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Stability and control of shear flows subject to stochastic excitations

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1. Flow and instability
2. Flow control
3. Stochastic methods
4. Some results from the thesis
5. Summary of the contributions

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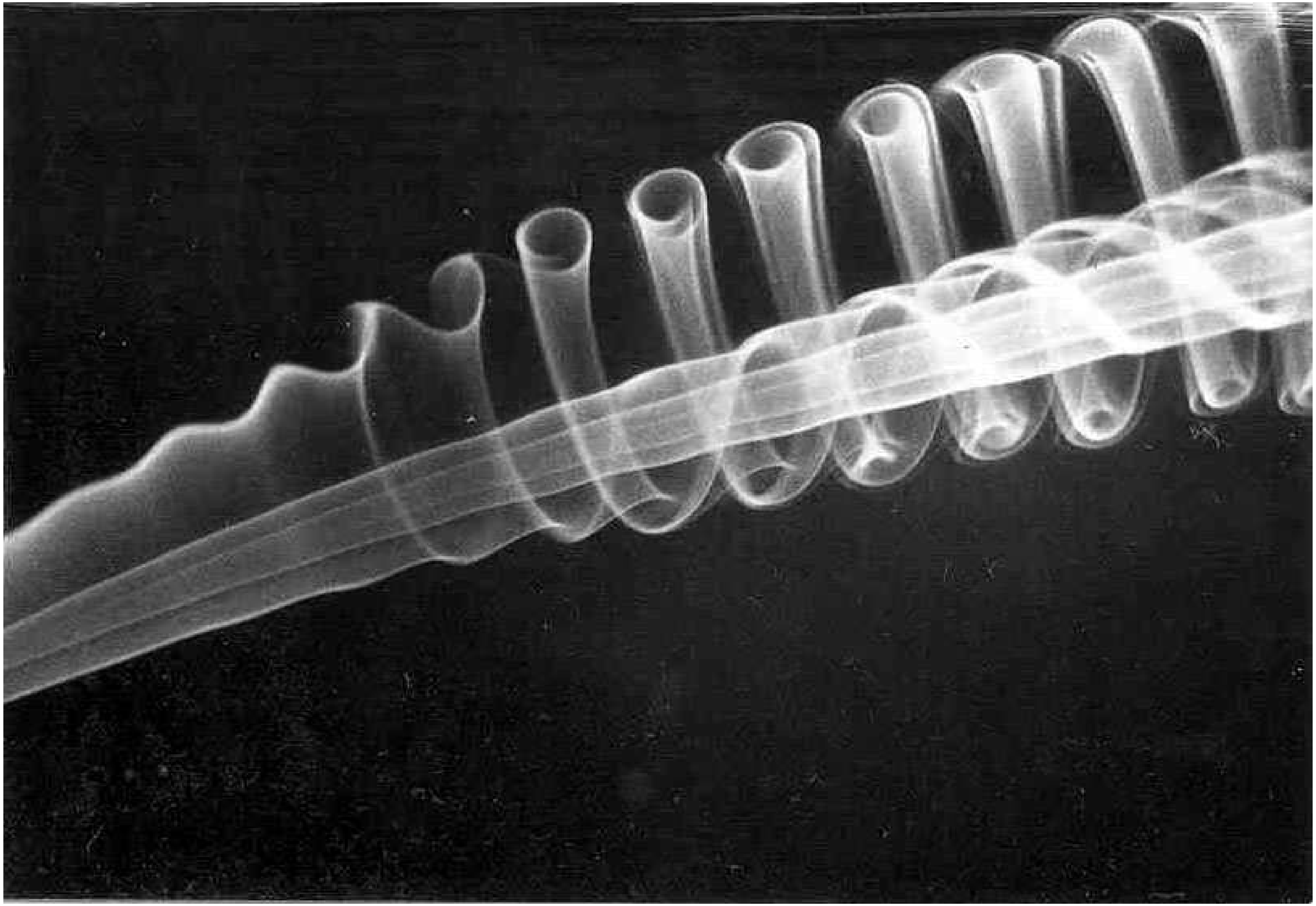

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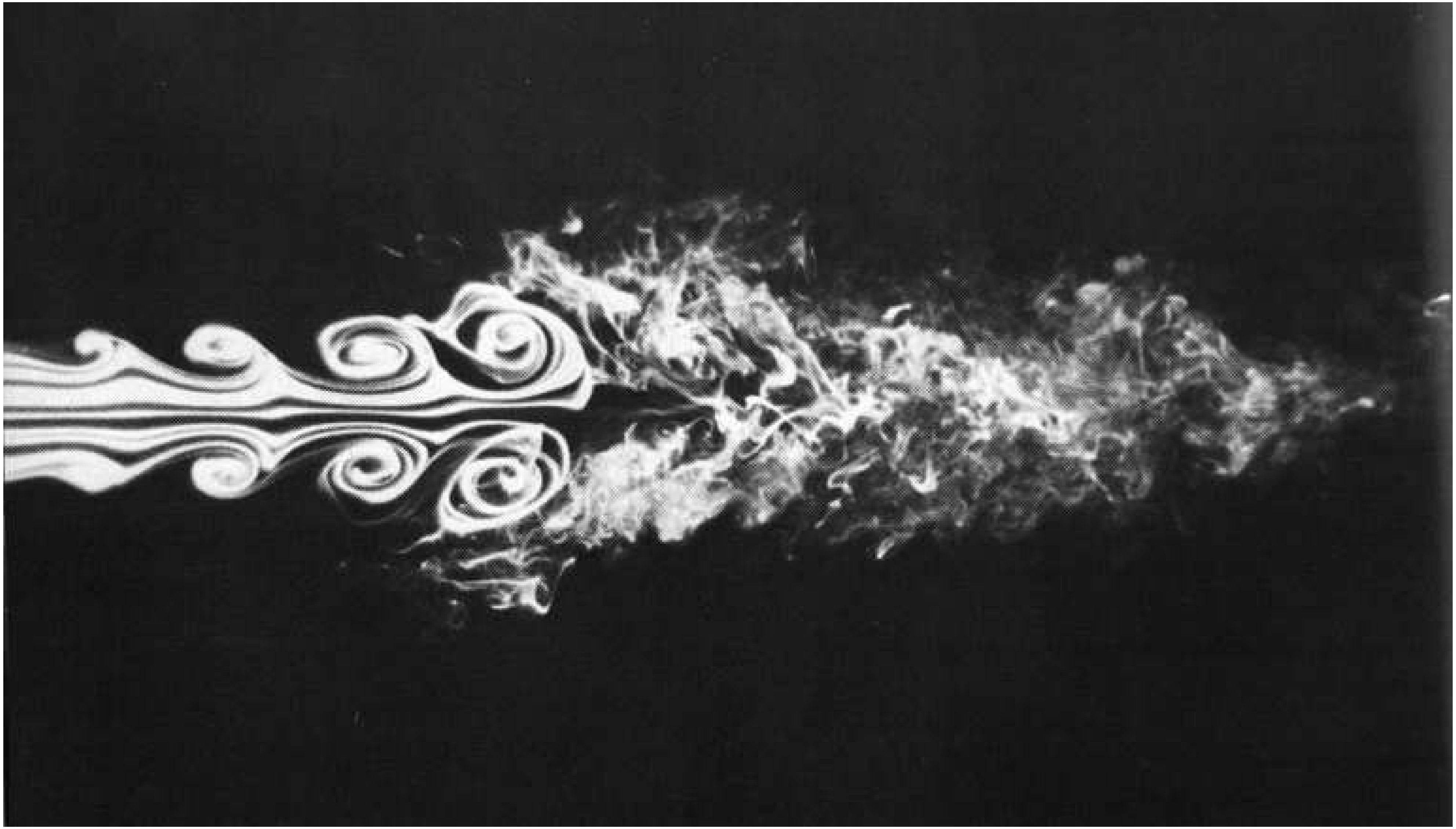

Navier–Stokes equations

Mathematical model for the motion of fluids:

$$\begin{cases} \partial_t u + u\partial_x u + v\partial_y u + w\partial_z u = -\partial_x p + \Delta u/Re, \\ \partial_t v + u\partial_x v + v\partial_y v + w\partial_z v = -\partial_y p + \Delta v/Re, \\ \partial_t w + u\partial_x w + v\partial_y w + w\partial_z w = -\partial_z p + \Delta w/Re, \\ \partial_x u + \partial_y v + \partial_z w = 0 \end{cases} \quad + \text{ boundary conditions}$$

u, v, w are the velocity components, p is the pressure.

