Flag Flutter: Potential Flow Around a Rectangular Plate

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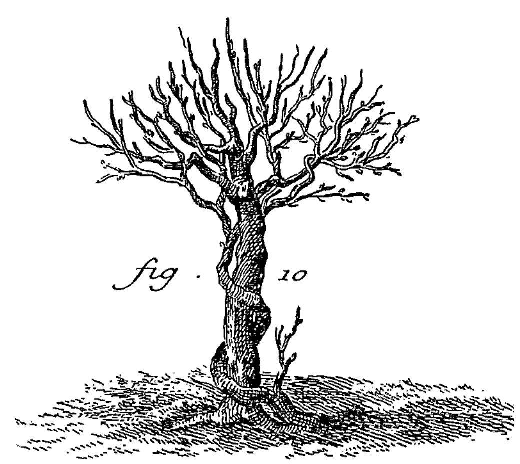
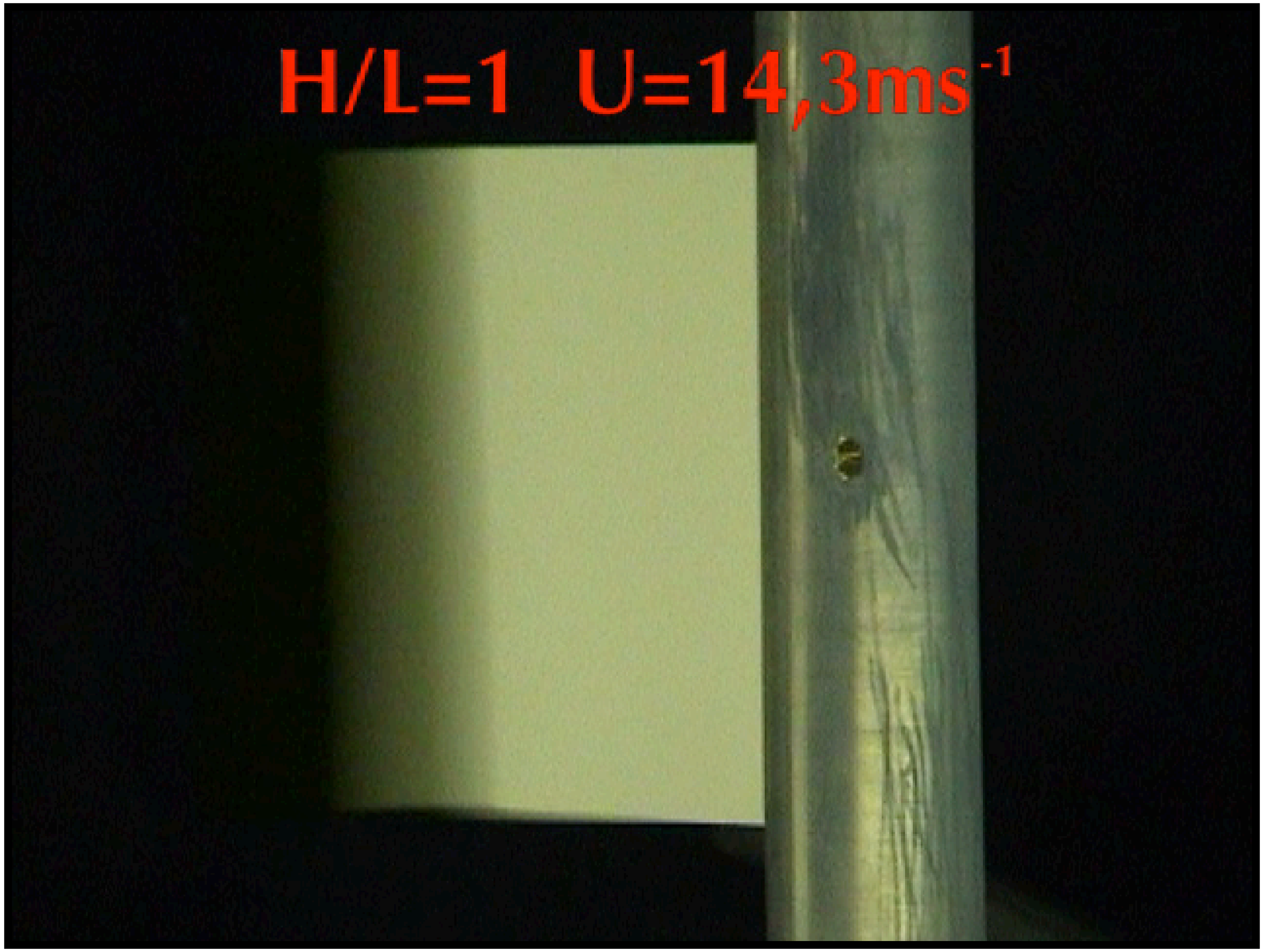
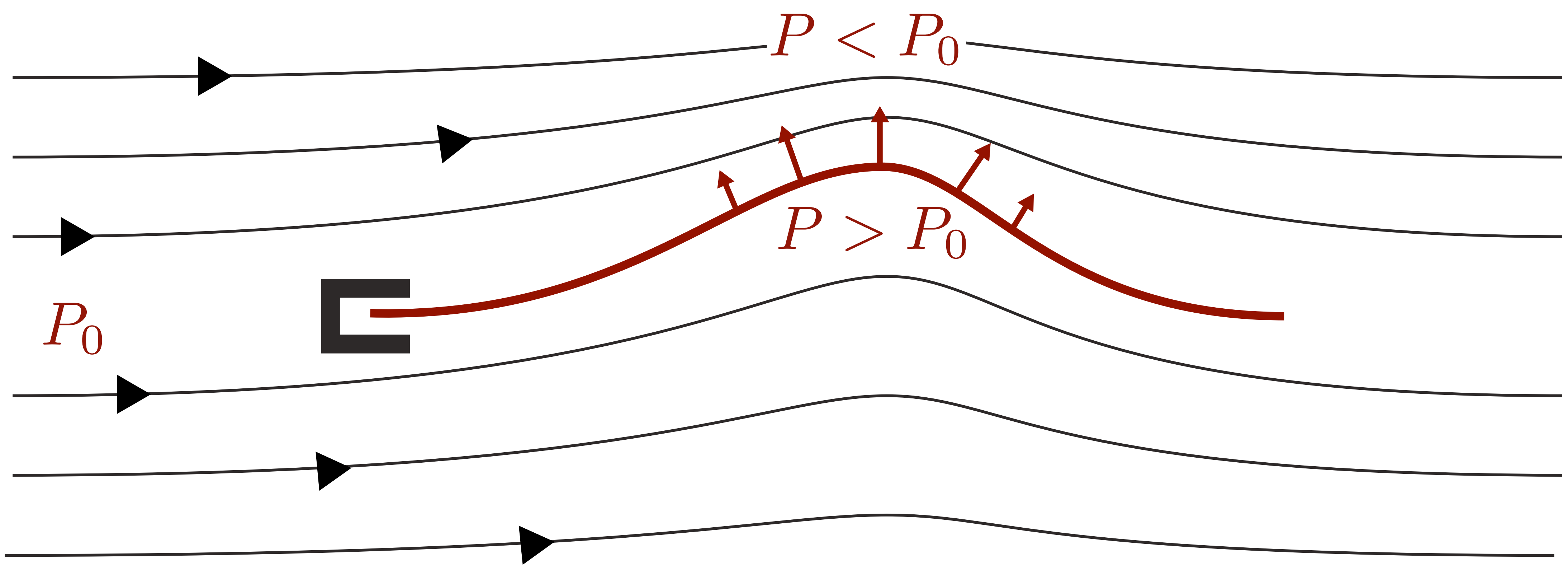


ANR Contract "DRAPEAU"

