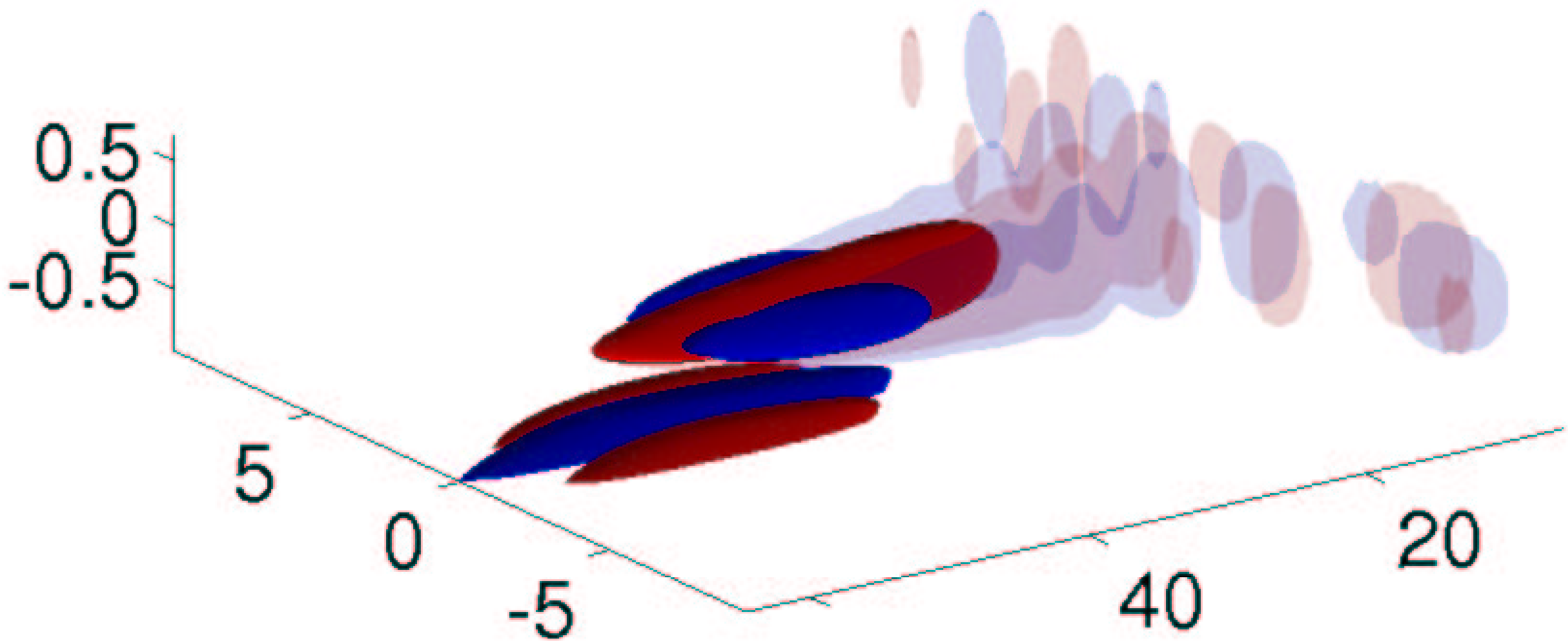
KTH Mechanics

Control and estimation of wall bounded flow systems



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Licentiate seminar

Team work

Follow-up of the PhD thesis of Markus Högberg.

In cooperation with Thomas Bewley (UCSD).

With Mattias Chevalier (PhD student),

and Espen Åkervik (Recently started PhD student).

Why using reactive control?

Act on the mean flow

And affect the stability of small perturbations

$\longrightarrow$ control effort of the order of magnitude of the mean flow

Act on the fluctuations

And prevent them from growing and disrupting the mean flow

$\longrightarrow$ control effort on the order of magnitude of the fluctuations

In a transitional case, the fluctuations are of much smaller amplitude than the mean flow.

Selected literature

- Hu H. H. & Bau, H.H. 1994 (Proc. R. Soc. Lond. A) *Feedback control to delay or advance linear loss of stability in planar Poiseuille flow*
  Use of proportional controller : $u(t) = Ky(t)$
- Högberg, M. 2001 (PhD thesis, KTH) *Optimal control of boundary layer transition*
  Decomposition of the feedback control into state estimation and full information control. Spatial localisation of the feedback law. DNS of parallel and spatially evolving flows.
- Walter, S., Airiau, C. & Bottaro, A. 2001 (PoF) *Optimal control of Tollmien Schlichting waves in a developing boundary layer*
  Open loop control acting on the flow fluctuations. Spatial framework (PSE). Use of the adjoint equations.
- Kim, J. 2003 (PoF) *Control of turbulent boundary layers*
  Review on the effort of feedback control for drag reduction.
- Lundell, F. 2003 (PhD thesis, KTH) *Experimental studies of bypass transition and its control*
  Suction through holes to hinder the streak's growth. Detection of the streaks by wall wire.

LQG (or $\mathcal{H}_2$) optimal feedback control

LQG for

Linear

Use of a linear model for the dynamics

Quadratic

A quadratic objective function

Gaussian

Stochastic disturbances to the flow

Fundamental achievement of control theory, further develloped into robust feedback control ($\mathcal{H}_\infty$)

The LQG control problem

Stochastic disturbances f, g, q_0

(External sources, sensor noise, unknown initial condition)

Actuation and sensing u, y

$$\begin{cases} \dot{q} = Aq + B_1 f + B_2 u, \quad q(0) = q_0, \\ y = Cq + g, \end{cases}$$

Feedback control

$$u = \mathcal{G}(y)$$

Which is the optimal mapping $\mathcal{G}$?

Solution of the LQG control problem

$$\text{Plant}\begin{cases}\dot{q} = Aq + B_1 f + B_2 u\\ y = Cq + g.\end{cases}$$

$$\text{Estimator}\begin{cases}\dot{\hat{q}} = A\hat{q} + B_2 u - v\\ \hat{y} = C\hat{q}.\end{cases}$$

$$\text{Feedback}\quad v = L\tilde{y} = L(y - \hat{y}), \quad u = K\hat{q}.$$

Decouple into an estimation problem and a full information problem. Solve two Riccati equations to get the optimal L and K.

