# Models for an alternative pole vault

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## Abstract

Pole vault is the athletic discipline of using a soft pole to jump as high as possible. The vaulter should run fast, and must carefully fit the length and stiffness of his pole to his size, strength and gymnastics ability. In this paper, we propose a spring-mass model for the vault as a tool to explore the possibilities in trajectories and performance.


## 1 Introduction

The scientific literature on pole vault is reviewed in [1, 2]. We can discern two main approaches: one based on experimental measurement of energies, forces, velocities on real vaulters, and the other one based on reproduction of real vaults using numerical models, using for instance optimization to account for the athlete muscular action. In the present paper, we wish to develop an alternative perspective: come back to a model of a simpler structure in order to enlighten structural aspects of the vault.

Running is a most straightforward manner to accumulate mechanical energy in athletics. Pushing each step forward the athlete may accelerate until all spent power eventually serves only to sustain the periodic running motion and cannot any longer be used for acceleration of the center of mass in its mean forward translation. A trained runner of weight 100 kg may develop about 500 joules at each step and reaches a speed of order 10m/s in about ten steps.

Mechanical energy accumulated in such a way may then be used for different purposes, such as for instance jumping. An ideal configuration for transformation of horizontal motion into height consists in running towards an incline. The runner reaches the incline at maximum speed $v$ and then slows down naturally as he ascends, reaching zero velocity at the height $h$ where the gravity potential energy equals its initial kinetic energy $mgh = mv^2/2$. Running at about 10 m/s, the incline may raise your center of mass at no extra energy cost to the height of 5 meters.

The usual way to think about the energy is based on the law for its conservation: the kinetic energy may be transfered to other types of energy like gravity potential energy for jumping. Unfortunately, the kinetic energy consists essentially in forward momentum: there is ideally no *energy*

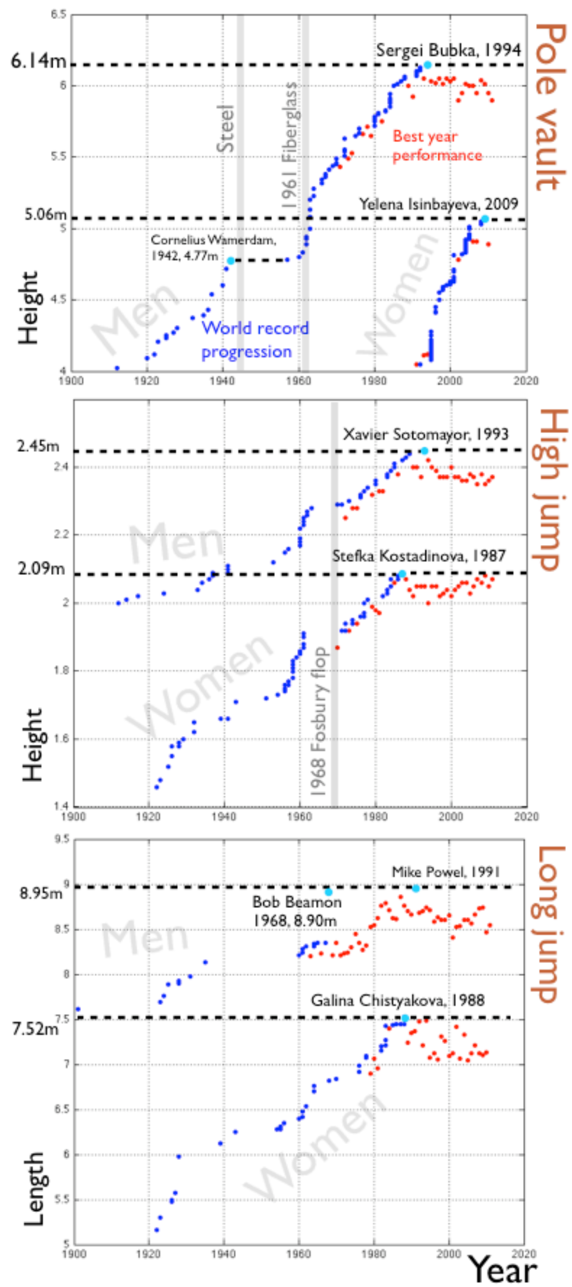


Figure 1: Evolution of the world record and best year performance.

cost for its redirection to vertical momentum, but there is the cost of a *force*. Humans are limited in energy and in power but also in force, so it is not straightforward to use the kinetic energy of running for the purpose of jumps: this is where discipline-specific skills and techniques come into play.