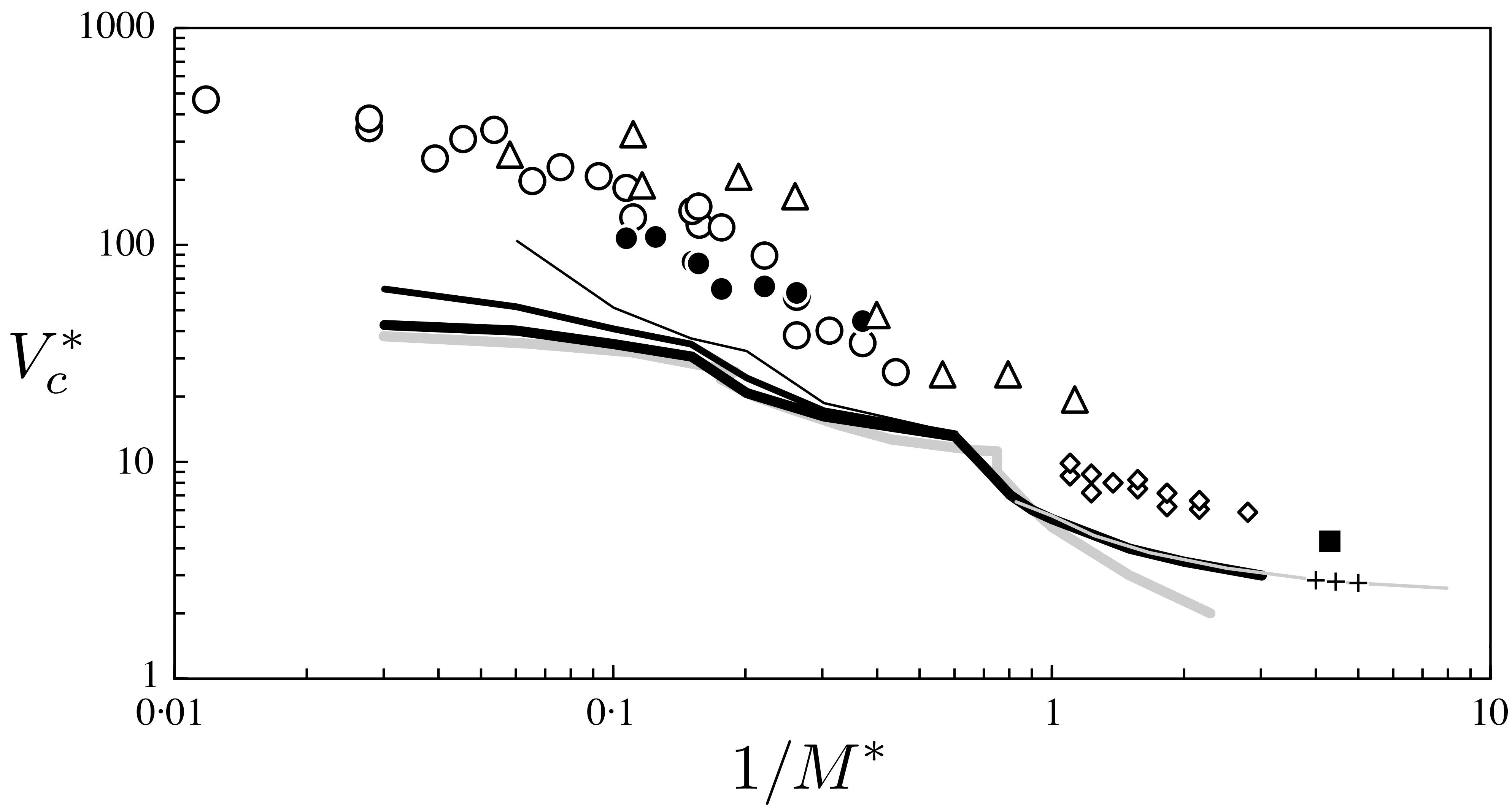

H/L=1 U=14,3ms-1

Instability Mechanism



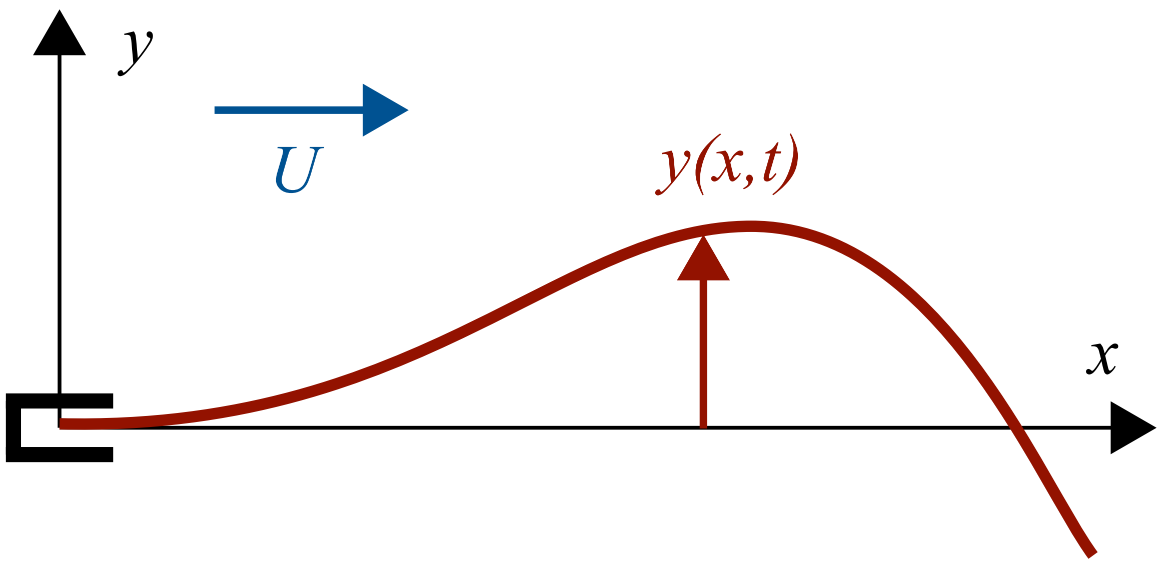
- **Pressure: destabilising**
- **Elasticity: stabilising**

Literature



- **[Ref]** Watanabe, Isogai, Suzuki & Sugihara, *J. Fluids Struct.* (2002)

Equation of Motion



EI : flexural rigidity
ρ : mass per unit area

- **Linearised Euler-Bernoulli beam equation**

$$\rho \partial_t^2 y + EI \partial_x^4 y = \Delta P(x)$$

Galerkin Method

- **Galerkin–Fourier expansion**

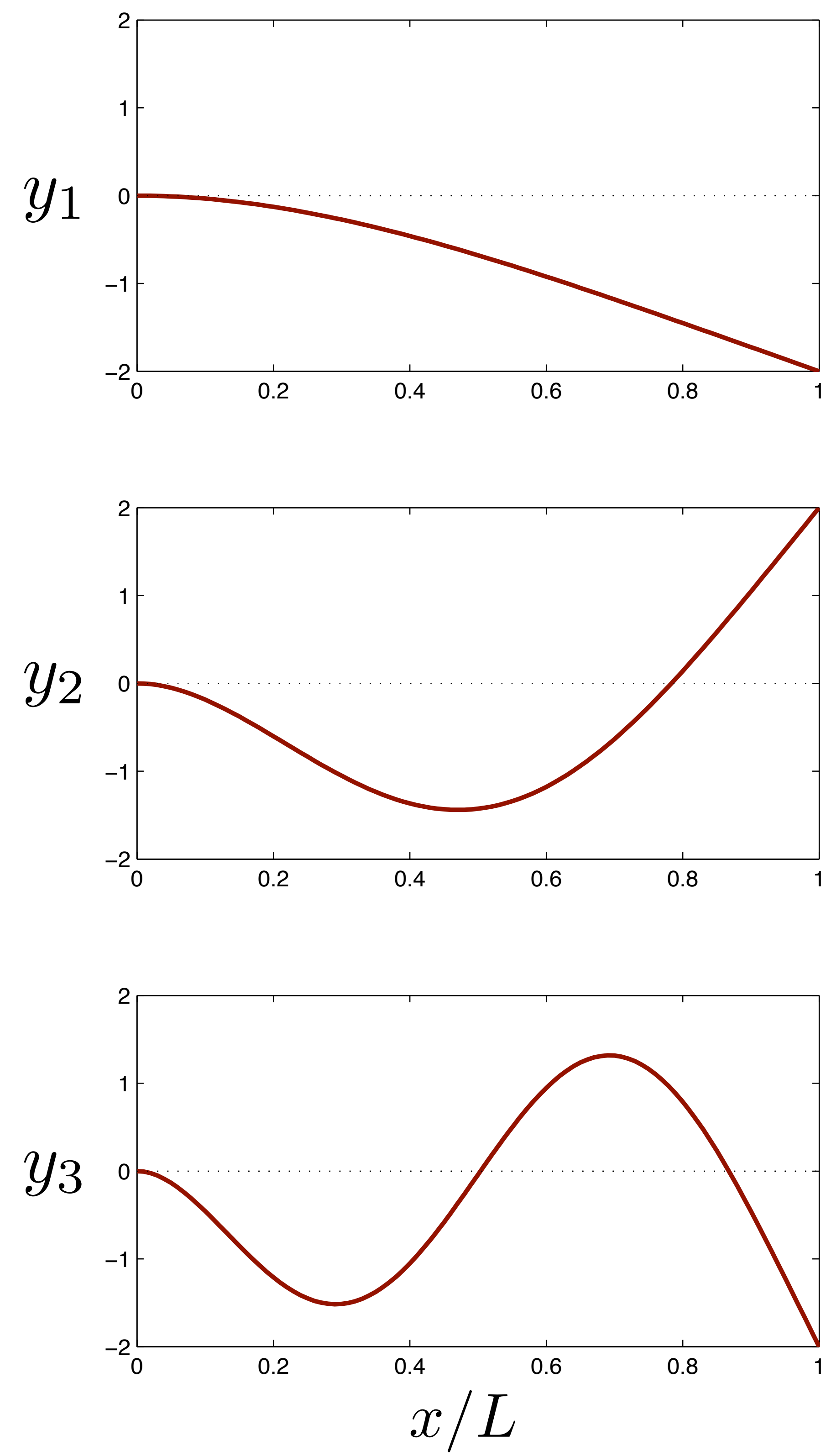
$$y(x,t) = \sum_n A_n y_n(x) \mathrm{e}^{\mathrm{i}\omega t}$$

- **PDE → eigenvalue problem**

$$\rho \partial_t^2 y + EI \partial_x^4 y = \Delta P(x)$$

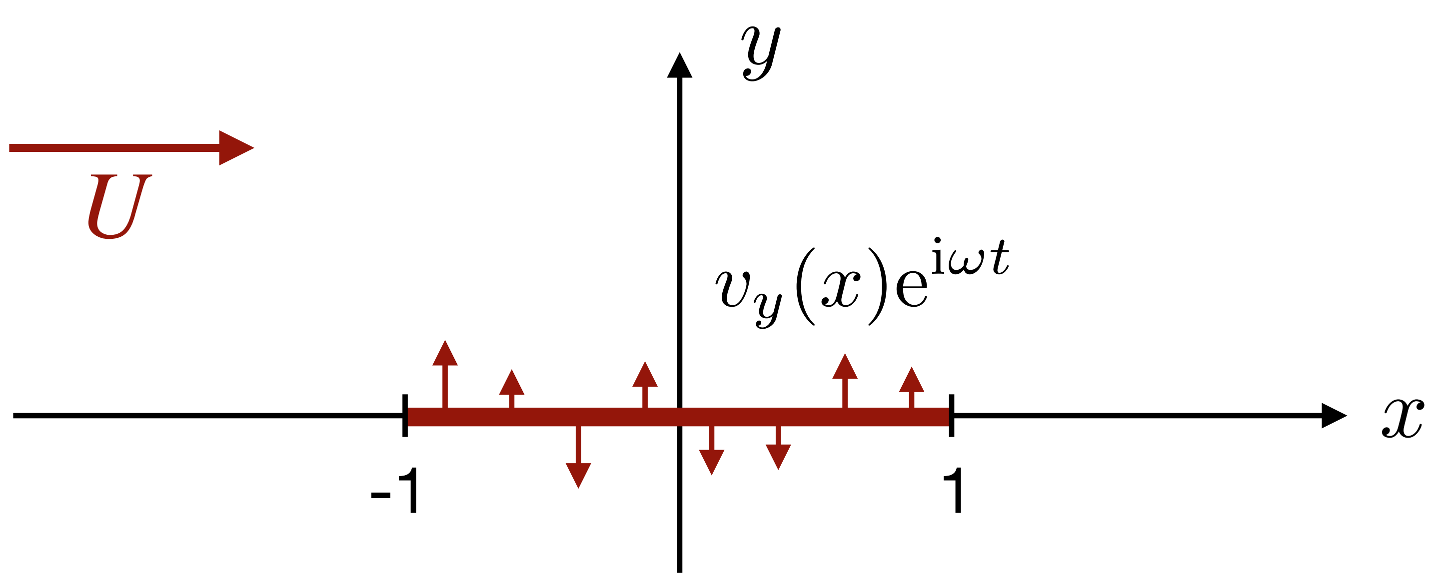
$$\downarrow$$

$$\left(-\rho\omega^2 \underline{\underline{I}} + EI \underline{\underline{K}} - \underline{\underline{P}}(\omega)\right) \underline{A} = 0$$



Flow Around the Plate

Potential Flow



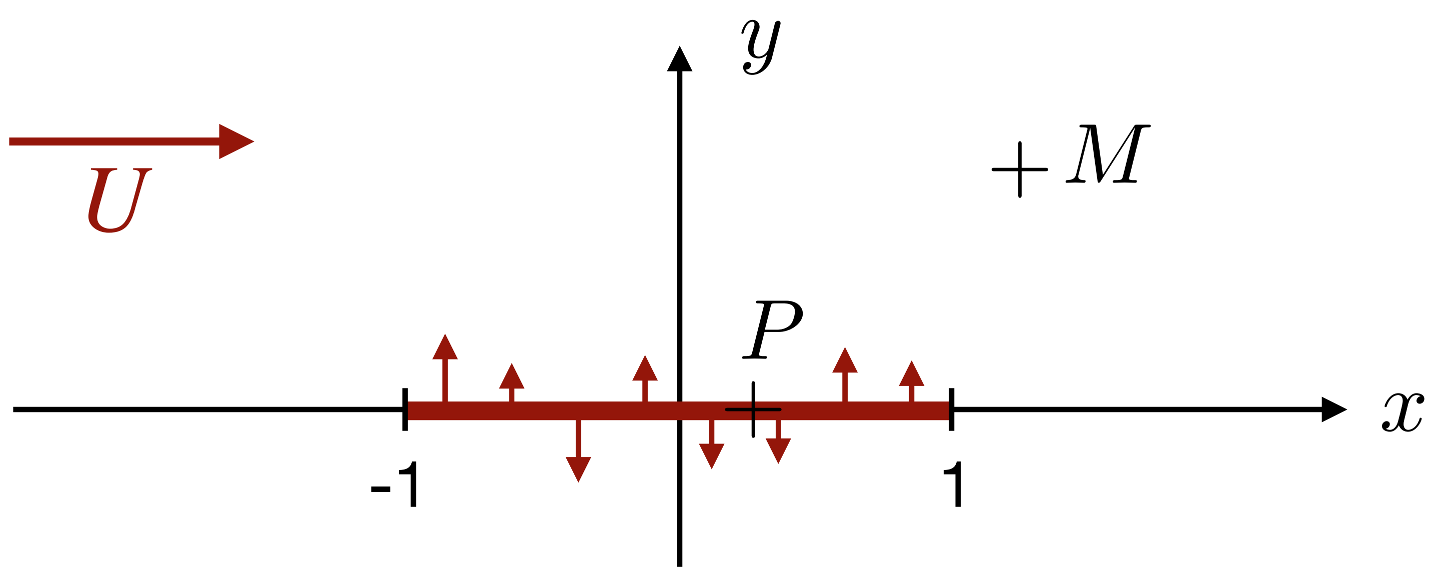
- **Perturbation potential:** $\phi(x, y)\mathrm{e}^{\mathrm{i}\omega t}$

$$\begin{cases} \Delta\phi = 0 \\ \partial_y \phi\big|_{y=0} = v_y = \left(\partial_t + U\partial_x\right) y_n \quad \text{for} \quad x \in [-1 \quad 1] \end{cases}$$