Linear dynamics

Linearised Navier–Stokes equations

- **low amplitude:** good model
- **Moderate amplitude:** nonlinear terms lumped into external disturbances.
- **high amplitude:** we rely on the linear driving energy growth mechanism...

Temporal dynamics for each wavenumber pair (Parallel flow)

Sensing and actuation

- **Sensors:** Two components of the skin friction and pressure fluctuation at the walls.
  $\longrightarrow$ Each measurement gives information about different types of flow structures.
- **Actuators:** Blowing and suction at the walls. (wall transpiration)
  $\longrightarrow$ a small component of wall-normal velocity can interact with the mean shear and affect $\mathcal{O}(1)$ flow disturbances.

Assume dense array of sensors and actuators.

Model reduction

Reduce the number of degree of freedom in the dynamic model.

- Possible low-dimensionality of the processes at hand. (specificity of the scenario, of the disturbances)
- The dynamics of the controller that are not affected by the input and doesn't affect the output can be removed.

Quadratic objective

Minimise the kinetic energy of the flow fluctuations.
Other objectives? minimise non-normality for inst.

A very specific objective is easy to achieve : ex kill one single dangerous wave, by transferring its energy to harmless processes.

(the question is then: what is dangerous and what is harmless?)

Gaussian disturbances

Describe the covariance of an expected stochastic volume forcing to the flow state.

External sources: acoustic waves, surface roughness : receptivity in general.

Non-modeled dynamics : nonlinear coupling, non parallel effects...

A very specific disturbance is easy to estimate : ex TS waves : simply estimate the amplitude and phase of a known wall normal variation.

Optimisation

- Accommodate sensor noise and disturbance amplitude
(signal to noise)
- Accommodate control objective and control cost
(outcome and expenditure)

The optimisation is based on a Lagrange multipliers technique.

Results

- An example of controlled flow
- Estimation of laminar and low-Reynolds turbulent flow.
  (papers 1 & 2)
- Model reduction for control and estimation.
  (paper 3)
- A transfer function formulation of the feedback law.
  (paper 4)

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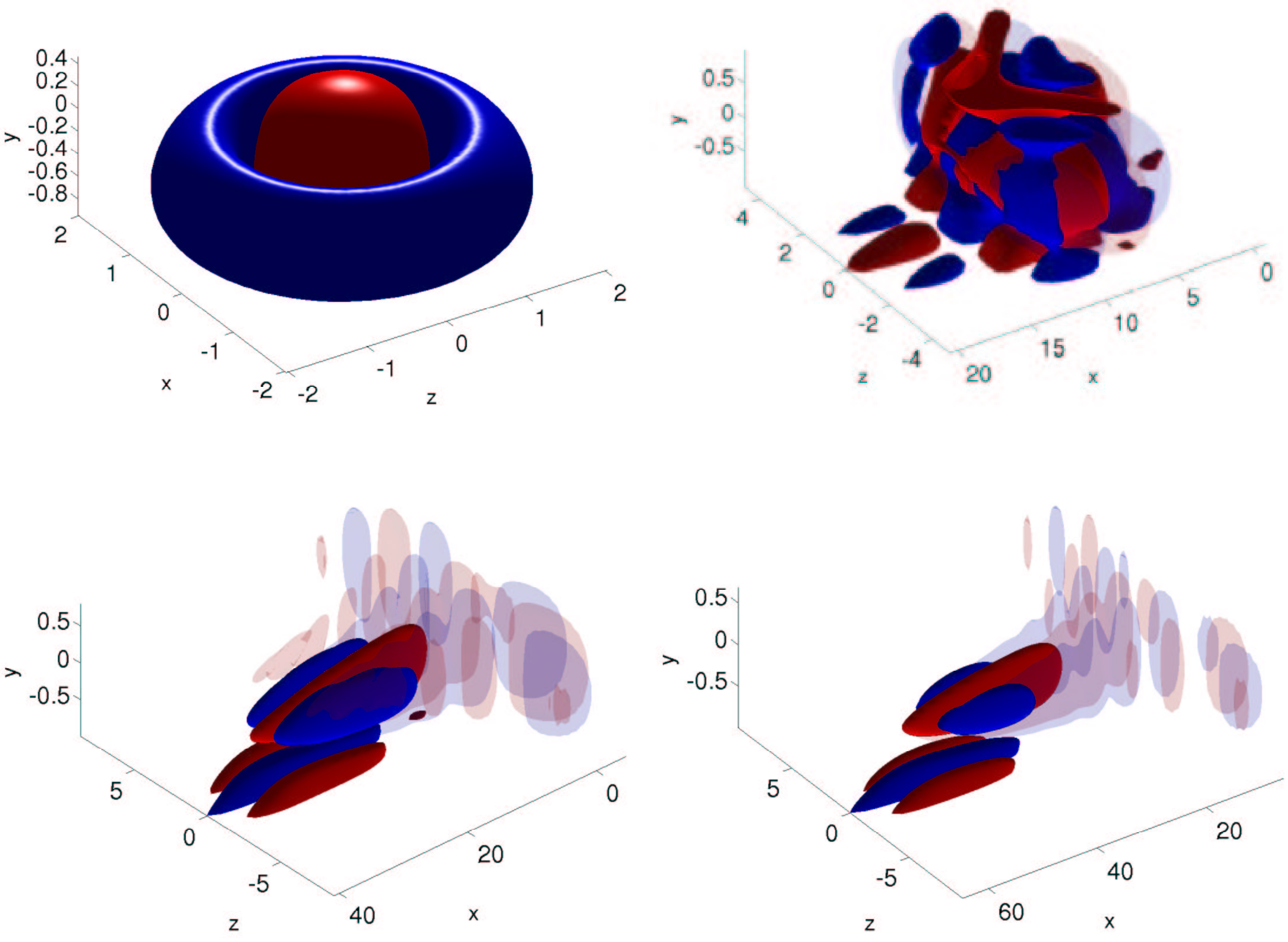
Feedback controlled initial condition

