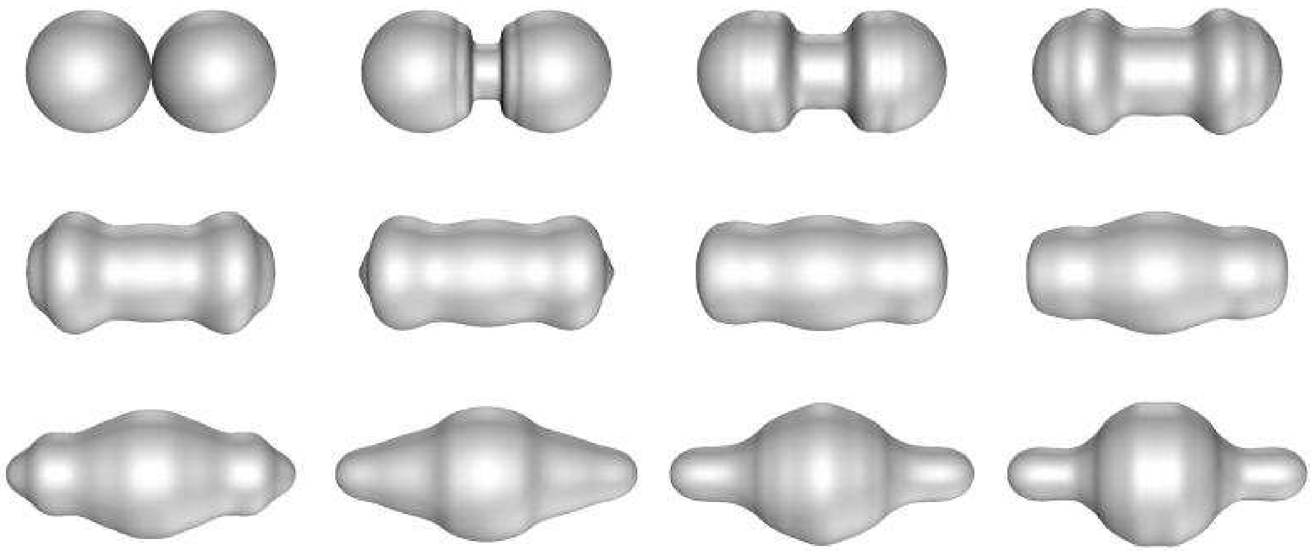
Laurent Duchemin, IRPHE, Marseille

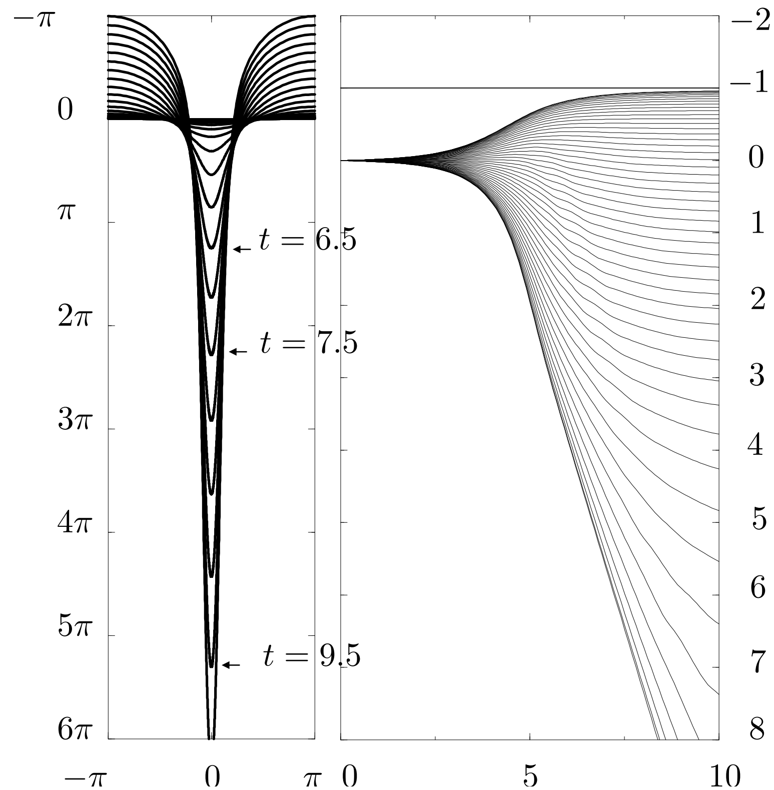
On some finite and infinite-time free-surface singularities



2D Rayleigh–Taylor instability

Christophe Josserand & Paul Clavin

- $A_T = \dfrac{\rho_h - \rho_l}{\rho_h + \rho_l} = 1$
- $t = T(s)\sqrt{gk}$
- $(x, y) = (X, Y)k$
- $\varphi = \phi\sqrt{k^3/g}$
- Bubble velocity : $\sqrt{g/3k}$



(L. Duchemin, C. Josserand & Paul Clavin, Physical Review Letters, 2005)

Theory

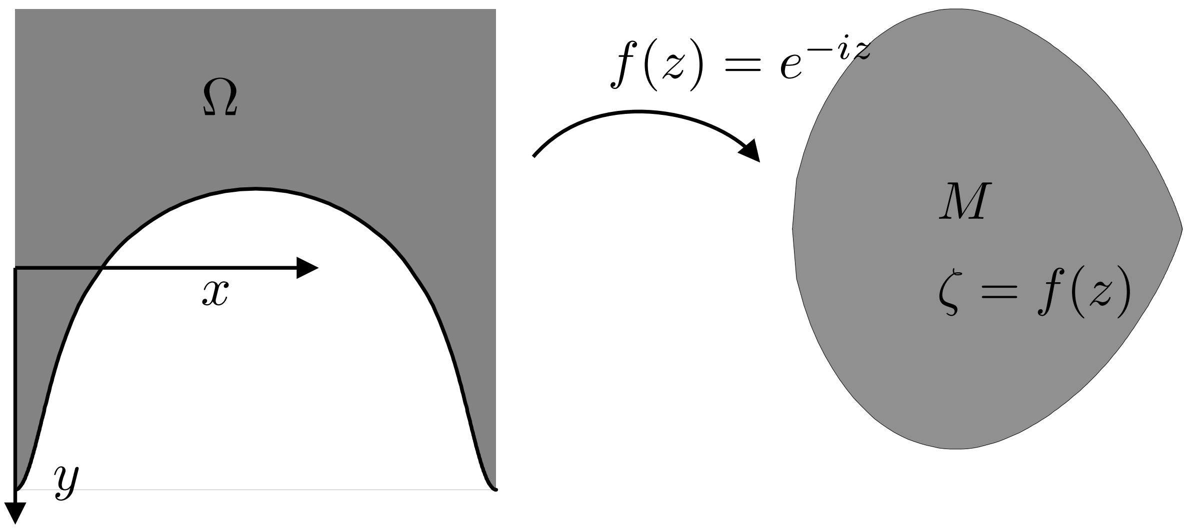
- Free surface : $y = \alpha(x,t)$
- Euler equations : $\dfrac{d\mathbf{U}}{dt} = -\nabla P + \mathbf{e_y}, \qquad \mathbf{U} = (u,v), \qquad u_x + v_y = 0$
- Kinematic condition : $\dfrac{\partial \alpha(x,t)}{\partial t} + u\dfrac{\partial \alpha(x,t)}{\partial x} = v$
- In the spike, quasi-parallel flow :

$$\boxed{v\ v_y \sim 1 \Rightarrow v \sim \sqrt{2y} \Rightarrow y_p \sim \frac{1}{2}t^2}$$