The long jumper runs as fast as possible and must apply during his last step the vertical force necessary for a free fall of a duration as long as possible. The effort of this last step is key to the exploitation of his acquired horizontal translation. A long-jumper running at a speed of 10m/s must fly for one second to reach a length of ten meters. For this, his center of mass initially at one meter above ground must raise to an apex at 1.8m and then down to the sand pit. Weighing 100 kg, this will cost his last step a price of 800 Joules in potential gravity energy. The arms and torso may participate to this effort in an upward projection just prior to the jump. The endeavor of the high jumper is essentially different: he may use its acquired kinetic energy only to the extent of its ability to redirect it to the vertical. He has no incline for this purpose: he is left with the structure of its skeleton and tendons as his only tool. He may use his last leg as a pivot to operate the rotation of his hip around the foot at the ground. To magnify this effect, he can operate a curved approach trajectory. The centrifugal force involved in this technique helps increasing the foot-hip angle to the vertical just before take off.

The pole vaulter on the other hand owns an elastic device to operate the transformation from horizontal momentum to vertical momentum. The strain stored in the bending of the pole is essentially an artifact to elongate the time lapse under which this transformation occurs. Elongating this time lapse reduces the intensity of the redirection forces needed from the jumper, and thus the lost power and potential for injury. On the other hand the many degrees of freedom in the motion of the pole and the time scales associated with its elastic behavior make it a device hard to master.

Figure 1 shows the evolution of the performance under the last century for pole vault, long jump and high jump. These three jump disciplines are similar in that the male world records were set about twenty years ago. Twenty years of training and thinking. The pole vault is maybe the most interesting one for scientists and engineers since it is the most technical. We know as well from history that great performance improvement can be achieved by a better choice of material of the pole: from wood to bamboo, then steel and now carbon and glass fibers [1, 2].

The first task of the jump is to run as fast as possible to acquire the largest horizontal kinetic energy just before take off. Holding the pole (about three kilos in weight and five meters in length) is a handicap to running fast, since the arms cannot play their role of balancing the body. The vaulter then plant the pole in the box and takes off. The *box* is the cavity where the athlete plants his pole. The take off angle is typically about 20 degrees to the horizontal. When planting, the vaulter applies a bending moment with its arms to avoid the large force and shock of the initial buckling of the pole. The stiffness of the pole is traditionally specified by its Euler buckling load, which typically corresponds to about the weight of the vaulter. Part of the running energy is lost in the shock of the pole: the body muscles react to the shock in a way which can be modeled as a heavily damped oscillator. Much of the performance improvement of the carbon and glass fiber poles can be traced back to this planting event, see [3]. The loss of velocity associated with the pole planting and take off amounts to about 2m/s, to be compared to about 10m/s of the running speed. The vaulter then strives along his flight trajectory, rotating his body about his hand grip in order to approach the crossbar with its feet first. He releases the pole once it has recoiled: all the bending energy has then been retrieved to the jumper. The vaulter exerts a strong effort during the flight, which amounts to about 20 percents of the final kinetic energy. For a description of the different phases of the jump, and experimental measurements of world-class vaulters energies and trajectories, see [2].

The pole has three roles. First it is used to redirect the velocity: from horizontal momentum to vertical momentum. It should be designed such that this redirection be as smooth as possible: shocks lead to damping of energy in the vaulter's muscles and vibrations in the pole. The planting should be smooth and the forces and momentum exerted on the vaulter during the flight should also be moderate. Shocks and large efforts also lead to injuries. A vaulter can master the technicalities of the jump only through a long training, which is hazardous if the required moves are a violence. The second role of the pole is less obvious: vaulters can improve their performance by running faster and vaulters are trained for this, but running fast is not specific to this discipline. The decisive part of the energy expenditure concerns the flight. The pole should be designed such that the flight sequence allows the strongest muscles to do their work, and to allow the coordinated use of the largest number of muscles. The third role of the pole is perhaps yet less obvious: the jump sequence should be *reliable*: the sequence should be chosen together with the pole characteristics such that a small imprecision in the parameters does not lead to a large error in trajectory. Accuracy is an issue: see [4].