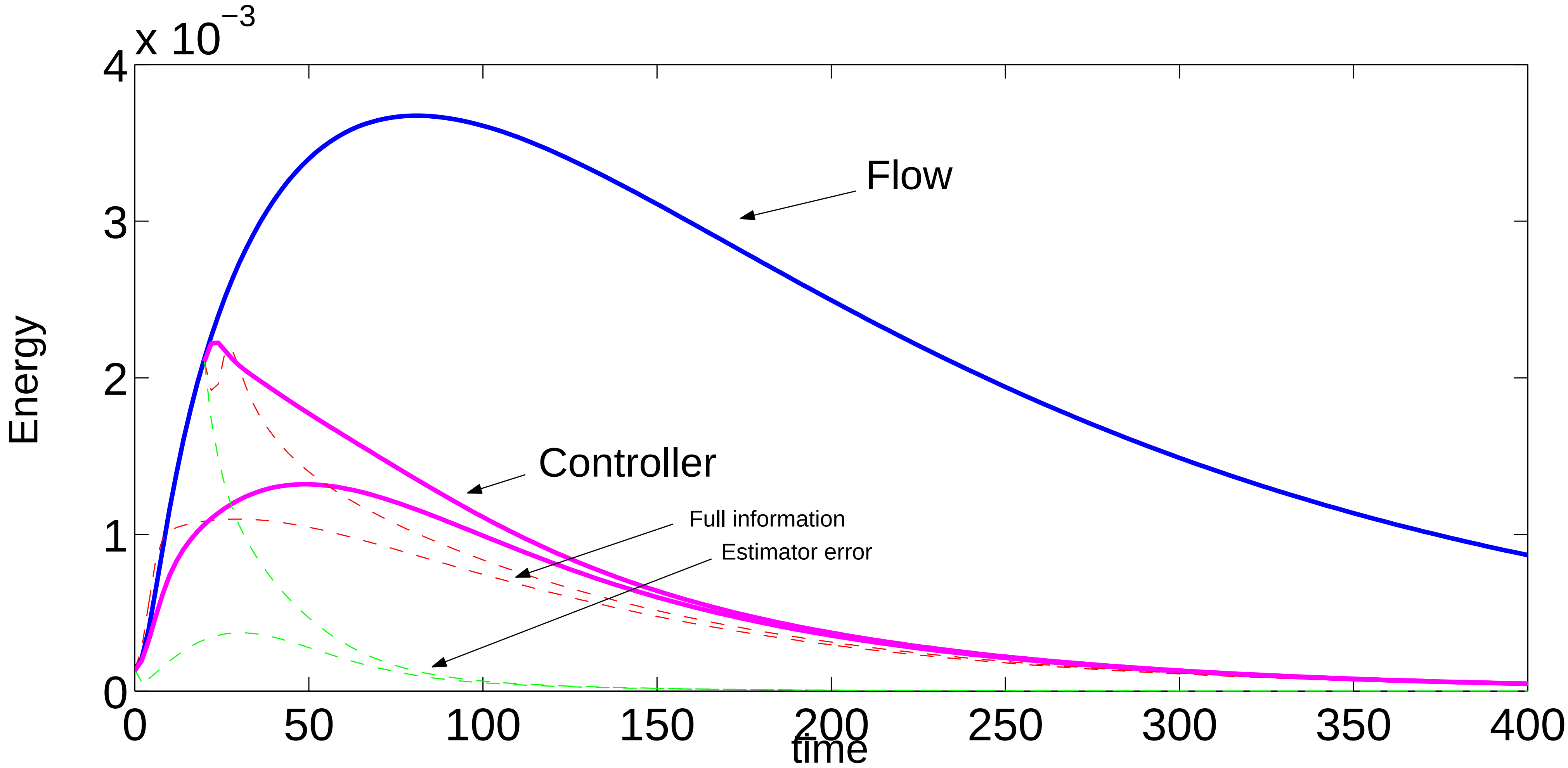
Axisymmetric localised initial condition

Wall normal velocity for original flow and controlled flow, Time 0, 10, 70, 90.