Theory

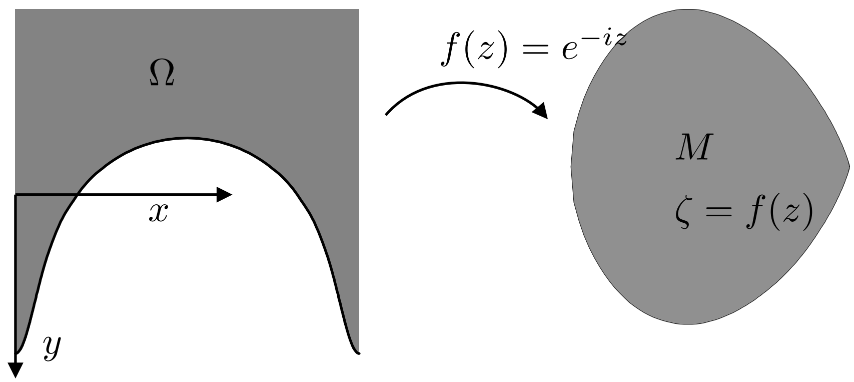
- $v \sim \sqrt{2y} \Rightarrow u \sim -\dfrac{x}{\sqrt{2y}}$
- $\dfrac{\partial \alpha(x,t)}{\partial t} - \dfrac{x}{\sqrt{2\alpha(x,t)}}\dfrac{\partial \alpha(x,t)}{\partial x} = \sqrt{2\alpha(x,t)}$
- $\gamma(x,t) \ll t/2 \qquad \Rightarrow \qquad \alpha(x,t) = t(\dfrac{t}{2} - \gamma(x,t)).$
- Self-similar solution : $\qquad \gamma(x,t) = \theta(xt).$
- $v(x,y,t) = \sqrt{2(y + f(x,y,t))}, \qquad f(x,y,t) \ll y$
- $\boxed{\alpha(x,t) = \dfrac{t^2}{2} - t\theta(xt), \qquad \kappa = t^3\theta''(0), \qquad \dfrac{d^2 y_s}{dt^2} = 1 + \dfrac{2}{t^5\theta''(0)}}$

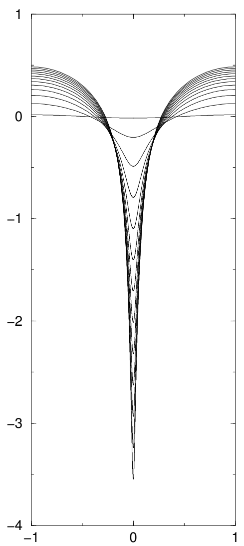
Boundary integral method



- $\Delta\varphi = 0$
- Boundary conditions : $\dfrac{\partial\varphi}{\partial t} = -\dfrac{1}{2}(\nabla\varphi)^2 + y, \qquad \dfrac{d\mathbf{x}}{dt}\cdot\mathbf{n} = \nabla\varphi\cdot\mathbf{n}$
- Complex potential : $\beta(z) = \varphi + i\psi$, Complex velocity : $\dfrac{d\beta}{dz} = u - i\,v$