We wish to identify the essence of using an elastic device to convert horizontal velocity into altitude: the details of this redirection. We do not model muscular action: our vaulter is a point mass with an initial horizontal velocity. The elastic device is as simple as possible: a linear spring with one end fixed at the ground. The goal of this paper is to guide the reader toward the diagram of figure 5 and give the elements of an interpretation of this diagram in terms

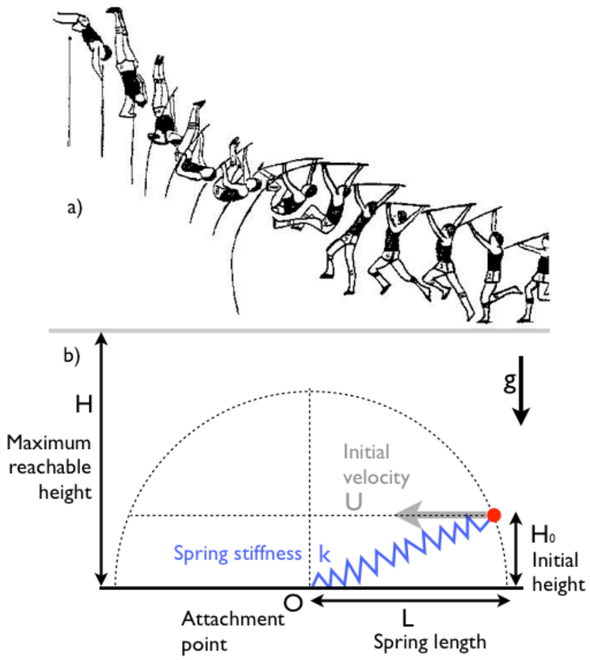


Figure 2: a) Typical sequence of a vault from [5], and b) our model system using a linear spring and a point mass with an initial velocity.

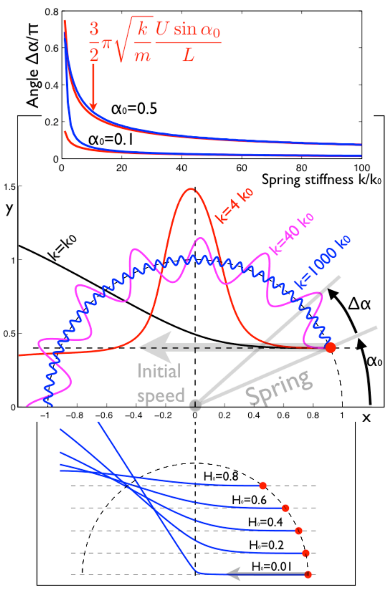


Figure 3: No weight: properties of the redirection with the spring. Center: trajectories while varying the stiffness; lower inset: trajectories while varying the initial height; upper inset: comparison in the deflection angle $\Delta\alpha$ with the time of rotation during three fourth of the natural period of oscillation of the spring/mass system.

of athletic performance.

## 2 Elastic redirection

The model system is illustrated in figure 2. A spring of free length $L$ and stiffness $k$, a mass $m$ with a initial speed $U$ at initial height $H_0$ above ground, under the action of gravity $g$. Six parameters with three independent dimensions give three dimensionless numbers [6]. It is useful to chose the three following numbers: the ratio of the spring length to maximum height reachable with the initial energy

$$\frac{L}{H} = \frac{L}{H_0 + U^2/2g},$$

the ratio of the spring stiffness $k$ to the stiffness $k_0$ such that complete compression of the spring stores all of the initial energy $(mgH_0 + mU^2/2 = k_0L^2/2)$

$$\frac{k}{k_0} = \frac{k}{(2mgH_0 + mU^2)/L^2},$$

and the ratio of the initial height $H_0$ to the spring length $L$

$$\frac{H_0}{L}.$$

Thus we decide to describe this system with two "heights" and one "stiffness".

The trajectory of this spring-mass system is integrated in time numerically from its initial condition using the equations

$$m\ddot{x} = k(L-r)\cos(\theta), \quad m\ddot{y} = k(L-r)\sin(\theta) - mg$$

with $r = \sqrt{x^2+y^2}$ and $\theta(t)$ the angle that the spring makes with the horizontal.

For more sophisticated numerical models of the pole vault, see [7, 8].

In a first step, we consider only the redirection of velocity and disregard the weight. There is thus only two dimensionless parameters. Figure 3 describes what may happen in this configuration. If the spring is very stiff, the mass follows a circular trajectory with rapid oscillations about the length $L$ of the spring. In terms of redirection, we are interested in the angle of the elastic deflection, represented on the figure as $\Delta\alpha$.

