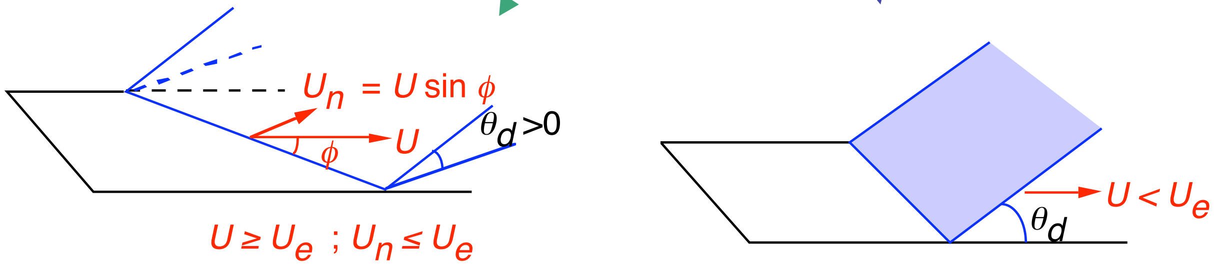


Interpretation:

$\theta_d = F(Ca) \dashrightarrow 0$
(or a critical value)







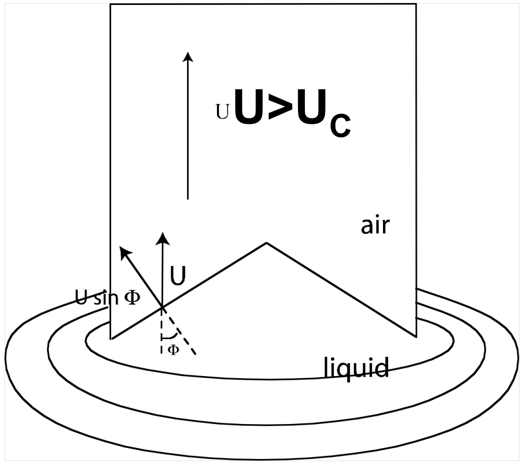
$\theta_d = F(Ca \sin\phi) \approx 0 \implies$

$$\sin\phi = \frac{Ca_c}{Ca}$$

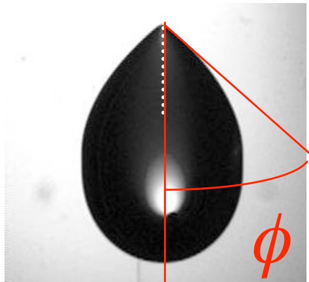
Blake, Ruschak(Nature 1979)

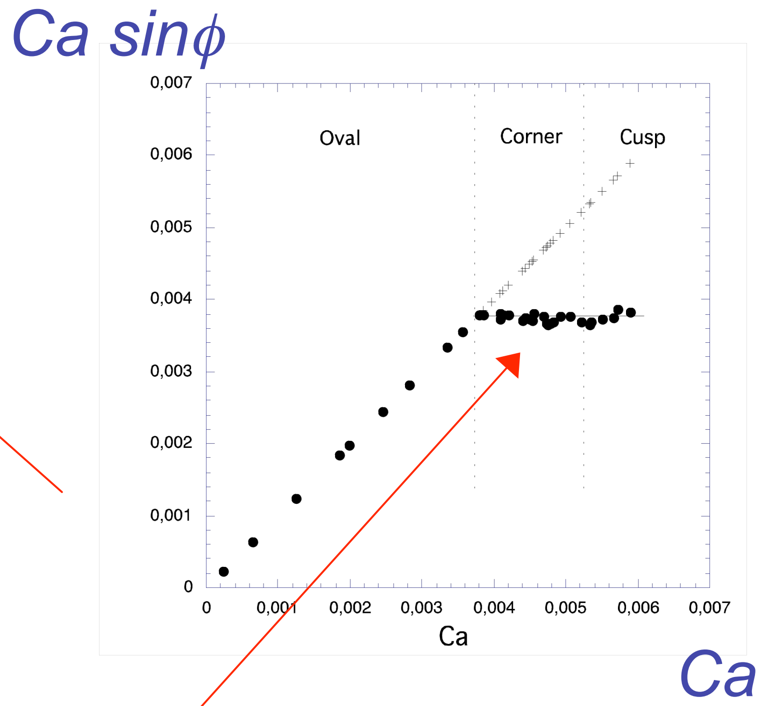
Other point of view: Drops avoid wetting by tilting contact lines

plate withdrawn
from bath



Blake and Ruschak '79





normal velocity: maximum speed=U_c

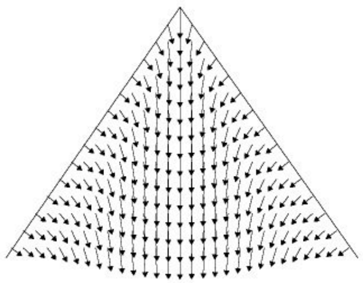
=> sinϕ ~ 1/Ca

//Mach cone

Outline:

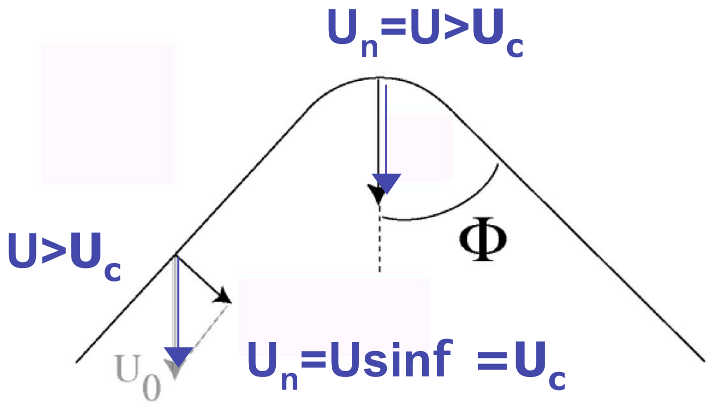
- Experiment - Observations

- 3D structure and flow in sliding drops
  - 3D geometry of sliding drops
  - flow structure - autosimilarity

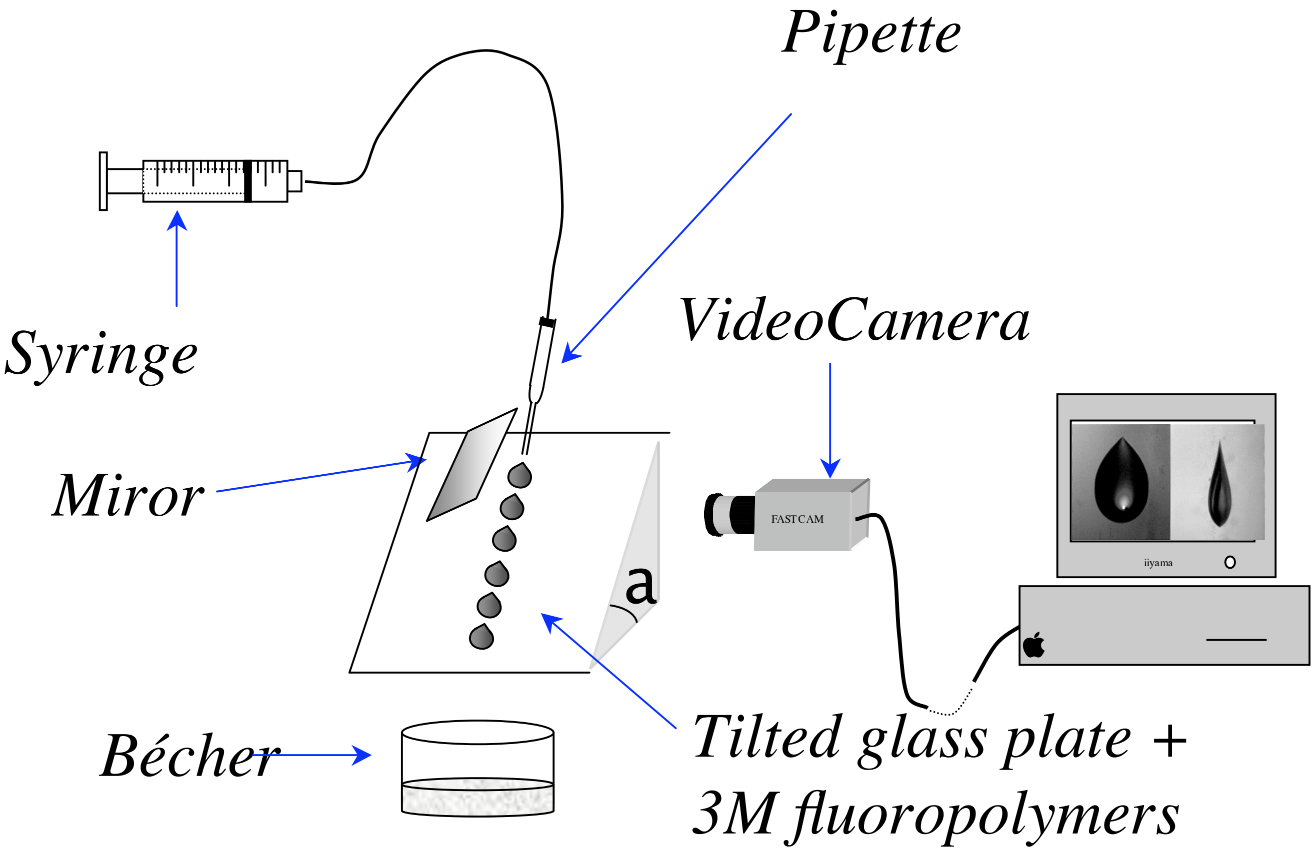


- Matching with contact lines
  - opening angle selection
  - pearling transition

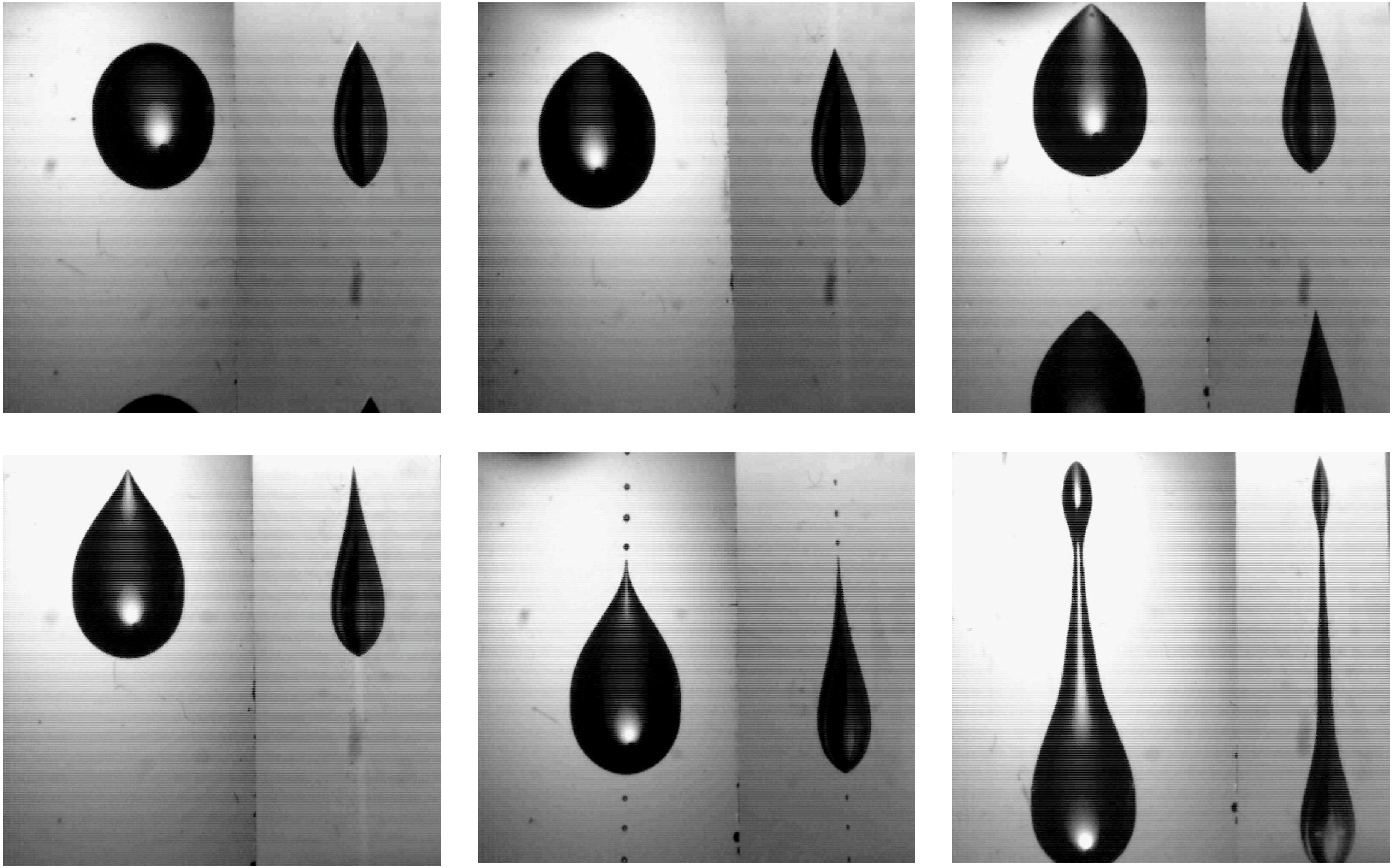
- Curvature at the tip
  - curved contact lines
  - divergence of curvature