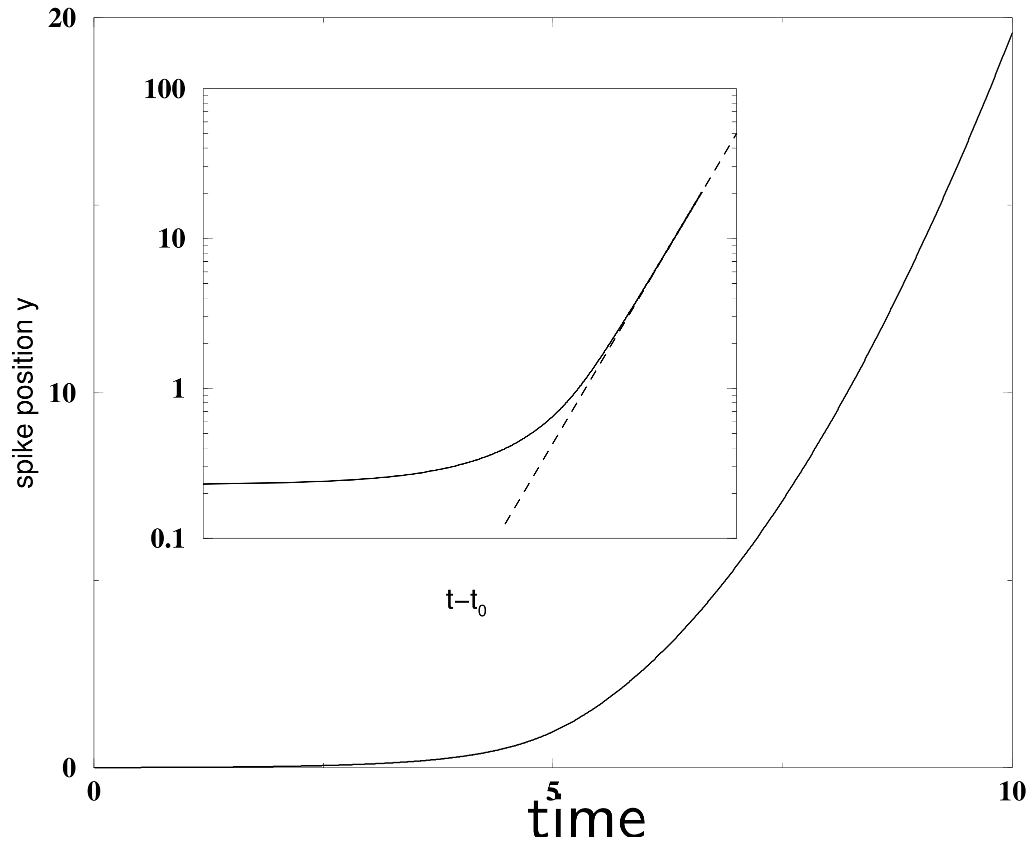
Boundary integral method





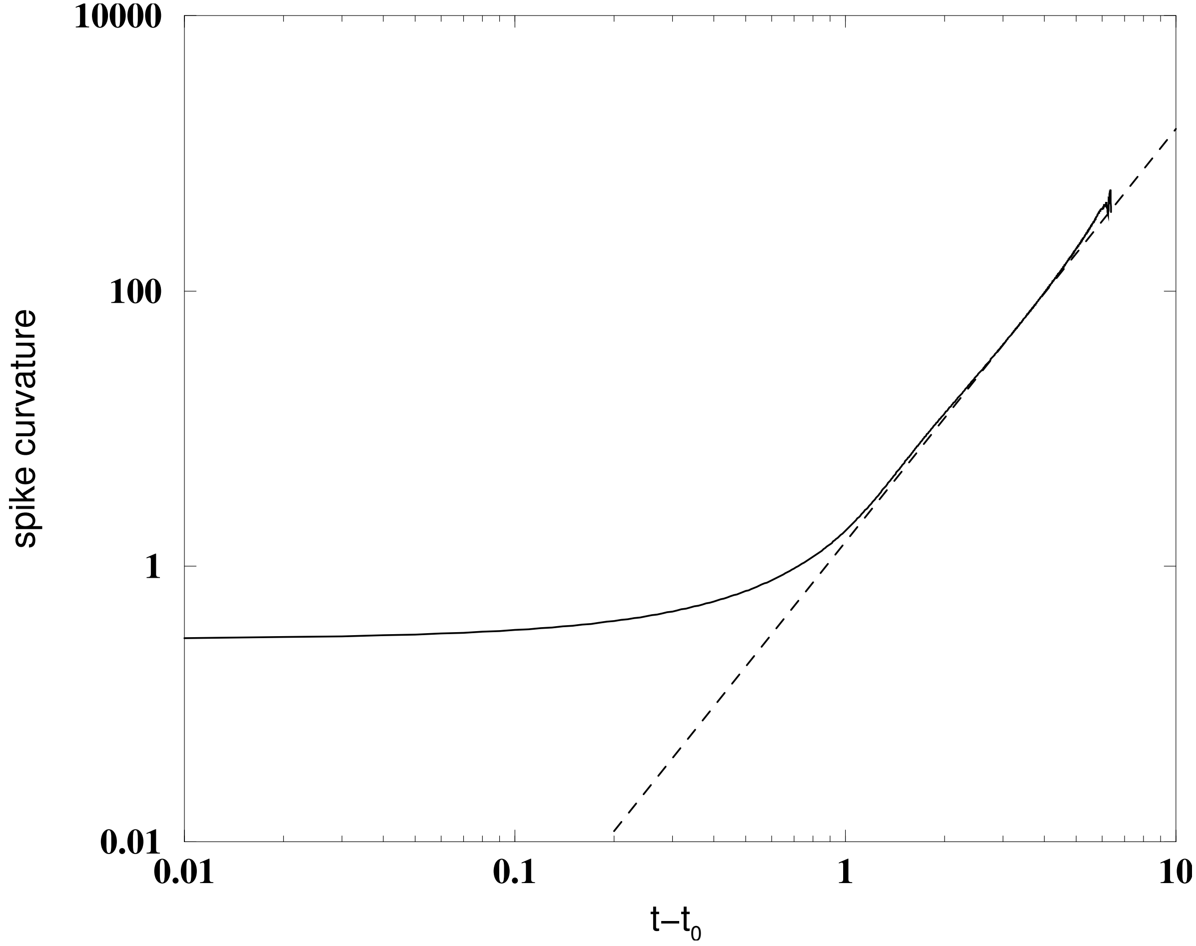
- Conformal map : $\zeta = f(z) = e^{-iz}$
- $\gamma(\zeta) = \beta(f(z))$
- Cauchy theorem : $Im\left(\int_{\mathcal{C}} \frac{\gamma(\zeta)}{\zeta - \zeta_e} dz\right) = 0$

$$y_s \sim \frac{1}{2}g(t - t_0)^2$$



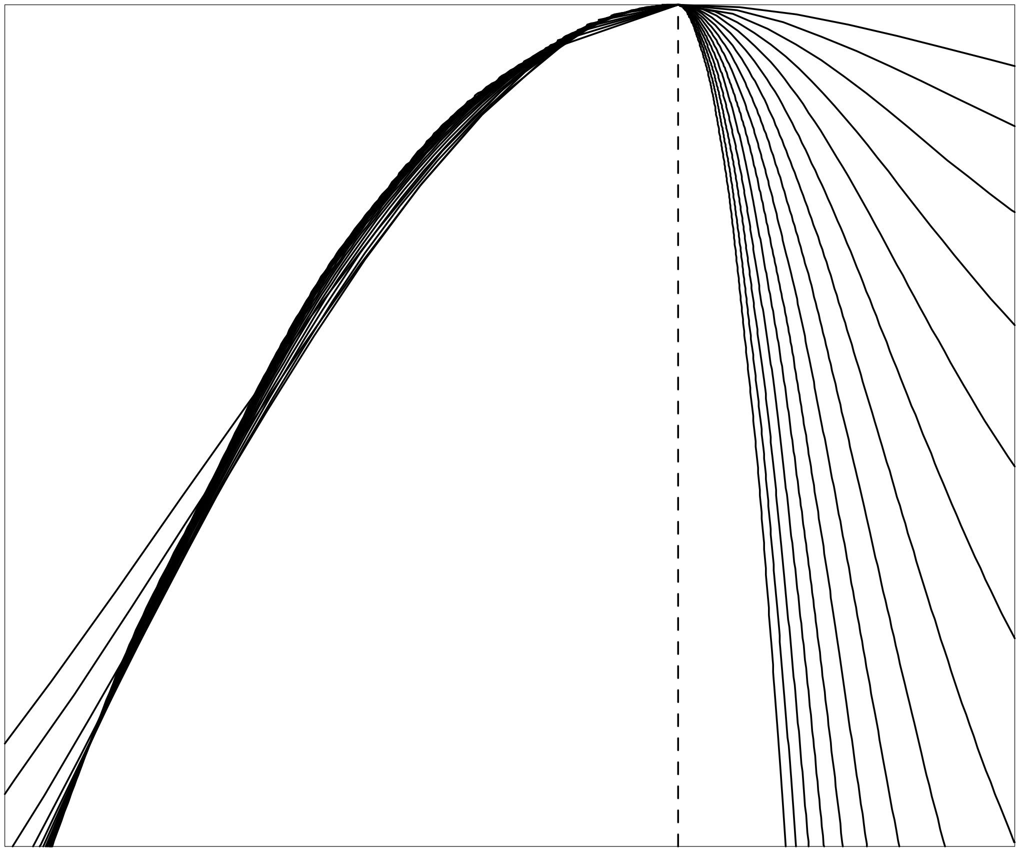
Vertical position of the spike as a function of time