This figures gives the order of magnitude of the stiffness that corresponds to the different behaviors: rapid oscillations for $k = 1000k_0$, ample motion with a redirection of about 90 degrees for $k = 4k_0$, and a smooth redirection for $k = k_0$

In the limit of large stiffness, the redirection amounts to rotation at the velocity initially tangent to the circular trajectory $U\sin(\alpha)$ during the times it takes to perform $3/4$ of the oscillation of the spring $T = 3/42\pi\sqrt{k/m}$. The inset on top of figure 3 shows the comparison of this model with the computed trajectory.

These results are shown for an initial height $H_0 = 0.4L$, a moderate initial angle $\alpha_0$. The inset below the figure shows a variation of the redirection trajectory when varying the initial height $H_0$. If $H_0 = 0$ the motion is a simple horizontal oscillation of the spring: no redirection. If on the other hand a very small initial height is prescribed, we see a spectacular redirection where the mass is progressively slowed down until it nearly reaches the attachment point of the spring, changes direction in this neighborhood, then an other straight trajectory. This limit is realized for low initial angle when the spring can just store all of the initial kinetic energy $k \approx k_0$: speed is nearly zero in the neighborhood of the attachment point. It is clear that in this regime, the deflection angle will be a very sensitive function of all system parameters. We will see below that this peculiar limit may play a role in pole vault.

# 3 The choice of the trajectory

The vaulter runs as fast as possible ($U$) and his center of mass has a given height ($H_0$). Once these parameters set, there is a height $H$ which he may hope to reach. To attain his goal, he must chose the correct combination of pole stiffness $k$ and pole length $L$. The poles are manufactured with standardized length of about 5m, so the notion of pole length is usually termed as *grip height* since the portion of the pole above the upper hand is immaterial to the jump, see [9].

In the present section we consider the effect of gravity, so we recover the full family of the three dimensionless numbers. In this setting, there is one specific trajectory which will be of interest for comparison: the free fall parabolic trajectory at zero spring stiffness. In the following figures, this free fall trajectory is drawn as a red curve. A quantity characteristic of this trajectory is the horizontal distance covered until touch-down $h = U\sqrt{2H_0/g}$. An interesting question is: touch-down happens before or after the spring attachment point?

In figure 4, we show the trajectory leading to the vertical for given $H = 1$ and $H_0 = 0.4$, and vary the pole length $L \in [0.65, 1]$. For given length, a pole too stiff will send the vaulter back to the track (a dangerous regime), and a pole too soft won't help deflecting its trajectory from free fall. There is thus an optimal value of $k$ where the action of the spring force all along the flight trajectory is just so that the final velocity is vertical once the spring has fully recoiled.

For a short pole, the trajectory is quick at almost constant high speed. The forces exerted on the vaulter are large, the time given to the athlete to spend his muscular power is short. For a longer pole, the optimal stiffness is

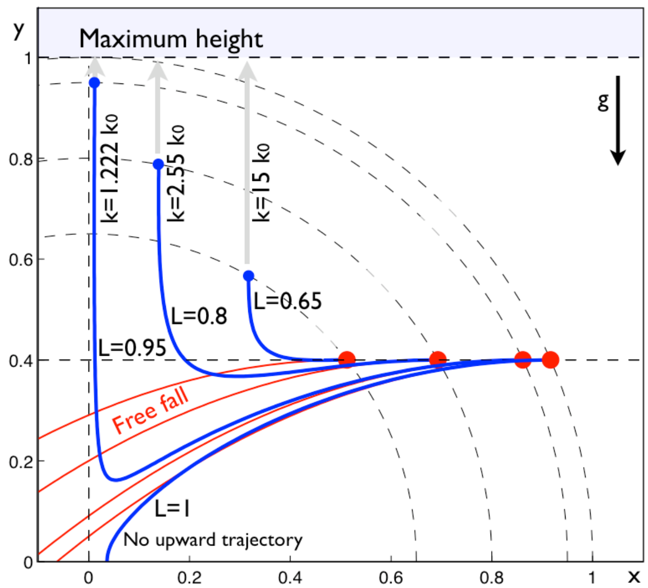


Figure 4: Optimal trajectories for four choices of the pole length. Here we keep the initial height and maximum height constant $H_0 = 0.4, H = 1$, and vary the pole length $L$.

lesser, and we see that the vaulter trajectory follows free fall for some time: his altitude declines, then raises when he gets closer to the attachment point.