- **Perturbation pressure (Bernoulli equation)**

$$P = \left(\partial_t + U\partial_x\right)\phi\,\mathrm{e}^{\mathrm{i}\omega t}$$

Inverse Problem



$$\begin{cases} \Delta\phi = 0 \\ \partial_y \phi\big|_{y=0} = v_y \end{cases}$$

- **Green's representation theorem**

$$\phi(M) = -\int_{P\in S} \delta\phi\, \partial_{y_p} G(|MP|)\mathrm{d}S_p$$

$$G(r) = \frac{1}{2\pi}\ln r \qquad \text{(in 2D)}$$

- **Inverse problem (Fredholm equation of 1st kind)**

$$v_y(x) = \text{FP}\!\!\int_{-1^-}^{1^+} \delta\phi(\xi)\frac{\mathrm{d}\xi}{2\pi(x-\xi)^2}$$

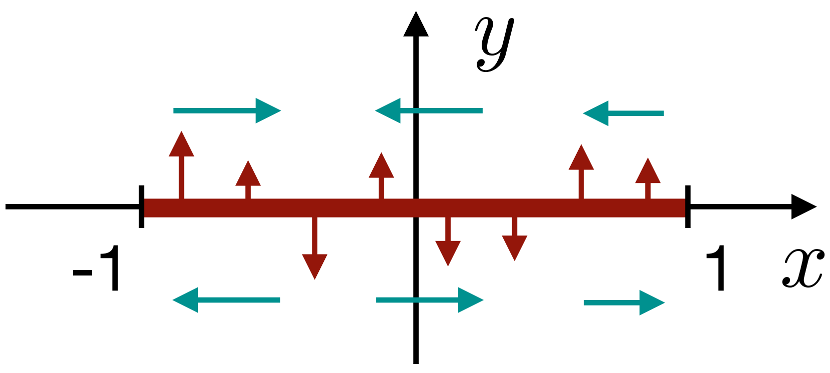
Vorticity Distribution

- **Symmetry of perturbation potential**

$$\phi(-y) = -\phi(y)$$

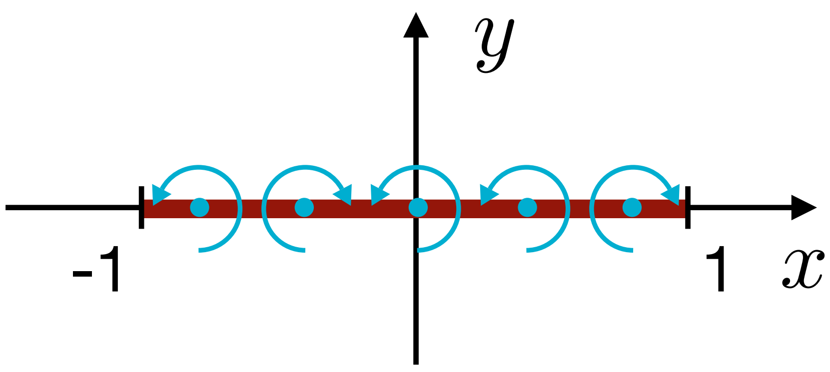
$$v_x = \partial_x \phi$$

$$v_x(-y) = -v_x(y)$$



- **Vorticity distribution**

$$\gamma(x) = -\partial_x \delta\phi$$



- **Inverse problem for vorticity**

$$v_y(x) = \mathrm{FP}\!\!\int_{-1^-}^{1^+} \delta\phi(\xi) \frac{\mathrm{d}\xi}{2\pi(x-\xi)^2}$$

$$v_y(x) = \mathrm{PV}\!\!\int_{-1}^{1} \gamma(\xi) \frac{\mathrm{d}\xi}{2\pi(x-\xi)}$$

Oscillating Plate

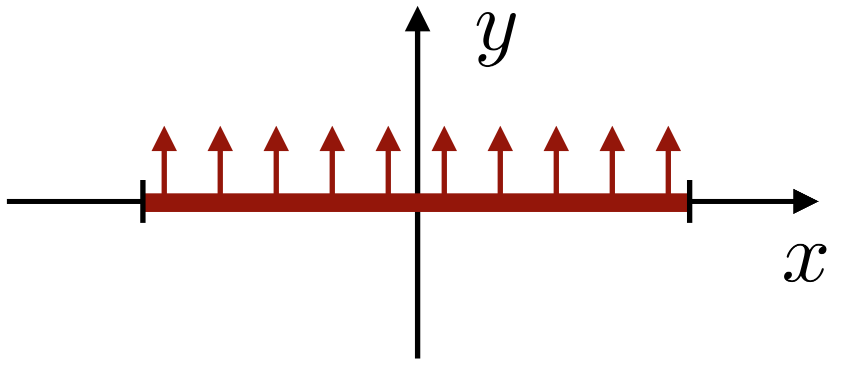
- **Inversion formula (Söhngen, 1939)**

$$v_y(x) = \oint_{-1}^{1} \gamma(\xi)\frac{\mathrm{d}\xi}{2\pi(x-\xi)}$$

$$\gamma(x) = -\frac{2}{\pi}\sqrt{\frac{1-x}{1+x}}\oint_{-1}^{1}\sqrt{\frac{1+\xi}{1-\xi}}\frac{v_y(\xi)}{x-\xi}\mathrm{d}\xi + \frac{\alpha}{\sqrt{1-x^2}}$$

- **Particular case:** $v_y = 1$

$$\gamma(x) = -\frac{2x}{\sqrt{1-x^2}} + \frac{\Gamma_o}{\pi\sqrt{1-x^2}}$$

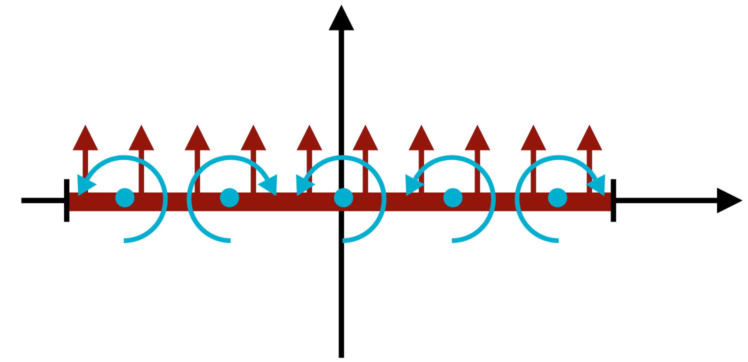


$$\Delta P(x) = -\gamma(x) - \mathrm{i}\,\omega\int_{-1}^{x}\gamma(\xi)\mathrm{d}\xi$$

- Γ_o **is the circulation around the plate:** $\Gamma_o = \int_{-1}^{1}\gamma(\xi)\mathrm{d}\xi$

- **Flow with no circulation**

$$\Delta P(x) = \frac{2x}{\sqrt{1-x^2}} - 2\,\mathrm{i}\,\omega\sqrt{1-x^2}$$

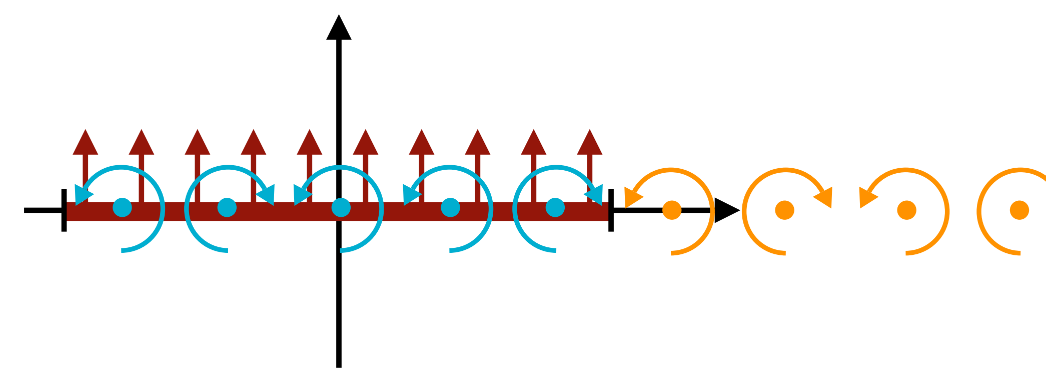


- **Flow with circulation** $\Gamma = \Gamma_o \mathrm{e}^{\mathrm{i}\omega t}$

$$\gamma|_{x=1^+} = -\frac{1}{U}\frac{\mathrm{d}\Gamma}{\mathrm{d}t} = -\frac{\mathrm{i}\,\omega}{U}\Gamma_o\,\mathrm{e}^{\mathrm{i}\omega t}$$ (Kelvin's circulation theorem)

- **Wake behind the plate advected at velocity** U

$$\gamma(x>1) = -\frac{\mathrm{i}\,\omega}{U}\Gamma_o\,\mathrm{e}^{\mathrm{i}\omega t}\mathrm{e}^{\mathrm{i}\omega(1-x)/U}$$



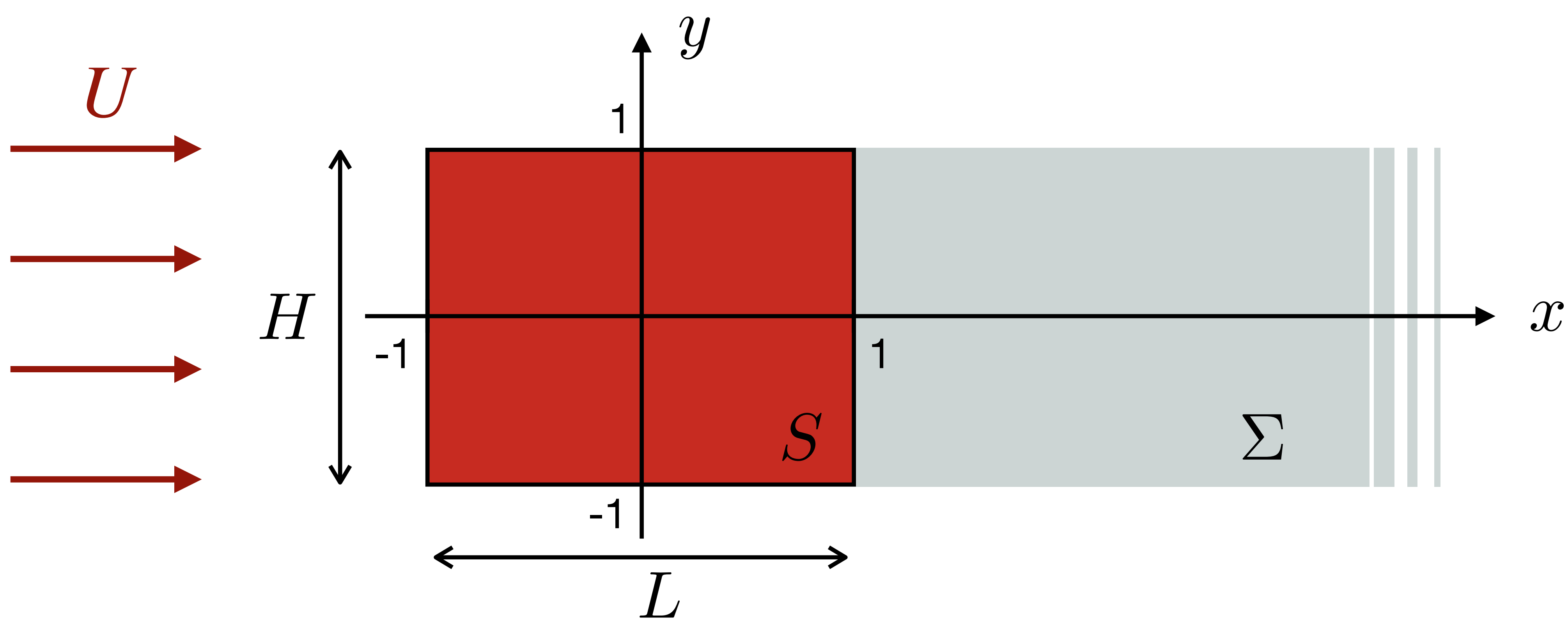
- **Using Kutta hypothesis**

$$\Delta P(x) = 2\,C(\omega)\sqrt{\frac{1-x}{1+x}} - 2\,\mathrm{i}\,\omega\sqrt{1-x^2}$$

$$C(\omega) = \frac{H_1^{(2)}(\omega)}{H_1^{(2)}(\omega) + \mathrm{i}H_0^{(2)}(\omega)}$$ (Theodorsen function)

3D Flow?