EXPERIMENT



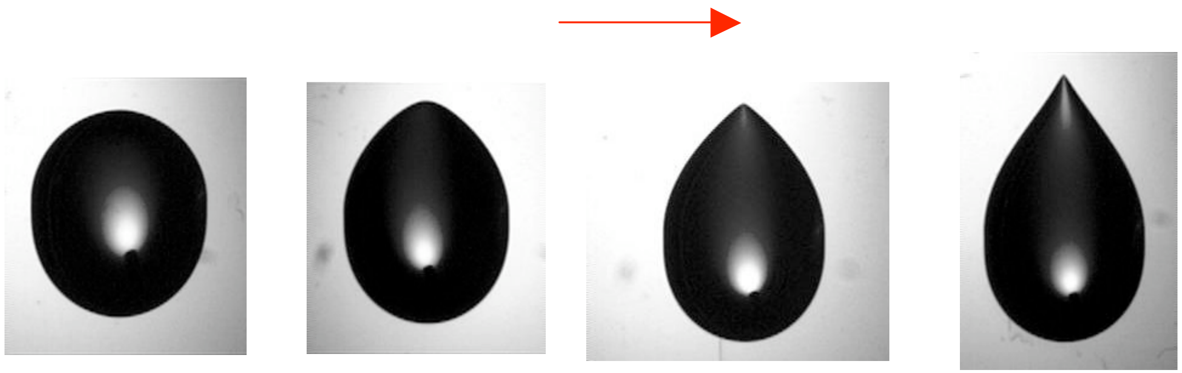
A. Daerr, N. Le Grand, LL, J. Fl. Mech. (2005)



sliding silicon oil drops...

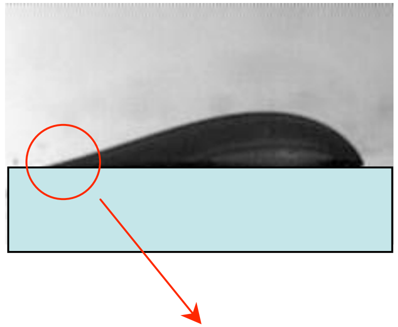
Podgorski, Flesselles Limat , PRL ’01
Le Grand, Daerr, Limat JFM ’05

increasing $Ca=\eta U_0/\gamma$

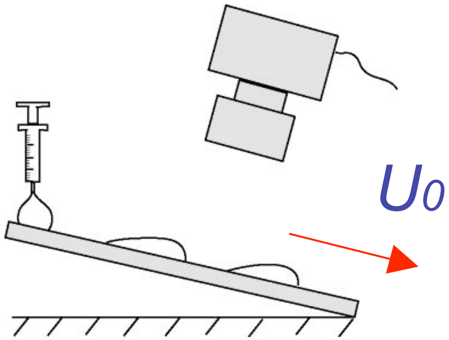


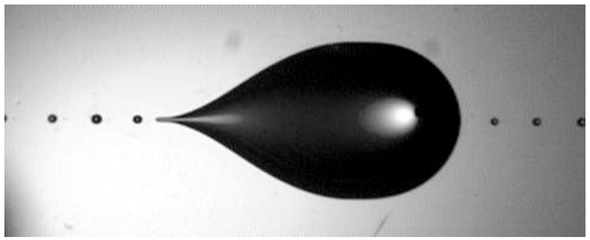
rounded corner cusp

Side views



conical singularity

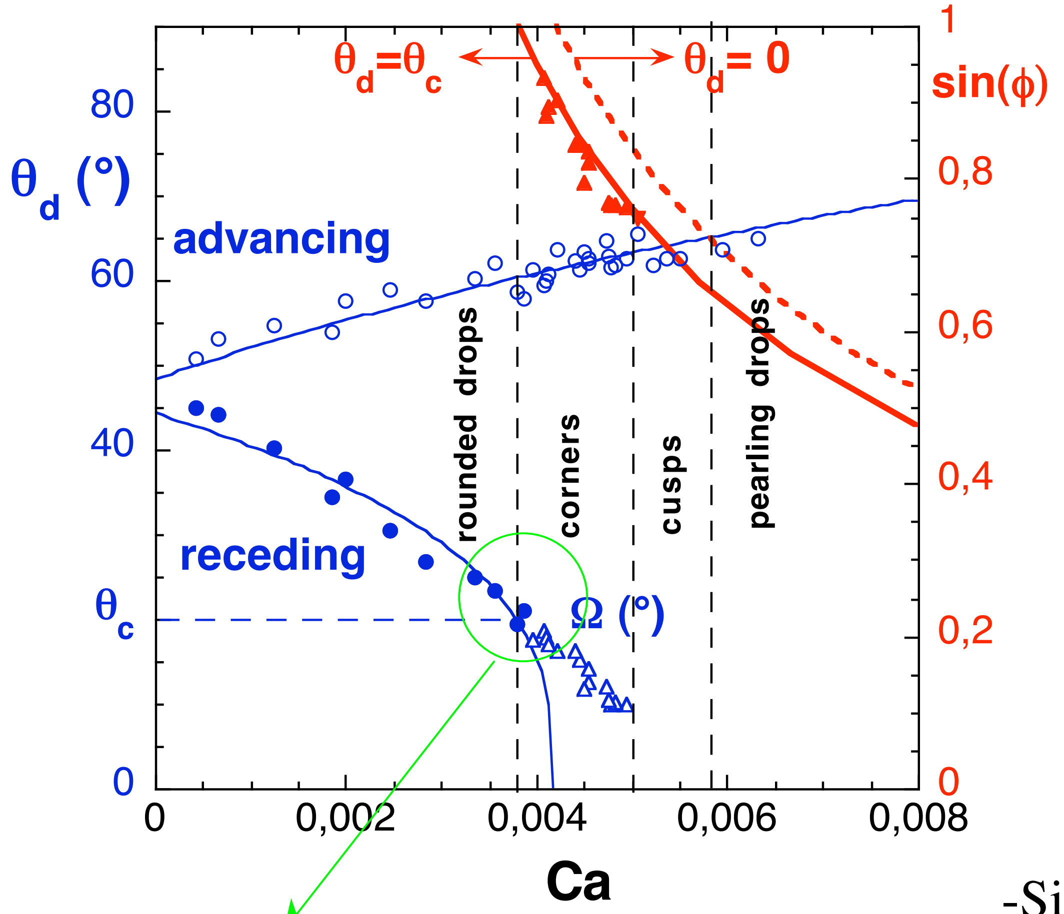




pearling

Stone, LL, Europhys. ‘04

Glass plate + fluoropolymer coating
Partial wetting: contact angle 45º

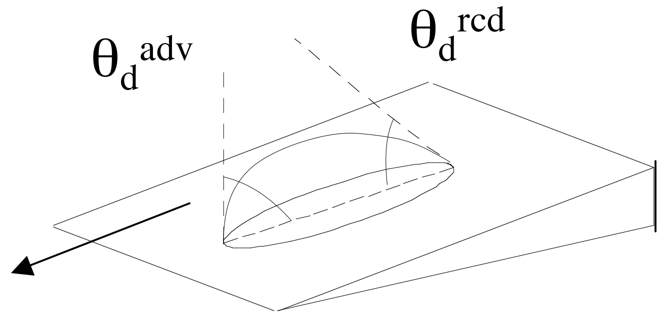


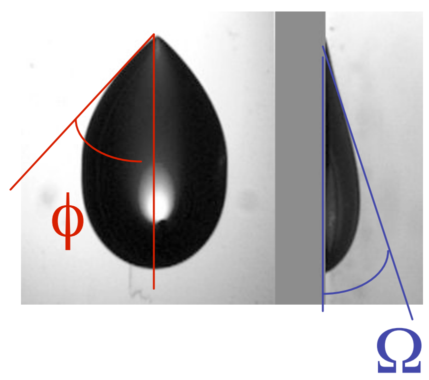
? Non-zero

// De Gennes…

// Raphaël, Golestanian…

Daerr, Le Grand, LL, J.Fl. Mech. 2005

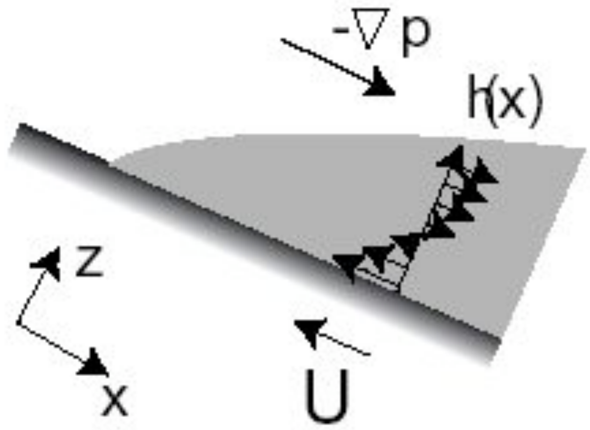




-Sinφ ~1/Ca, but

- non-zero critical angle θ_c

-Conical interface : θ, Ω non-zero



Voïnov-Cox model

- Mass conservation:

$$Uh = \frac{h^3}{3\eta}\gamma(h_{xx})_x$$

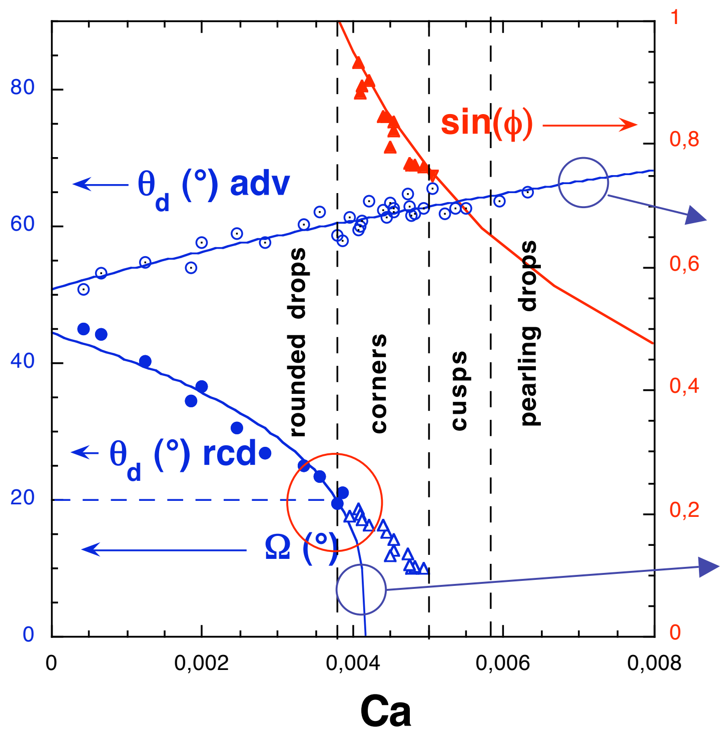
$$3\frac{Ca}{h^2} = h_{xxx}$$

everything is singular for h=0

- To be completed with :