Progressively increasing the pole, we see that we progressively tend toward a specific limit, where just as in figure 3-lower inset, the complete redirection happens in a small neighborhood of the attachment point. The optimal stiffness $k$ tends toward the critical stiffness $k_0$. See figure 4 for $L = 0.95$, the trajectory shows three successive regimes: a first flight nearly along free fall, redirection near $(0, 0)$, then a vertical raise. This critical limit seems to be linked with the approach of the free fall trajectory toward the origo.

For the present choice of $H = 1$ and $H_0 = 0.4$, the critical value of pole length $L$ is between 0.95 and 1, indeed, here for $L = 1$ there is no stiffness leading to an upward trajectory.

The landscape of this critical limit is shown from a bird's eye in figure 5, where we distinguish the qualitative properties of the jump when varying the initial height $H_0/L$ and the stiffness $k/k_0$. For this figure, we have chosen $H = L = 1$.

The red line is the optimal choice of stiffness: that leading to a vertical raise. The red region gathers good choices of the stiffness, leading to a final height above 0.8 times the maximum height. We observe that for $L$ larger than about $0.43H_0$ the device we gave ourself to fly above the crossbar allows no longer any upward trajectory. In addition, this region of the good choice gets thinner when the

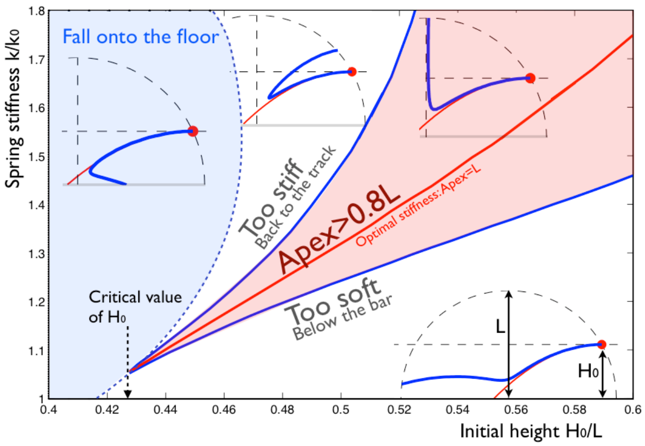


Figure 5: Landscape of the jump. Here we keep $L = 1$ constant, and vary the initial height $H_0$. The red zone correspond to choices of length and stiffness leading to good performance. In the blue zone, the trajectory leads to the floor.

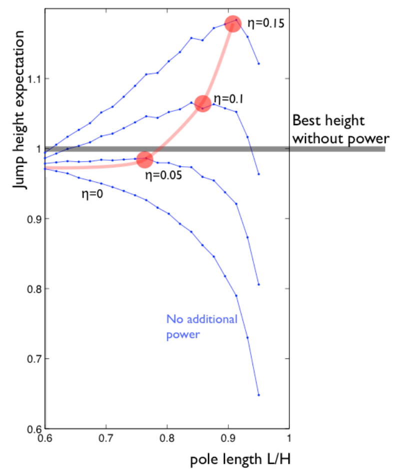


Figure 6: Expectation of apex height with imperfect vault and power spent during the flight.

pole gets longer: the trajectory becomes extremely sensitive to the choice of stiffness and pole length. Just as for the trajectory in 3, the decisive sequence of the deflection happens in a neighborhood of the attachment point. The great sensitivity to the parameters is a consequence of the smallness of this decisive region. On the other hand, for a choice of shorter pole (larger $H_0/L$), the window of good choice is large: the trajectory is safe.

# 4 Imperfect vaults

We may already attempt to draw a few athletic interpretations of figure 5. A soft and long pole is good because the flight is long (longer time for the athlete to spend his power) and the deflection forces are low (less damping, less injuries). On the other hand we see here a fundamental limit to how soft the pole can be. Our model suggests that this limit is closely linked to the horizontal reach of the free fall trajectory: is it below the box or after the box? Here, we chose a velocity $U$ initially horizontal, but vaulters instead chose not to rely entirely on the pole for redirection: they jump at an angle of about 20 degrees to reduce energy loss in the shock of planting the pole. This angle comes from the dilemma wether to lose energy in the planting shock or in the jumping shock (see [3]). Another consequence of the initial angle is—just as for long jump—to increase the reach of the free fall trajectory.