Control

If a flow is stable when we would like it unsteady and erratic, or if a flow is unstable when we would like it regular and well ordered, we should be able to alter its dynamics. This is the concern of flow control.

Flow control

Act on the flow so that it is stable:

- Prevent transition on aeroplane wings: reduce drag
- Prevent generation of large amplitude acoustic waves
- Reduce flow related vibrations

Interesting problems on the way:

- Flow modeling
- Optimization of PDE
- Numerical methods for large scale systems

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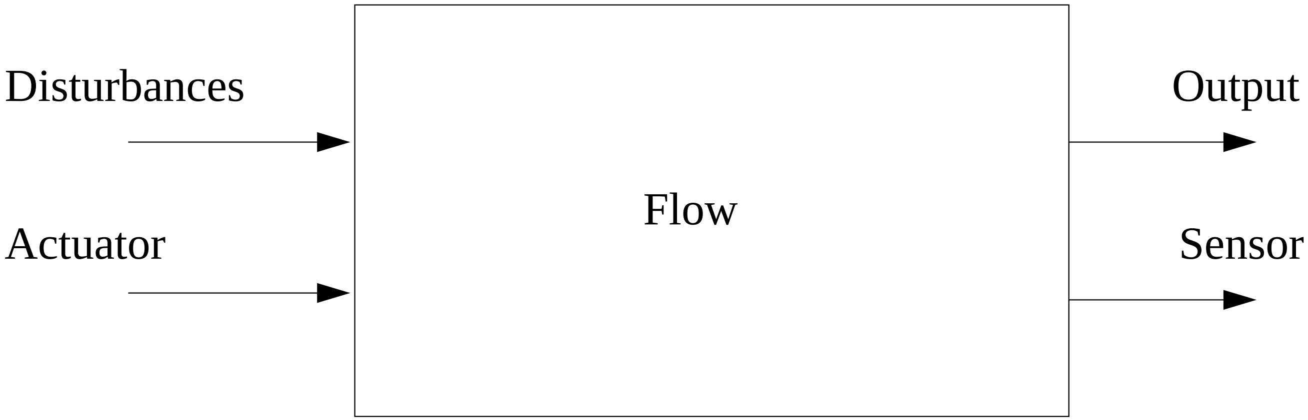
Actuators and sensors



Actuators to act on the flow state:

- Blowing and suction at the wall
- Wall deformation
- . . .

Sensors to measure the flow state:

- Skin friction
- Pressure
- . . .

Control theory

Design problems for flow control:

- What are the disturbances?
- Which type of model should we use?
- What actuators should we use?
- What sensors should we use?

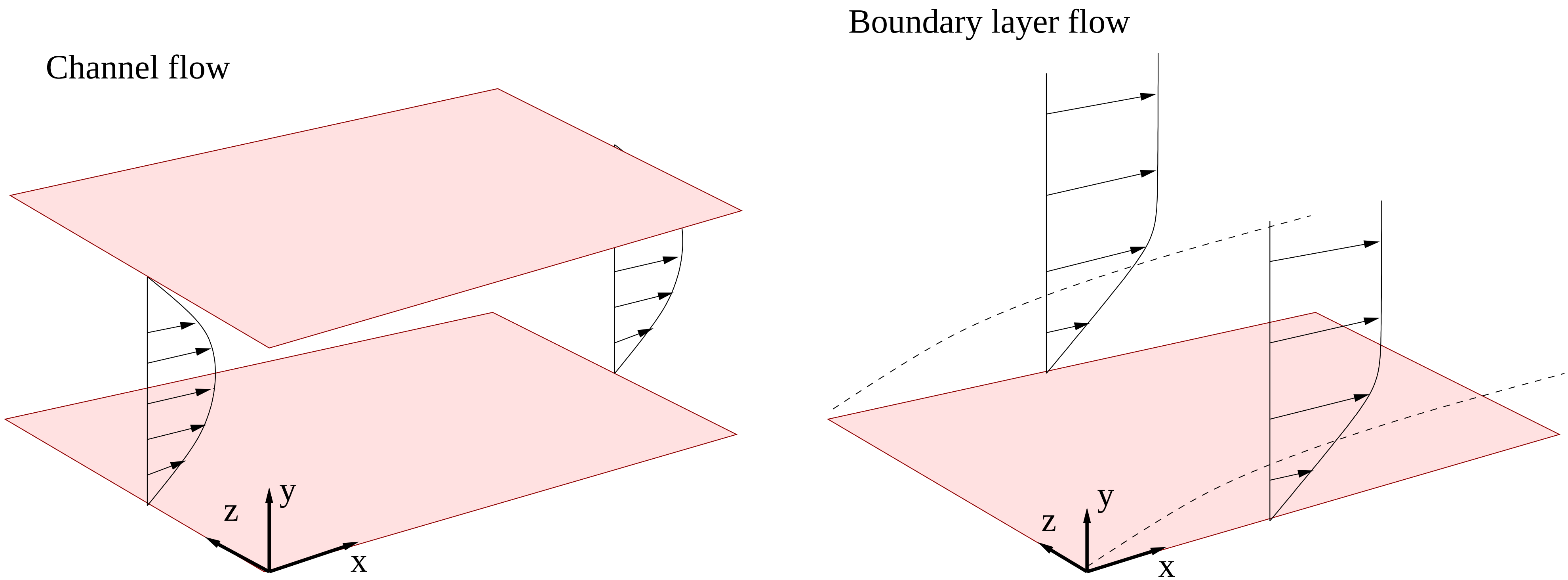
Technical problems:

- PDEs: more care is needed
- Discretized system are very large
- Little experience in describing disturbances, designing sensors and actuators

Spatial invariance

Streamwise x and spanwise z directions are invariant

(Dynamics, sensing, actuation, cost function, disturbances)