3D Problem



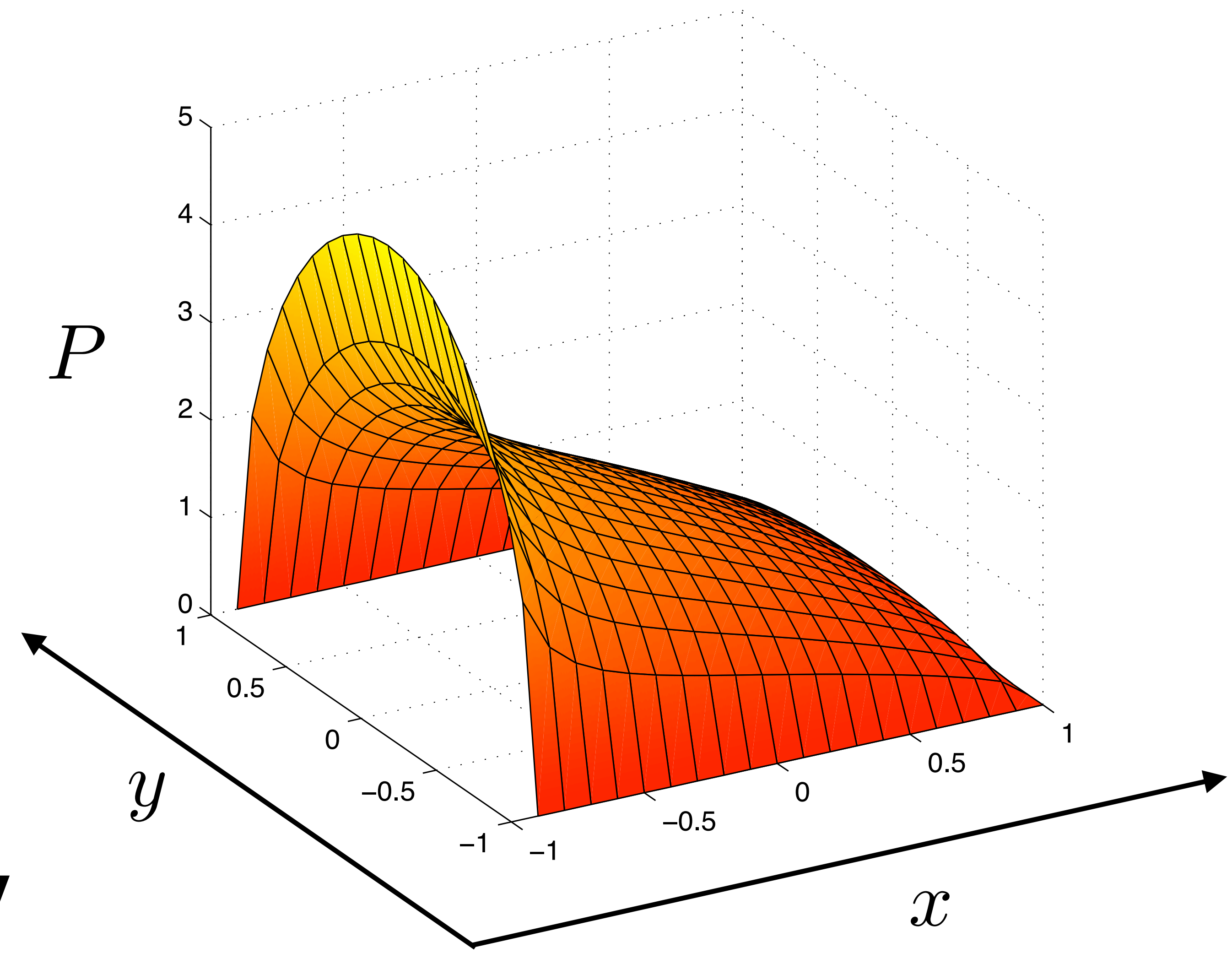
- **Inverse problem (Fredholm equation of 1st kind)**

$$v_z(x,y) = \frac{A}{4\pi} \iint_{S+\Sigma}^{\text{FP}} \frac{\gamma_y(\xi,\eta)\,\mathrm{d}\xi\mathrm{d}\eta}{(y-\eta)^2} \left(1 + \frac{A(x-\xi)}{\left[A^2(x-\xi)^2 + (y-\eta)^2\right]^{1/2}}\right)$$