Energy evolution

Turn on the controller at time 0 and time 20



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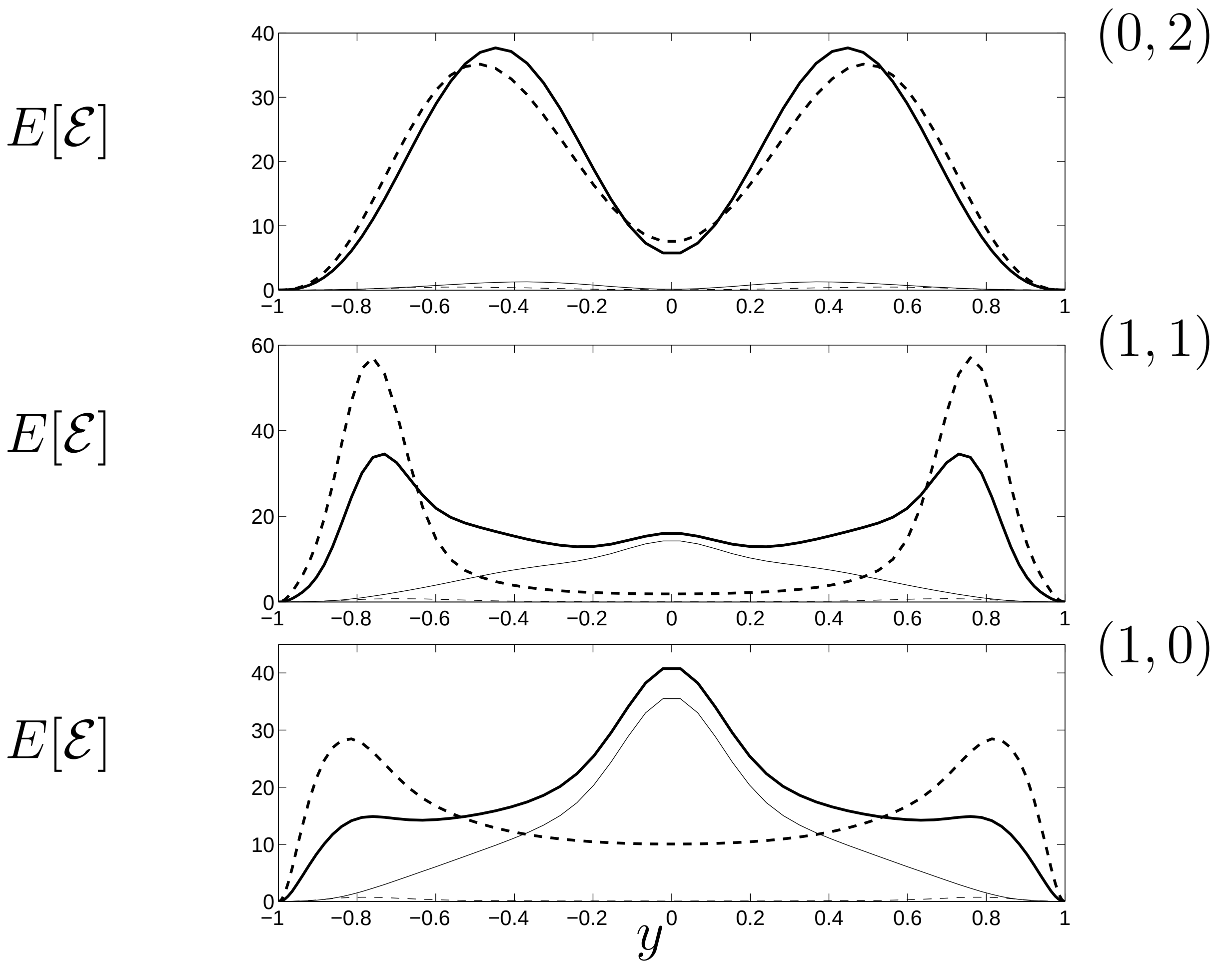
Paper 1 : laminar flow estimation



Flow : Thick lines

estimation error : thin lines.

Estimation performance for disturbances in all the domain (solid lines)

and close to the walls (dashed lines).

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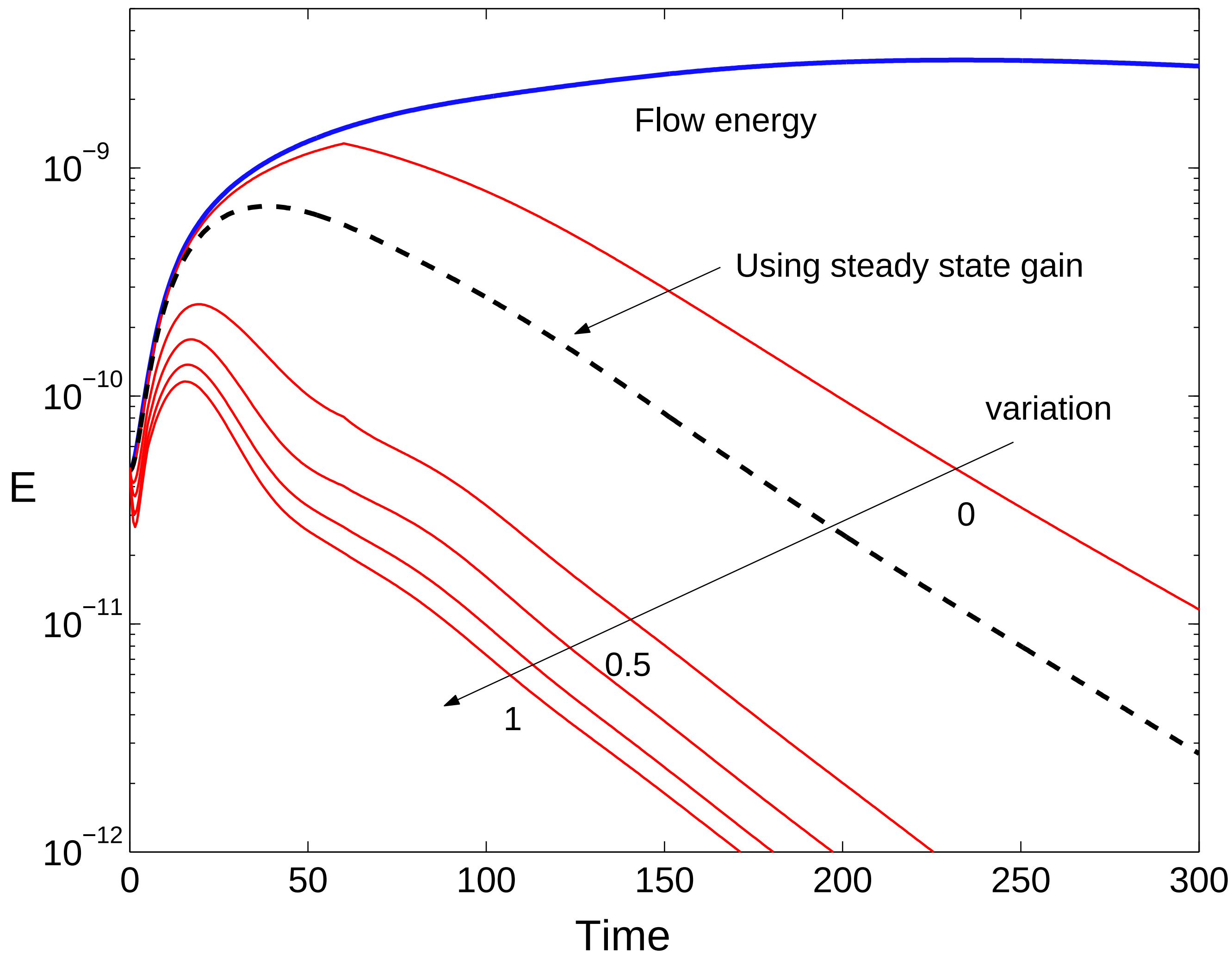
Partial knowledge of the initial condition

Red : estimation error using time varying gains.