Curvature of the spike : $t^3\theta''(0)$



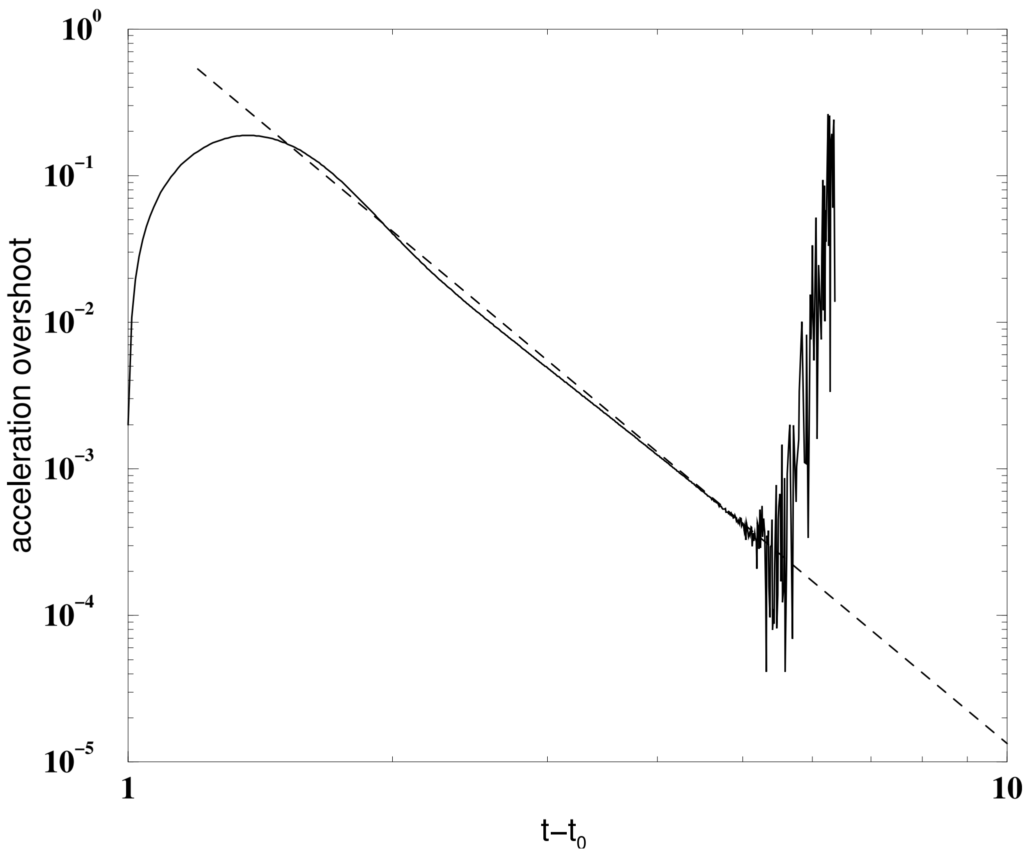
Spike curvature as a function of time in a log-log plot

Self-similar profiles : $x \sim 1/t$, $y \sim t$



Left : Rescaled profiles. **Right :** Successive profiles.

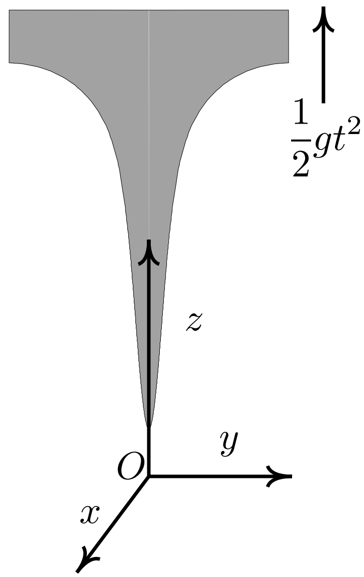
Overshoot in acceleration : $\frac{2}{t^5\theta''(0)}$



Overshoot in acceleration as a function of time in a log-log plot.

General theory

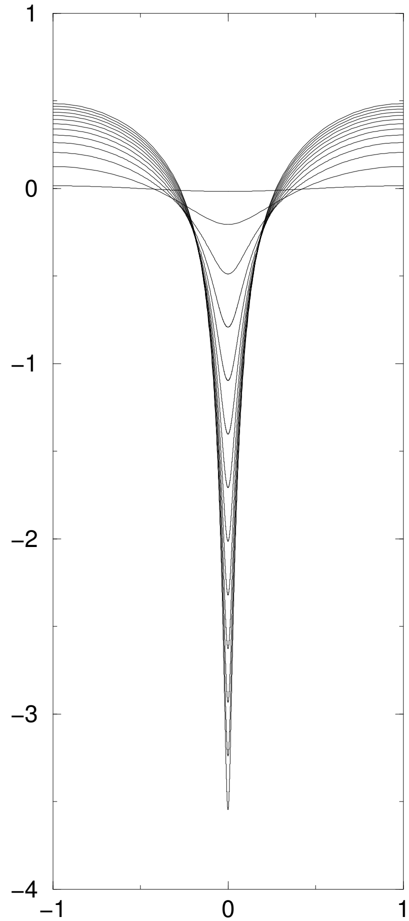
- $\Delta\varphi = 0$
- $\varphi = \varphi_0(z,t) + x^2\varphi_2(z,t) + y^2\varphi_2'(z,t)$
- $\Rightarrow \varphi = \frac{1}{2}\left(\frac{\dot{a}}{a}x^2 + \frac{\dot{b}}{b}y^2 + \frac{\dot{c}}{c}z^2\right)$
- $\Delta\varphi = 0 \Rightarrow abc = M$
- $-2p = 2\varphi_t + \varphi_x^2 + \varphi_y^2 + \varphi_z^2 + f(t) = 0$
- $\frac{d}{dt}(-2p) = (\partial_t + \nabla\varphi \cdot \nabla)(-2p) = 0$
- Dirichlet hyperboloid : $-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{h^2(x,y,t)}{c^2} = 1$



General theory

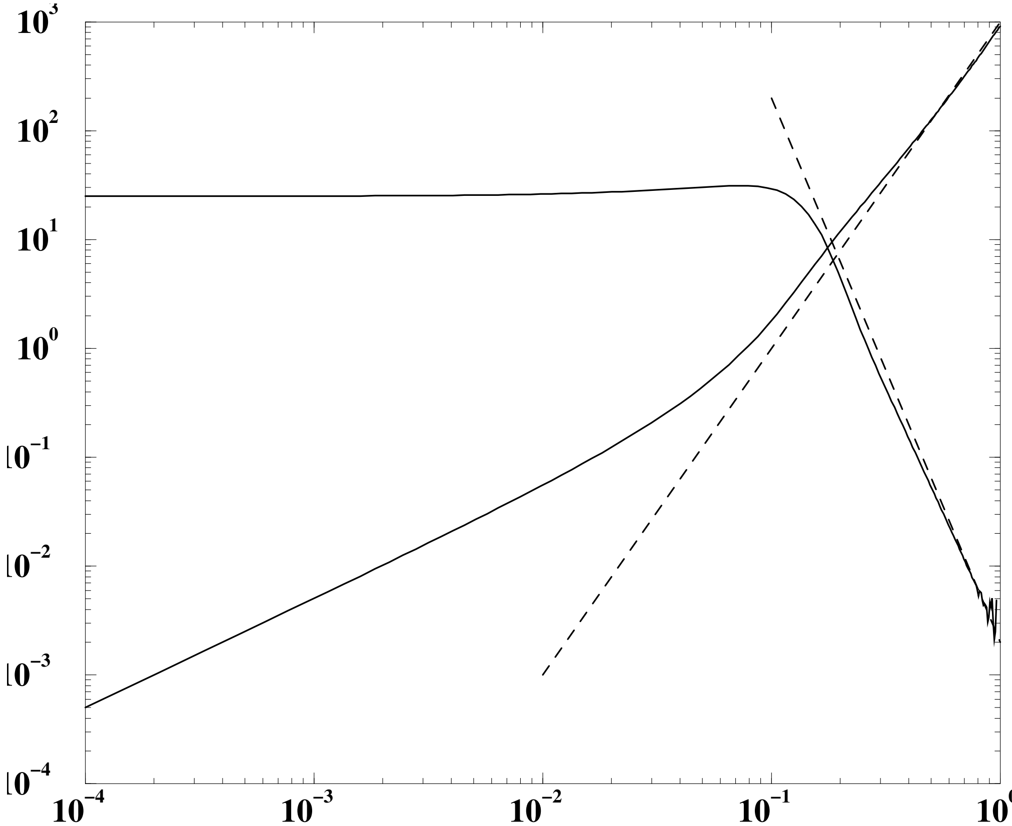
- $c(t) = c_0 t - \frac{M}{8c_0^2 t^2} + o(t^{-2})$
- In 3D : $\kappa_0 = 2\partial_{xx} h(0,0,t) = 2\frac{c}{a^2} = 2\frac{c^2}{M} \sim 2\frac{t^2}{M}$
- $a_t = \partial_{tt} h(0,0,t) = \ddot{c} = g - \frac{3M}{4t^4} + o(t^{-4})$
- In 2D : $\kappa_0 = \partial_{xx} h(0,t) = \frac{c^3}{M^2} \sim \frac{t^3}{M^2}$
- $a_t\big|_{R'} = g - \frac{2M^2}{t^5} + o(t^{-5})$