$$A = L/H$$

- **Expansion in powers of A or A^{-1} (lifting-line theory)**

Singularities in the Pressure Field



- **Leading-edge singularity**

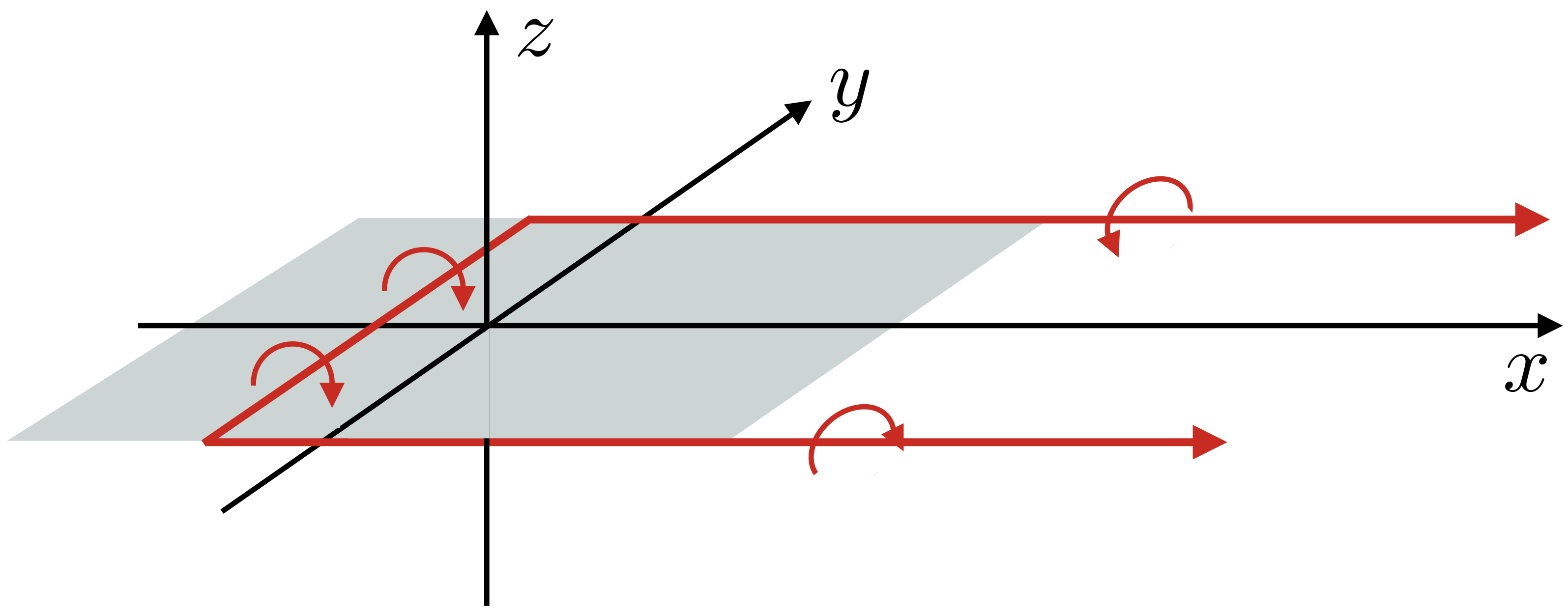
$$P \sim \frac{1}{\sqrt{n}}$$

- **Trailing-edge and side edges**

$$P \sim \sqrt{n}$$

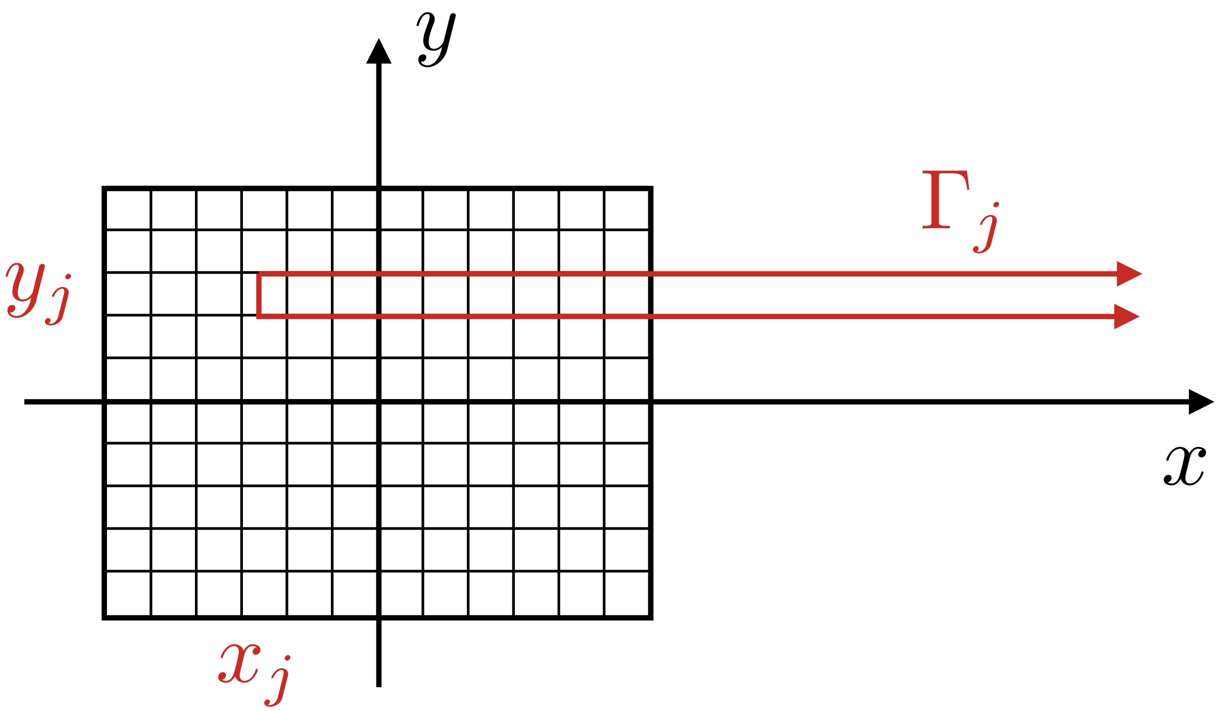
Vorticity Distribution

- **Vorticity lines**



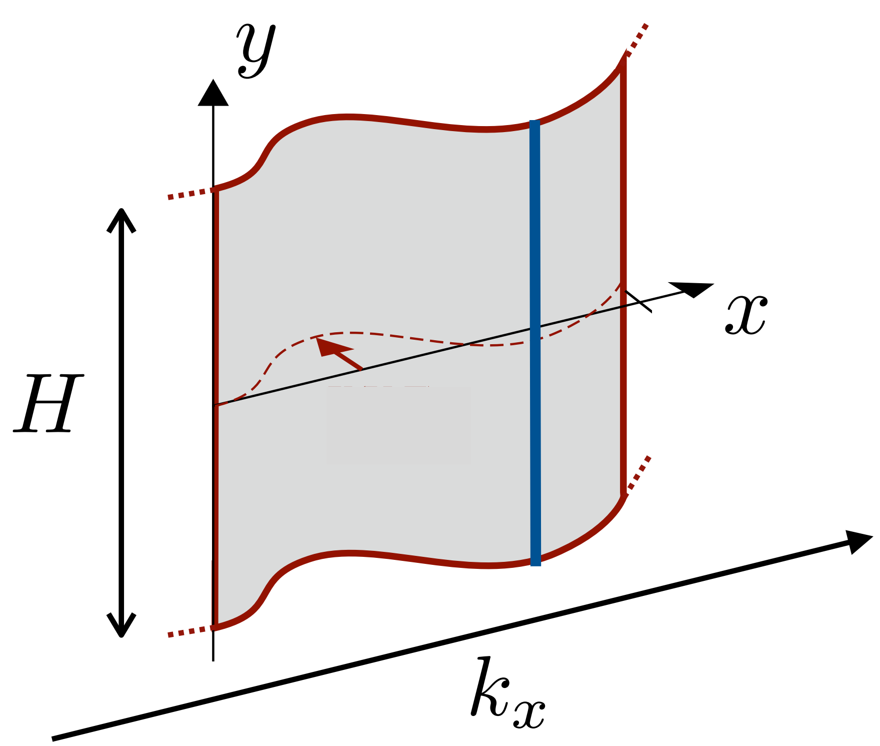
- **Vortex lattice method**

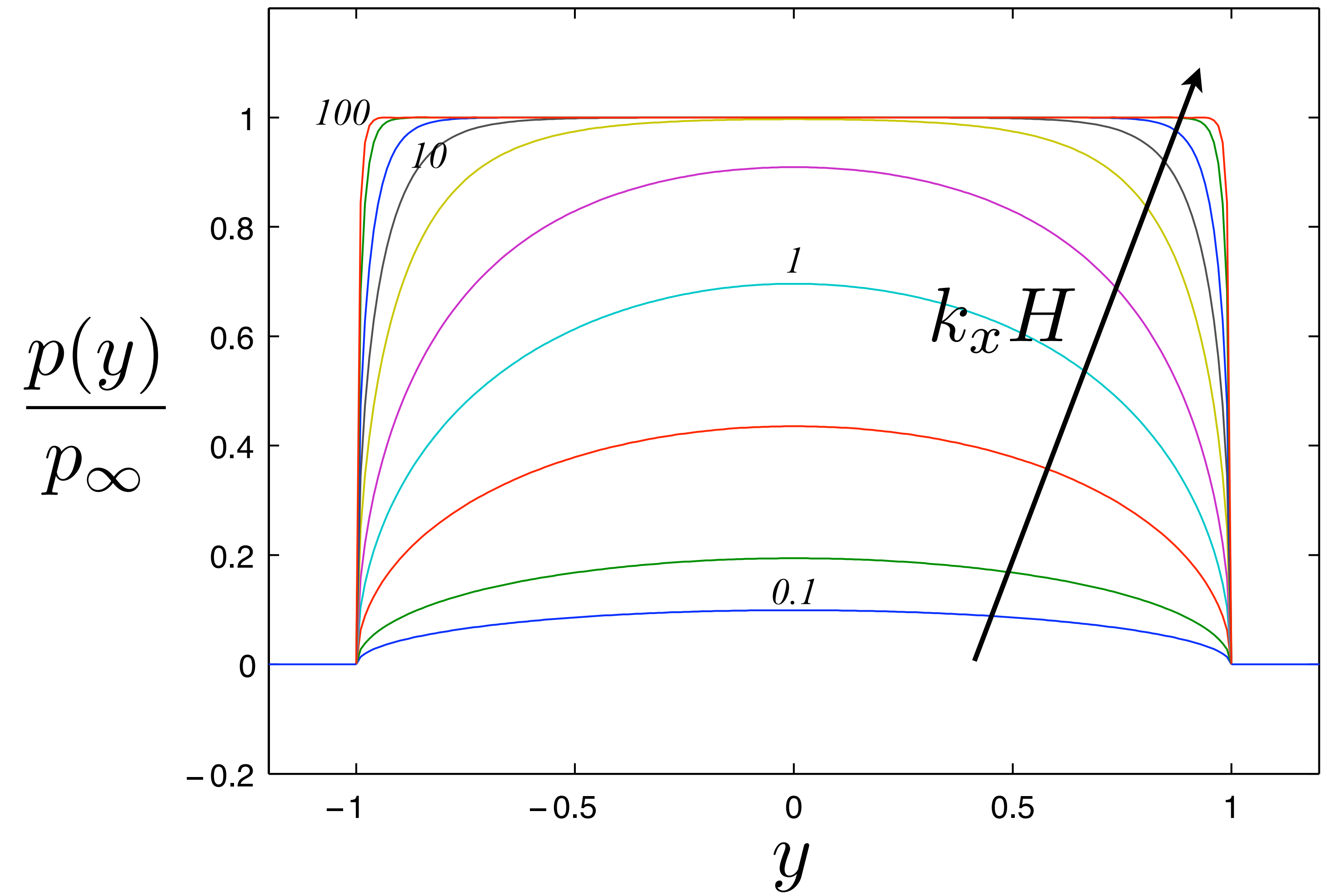
$$v_i = \sum_j K_{ij}\Gamma_j$$



[Ref] Tang, Yamamoto & Dowell, *J. Fluids Struct.* (2003)

Fourier Space





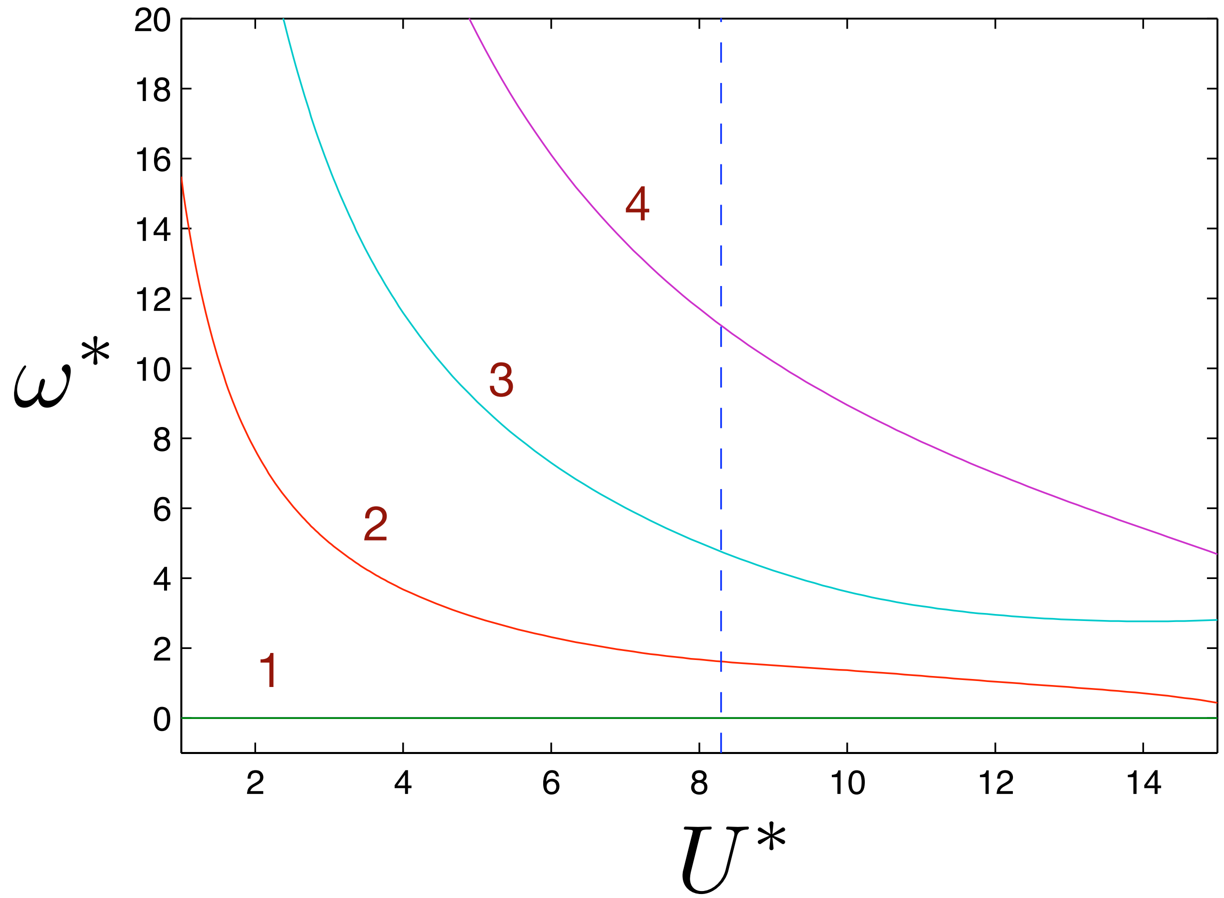
- **Assuming** $v_z \sim \mathrm{e}^{\mathrm{i}k_x x}$
- **The pressure along** y **can be determined**