$\theta = \theta_e$ for x=a

and a matching with an outer solution for x -> ∞



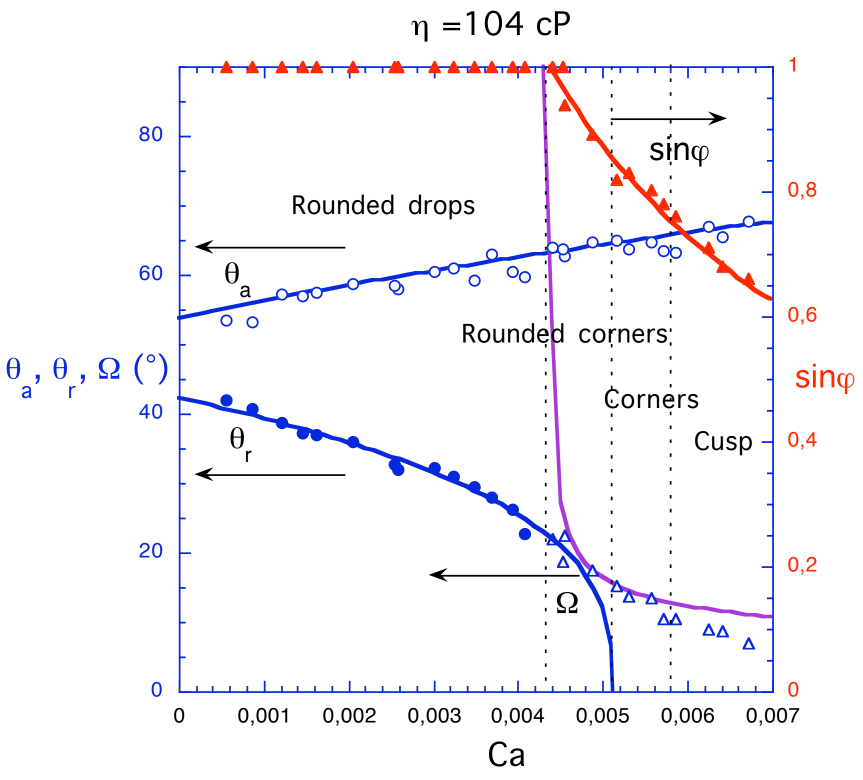
9Log(b/a)=130

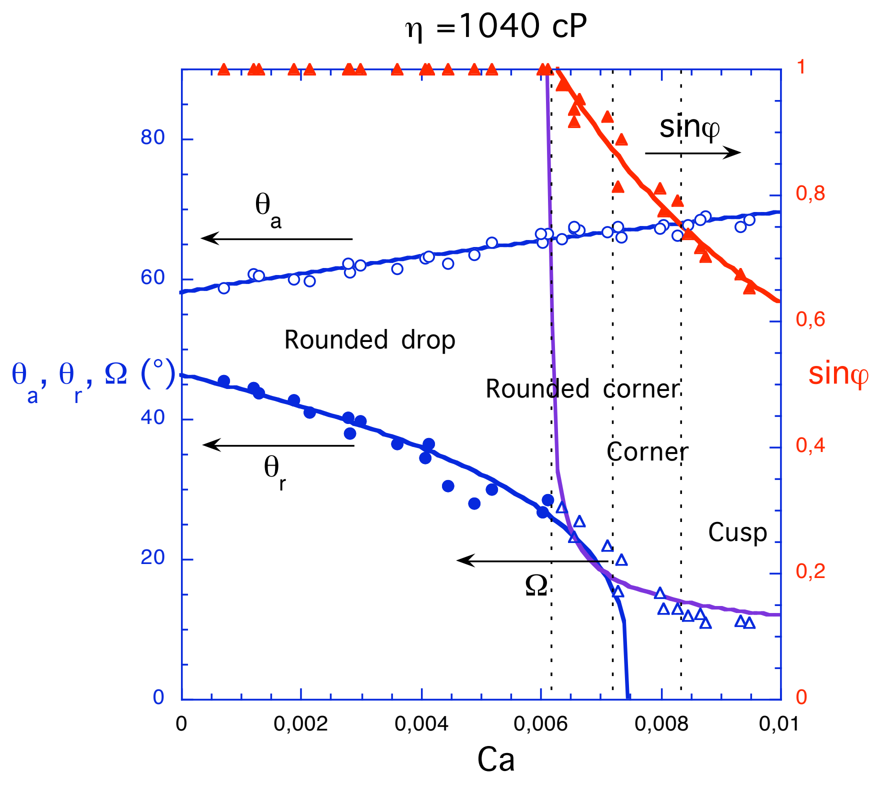
9Log(b/a)=130

macroscopic microscopic

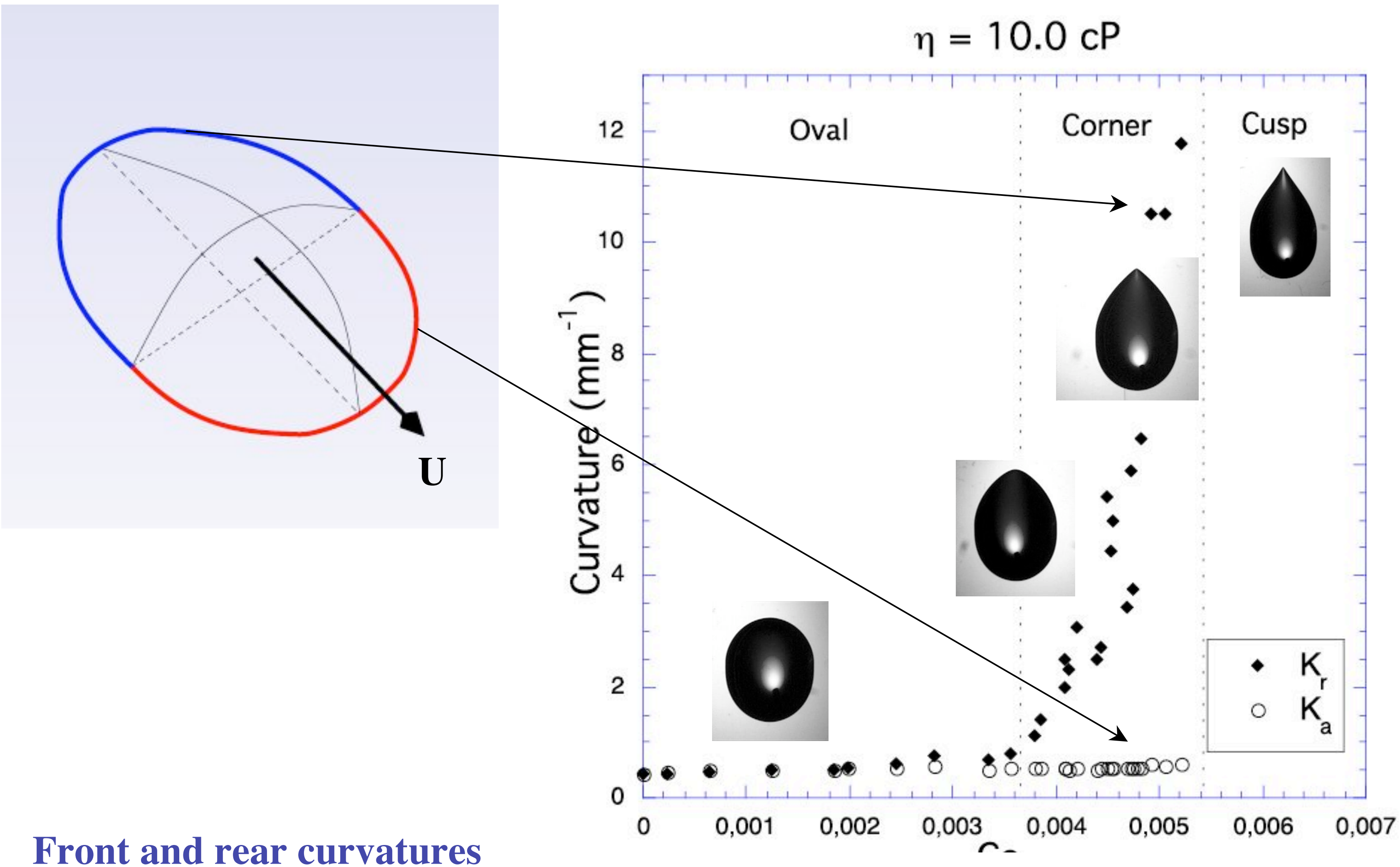
$$\theta^3(b) = \theta^3(a) \pm 9CaLog(b/a)$$

θ_e
(equilibrium angle)





Same with two other viscosities….



Ca=ηU/γ

Front and rear curvatures

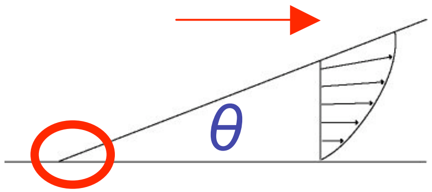
Structure of the singularity

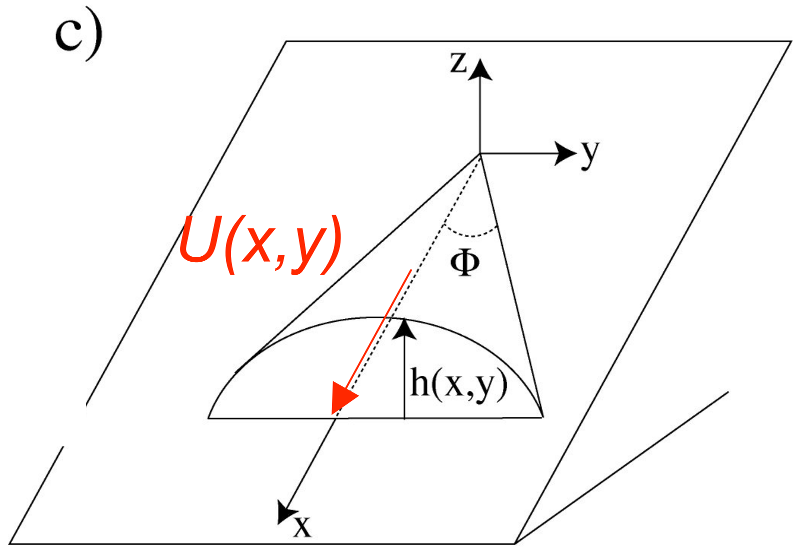
$h(x,y)$?

$\vec{U}(x,y) = \langle \vec{v}(x,y,z) \rangle_{/z}$?

$$-\vec{\nabla} p + \eta \Delta \vec{v} + \rho \vec{g} = \vec{0}$$

$$\vec{U}(x,y) = -\frac{h^2}{3\eta} \nabla P$$





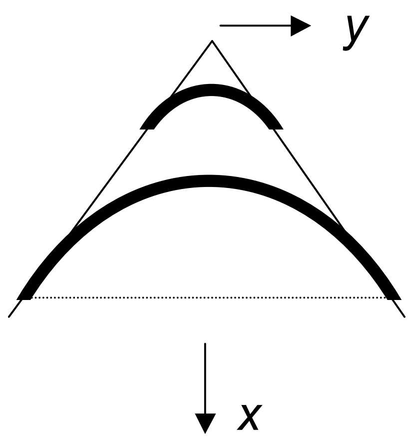
Flow driven by capillary pressure

$$p = p_0 - \gamma \Delta h$$

Stone + LL, Europhys '04
Snoeijer et al., Phys. Fluids '05

corner model

$$\begin{cases} \partial_t h + \vec{\nabla} \cdot \left(h \vec{U} \right) = 0 \\ \vec{U}(x,y) = \dfrac{h^2}{3\eta} \nabla(\Delta h) \end{cases}$$



+ steady state solutions:

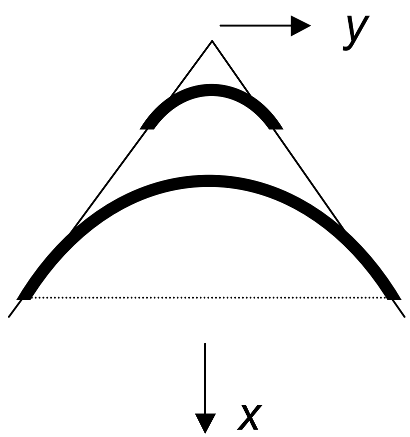
$h(x - U_0 t, y)$

$$3Ca \frac{\partial h}{\partial x} = \nabla . \left[h^3 \nabla(\Delta h) \right]$$

$Ca = \eta U_0 / \gamma$

corner geometry

$$3Ca\frac{\partial h}{\partial x} = \nabla.\left[h^3\nabla(\Delta h)\right]$$



equation allows similarity solutions:

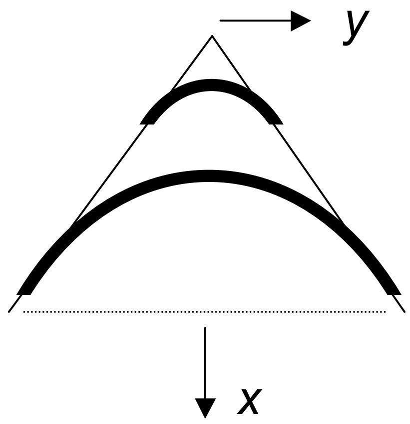
$h(x,y) = Ca^{1/3}x\, H(y/x)$

$$\left(1+\zeta^2\right)^2\left(H^3H_{\zeta\zeta\zeta}\right)_\zeta + 3\zeta\left(1+\zeta^2\right)\left(H^3H_{\zeta\zeta}\right)_\zeta + 2\zeta\left(1+\zeta^2\right)H^3H_{\zeta\zeta\zeta} + \left(1+3\zeta^2\right)H^3H_{\zeta\zeta} = 3\left(H-\zeta H_\zeta\right).$$