This demonstrates how an accurate estimate of the assumed statistics of the initial conditions (λ_2) can improve the estimator behaviour



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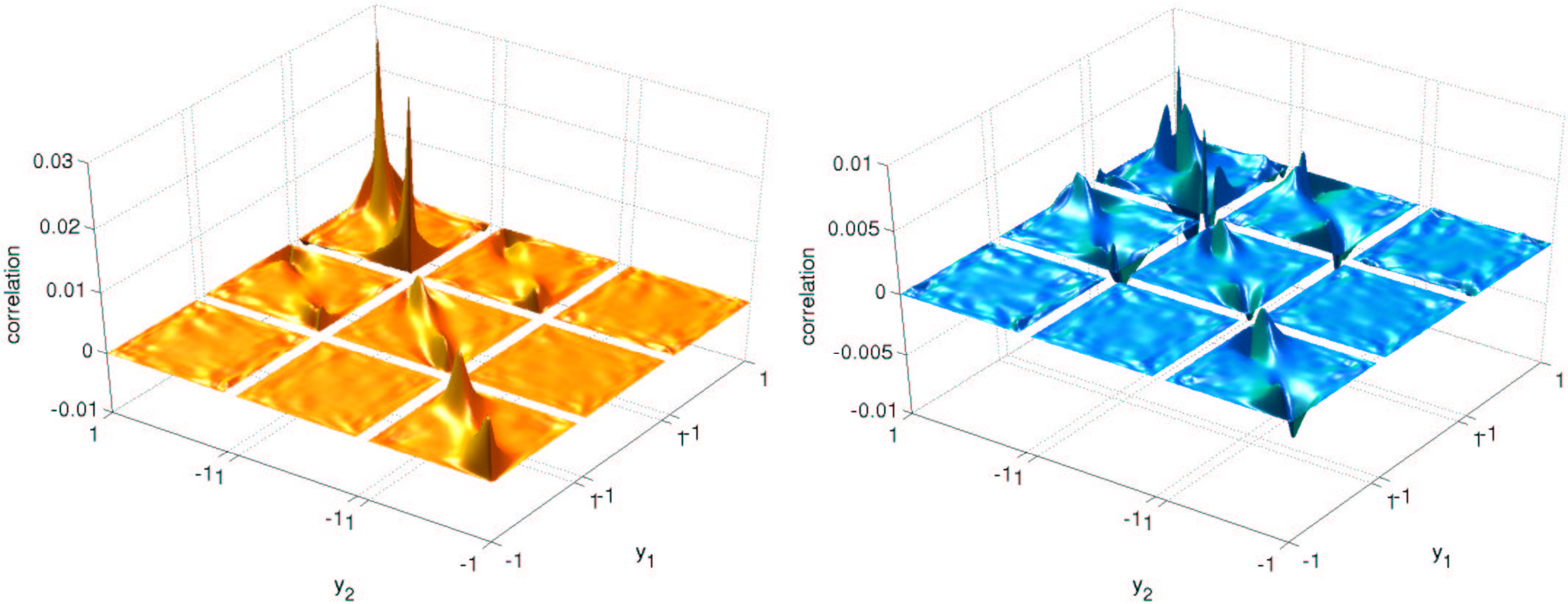
Paper 2 : turbulent flow estimation

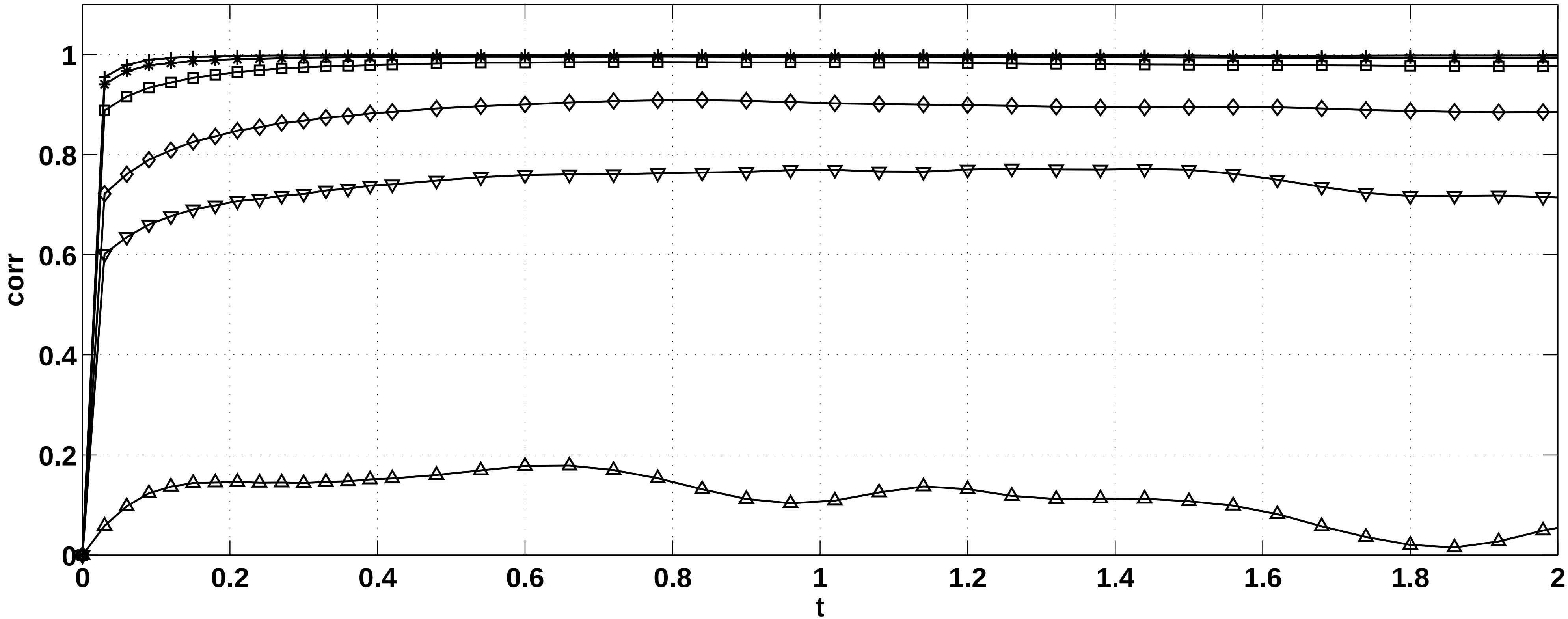
Assume the nonlinear terms are a forcing to the state→ external disturbances