3D problem $\rightarrow$ many parameterized 1D problems

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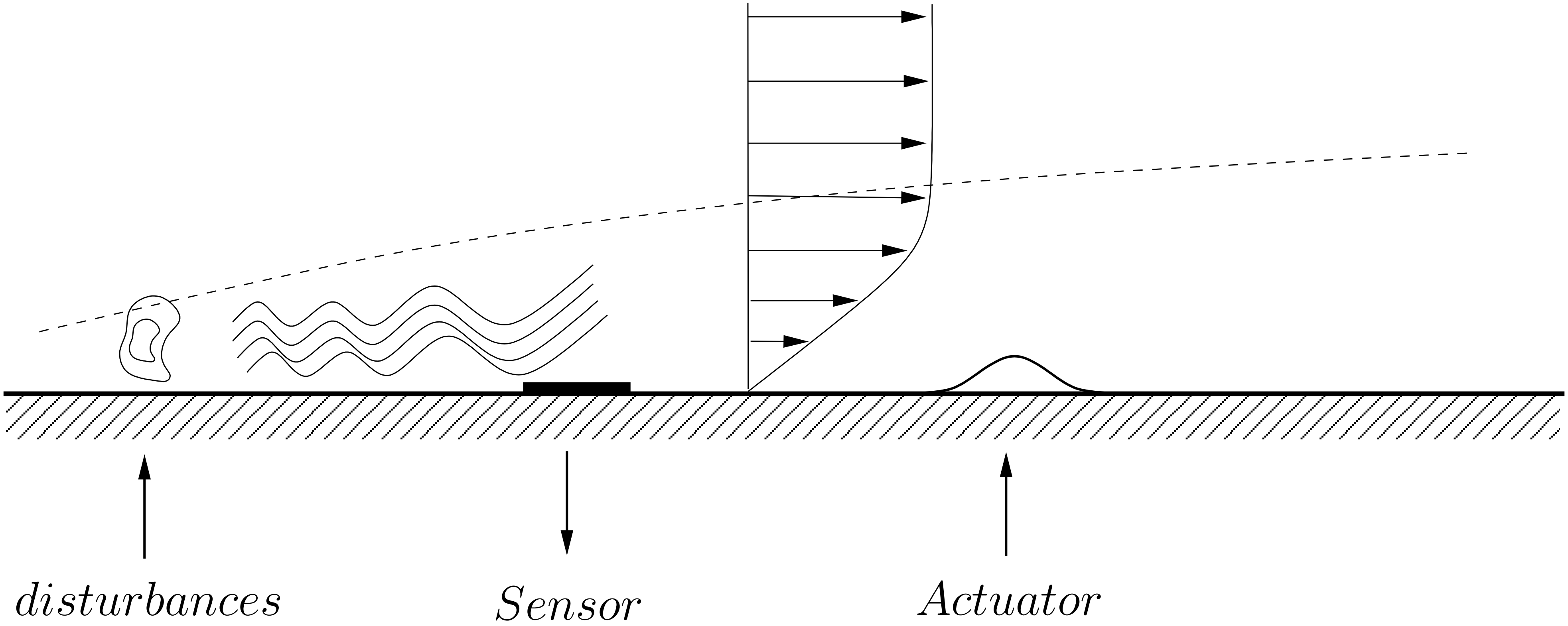
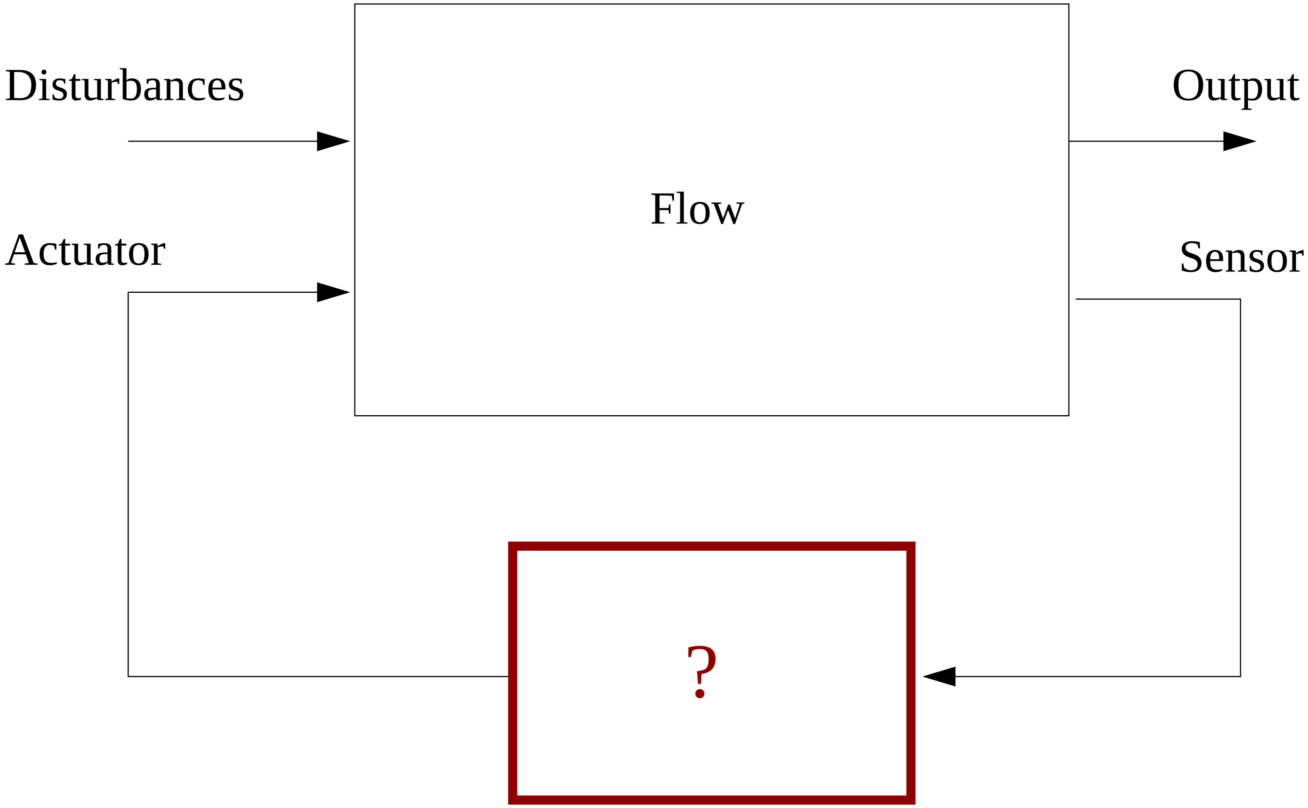


Feedback



Use optimization for the feedback law

Control and estimation

$$\text{system}\begin{cases}\dot{q} = Aq + B_1 w + B_2 u, \\ y = Cq + g\end{cases}, \quad \text{estimator}\begin{cases}\dot{\hat{q}} = A\hat{q} - L(y - \hat{y}), \\ \hat{y} = C\hat{q}\end{cases}$$

Full information control:

Feedback: $u = Kq$

Closed loop: $\dot{q} = \underbrace{(A + B_2 K)}_{A_c} q + B_1 w$

A_c **is stable?**

Estimation:

Estimation error $\tilde{q} = q - \hat{q}$:

$\dot{\tilde{q}} = \underbrace{(A + LC)}_{A_e} \tilde{q} + B_1 w - Lg$

A_e **is stable?**

Output feedback control: $u = K\hat{q}$.

Upstream of this thesis

- Bewley&Liu, (1998) :

  Optimal and robust control and estimation of linear paths to transition.

- Högberg, Bewley& Henningson (2003) :

  Linear feedback control and estimation of transition in plane channel flow.

Obtain feedback control law for 3D channel and boundary layer, apply to DNS

Good performance for full information control, but estimation to be improved...

In this thesis: 1)

- HŒPFFNER, CHEVALIER, BEWLEY, & HENNINGSON (2005) :

  State estimation in wall-bounded flow systems. Part 1. Perturbed laminar flows.

- CHEVALIER, HŒPFFNER, BEWLEY & HENNINGSON (2006) :

  State estimation in wall-bounded flow systems. Part 2. Turbulent flows.

- CHEVALIER, HŒPFFNER, ÅKERVIK, HENNINGSON (SUBMITTED) :

  Linear feedback control and estimation applied to instabilities in spatially developing boundary layers

Improve the estimation, using stochastic description of external disturbances

External disturbances

Flow instability/sensitivity + external disturbances $\rightarrow$ waves, patterns, turbulence...

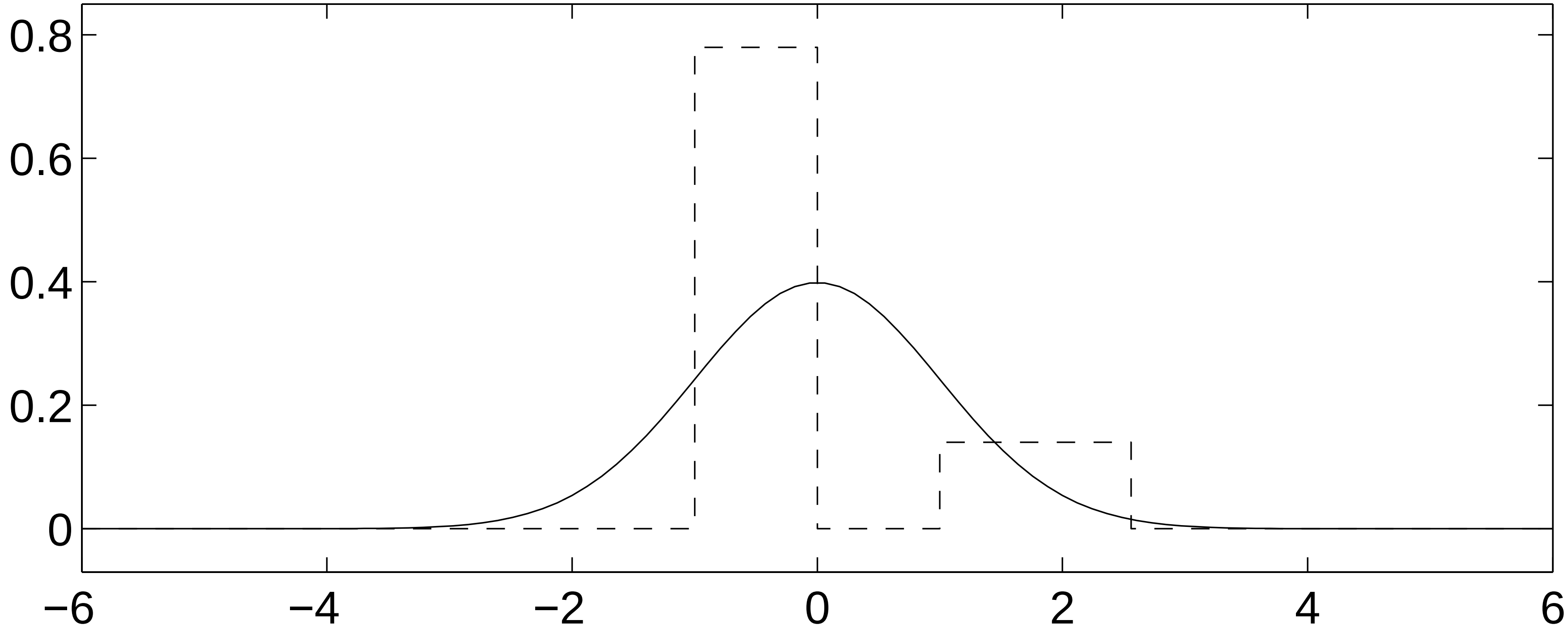
Typical external disturbances:

- Wall roughness
- Acoustic waves
- Free-stream turbulence
- ...