2D Richtmyer–Meshkov instability



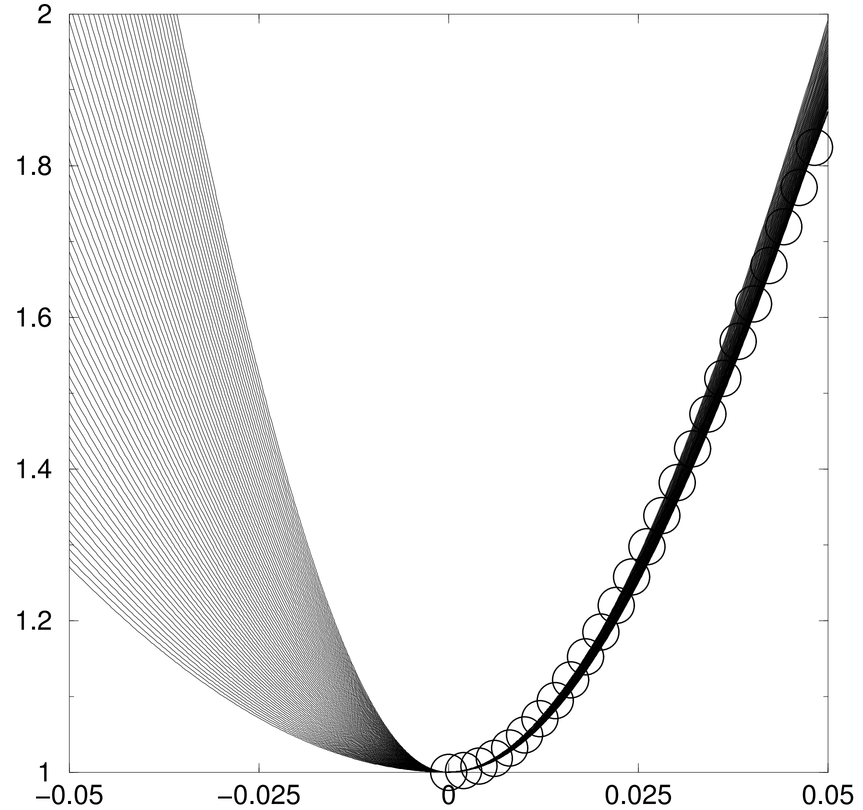
Successive profiles

Curvature and acceleration



$$\kappa \sim \frac{t^3}{M^2} \qquad a_t \sim -\frac{2M^2}{t^5}$$

Self-similar profiles

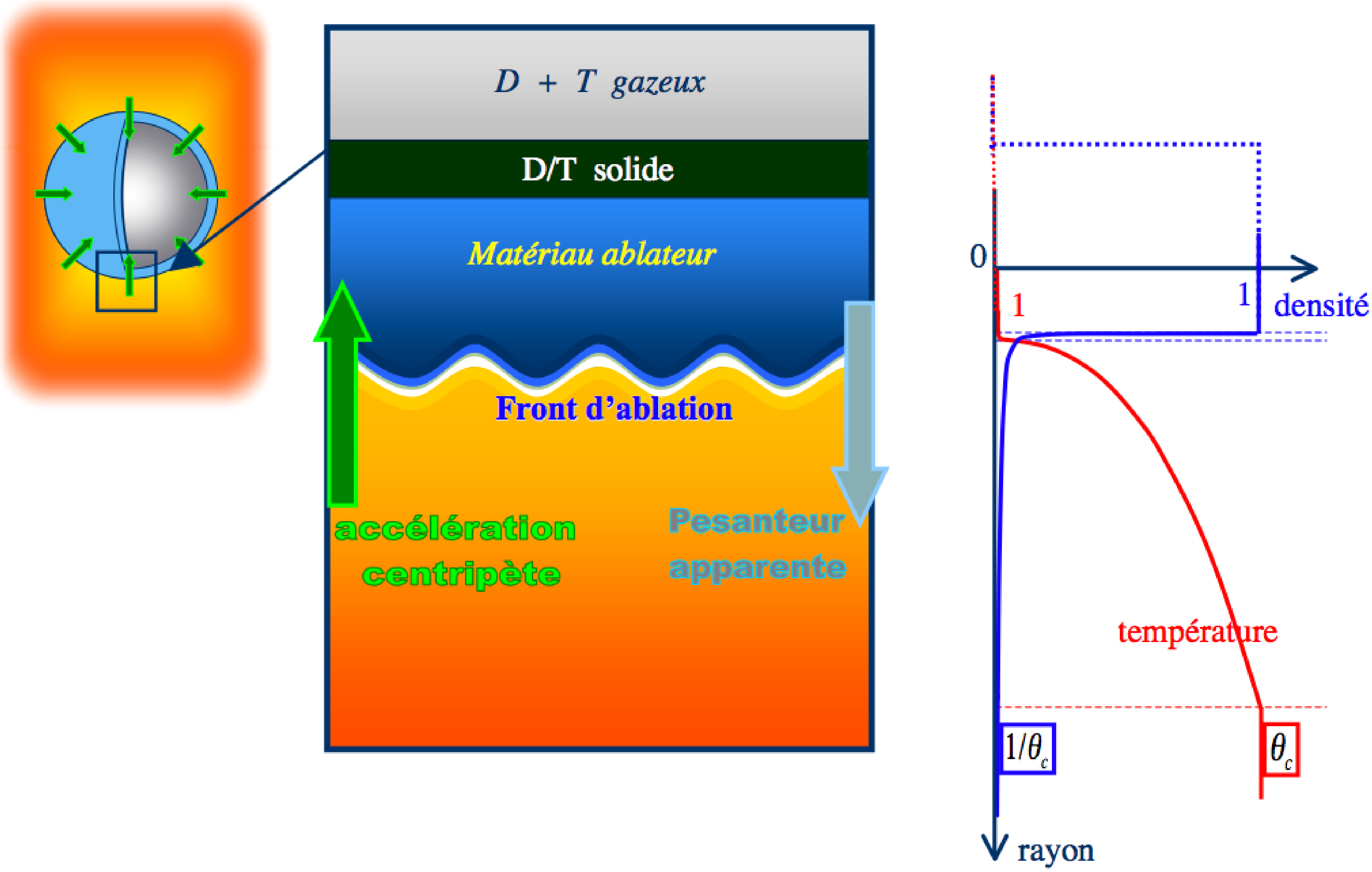


Left : Successive profiles. **Right :** Rescaled profiles and self-similar curve.