We measure the covariance of this forcing (DNS)



Use this for the covariance of the disturbance in the optimisation of the estimator

Flow–estimate correlation



Estimation transient for the streamwise velocity u at $y^+ = 1.5$, $y^+ = 5.5$, $y^+ = 9.7$, $y^+ = 19.8$, $y^+ = 31.5$, and along the channel centreline

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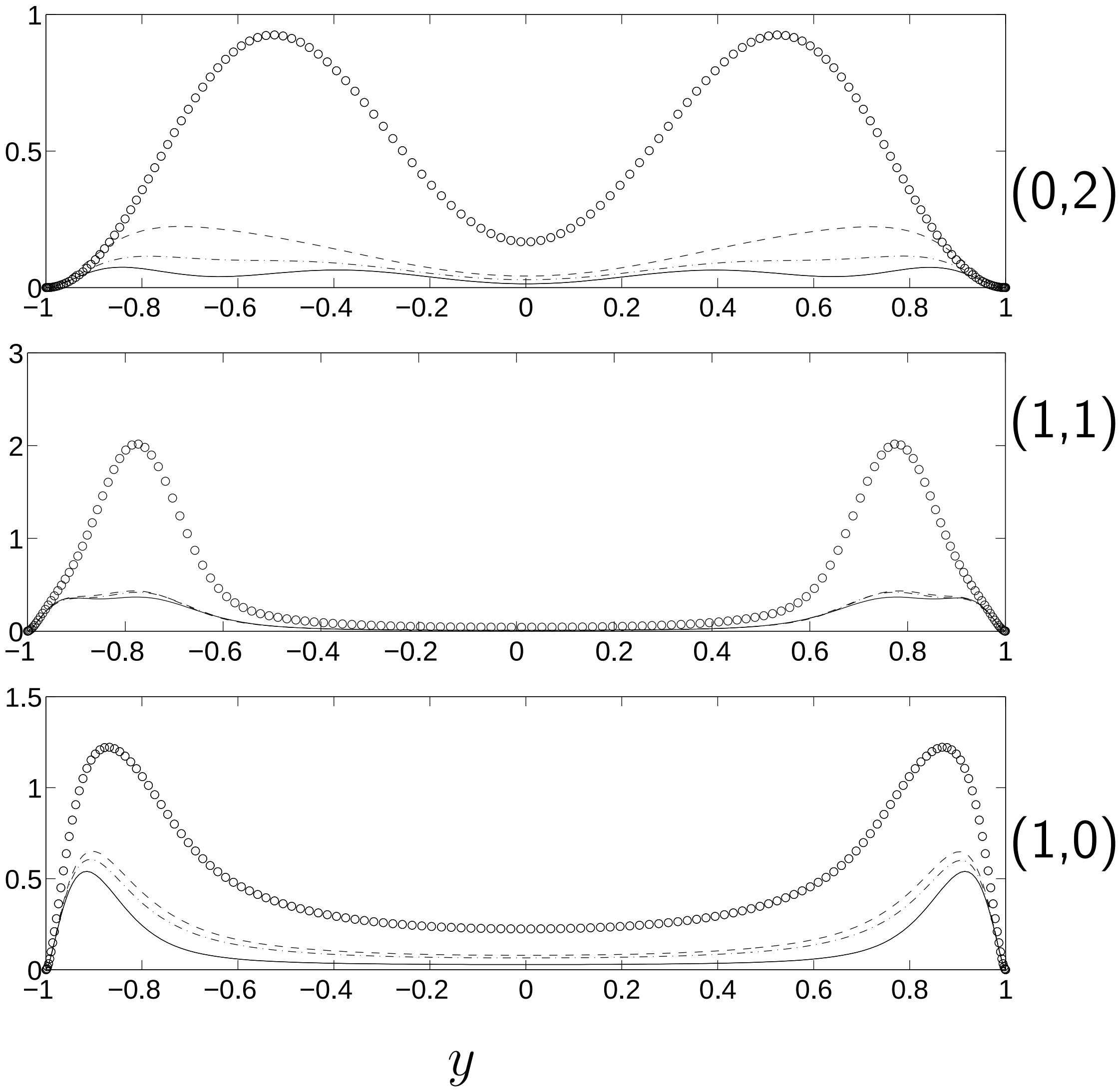
Paper 3 : Model reduction



The flow (○) .