Note: Ca has disappeared

corner geometry

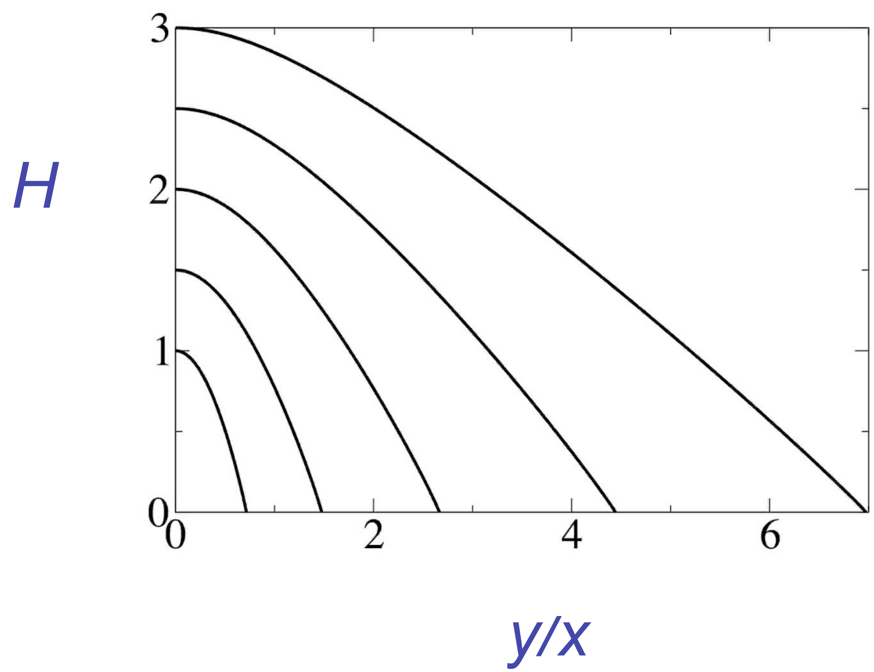
$$3Ca\frac{\partial h}{\partial x} = \nabla.\left[h^3\nabla(\Delta h)\right]$$



equations allow similarity solutions:

$$h(x,y) = Ca^{1/3} x\, H(y/x)$$

1-parameter family:

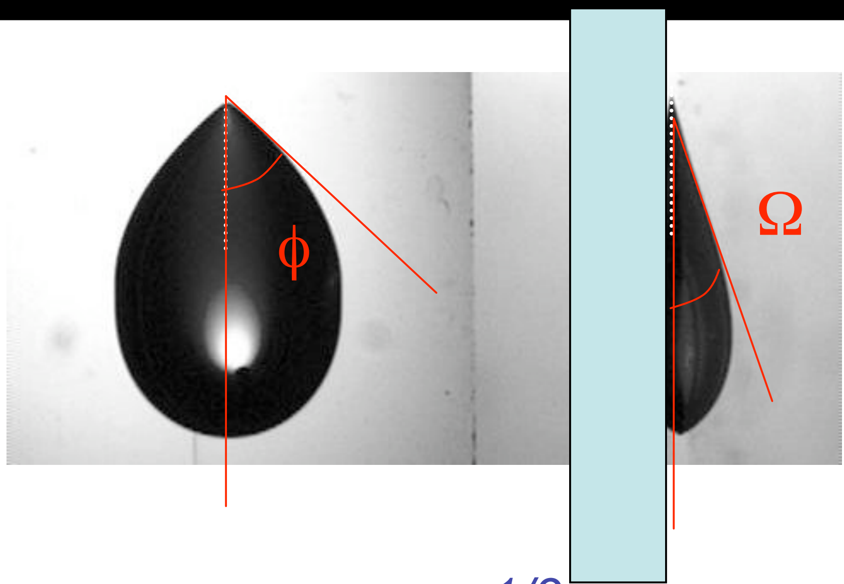


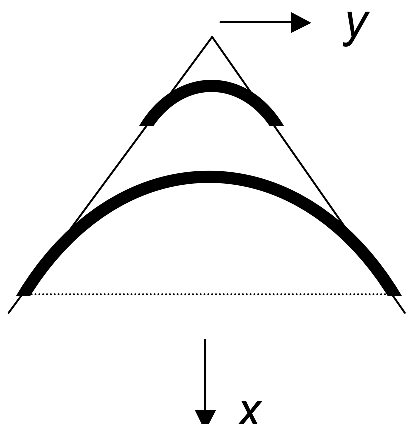
b.c: $H' = H''' = 0$ (symmetry)

$H_0 \longrightarrow H''$ (continuity)

Note: Ca has disappeared

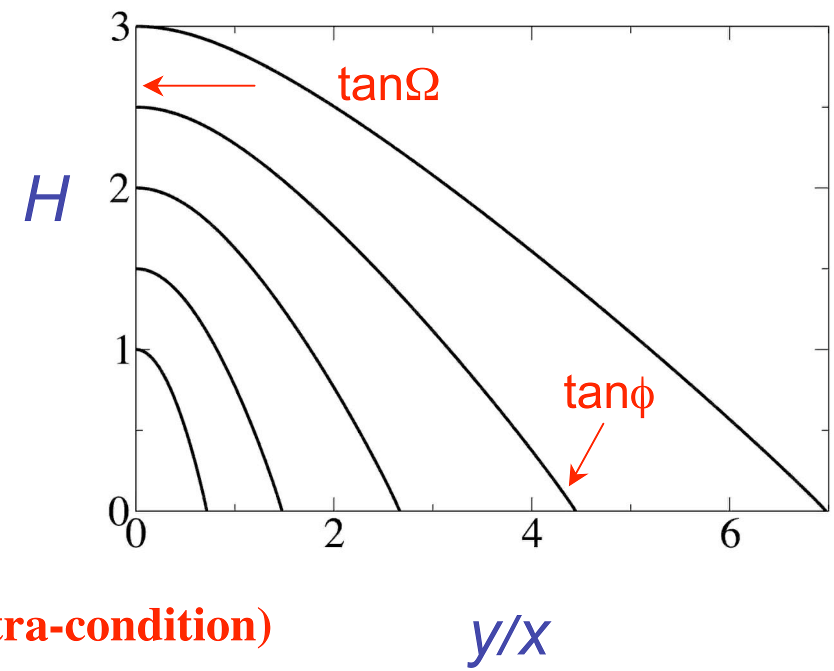
corner geometry





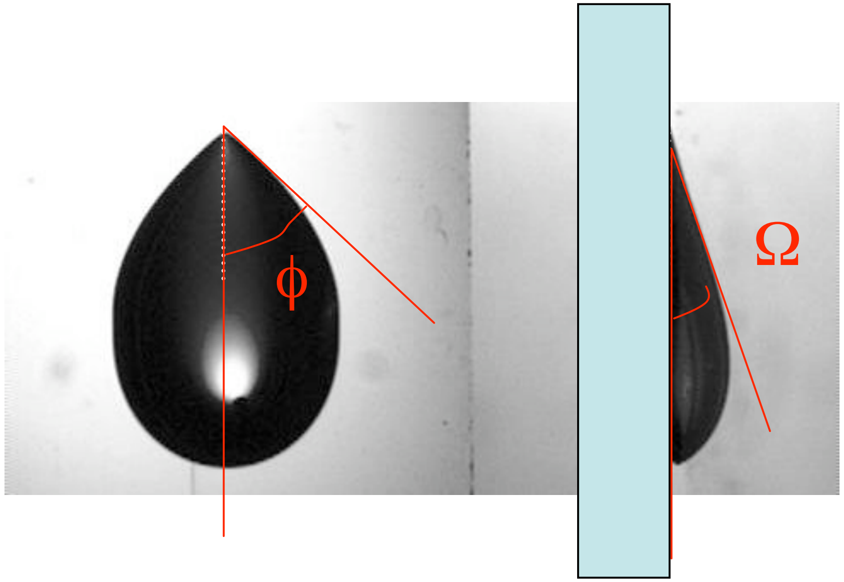
$$h(x,y) = Ca^{1/3}\, x\, H(y/x)$$

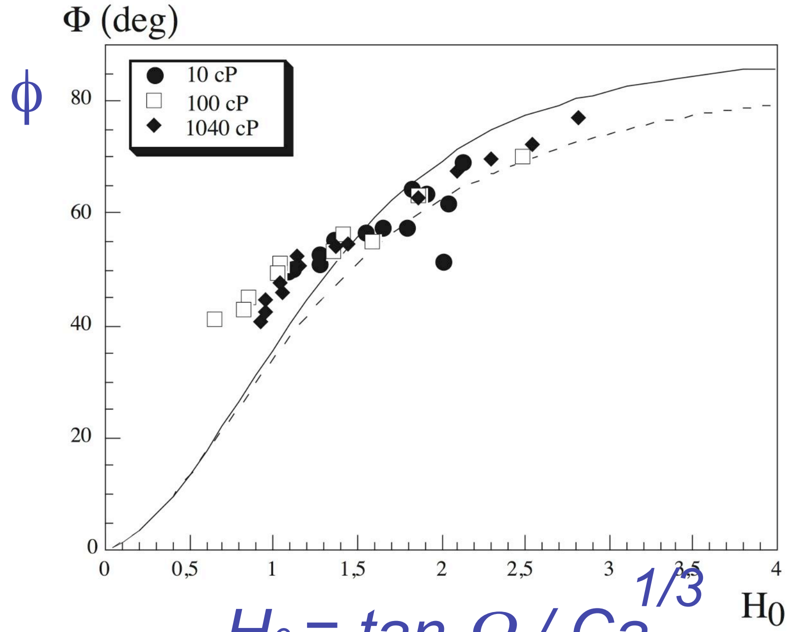
$$H_0 = \tan \Omega / Ca^{1/3}$$



Infinite number of solutions,
irrespectively of Ca (we will need later an extra-condition)

relation Ω and ϕ





$H_0 = \tan\,\Omega\,/\,Ca^{1/3}$

(no adjustable parameters)

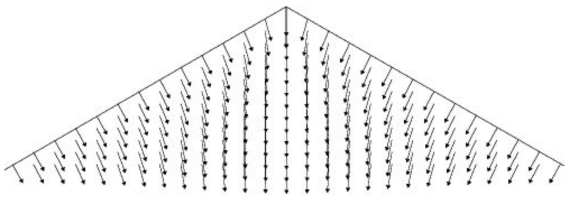
dashed: $(\tan\Omega)^3 = \frac{35}{16}\ Ca\ (\tan\phi)^2$

velocity field

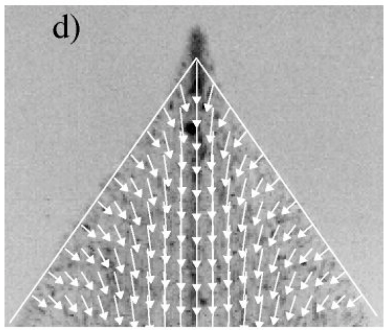
h(x,y) → $\vec{U}(x,y)=U_0\,\vec{F}(y/x)$