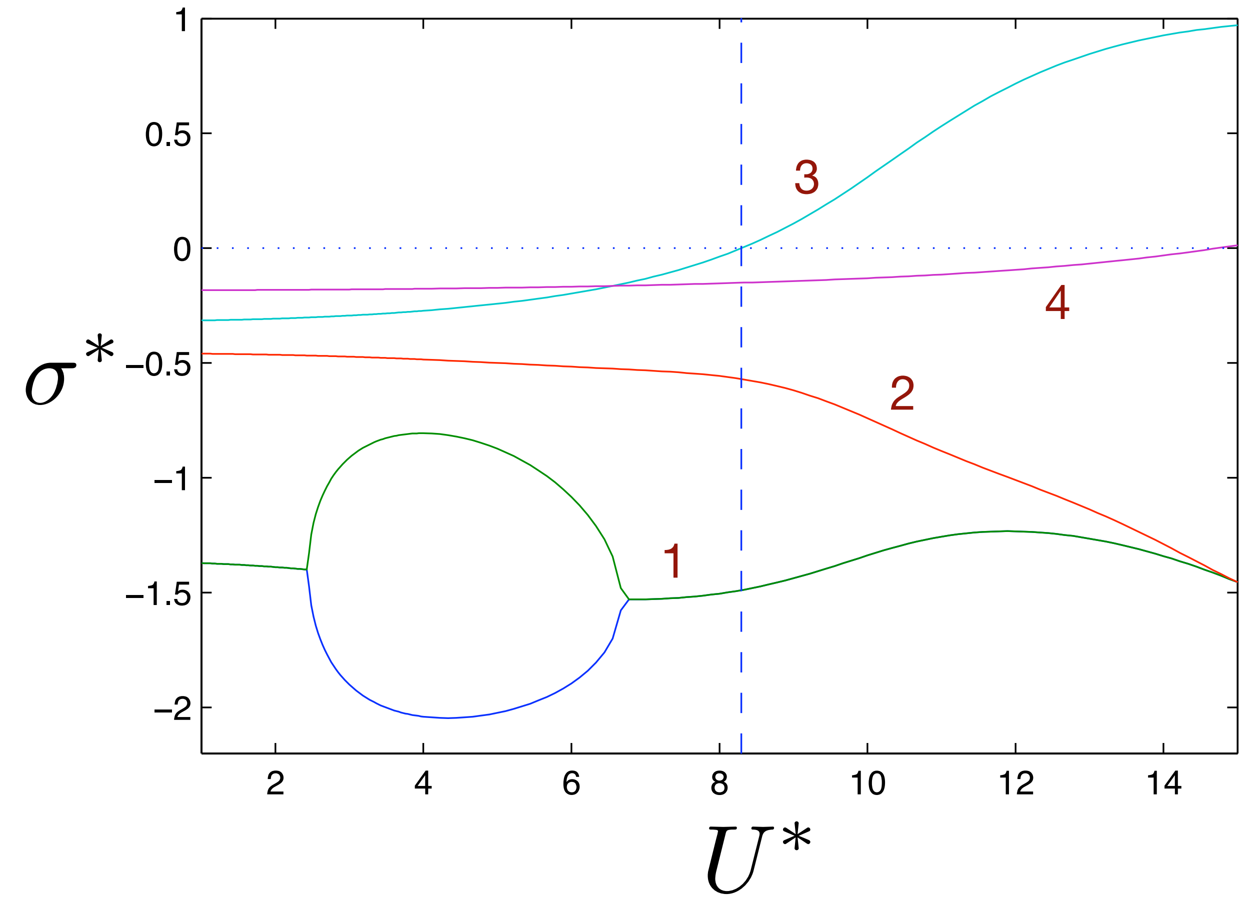
$$\langle p(y)\rangle = p_\infty \left(1 - \frac{1}{k_x H} + \mathcal{O}\left(\mathrm{e}^{-k_x H}\right)\right)$$

Results of the Stability Analysis

Flutter Modes

- **Parameters:** $H^* = \infty \quad M^* = 2.5$

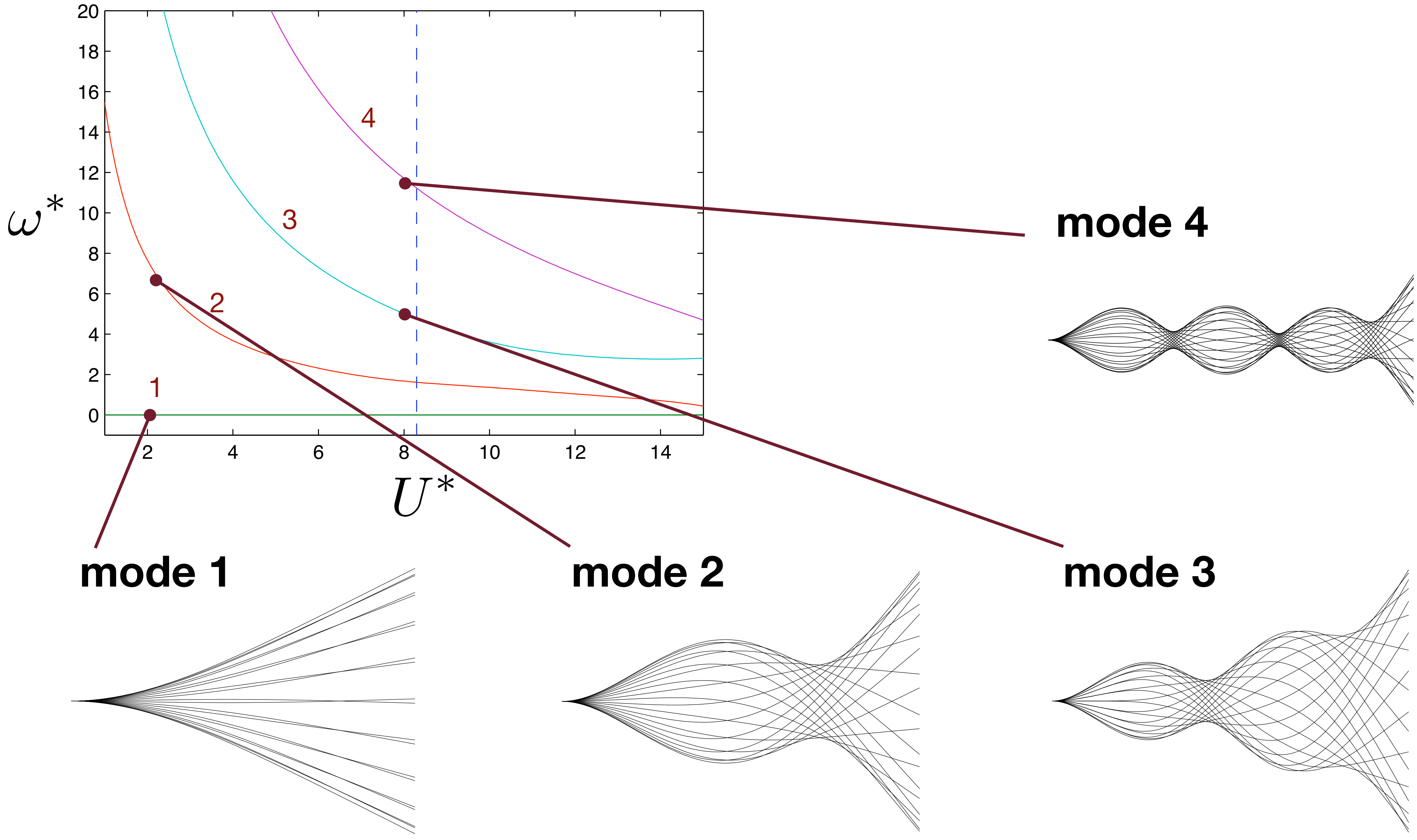




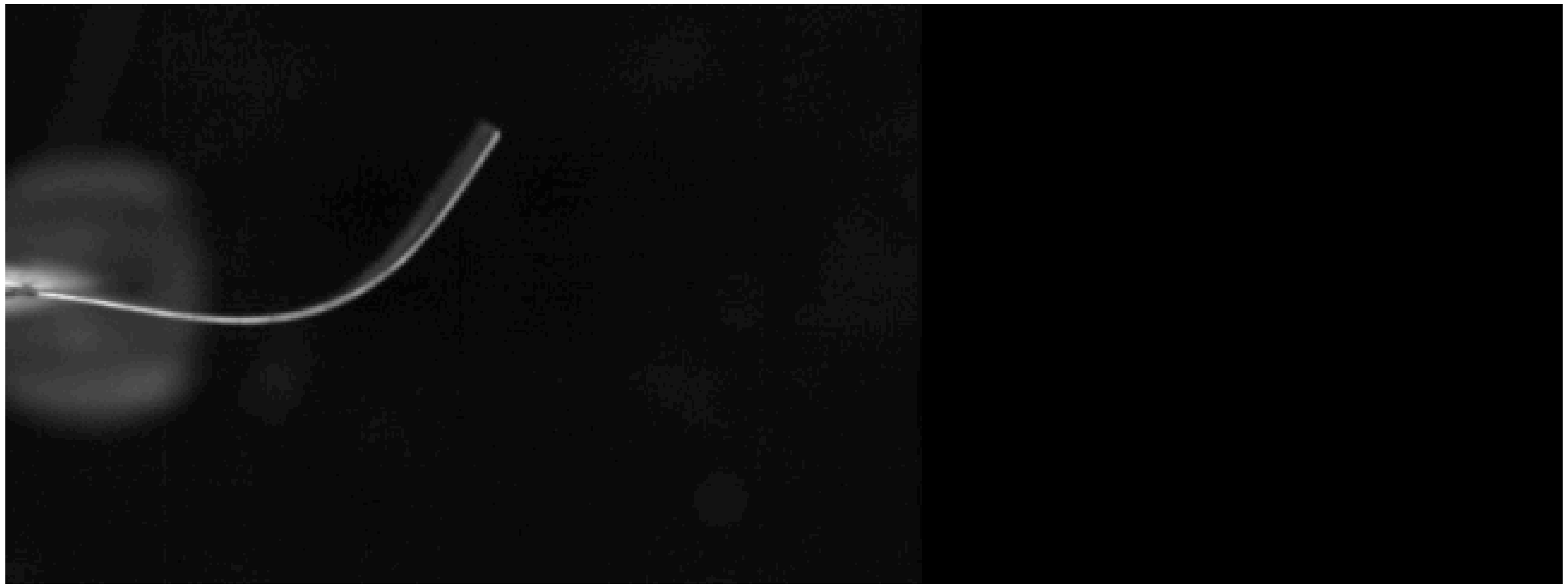
- **Dimensionless numbers**

$$U^* = \sqrt{\frac{\rho}{EI}} LU \qquad M^* = \frac{\rho_{\text{air}} L}{\rho} \qquad H^* = \frac{H}{L} \qquad \omega^* = \frac{L\omega}{U}$$

Shape of Flutter Modes



Unstable modes (50x slower)