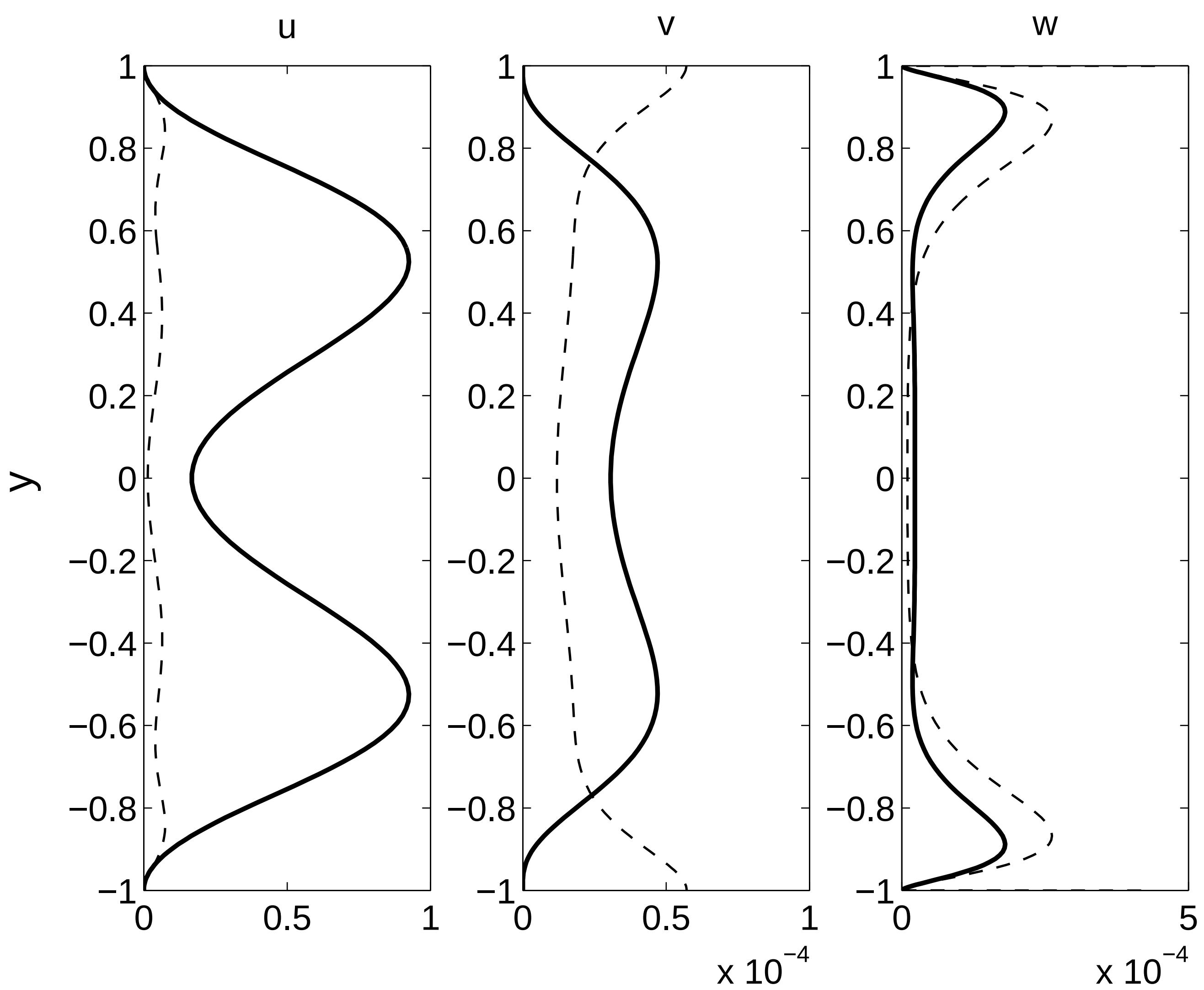
full order controller (solid)

reduced to 10 modes (dash-dotted)

and 5 modes (dashed).

Expected energy of the flow perturbation when excited by a stochastic forcing.

Control strength



wavenumber pair (0,2)

for no control (solid)

full order control (dashed)

Paper 4 : Transfer function formulation

$$\text{Plant} \begin{cases} \dot{q} = Aq + B_1 f + B_2 u \\ y = Cq + g. \end{cases}$$

$$\text{Controller} \begin{cases} \dot{\hat{q}} = (A + LC + B_2 K)\hat{q} - Ly \\ u = K\hat{q} \end{cases}$$

Transfer function mapping input and output:

$$u(t) = \int_0^\infty \underbrace{-K\mathrm{e}^{(A+B_2K+LC)\tau} L}_{G(\tau)} \, y(t-\tau)d\tau.$$

Transfer function in the channel

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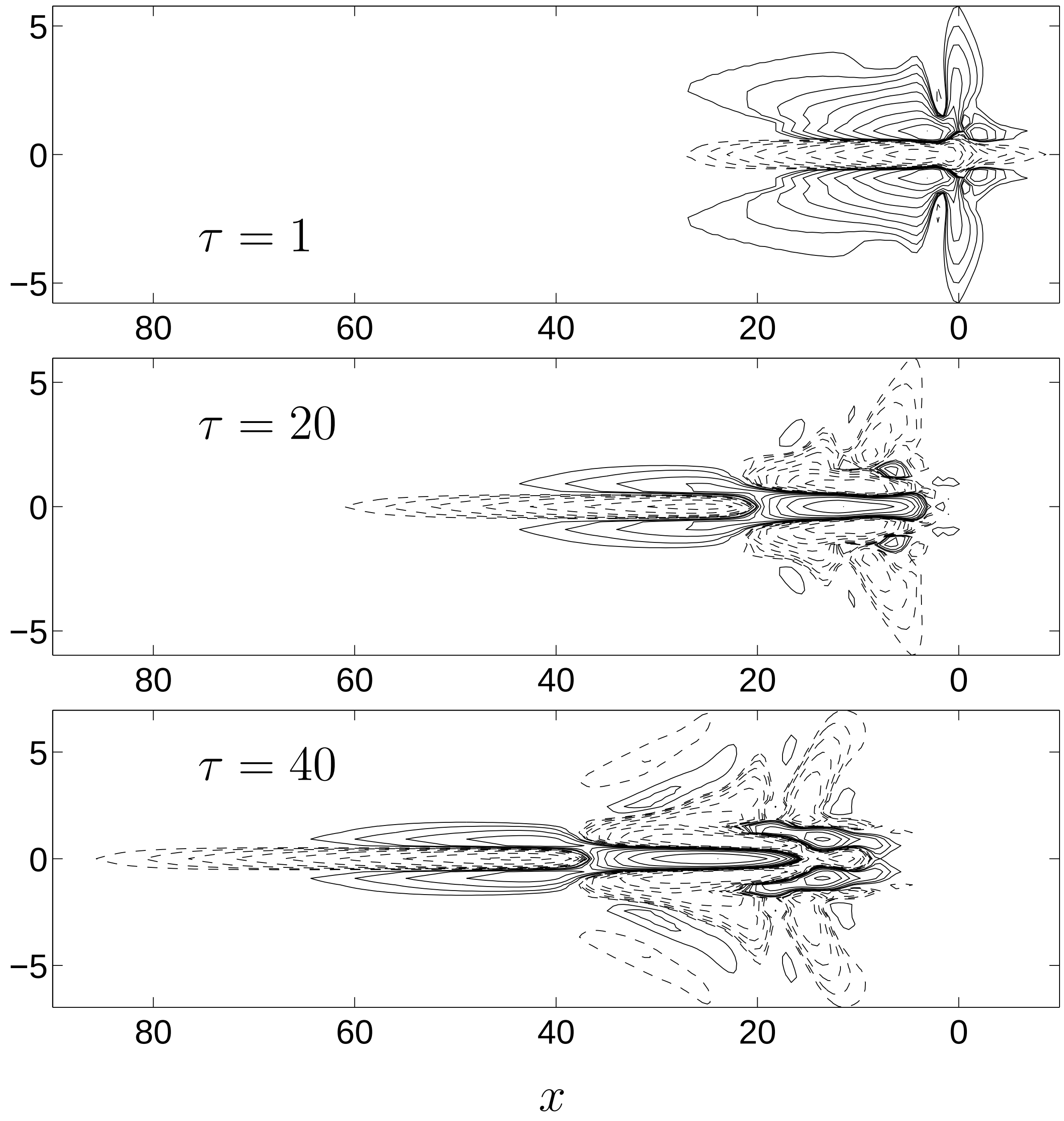
Transfer function relating streamwise skin friction measurement to the actuation.