Comparison with experiments?

Antkowiak, A., Bremond, N., Le Dizèse, S. & Villermaux, E., 2007 Short-term dynamics of a density interface following an impact. *J. Fluid. Mech.* **577**, 241–250.

RTI in ICF

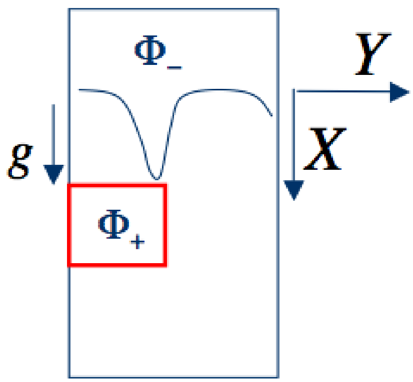

D + T gazeux
D/T solide
Matériau ablateur
Front d'ablation
accélération centripète
Pesanteur apparente
0
1
1
densité
température
$1/\theta_c$
θ_c
rayon

2D ablation front

X_f la coordonnée du front d'ablation
Φ le potentiel de vitesse et on a $\Phi_+ = \theta$

$X < X_f$ $\quad \Delta\Phi_- = 0$

$X \to -\infty$ $\quad \Phi_- = 0$



$$X = X_f \quad \begin{cases} \dfrac{\partial \Phi_-}{\partial \tau} + \dfrac{1}{2}(\vec{\nabla}\Phi_-)^2 \boxed{+ \dfrac{1}{2}(\vec{\nabla}\Phi_+)^2} - X_f = 0 \\ \dfrac{dX_f}{d\tau} = \nabla\vec{\Phi}_-.\vec{e}_x \text{ et } \dfrac{dY_f}{d\tau} = \nabla\vec{\Phi}_-.\vec{e}_y \\ \boxed{\Phi_+ = 0} \end{cases}$$

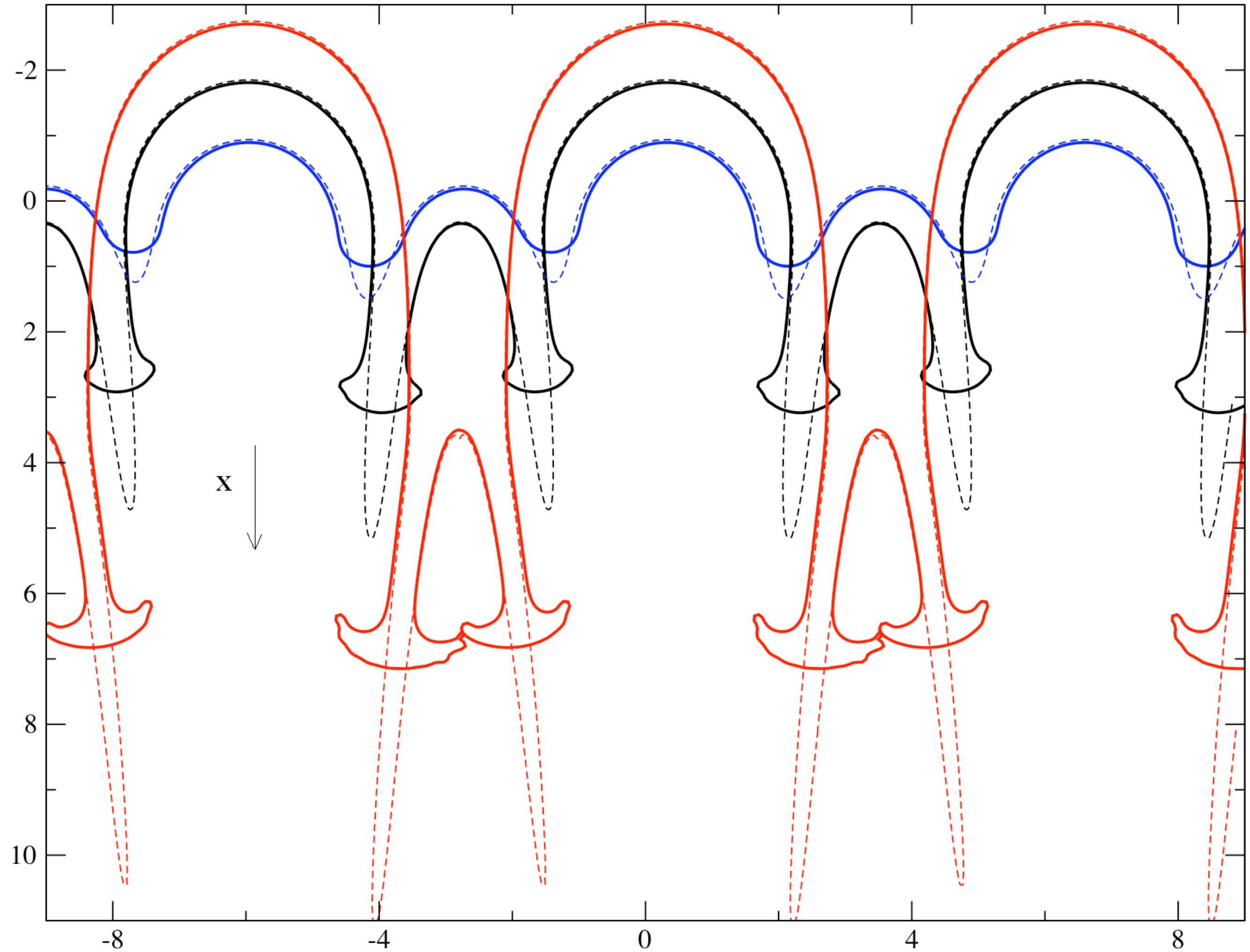
Nouveaux termes par rapport
à Rayleigh-Taylor pur (At=1)