- **mode 2**

$M^* = 0.74$

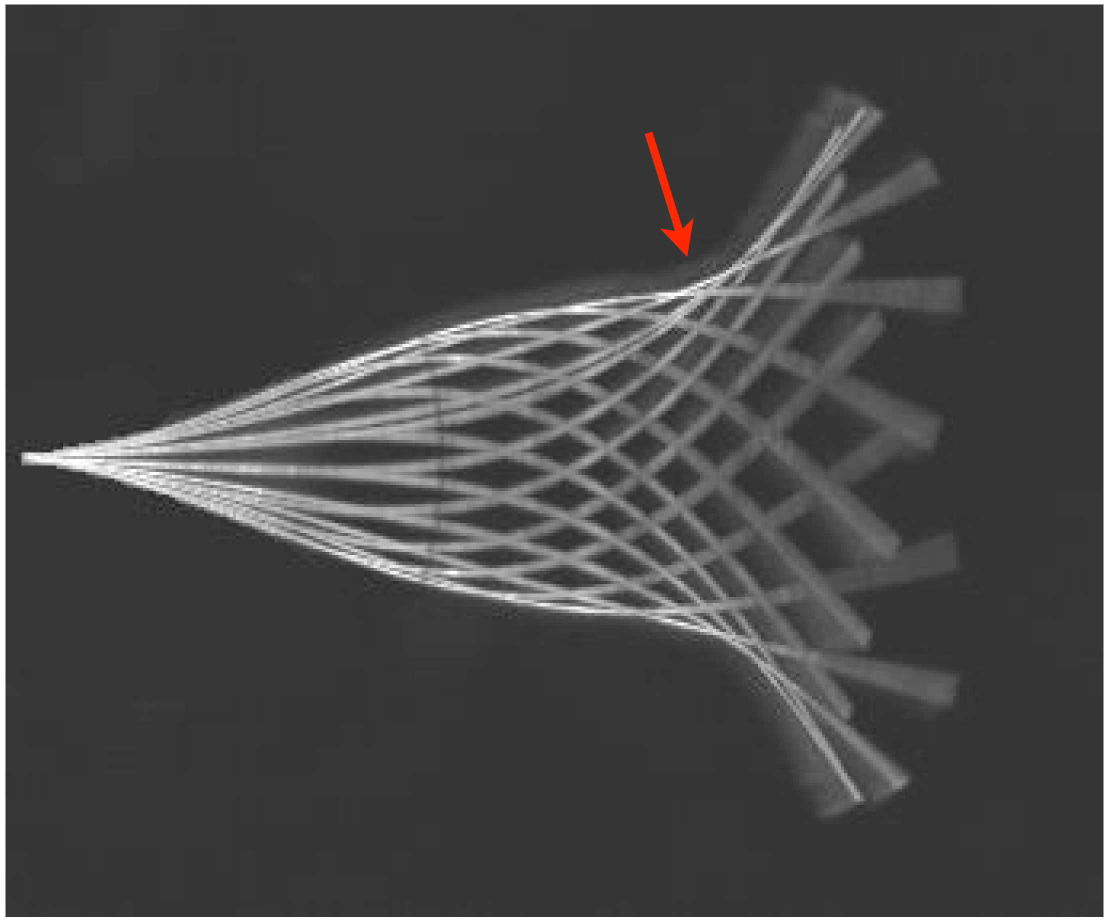
$L = 8\,\mathrm{cm}$

- **mode 3**

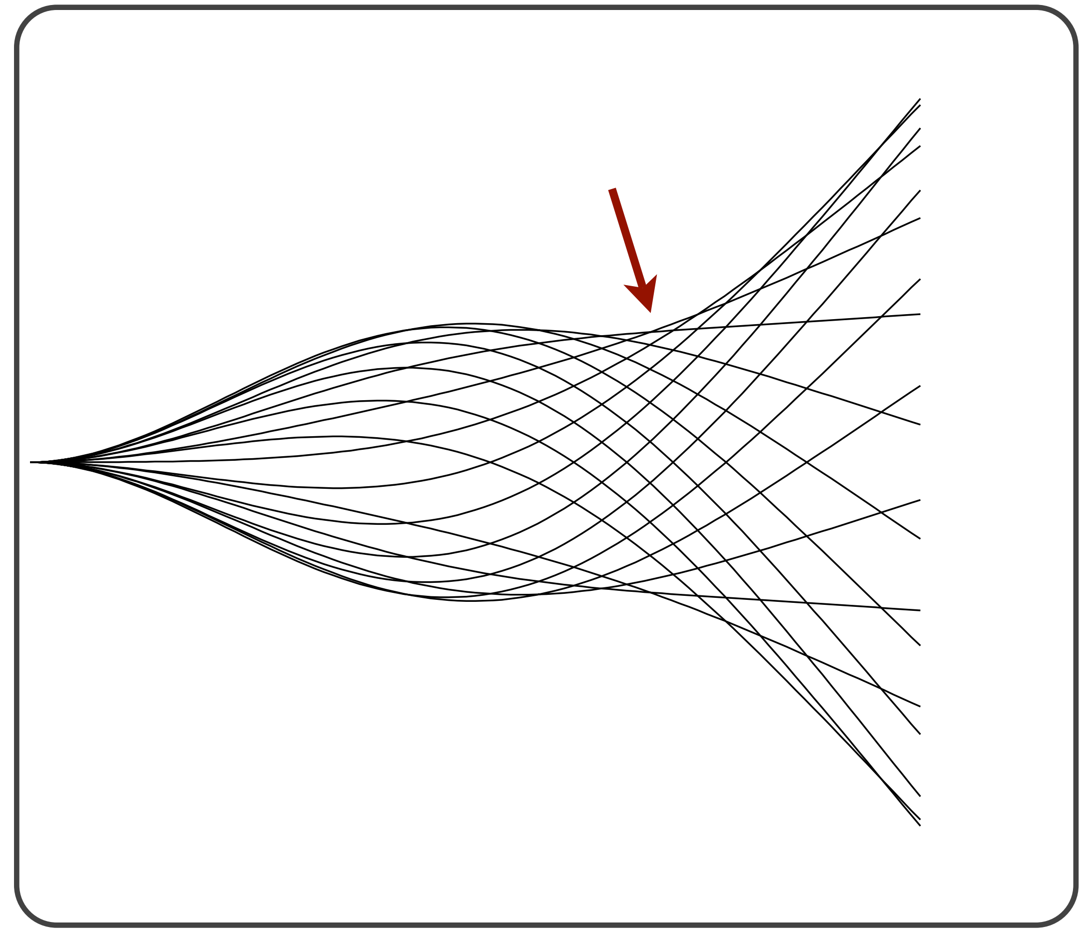
$M^* = 1.94$

$L = 21\,\mathrm{cm}$

Mode 2

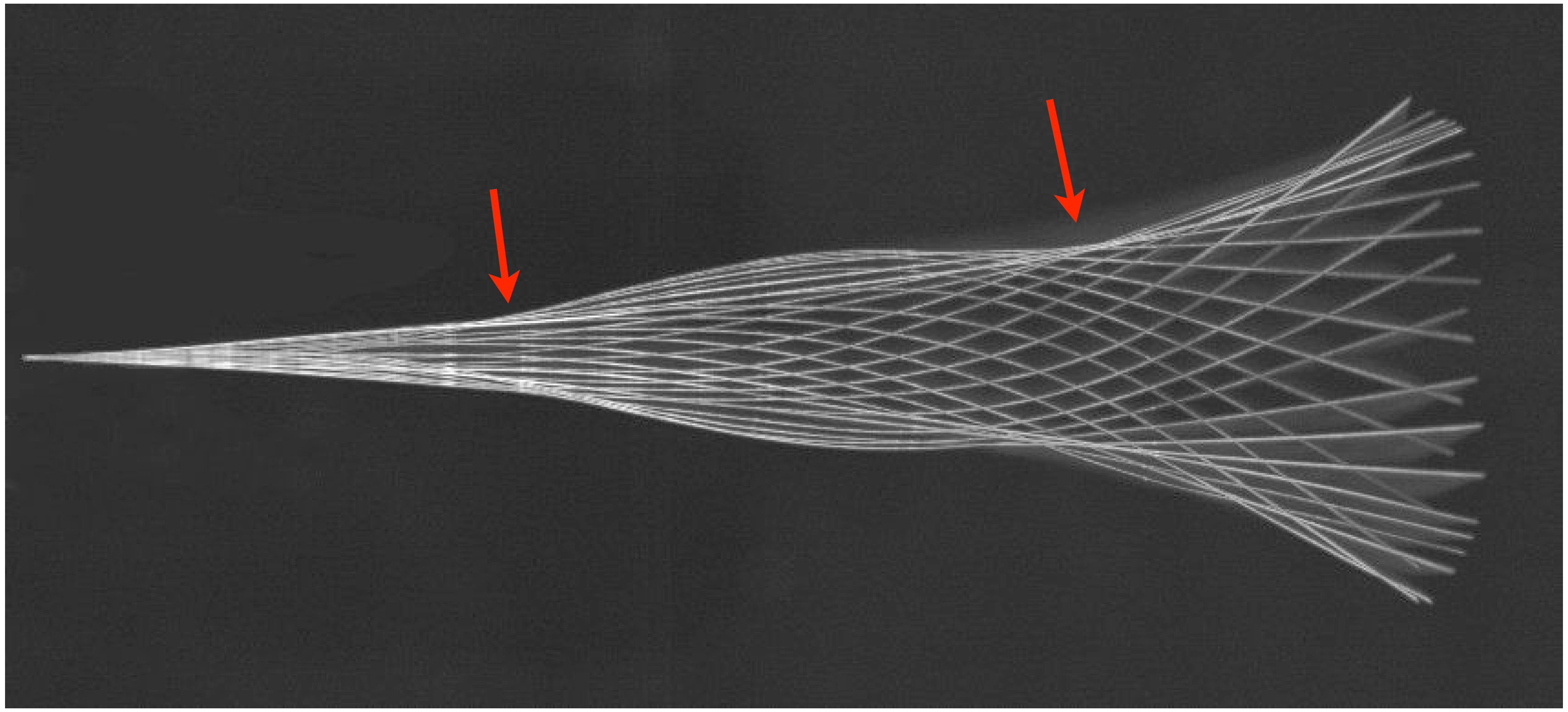


$\omega^* = 1.82$

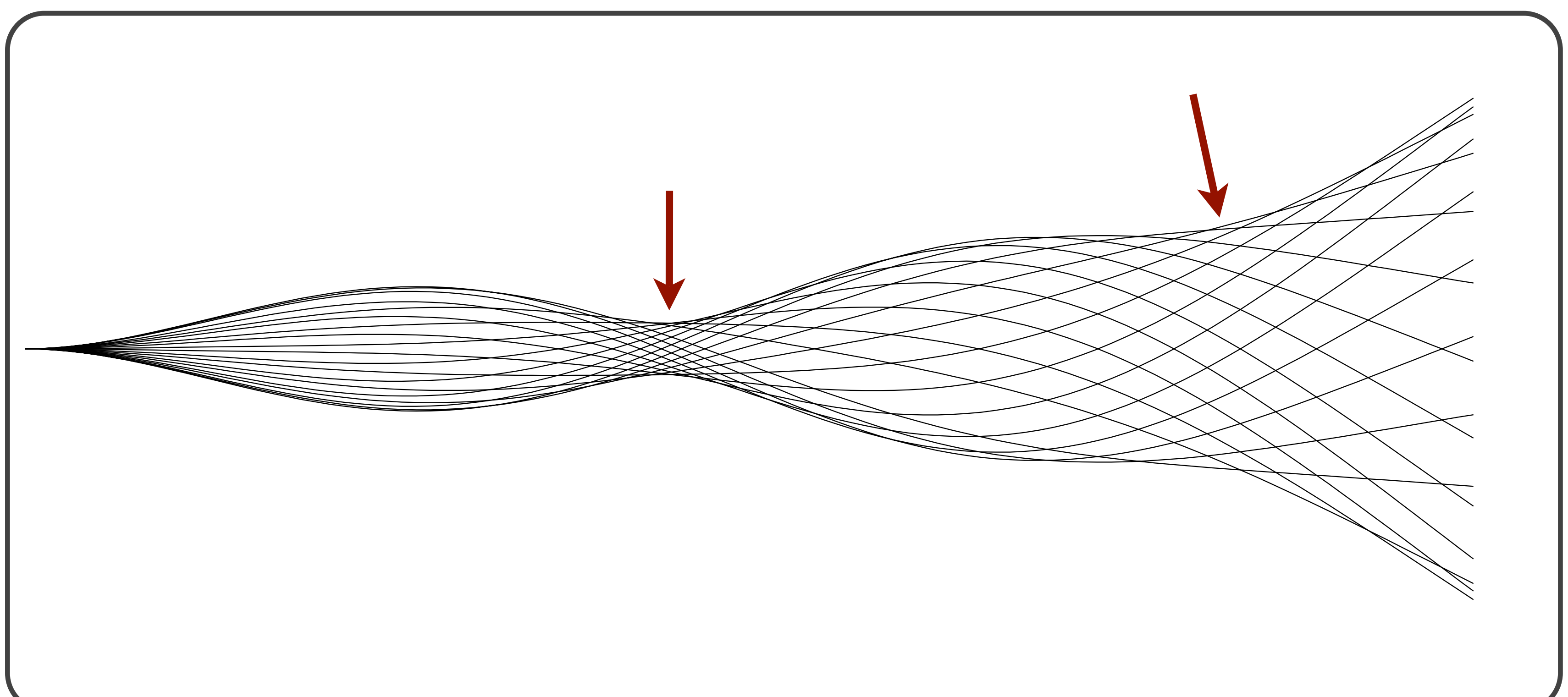


$\omega^* = 2.01$

Mode 3

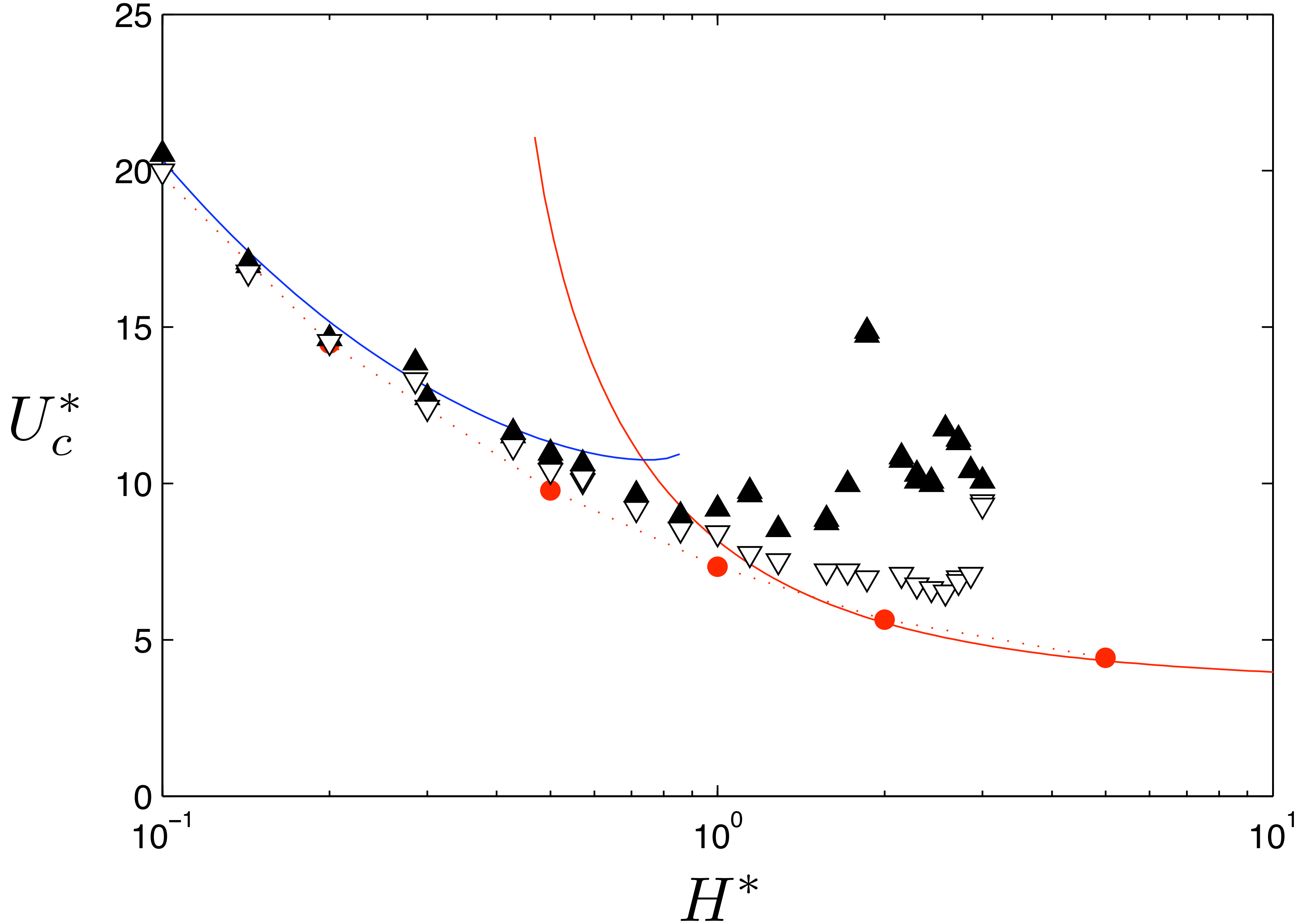