Stochastic approach

Disturbances are unpredictable, erratic → use **probability/statistics**

Probability density function:



The disturbances are stochastic → the state is stochastic

Lyapunov equation

For a linear system

$$\begin{cases} \dot{q} = Aq + Bw \\ y = Cq \end{cases}$$

with external disturbance w with covariance W, the state q has covariance P

$$AP + PA^{+} + BWB^{+} = 0$$

And the covariance M of the output is

$$M = CPC^{+}$$

From statistics of the disturbances, get statistics of the state

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Extract the mean energy

From flow statistics P, extract the **mean energy**

$$E_K = \mathrm{Tr}(P)$$

System is sensitive or unstable $\longrightarrow$ large energetic response to external disturbances

Control:

$$\dot{q} = \underbrace{(A + B_2 K)}_{A_c} + B_1 w$$

Lyapunov:

$$A_c^+ P + P A_c + B_1 W B_1^+ = 0$$

Estimation:

$$\dot{\tilde{q}} = \underbrace{(A + LC)}_{A_e} \tilde{q} + B_1 w - Lg$$

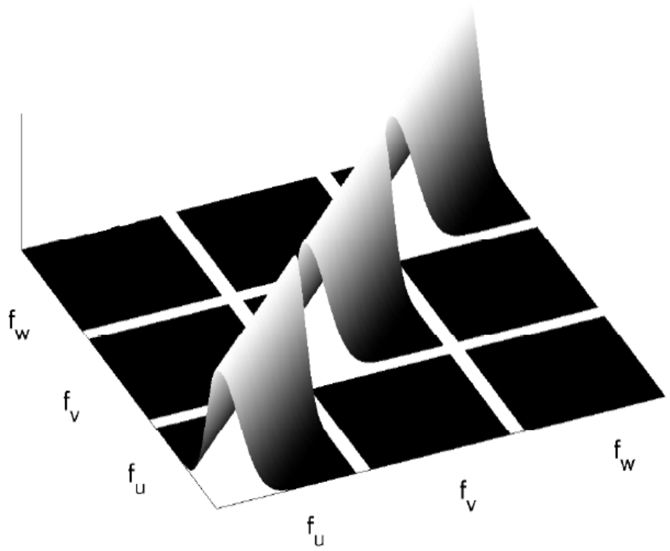
Lyapunov:

$$A_e \tilde{P} + \tilde{P} A_e^+ + B_1 W B_1^+ + LGL^+ = 0$$

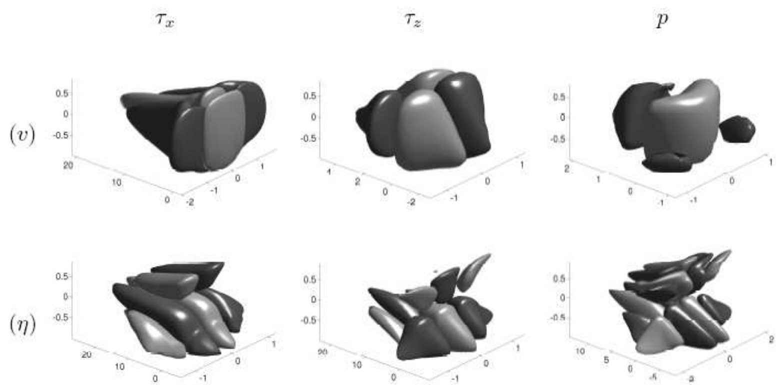
Optimal feedback K and L by solving two Riccati equations

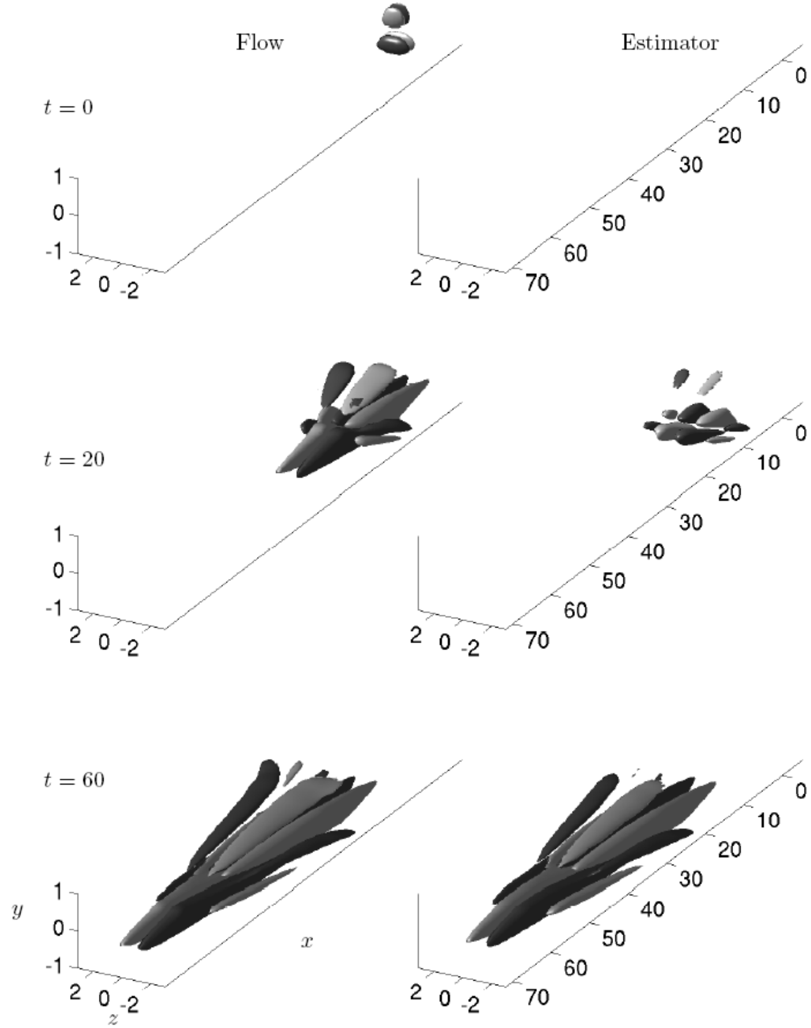
Estimation in laminar channel flow

Simple covariance model:



Estimation convolution kernels:

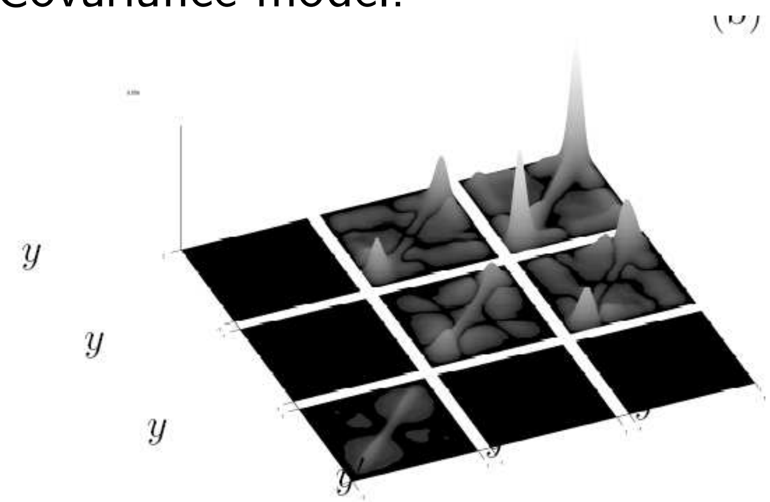




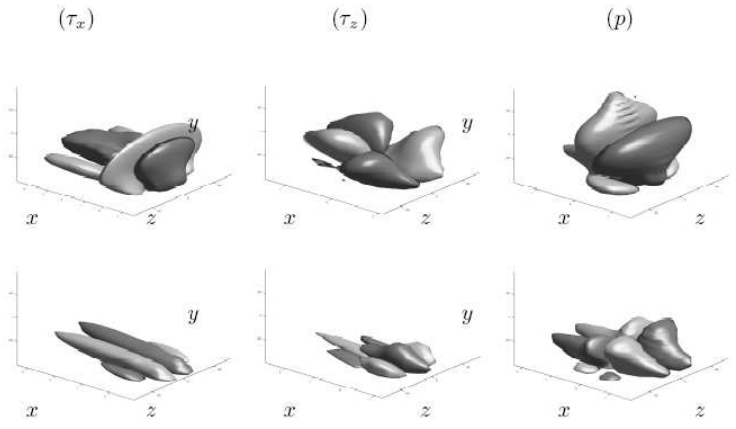
Estimation of initial condition

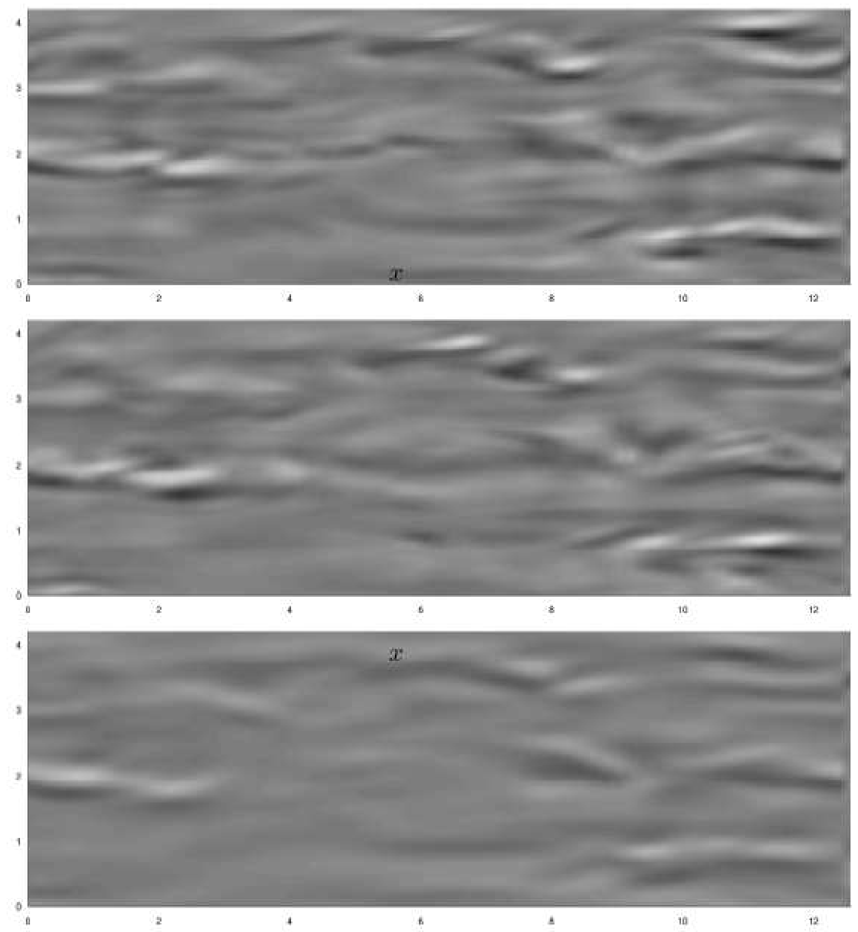
Estimation in turbulent channel flow

Covariance model:



Estimation convolution kernels:





Snapshot of flow/estimated flow

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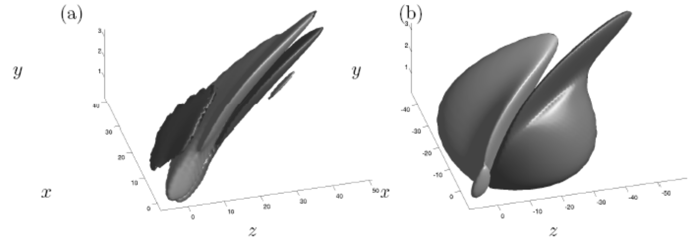
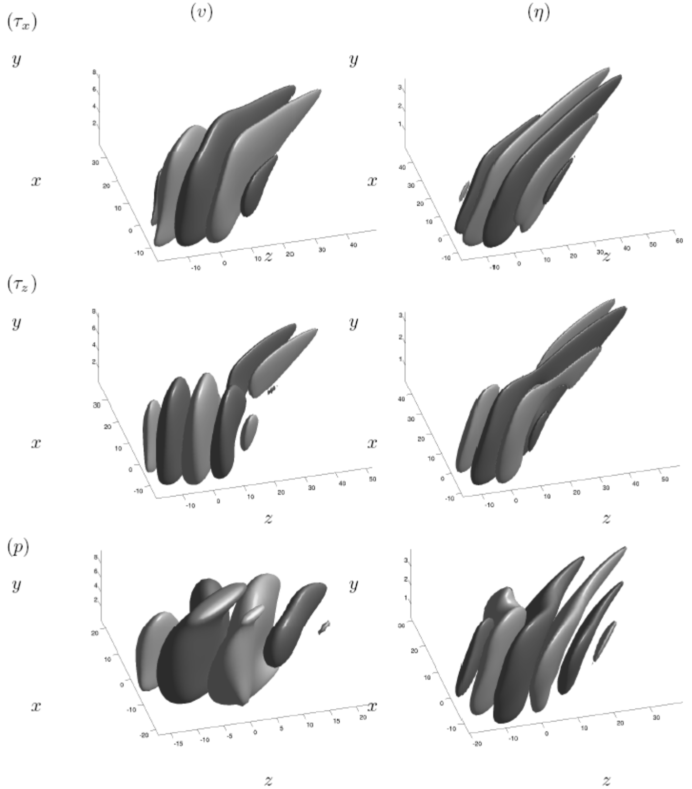
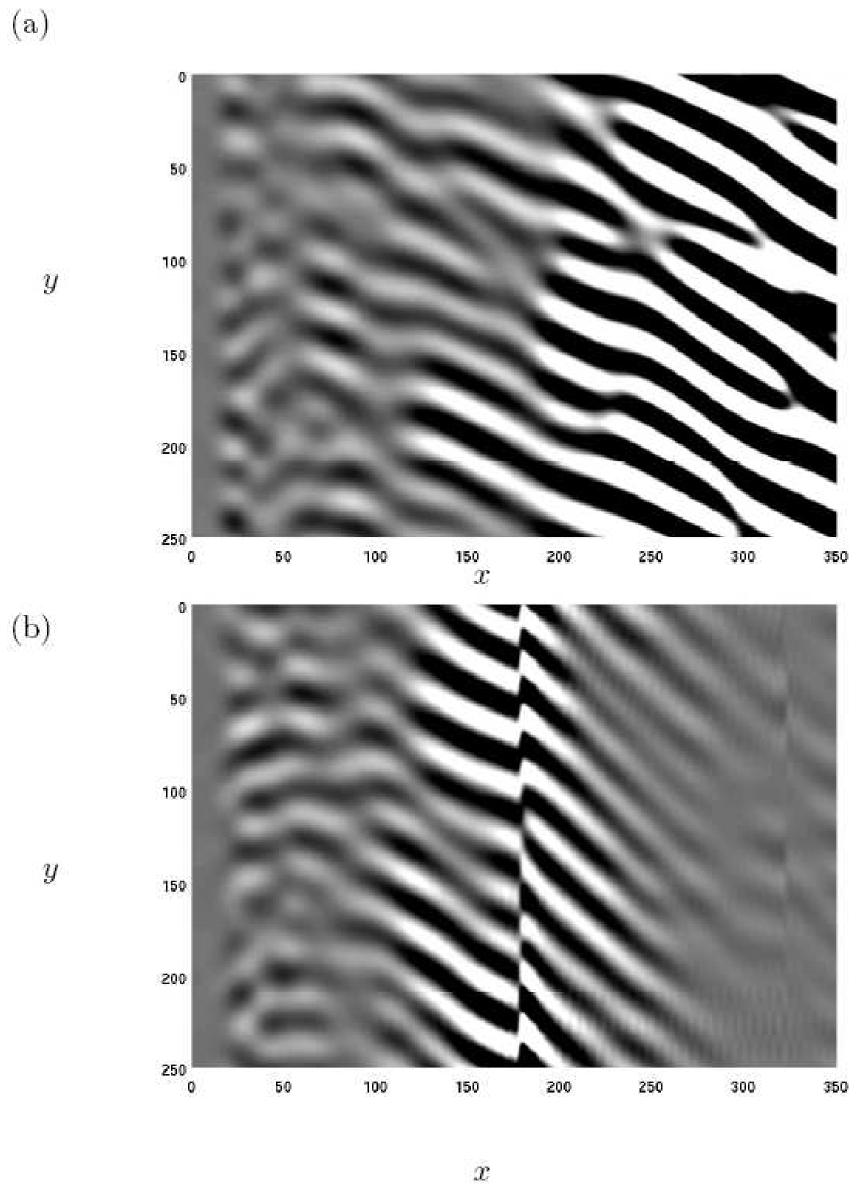
Estimation/Control of swept boundary layer