$X > X_f$ $\quad \Delta\Phi_+ = 0$

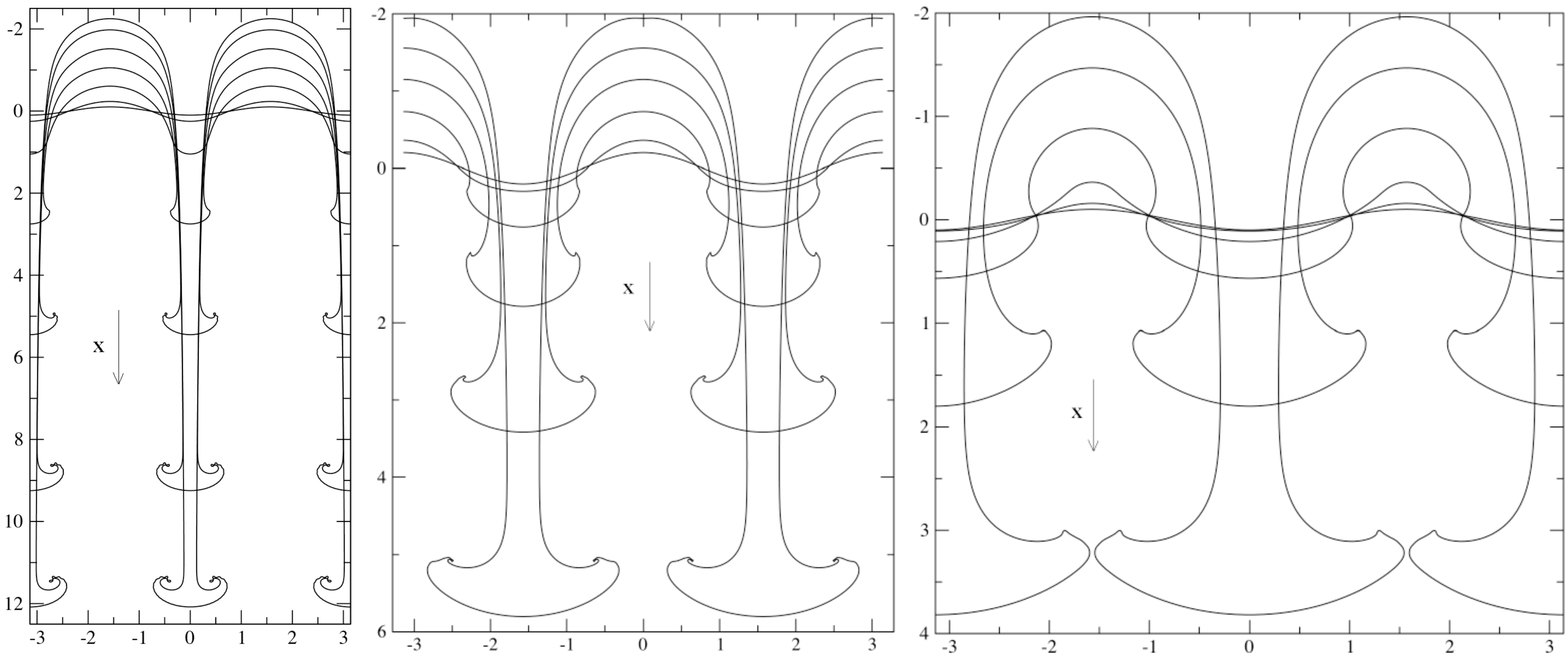
$X \to +\infty$ $\quad \Phi_+ \sim X$

Almarcha, Clavin, Duchemin, Sanz JFM 2007
Clavin, Almarcha C.R. Méc 2005

Comparison with pure RTI

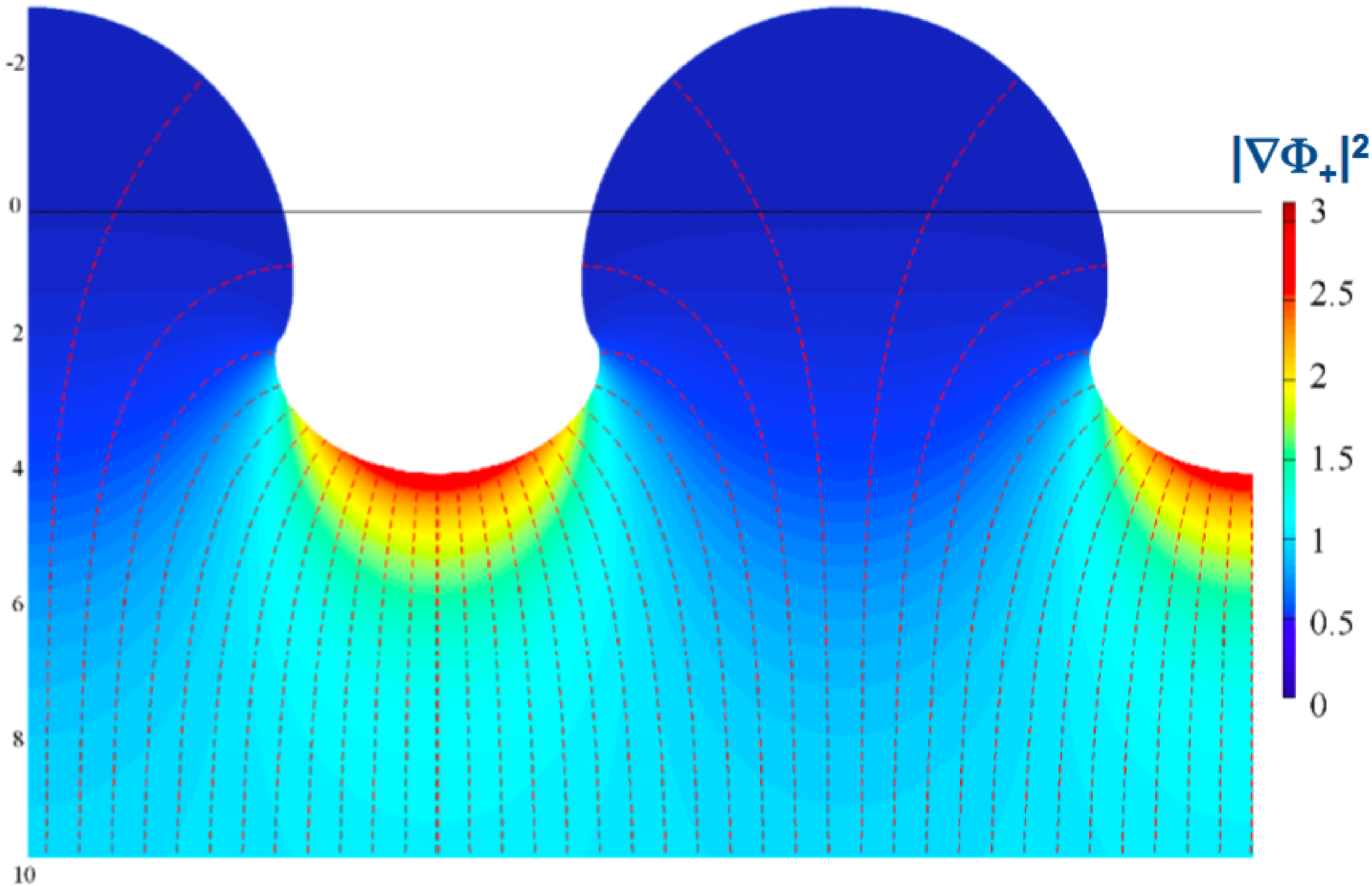


Influence of the wave number

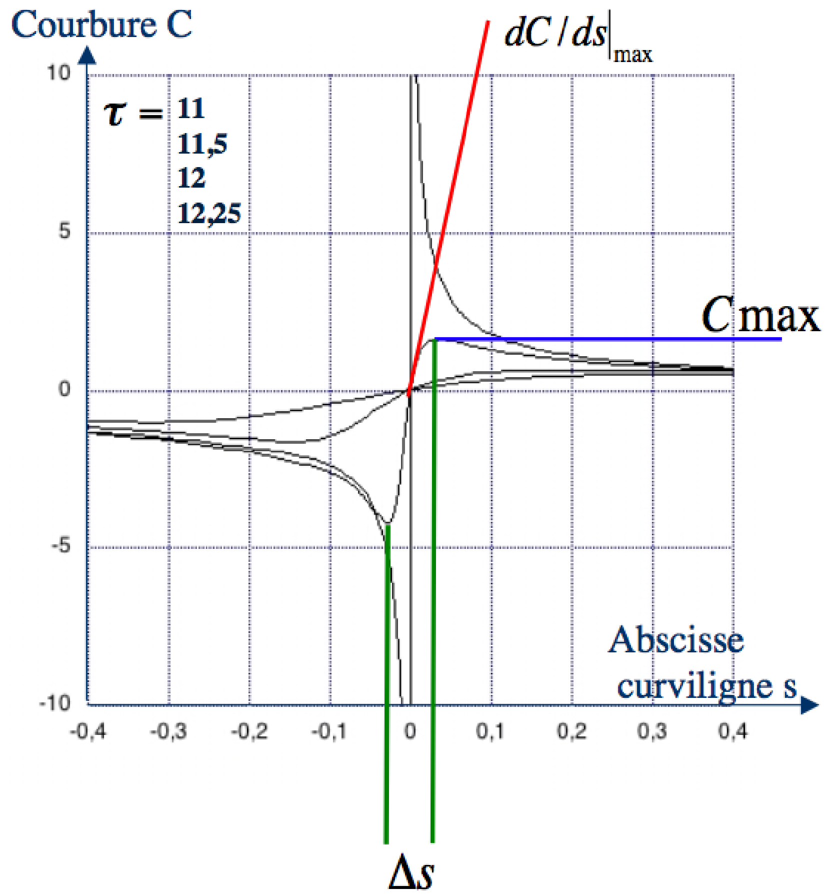


Different shapes for $k = 0.1, 0.5$ and 0.9 (from left to right)

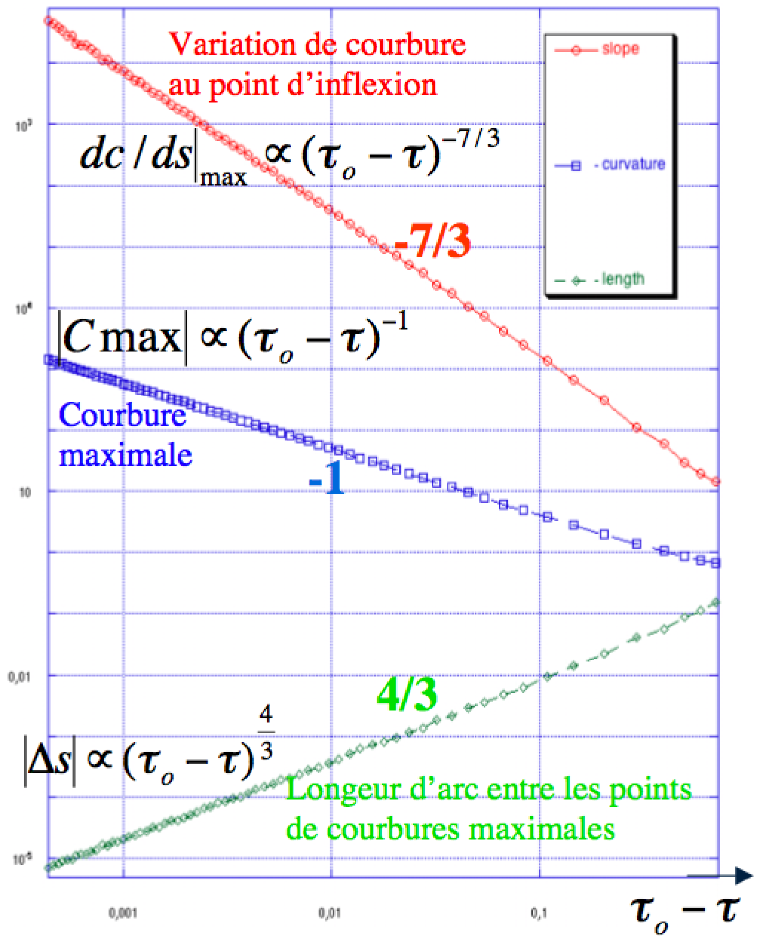
Streamlines in the hot gas