F deduced from H

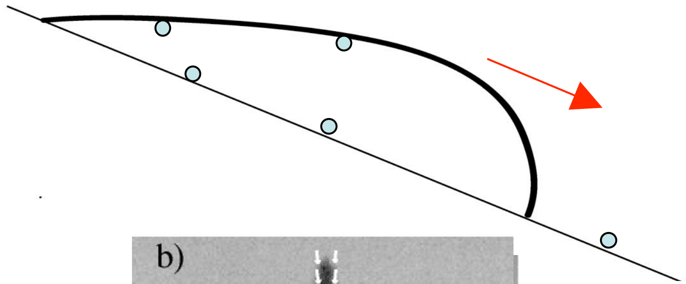
PIV experiment: tracers

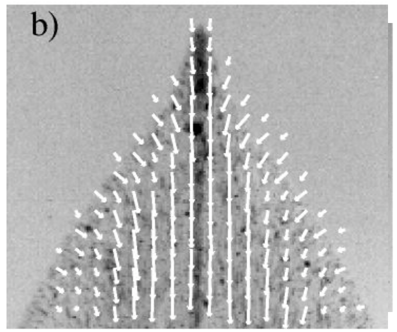


Φ=60°



Φ=35°

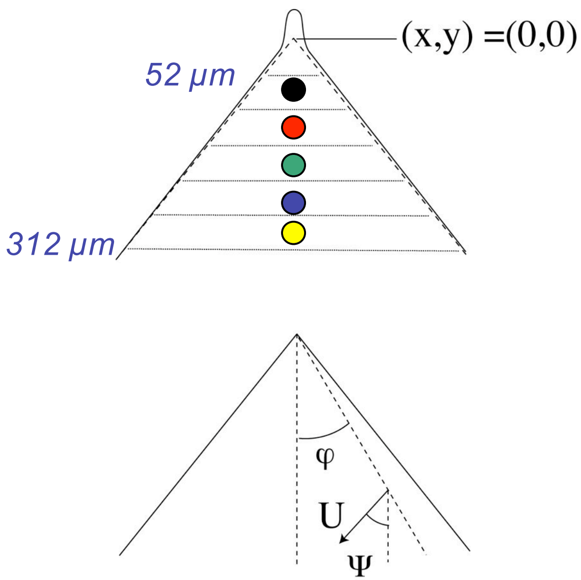




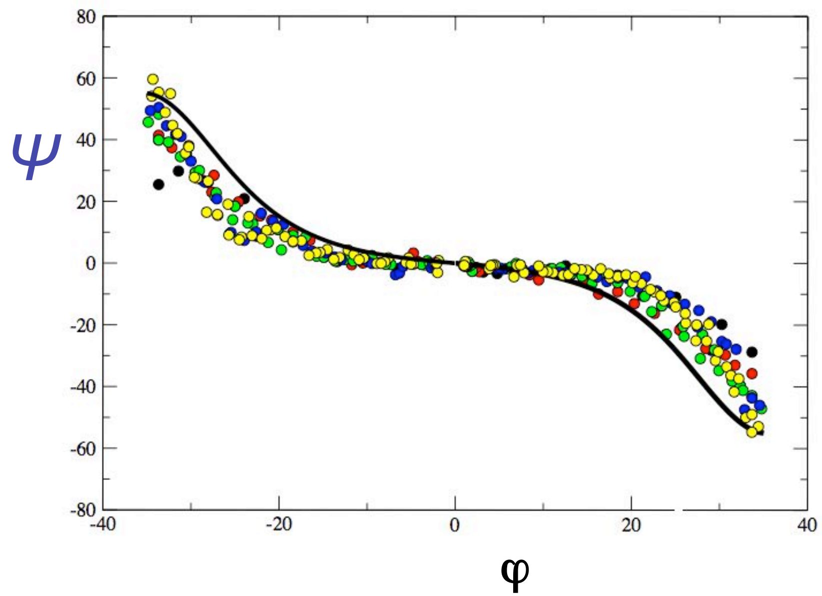
Rio, Andreotti, Daerr

velocity field

Theory: velocity self-similar ⟶ $\vec{U} = \vec{U}(y/x)$



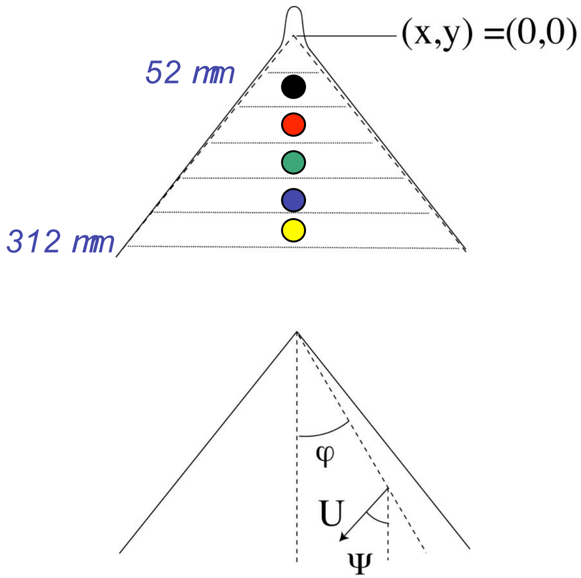
flow direction

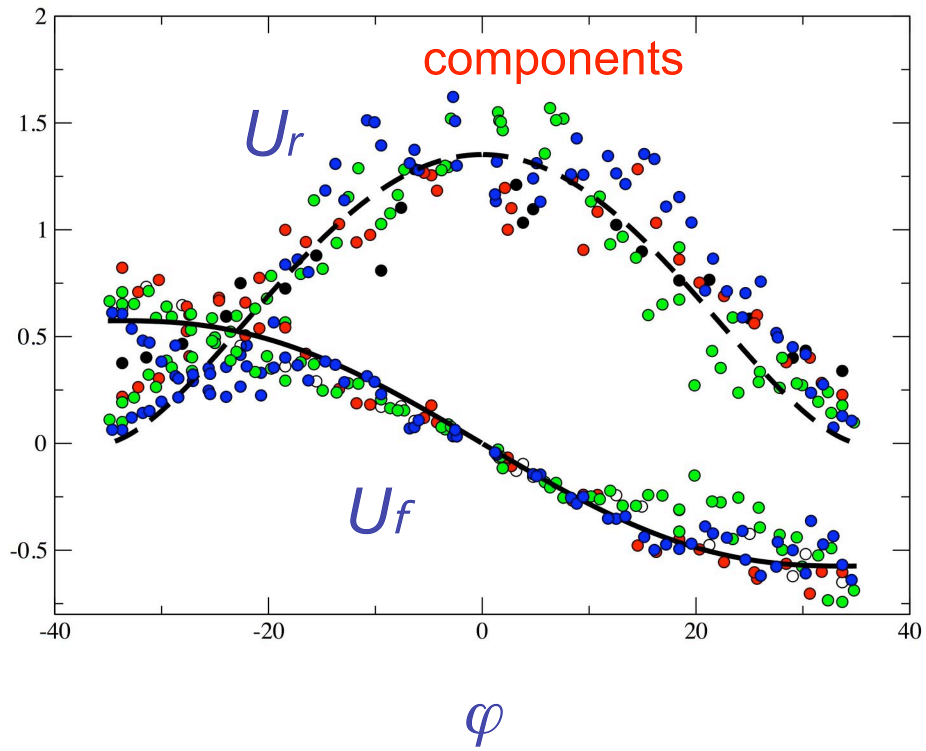


Snoeijer *et al.* Phys Fluids '05

velocity field

Theory: velocity self-similar → $\vec{U} = \vec{U}(y/x)$





Snoeijer *et al.* Phys. Fluids ’05

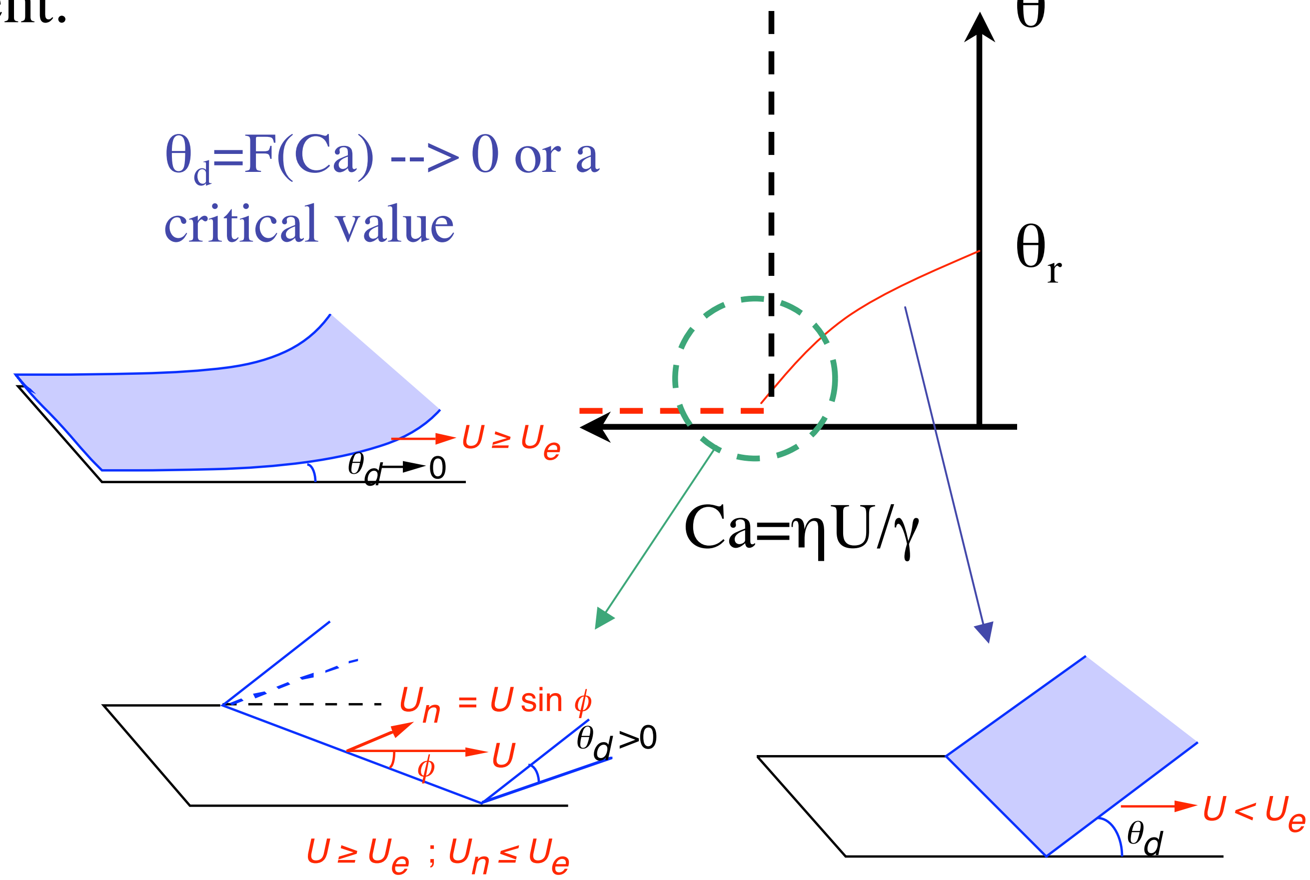
But….

- Selection of ϕ ?
- Prediction of ‘pearling’ instability ?

How to manage with the singularity at contact line?

Classical argument:

θ_d=F(Ca) --> 0 or a critical value

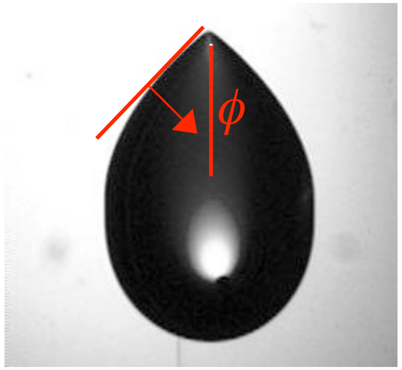


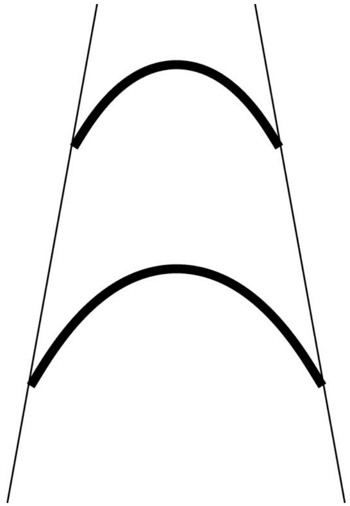
θ_d=F(Ca sinφ) ≈ 0 =>

$$\sin\phi = \frac{Ca_c}{Ca}$$

Blake, Ruschak(Nature 1979)
Podgorski et al (PRL 2000)

Low ϕ paradox:





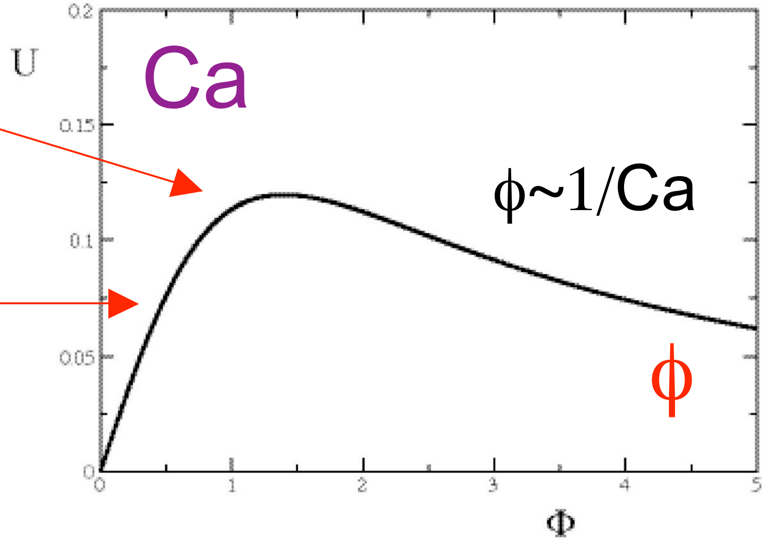
pressure gradients become weaker...

velocity → 0

$\phi \to 0$ implies drop speed → 0

maximum speed !!