Control convolution kernels



Estimation convolution kernels





Wave growth: no control /control

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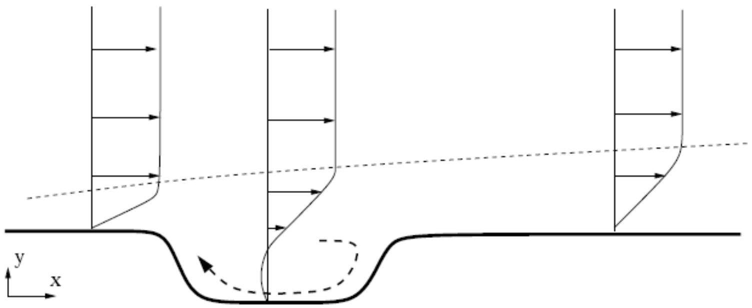
In this thesis: 2)

Hœpffner, Åkervik, Ehrenstein, Henningson. (2006):

Control of cavity-driven separated boundary layer.

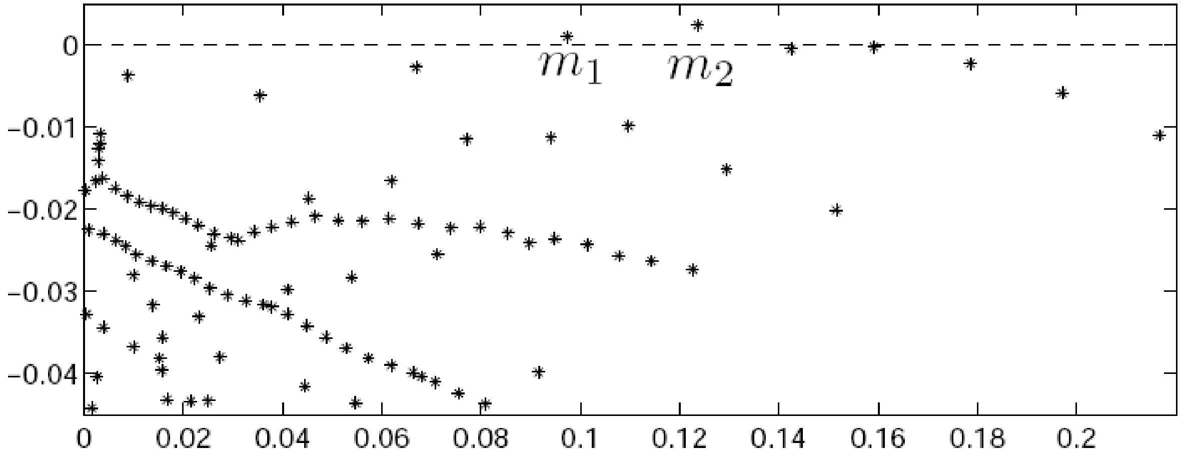
Control and estimation in 2D flow without spatial invariance: Model reduction using flow eigenmodes.

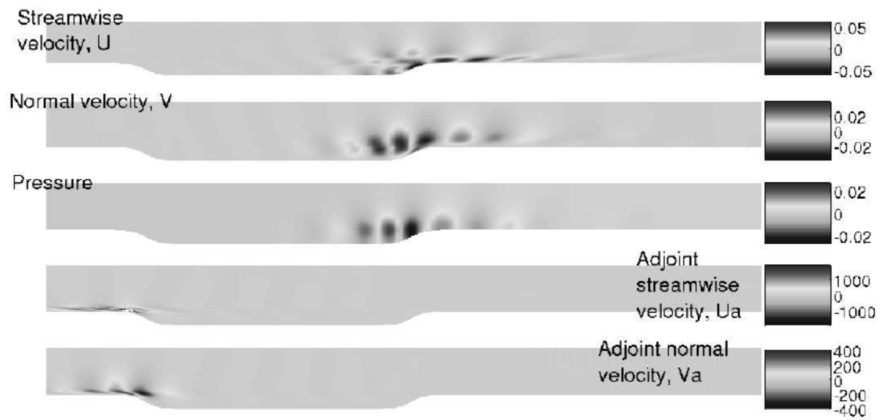


A first step for model reduction for control and estimation of large flow systems

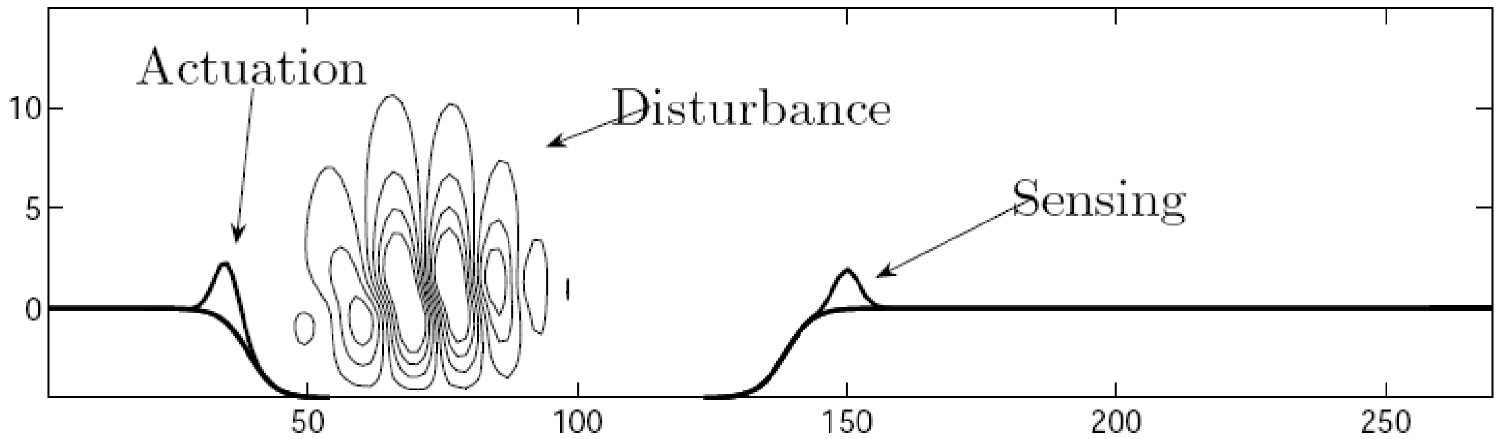
Cavity flow

Spectra / Least stable eigenmode:



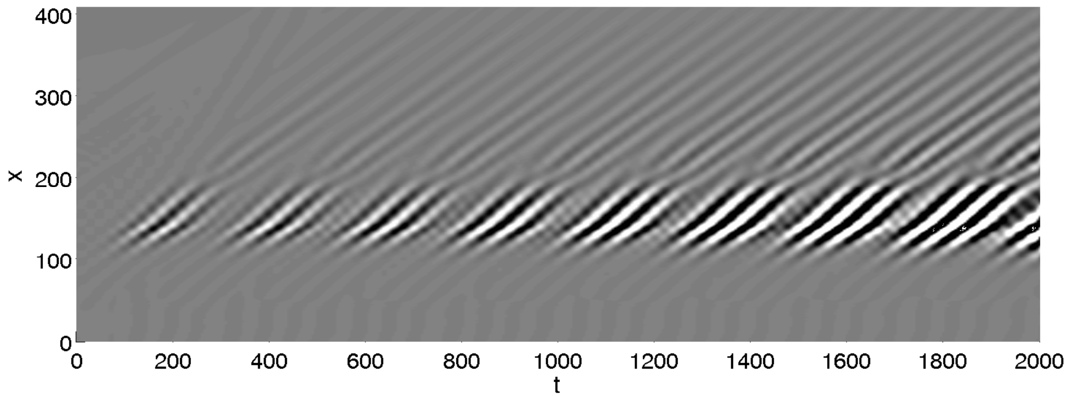


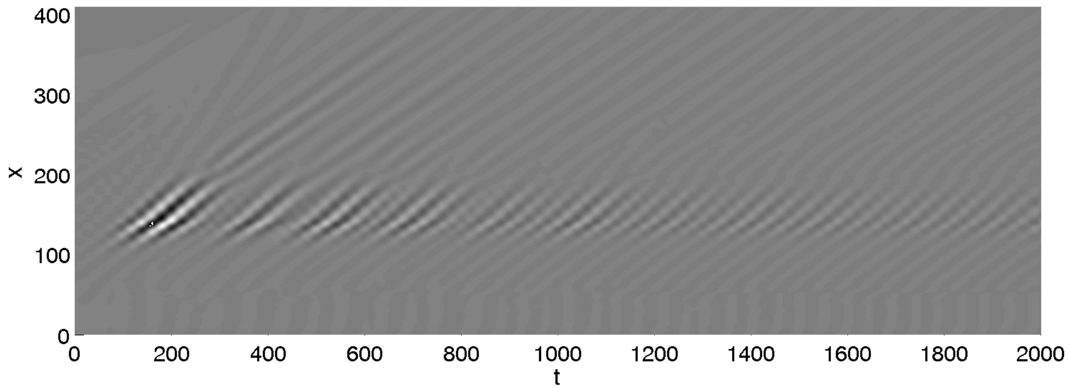
Control setting:



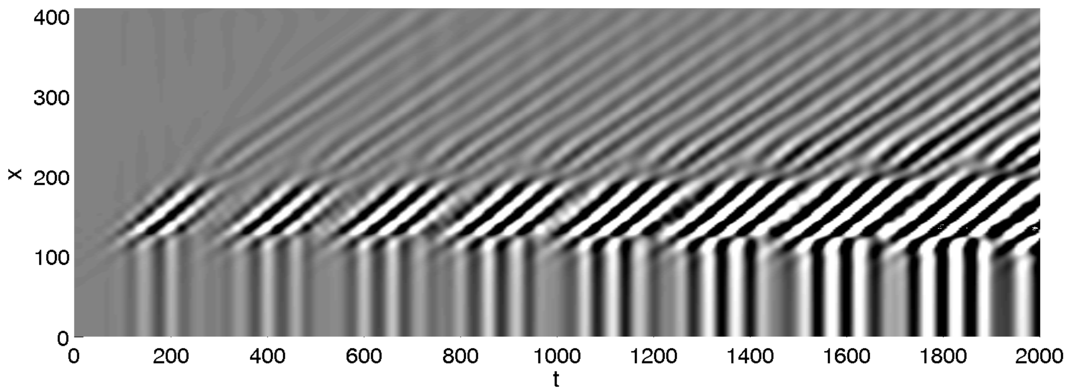
Controlled cavity flow

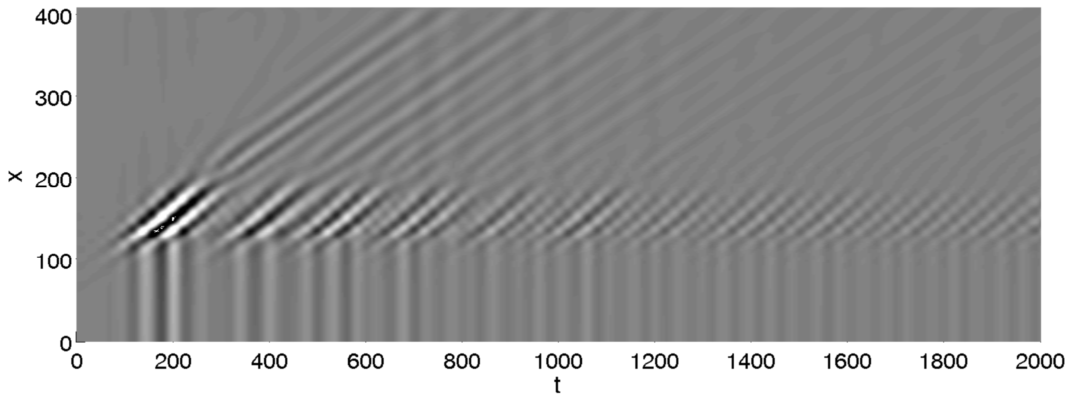
Wall normal velocity: no control / control





Pressure: no control / control





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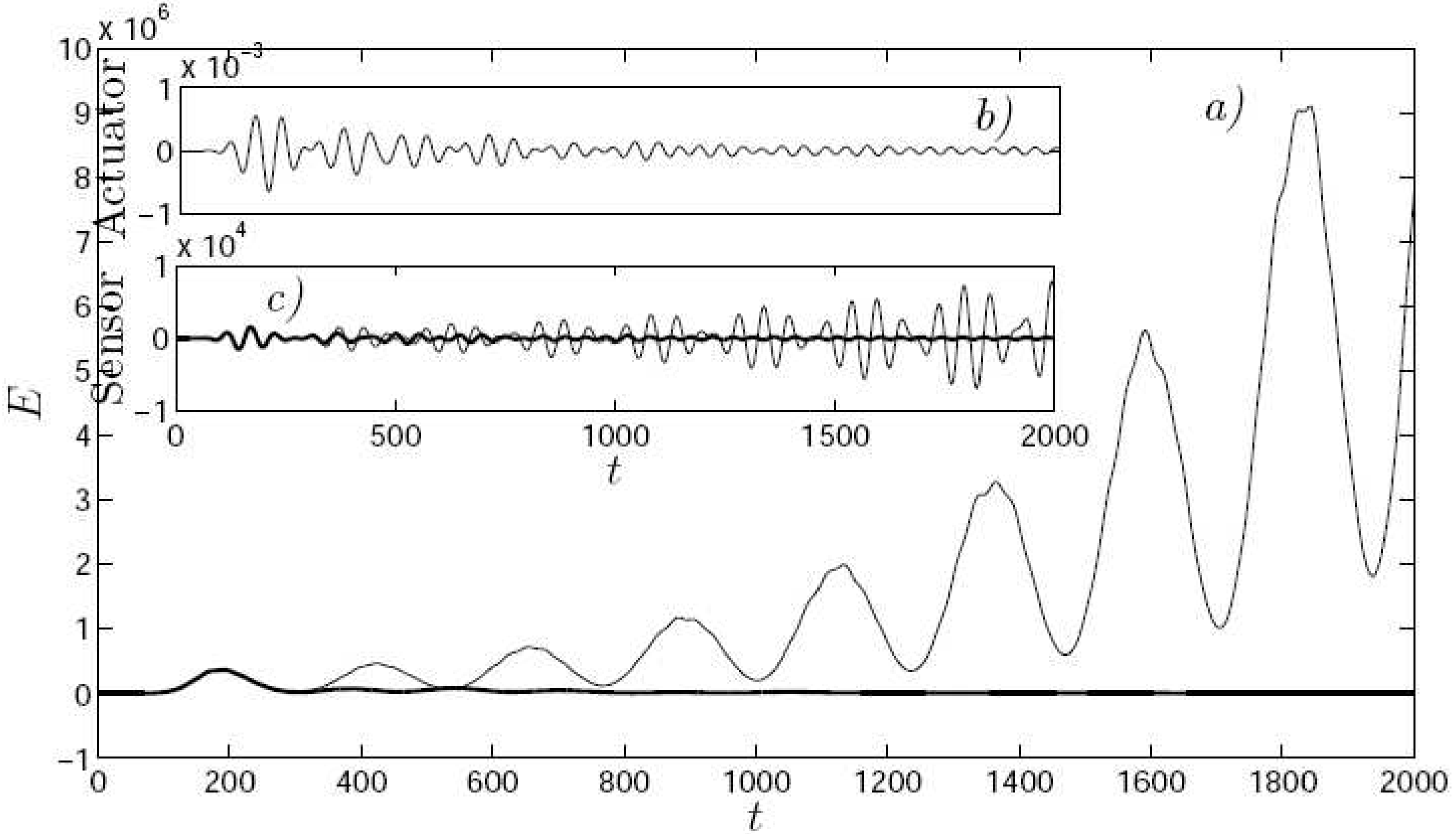
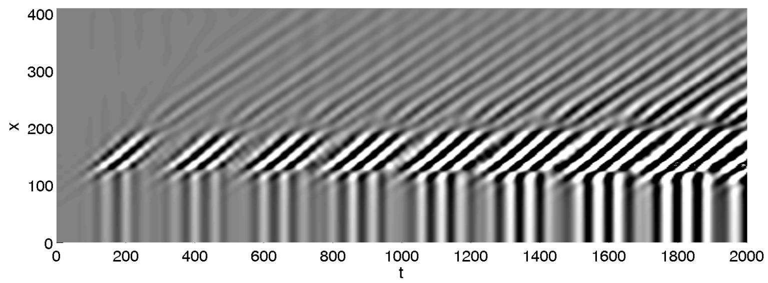
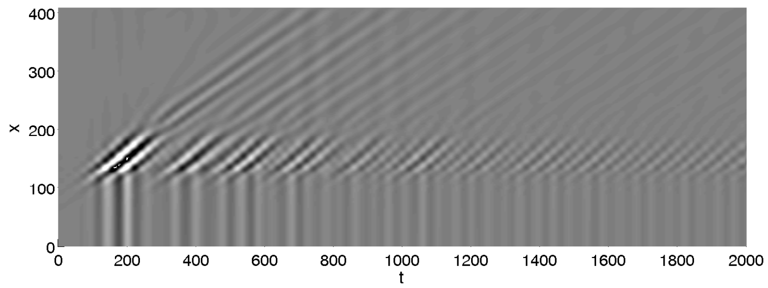
Controlled cavity flow