We need at this stage a quantitative tool to evaluate the contradicting effects of pole length on the expectations of a good performance. To account for the precision needed when using long and soft poles, we may describe the vaulter's action using probability. For instance here we assume that the spring stiffness $k$ is subject to a small inaccuracy in the form of a gaussian random variable with mean value the optimal stiffness and standard deviation $\sigma = (k_{opt} - k_0)/2$. We may then compute the expected jump height by integrating over the subsequent variation of $k$.

$$\overline{H} = \int_k P(k)H(k)dk$$

where $\overline{H}$ is the expected jump height, $P(k)$ is the probability density function of $k$ and $H(k)$ is the jump height for the stiffness $k$.

This way we may account for the imperfection of the trajectory. The expected jump height shall be of course lower than the optimal one. On the other hand a longer flight is a longer occasion for the vaulter to spend his power. Parameterizing this power by the variable $\eta$, the final jump height should then be $H_\eta = \overline{H} + \eta T$ where $T$ is the flight duration.

The results of these calculations are shown in figure 6. For $\eta = 0$, the maximum jump height is found for a short pole: when the pole is short, the trajectory is not much sensitive to imperfections. On the other hand, jump height expectation drops rapidly for larger pole length: with no added power and imperfections, a long pole is penalizing. On the other hand, we see when increasing $\eta$ that the pole length for best jump increases. This is simply because a long pole gives a longer trajectory and the spent power can then compensate for the increased sensibility to inaccuracy.

# 5 An alternative vault

The vaulter produces about 20 percent of the total energy during the flight (that is, not by running). The sequence of

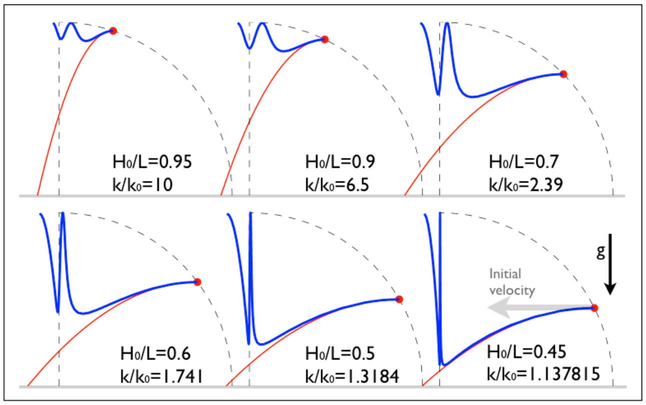


Figure 7: The alternative vault with rebound, using a pumping strategy to increase the vault duration.

moves is thus decisive in the final performance. One alternative for improvement of the performance would thus be to conceive a combination of the pole material and sequence of moves such as to maximize the flight time. The typical evolution in time of the athletic performance (figure 1) results from a continuous and progressive improvement of the sequence known as the best one. For pole vault, the most spectacular changes were made by an evolution in the material properties of the pole, going from stiff wooden poles to soft glass fiber. Introduction of fibers may be thought of as a change of paradigm when considering the quick evolution of the average technique and performance that came with it. Observing the best year performances of the last 20 years, it appears that we are in the need for a new paradigm in pole vault: any suggestion, possibly humble or unpractical might be considered with care in our present situation.

When an oscillator is forced periodically near its frequency of resonance, a great energy can be transfered through a small accumulation during each period of the oscillations. We may think of the present vaulting paradigm as three fourth of one period of the excitation (see figure 3-upper inset). In the geometry of our model, we may conceive a trajectory with seven fourth of periods of excitation as illustrated in figure 7. The difficulty of this sequence—just as for the limit of long poles described above—originates from the great accuracy required from the athlete.

## 6 Conclusion

In this paper, we have used a simple spring-mass model system as a tool to discuss the properties of the pole vault flight sequence. The spring plays the role of redirecting upward the initial momentum, and at the same time counteracts the effect of the weight. If the initial height of the vaulter is too low, the spring angle to the horizontal is small. In this situation, the vertical component of the spring force cannot counteract the weight along the trajectory: the jumper falls onto the floor. When aiming for higher jumps, the initial height become low in comparison to the pole length. If the length of the pole is increased in accordance with the ambition of the jump, then the flight trajectory tends toward a limit where a very large precision is needed at take off and during the flight. Indeed, in the regime of a long pole, the trajectory becomes increasingly sensitive to imprecision: such vault sequences require a high gymnastic ability of the athlete.

When the pole is short, the flight is short and reliable, but the planting shock is violent and the forces applied on the athlete are large. Also the duration of the flight is short, leaving little time to the athlete to perform his muscular action. On the other hand, if the pole is long, the stiffness of the pole can be accordingly lowered. This has a moderating effect on the planting shock and on the magnitude of the forces applied to the athlete during flight.

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