Time lag τ



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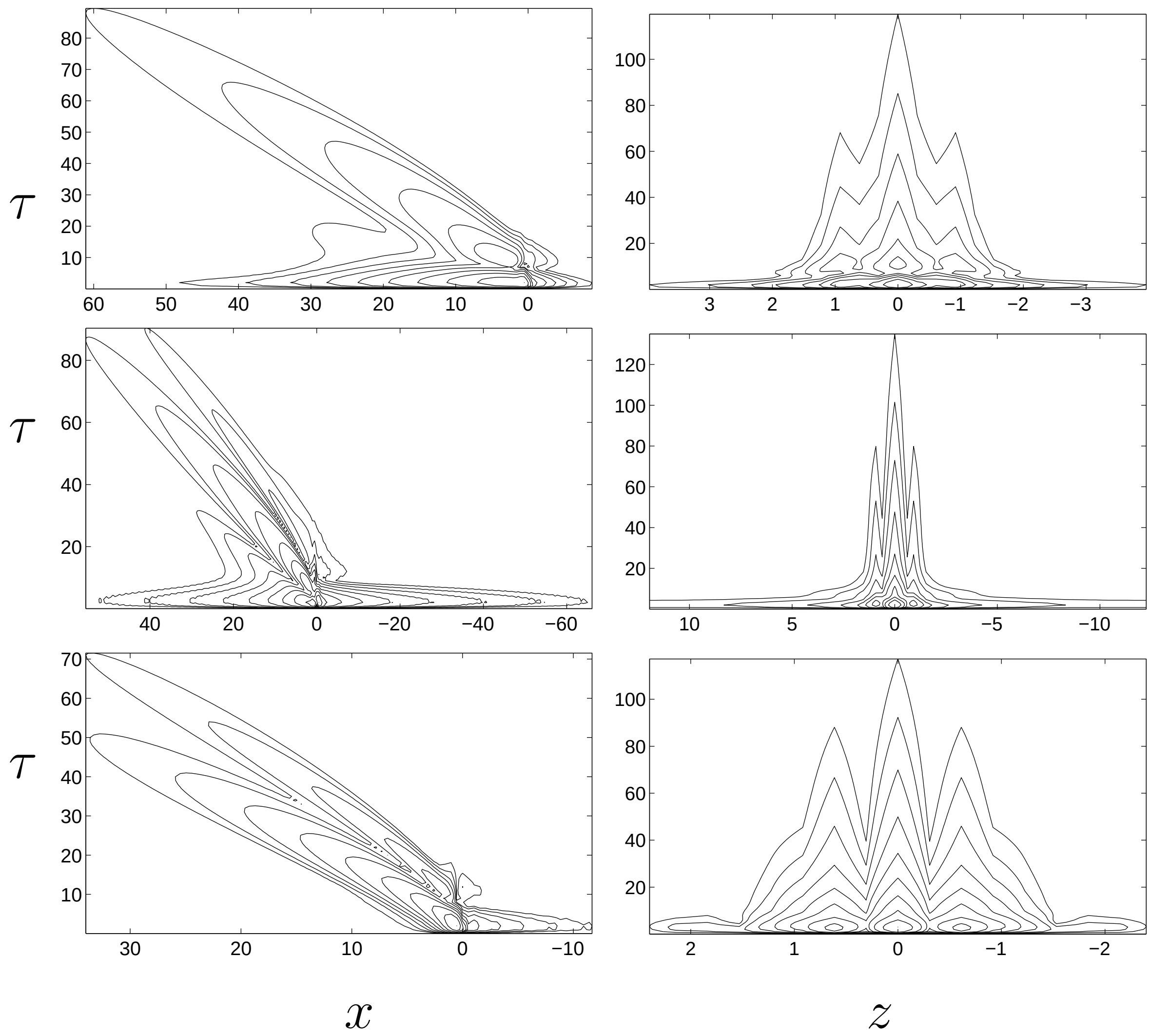
Integrated TF



The transfer function in time, integrated in the streamwise direction (left column) and spanwise direction (right column) for the three measurement, streamwise skin friction (top), spanwise skin friction (middle) and pressure (bottom).

Conclusions

- A good flow control case is a dynamics involving little degree of freedom (one dominant instability mechanism) excited by a clearly defined external disturbance (one dominant receptivity mechanism).
- A proper design : relevant disturbance model and properly targeted objective is key to the performance.
- The complexity of the controller increases with the complexity of the process to be controlled. (i.e. amount of needed information, control authority requirement, amount of data to be treated)
- A natural representation for the feedback law is the transfer function.