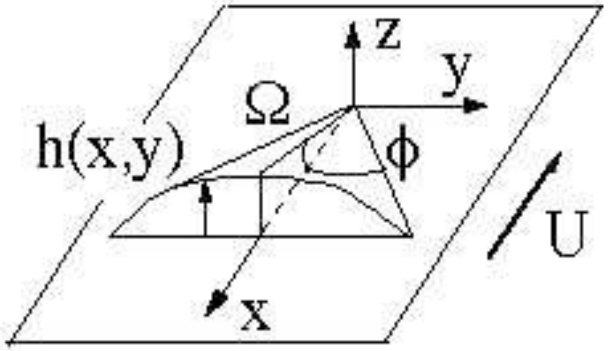
Breakdown of 1/Ca law



ϕ-selection : mixing contact line and bulk flow

$\tan^3\Omega \approx (35/16) Ca \tan^2\phi$

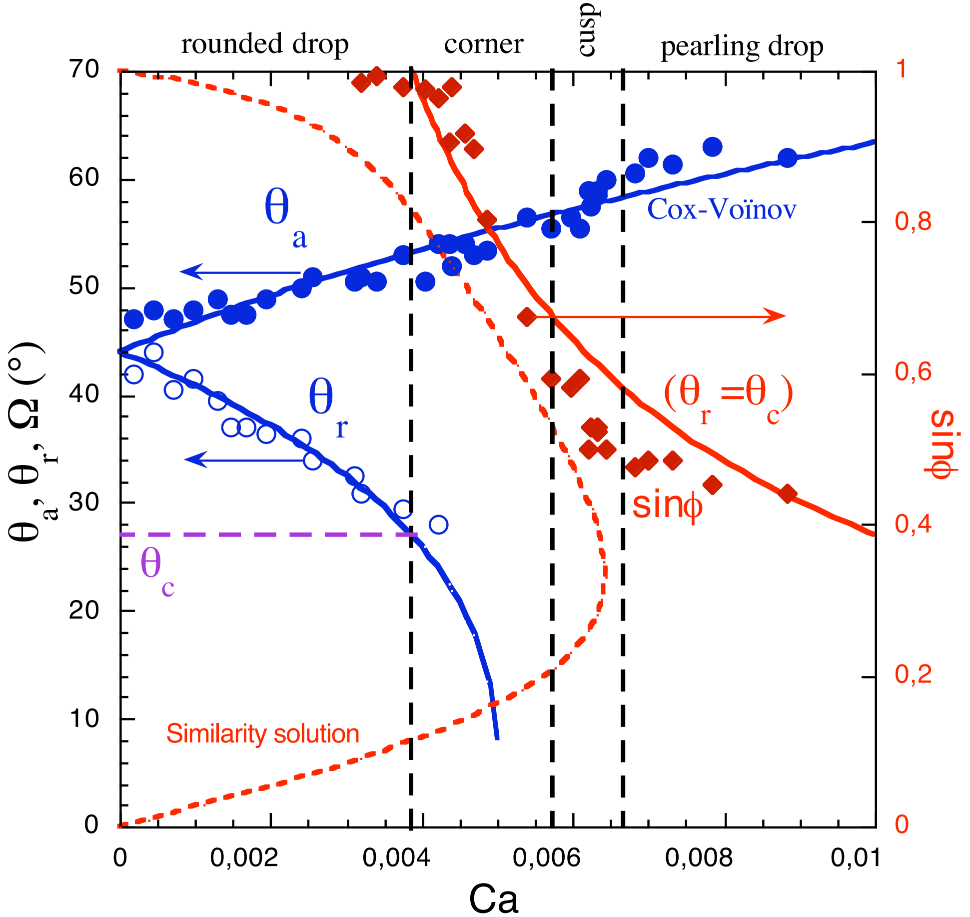
and:



$\theta \approx 2\,\Omega/\sin\phi$

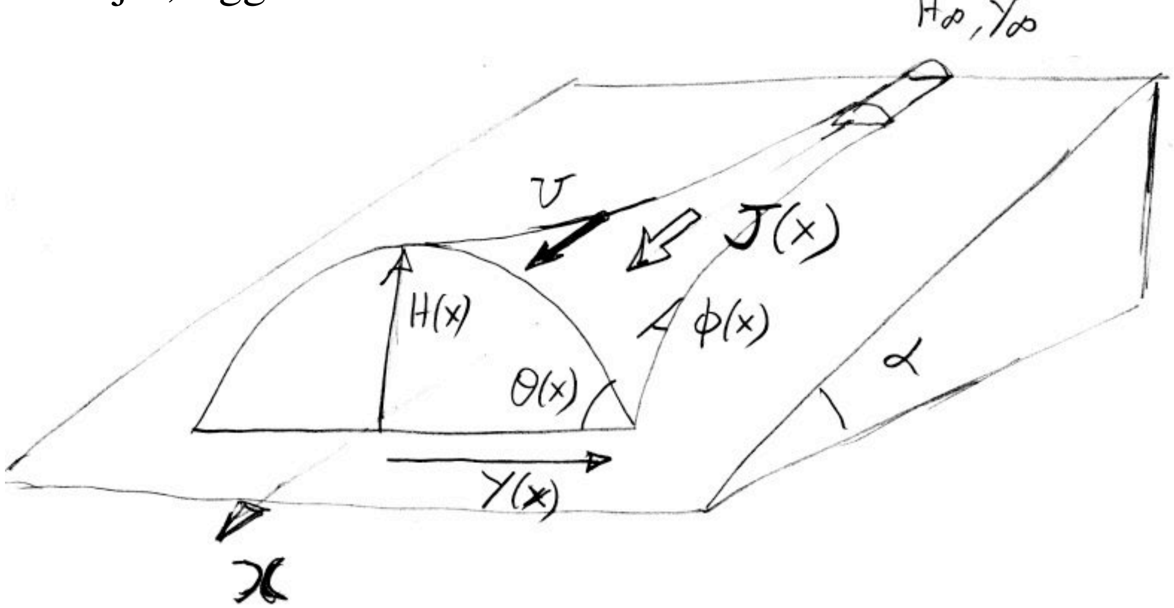
and:

$$\theta^3 = \theta_e^{\ 3} - 9(Ca\sin\phi)Log\left(b/a\right)$$



$$\frac{Ca}{\theta_e^{\ 3}} = \frac{1}{\sin\phi}\frac{1}{\left[9Log(b/a) + \frac{70}{(\sin 2\phi)^2}\right]}$$

Snoeijer, Eggers



Ca=ηU/γ

$$\frac{d}{dx} = \varepsilon \frac{d}{dX}$$

small

- Lubrication + parabolic approx.

$$Ca\left[HY - H_\infty Y_\infty\right] \approx -\frac{16}{35} H^3 Y \varepsilon \left(\frac{H}{Y^2}\right)_X$$

Flowing down

Left behind

Viscous capillary flux

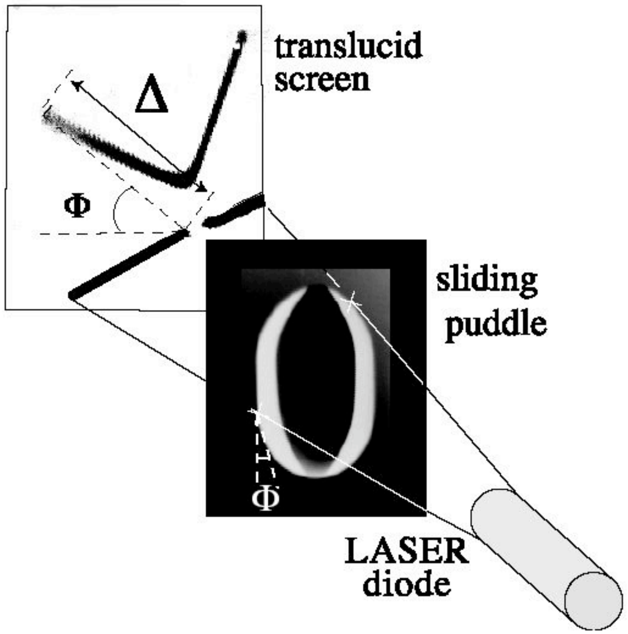
- Directional Cox-Voïnov law:

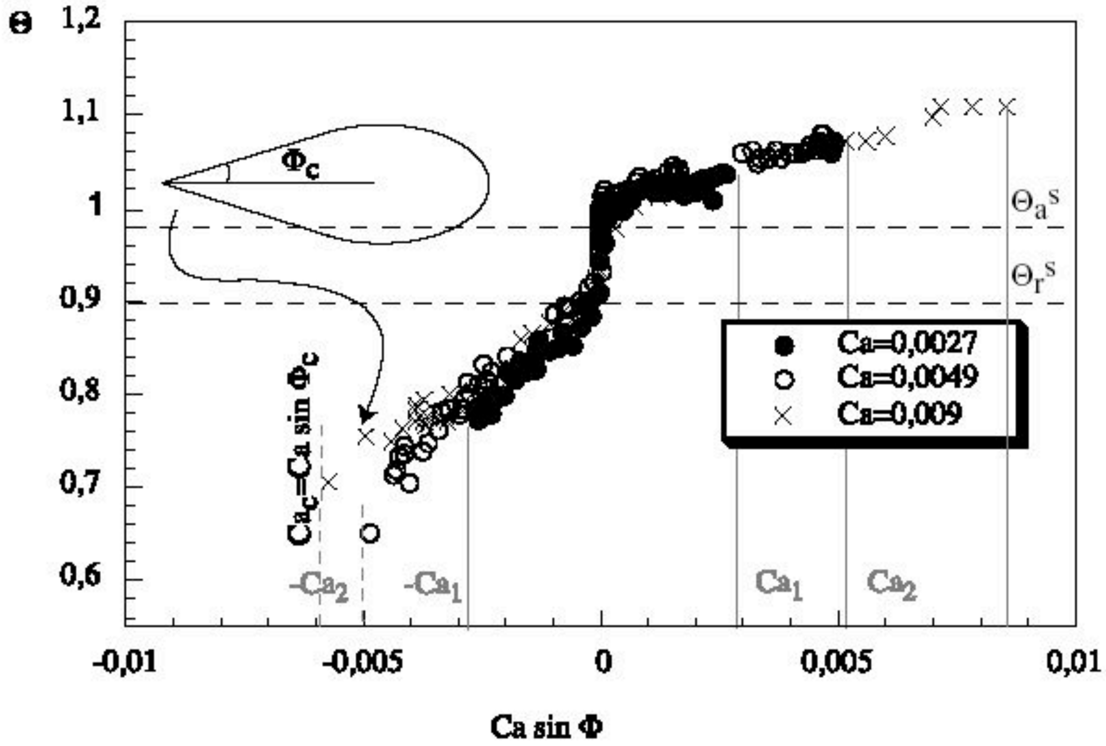
$$\theta^3 \approx \left(2\frac{H}{Y}\right)^3 \approx \theta_e^3 - 9Ca(\varepsilon Y_X) Log(b/a)$$

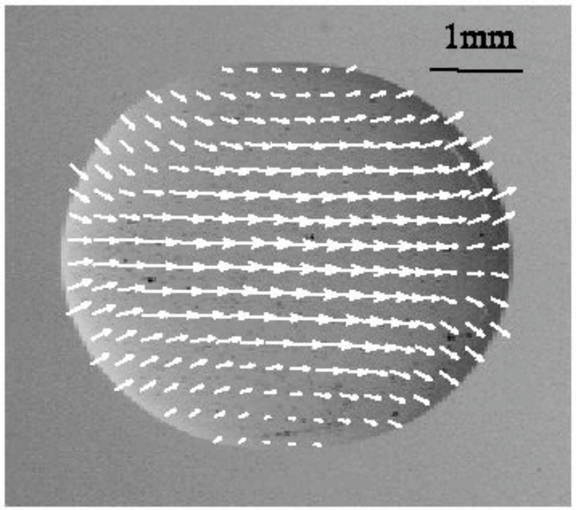
Ca sinϕ

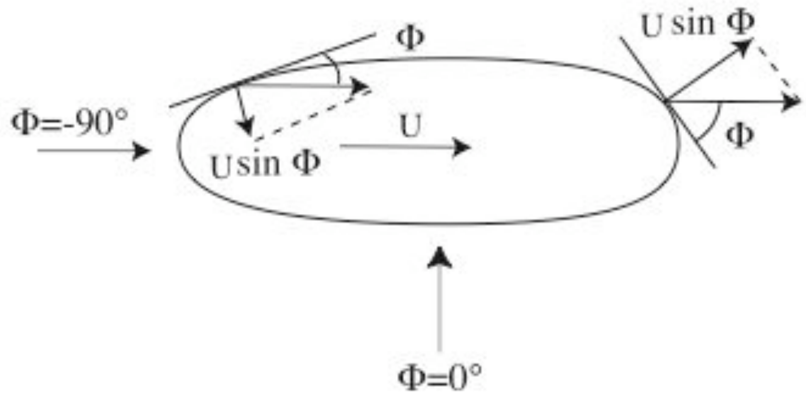
Consistency: Ca ε Log(b/a) ~Ca/ε~1

About directional Cox-Voïnov law







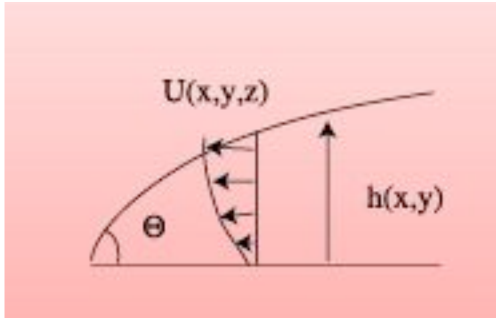


E. Rio, A. Daerr, B. Andreotti, LL, Phys Rev Lett 2004

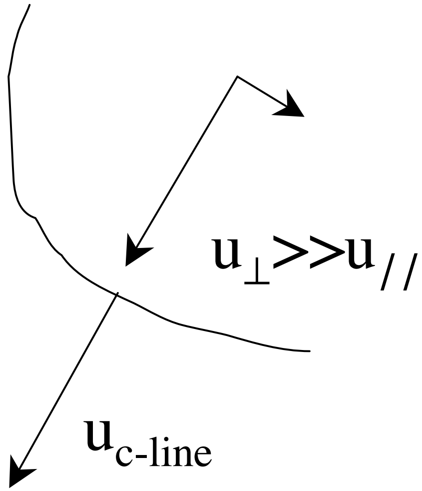
Lubrification: $$< \vec{u} >= -\frac{h^2}{3\eta}\nabla P$$