Energy evolution / actuation / sensing:







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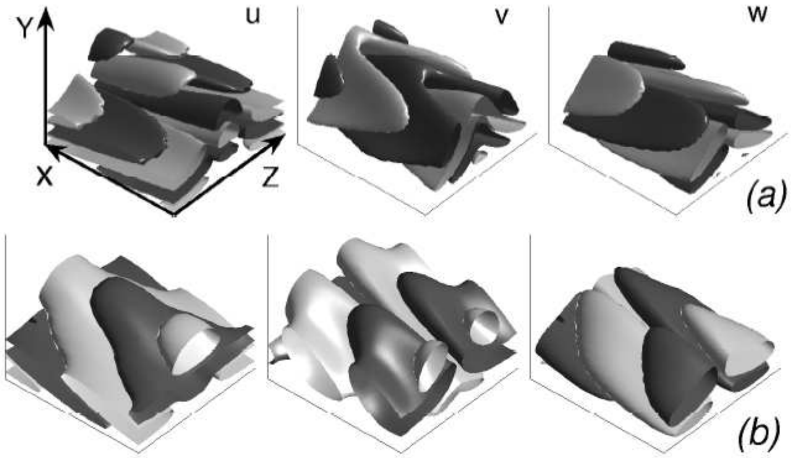
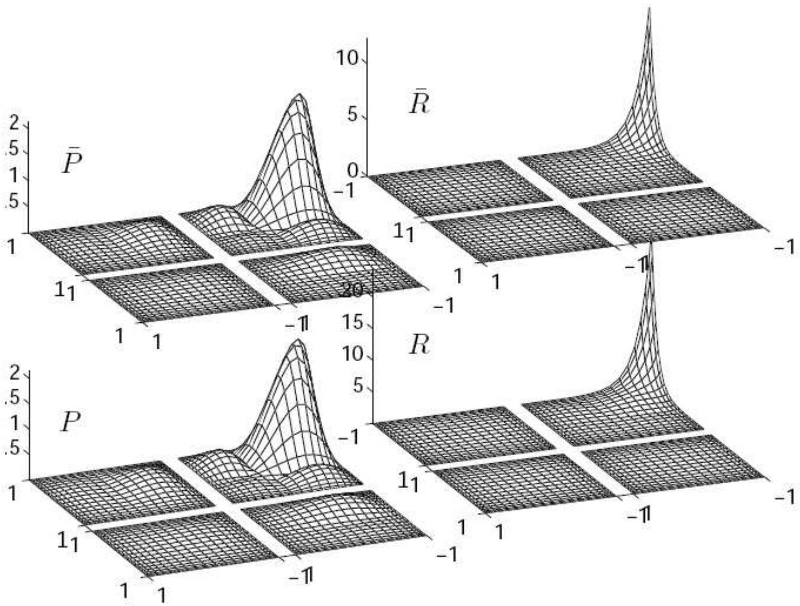
Additional results

Secondary transient growth: (paper 5)



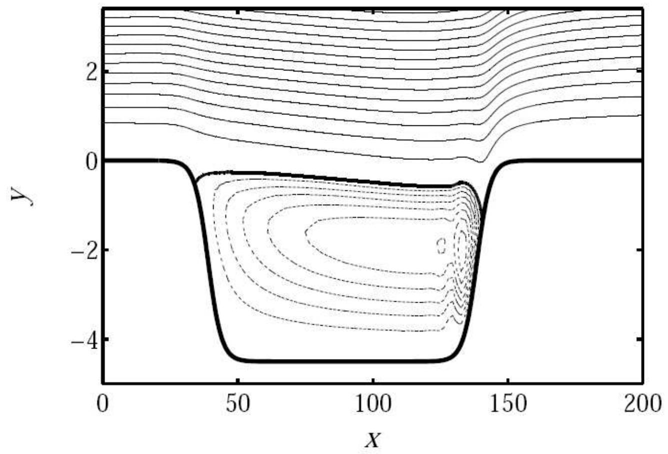
Covariance modeling using LMI: (paper 6)



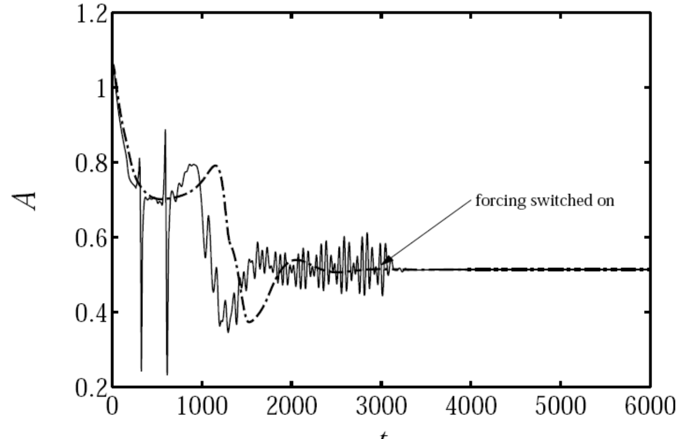
Steady solutions of NS: (paper 7)

$$\left.\begin{aligned} \dot{q} &= f(q) - \chi(q - \overline{q}) \\ \dot{\overline{q}} &= (q - \overline{q})/\Delta \end{aligned}\right\} .$$

Steady flow for cavity:



Convergence:



Conclusion

Main achievements:

- Improved the estimation, using stochastic description of external disturbances (paper 1,2)
- Applied to laminar and turbulent channel flow, + control of transitional boundary layer (paper 1,2,3)
- Computation of 2D cavity flow eigenmodes using Arnoldi method (paper 4)
- Model reduction based on flow eigenmodes for cavity flow control (paper 4)
- Secondary transient growth in streaky boundary layer (paper 5)
- Model external disturbance covariance using LMI (paper 6)
- Numerical method for computation of steady solutions of Navier–Stokes equations (paper 7)

Outlook:

- Better model reduction
- More numerical methods for control of large scale systems
- Robustness analysis and design
- investigate flow actuators
- Apply control in flow experiment