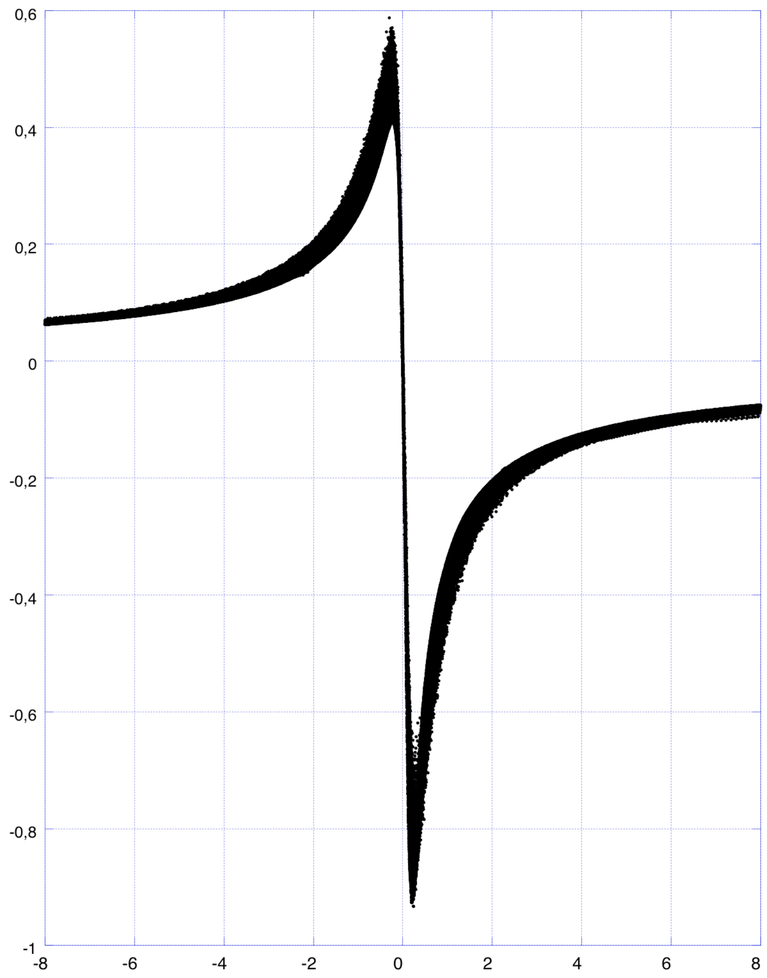
Finite-time curvature singularity



Scalings around the singularity



Rescaled curvature



Lack of symmetry in the scalings : hypothesis

- $\Delta\varphi = 0$, but x and y scale differently.
  Possible issue : different self-similar order for x en y (or s and n).

- Second type singularity/self-similar solution.

- Numerical accuracy $\Rightarrow 3^{rd}$ degree interpolation of the complex potential.