$$P \approx -\gamma \Delta h$$

Singular at contact line



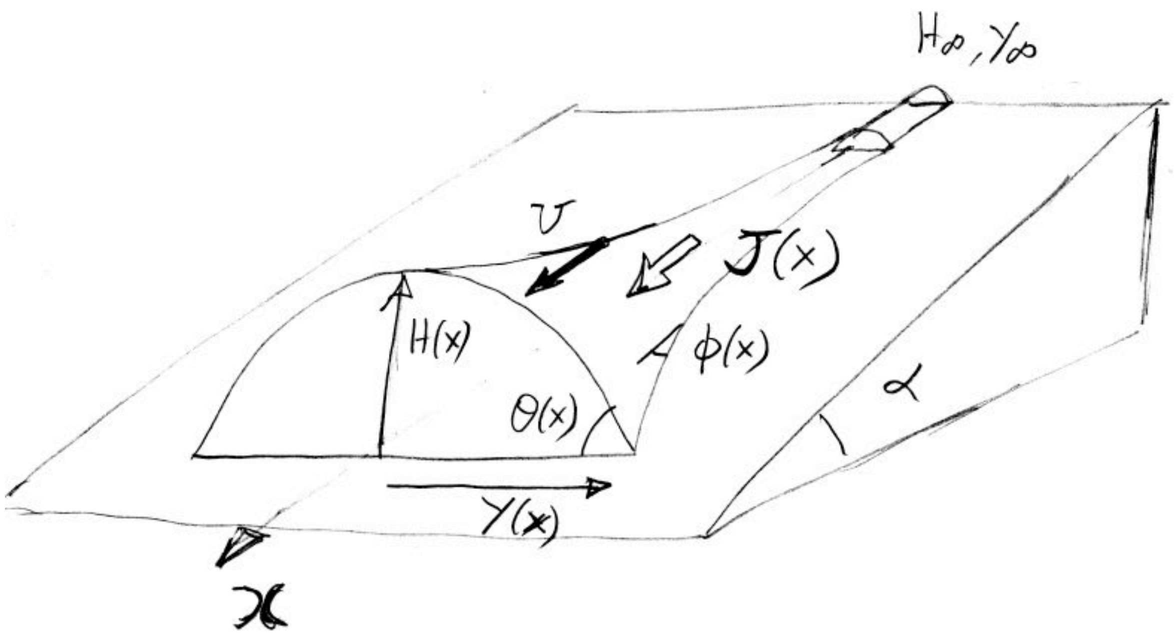
$$\partial / \partial x_\perp >> \partial / \partial x_{//}$$



Mass conservation:

$$<u_\perp> = u_{c\text{-line}}$$

-> local 2D situation



Ca=ηU/γ

$$\frac{d}{dx} = \varepsilon \frac{d}{dX}$$

small

- Lubrication + parabolic approx.

$$Ca\left[HY - H_\infty Y_\infty\right] \approx -\frac{16}{35} H^3 Y \varepsilon \left(\frac{H}{Y^2}\right)_X$$

Flowing down

Left behind

Viscous capillary flux

- Directional Cox-Voïnov law:

$$\theta^3 \approx (2\frac{H}{Y})^3 \approx \theta_e^3 - 9Ca(\varepsilon Y_X)Log(b/a)$$

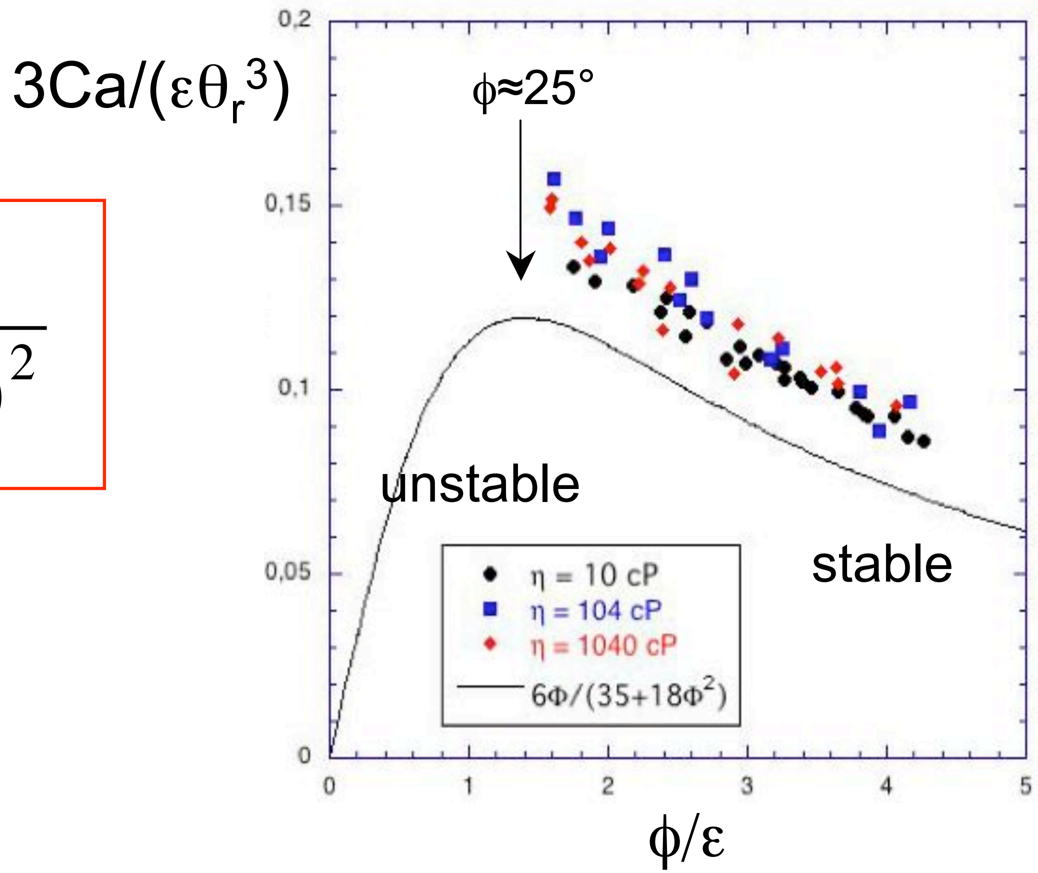
Ca sinϕ

Consistency: Ca ε Log(b/a) ~Ca/ε~1

For $\varepsilon = 1/[\mathrm{Ln}(b/a)]^{1/2}$ small

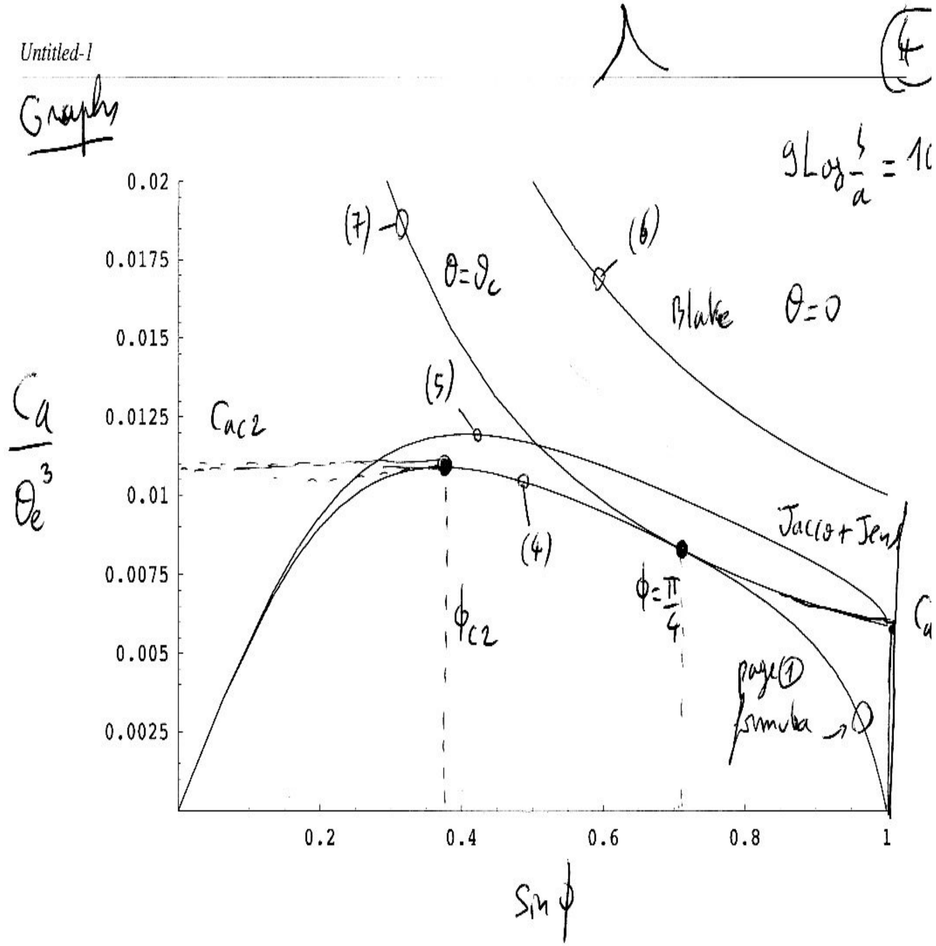
there exist corner solutions (H~Y~X) with:

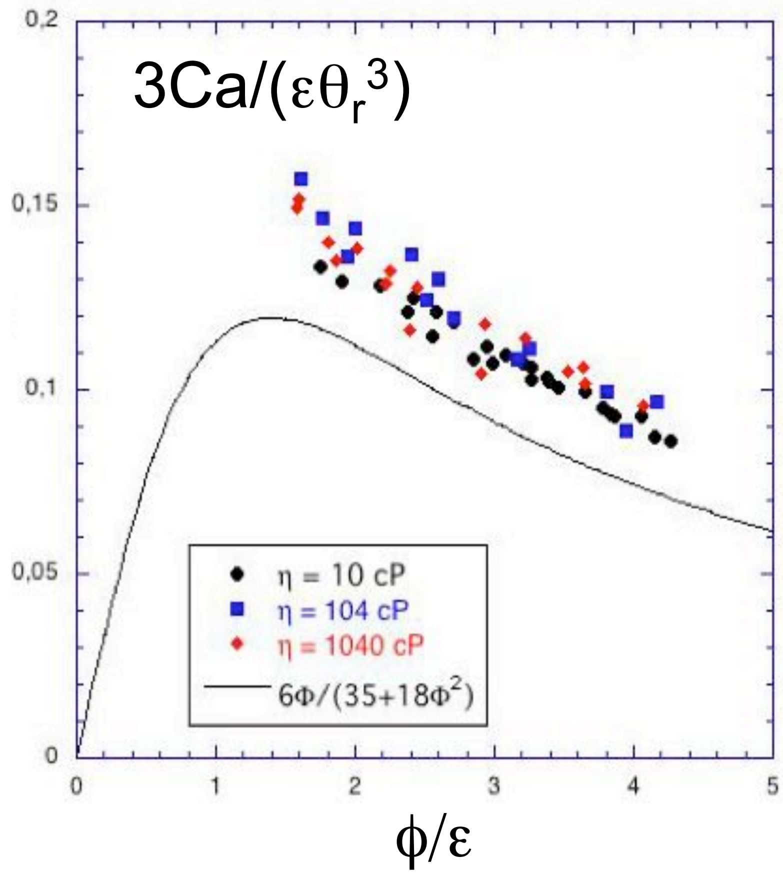
$$\frac{3Ca}{\varepsilon\theta_e^3} \approx \frac{6(\phi/\varepsilon)}{35+18(\phi/\varepsilon)^2}$$



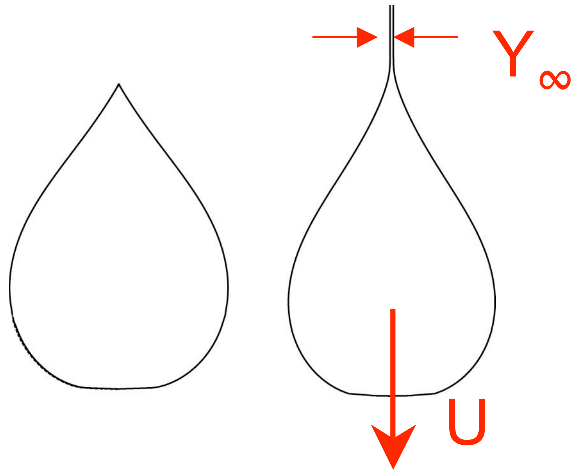
Oval drops -> Log(b/a)~10

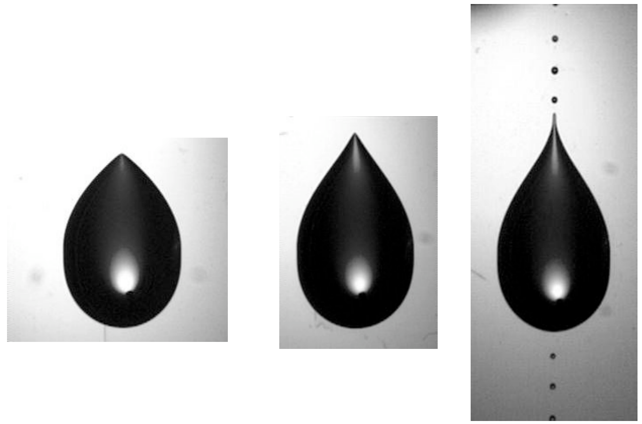
$\varepsilon \sim 0.3$


Untitled-1
Graphs
0.02
0.0175
0.015
0.0125
0.01
0.0075
0.005
0.0025
0.2
0.4
0.6
0.8
1
(7)
(6)
(5)
(4)
Blake θ=0
Jacco + Jens
Sin φ