$\omega^* = 2.5$



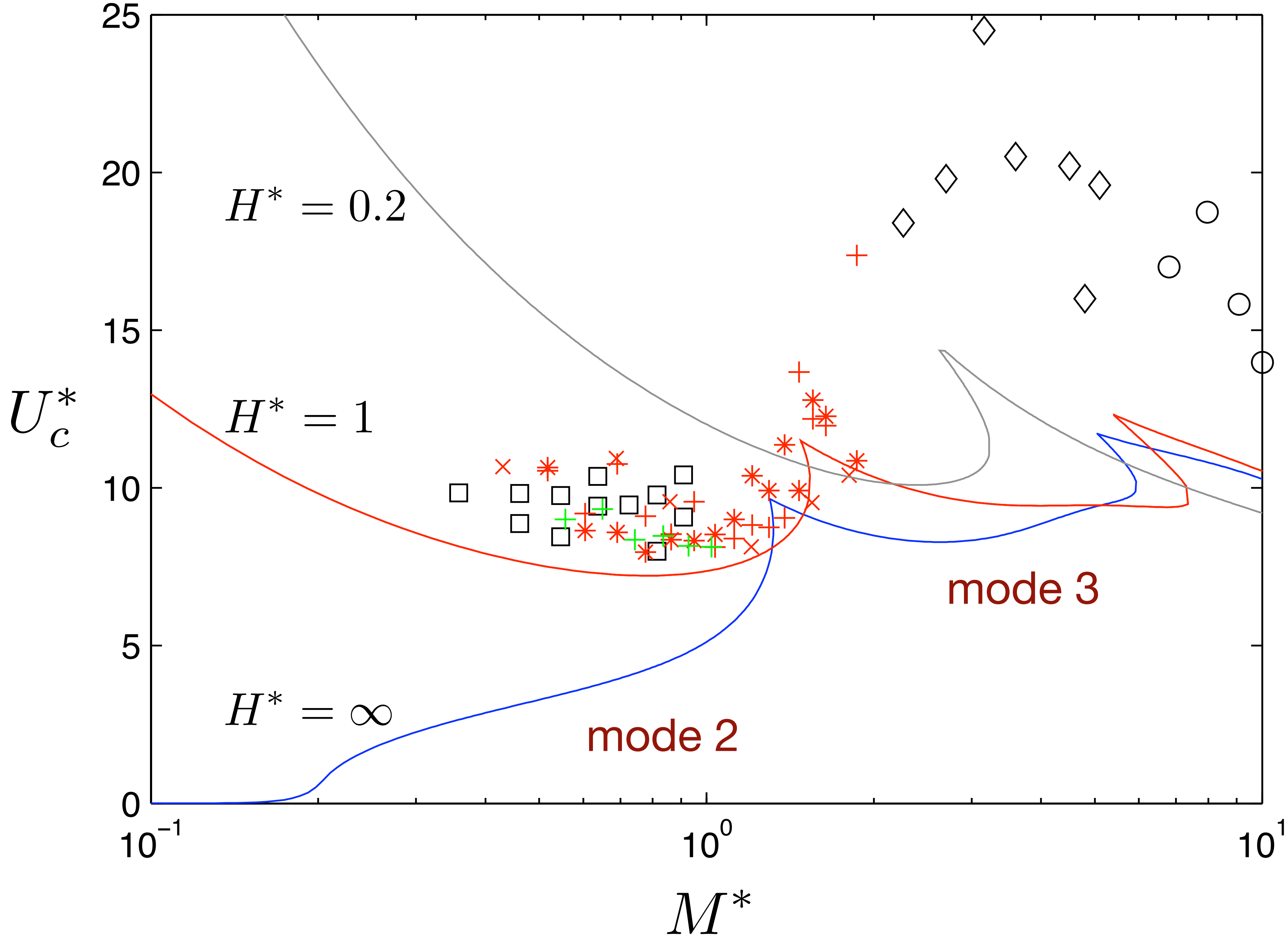
$\omega^* = 3.5$

Stability curve



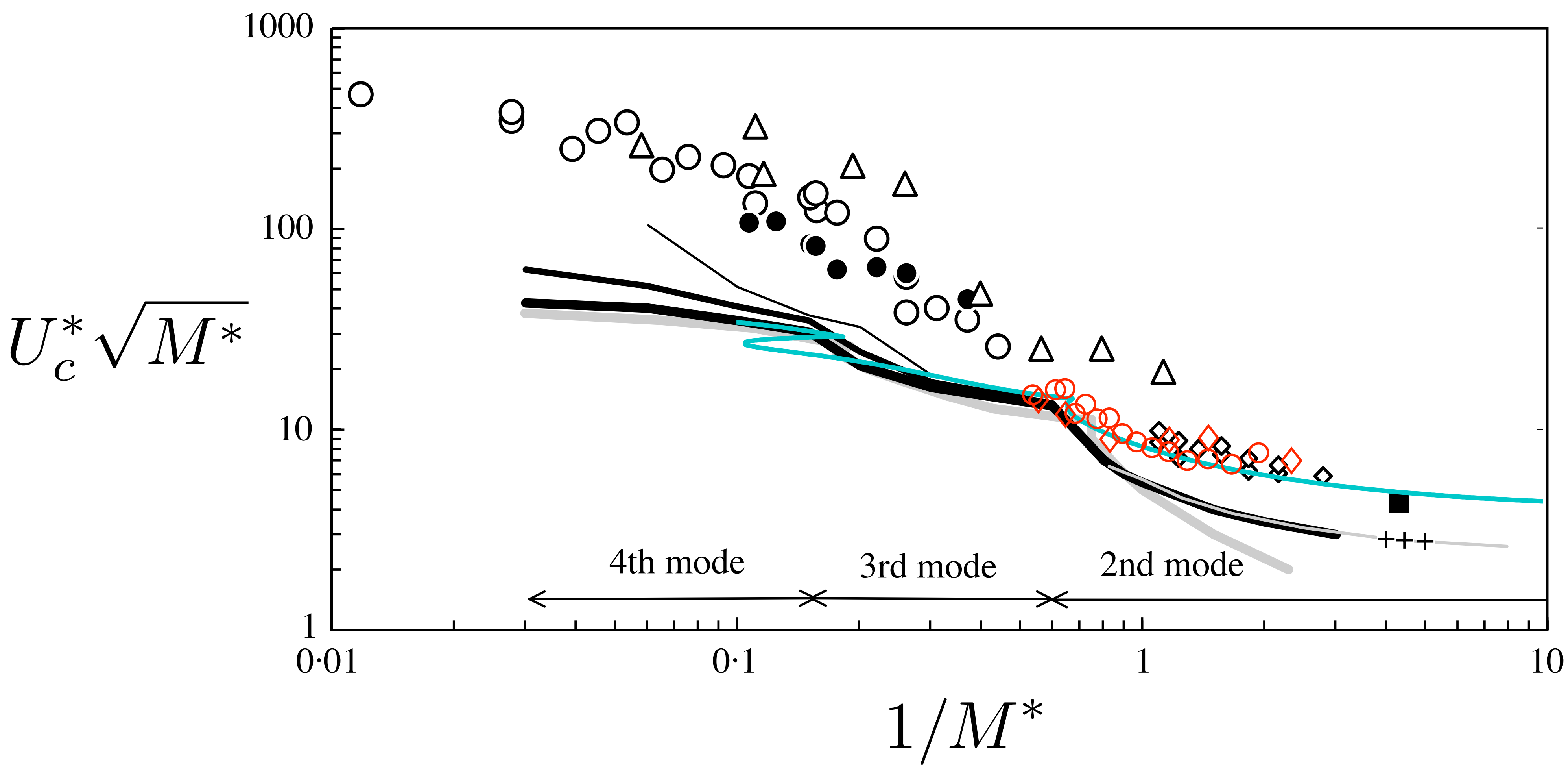
- **Mass ratio:** $M^* = 0.6$

Stability curve



- **Experiments for** $H^* = 1$ **and** $H^* = 0.25$

Conclusion



- **Flow inherently singular**
- **Good agreement for 1D mode + 3D flow**
- **Importance of 3D effects**