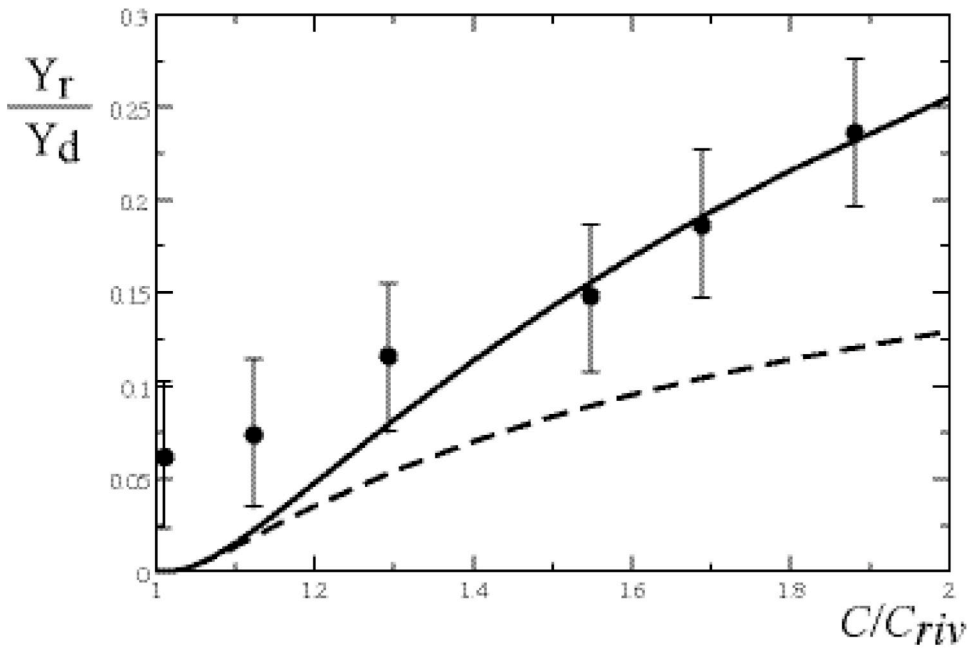

3Ca/(εθ_r^3)
0,2
0,15
0,1
0,05
0
0
1
2
3
4
5
η = 10 cP
η = 104 cP
η = 1040 cP
6Φ/(35+18Φ^2)
φ/ε

- beyond maximum speed: rivulet solutions



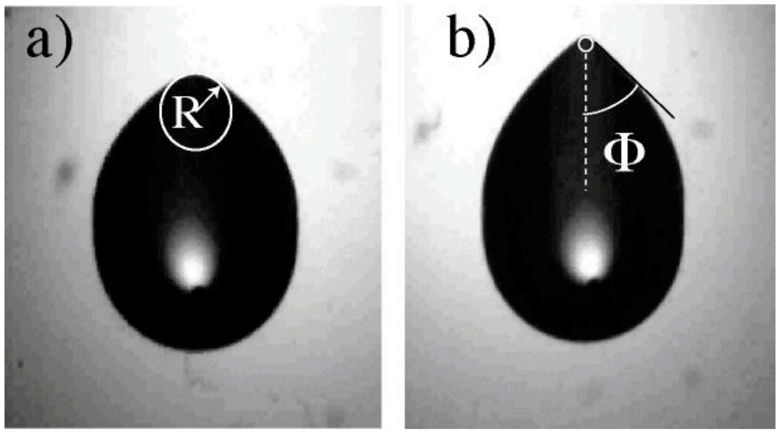


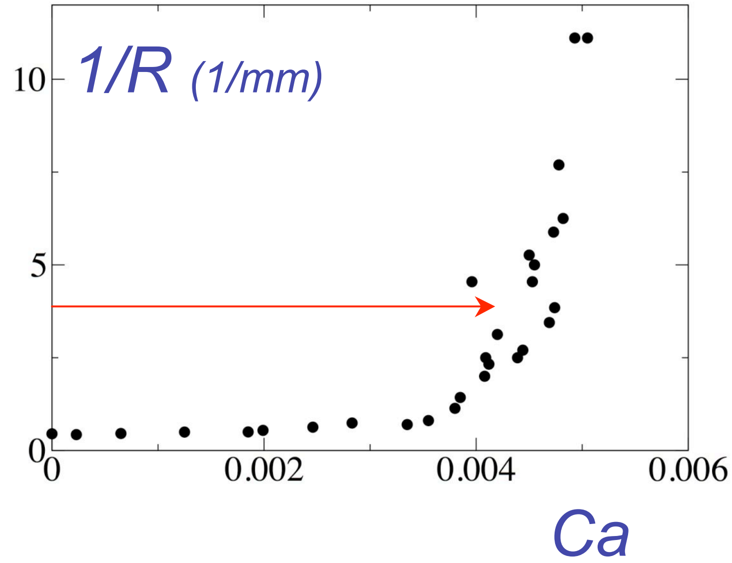
- rivulet width Y_∞ vs U

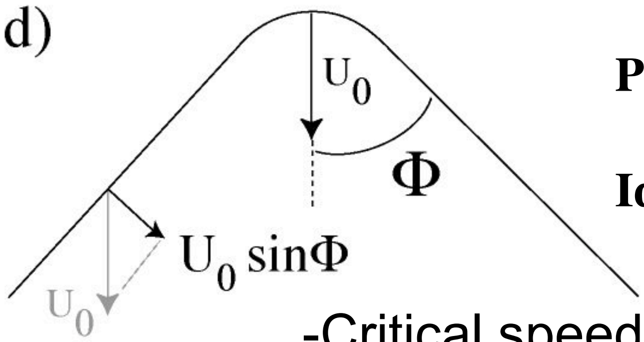


Curvature at the tip...

Le Grand *et al.* JFM '05







Problem: $U_0 \sin\phi = U_c$ => $U_0 > U_c$!!

Idea: $U_c = f(R)$ => $U_0 \sin\phi = U_c(R=\infty)$
et $U_0 = U_c(R)$

-Critical speed *Ca(1/R)* ?=> calc. for « large » R

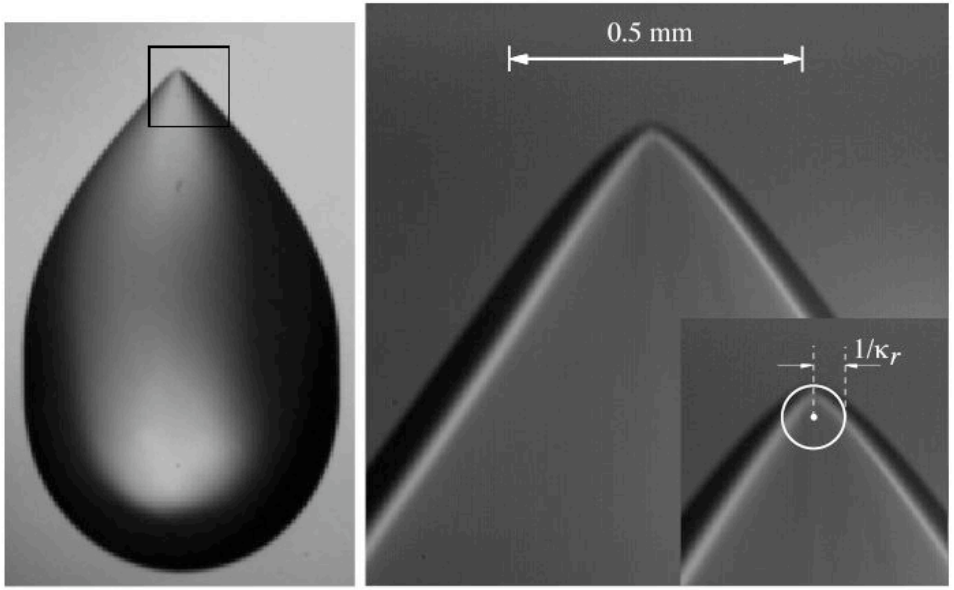
-> Snoeijer et al. Phys. Fl. 2005

-Other question: R(Ca)? Scaling laws for tip equilibrium

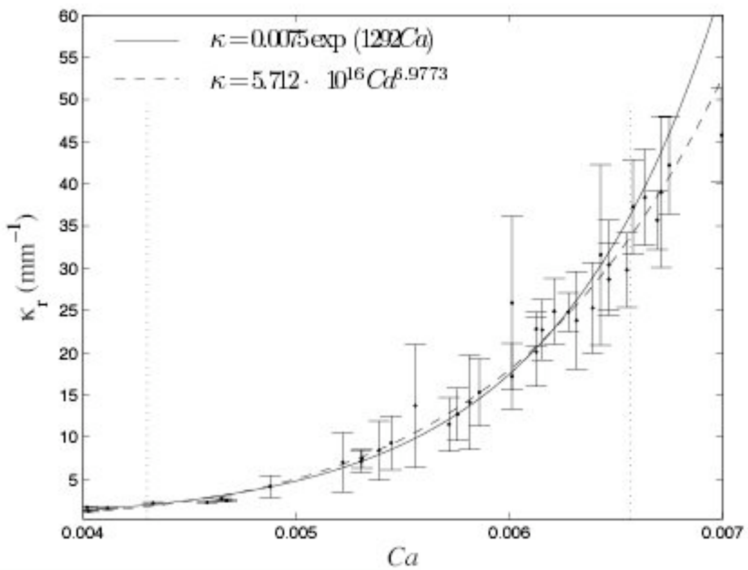
An open issue: divergence of 1/R

Ivo Peters

Adrian Daerr

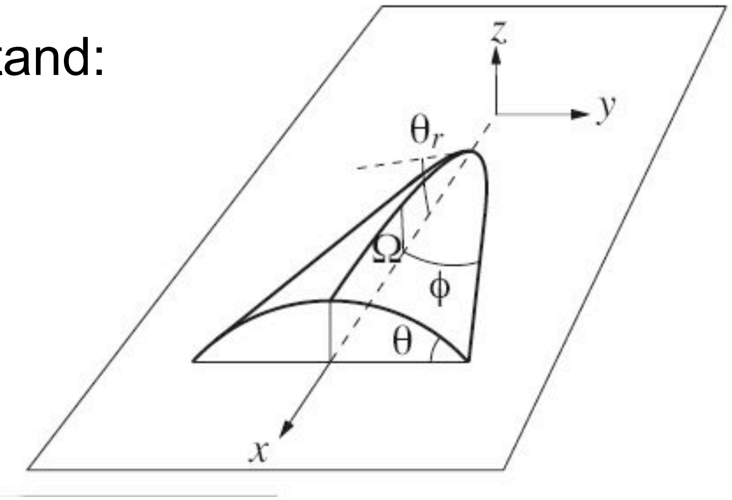


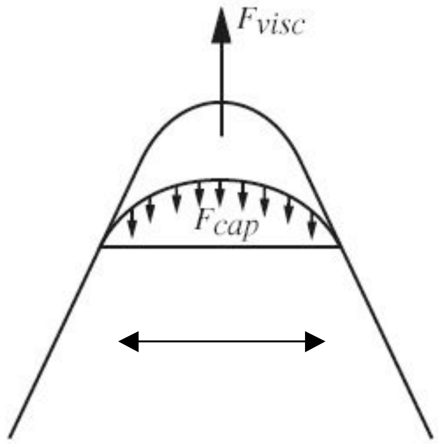
$1/R \sim A\exp(B\ Ca)$



Similar to Lorenceau

Difficult to understand:





$F_{cap} \sim L\theta^2\gamma \sim F_{visc} \sim L\ \eta\ (U/\theta)\ \mathrm{Log}(R/a)$

-> $1/R \sim (1/a)\ \exp(\theta^3/Ca)$

To get $1/R \sim (1/a)\ \exp(Ca/\theta^3)$

One would need $F_{visc} \sim L\ \eta\ (U/\theta)\ /\ \mathrm{Log}(R/a)$ ….

Discussion

- drops have singular shapes to avoid wetting
  - inclined: reduces normal speed
  - curved: additional capillary forces

- 3D structure governed by similarity solutions.

- Matching with contact line (where the flow is perpendicular) selects opening angle.

- Matching becomes impossible above a critical Capillary number -> pearling

- Behavior of 1/R at the tip remains to be Understood…

Rio *et al.* PRL '05