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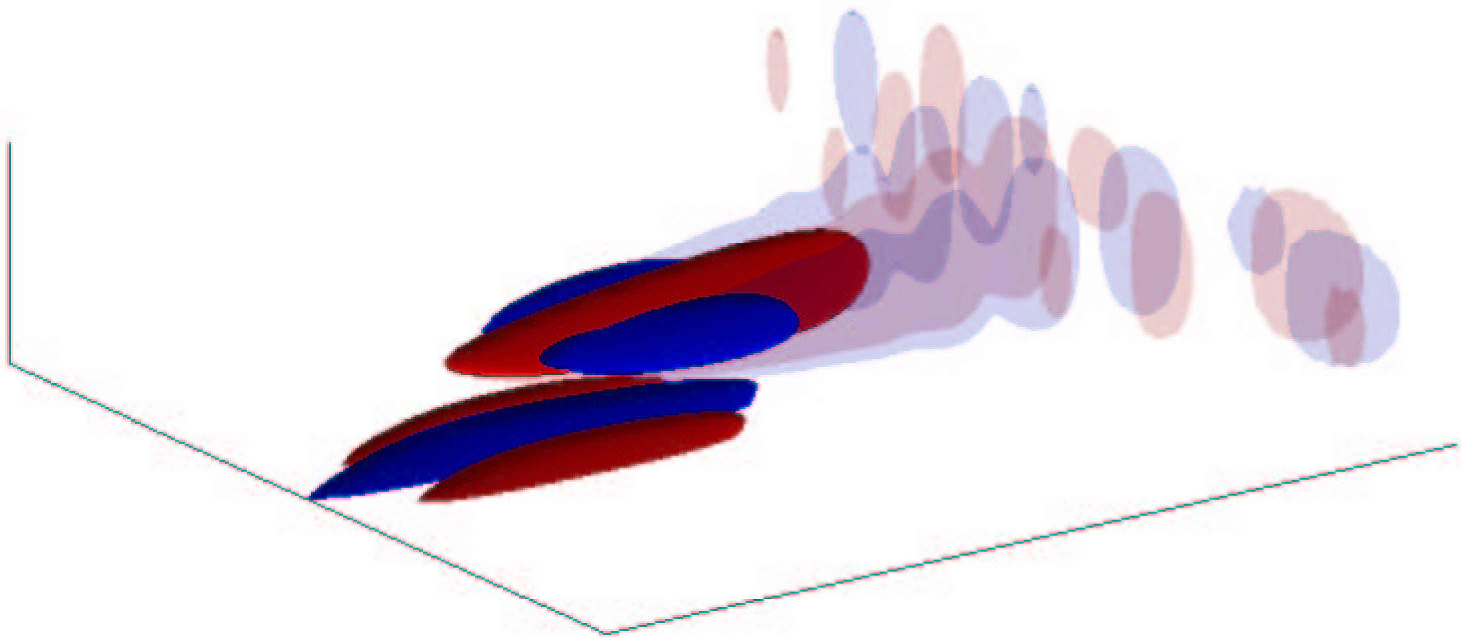
Linear feedback control of transition in shear flows



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Optimal linear control

- State model using linearized Navier-Stokes (LNS)
- OS-SQ formulation assuming parallel mean flow
- External sources of disturbances
- Control at the wall with blowing/suction
- Minimize disturbance energy

Shear flows are highly sensitive to external sources of disturbances because of the non-normality of the underlying dynamical operator: OS-SQ

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OS-SQ equations

Dynamics of small perturbations q

about the laminar base flow profile U

Flow state $q = (v, \eta)^T$ and dynamics A

$$\underbrace{\begin{pmatrix} \dot{v} \\ \dot{\eta} \end{pmatrix}}_{\dot{q}} = \underbrace{\begin{pmatrix} \mathcal{L}_{OS} & 0 \\ \mathcal{L}_C & \mathcal{L}_{SQ} \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} v \\ \eta \end{pmatrix}}_{q} + \underbrace{\begin{pmatrix} f_v \\ f_\eta \end{pmatrix}}_{f}, \quad \underbrace{\begin{pmatrix} v(0) \\ \eta(0) \end{pmatrix}}_{q(0)} = \underbrace{\begin{pmatrix} v_0 \\ \eta_0 \end{pmatrix}}_{q_0}.$$

$$\begin{cases} \mathcal{L}_{OS} = \Delta^{-1}(-ik_x U \Delta + ik_x U'' + \Delta^2/Re), \\ \mathcal{L}_{SQ} = -ik_x U \Delta/Re, \\ \mathcal{L}_C = -ik_z U', \end{cases}$$

External source of disturbances f as a volume forcing

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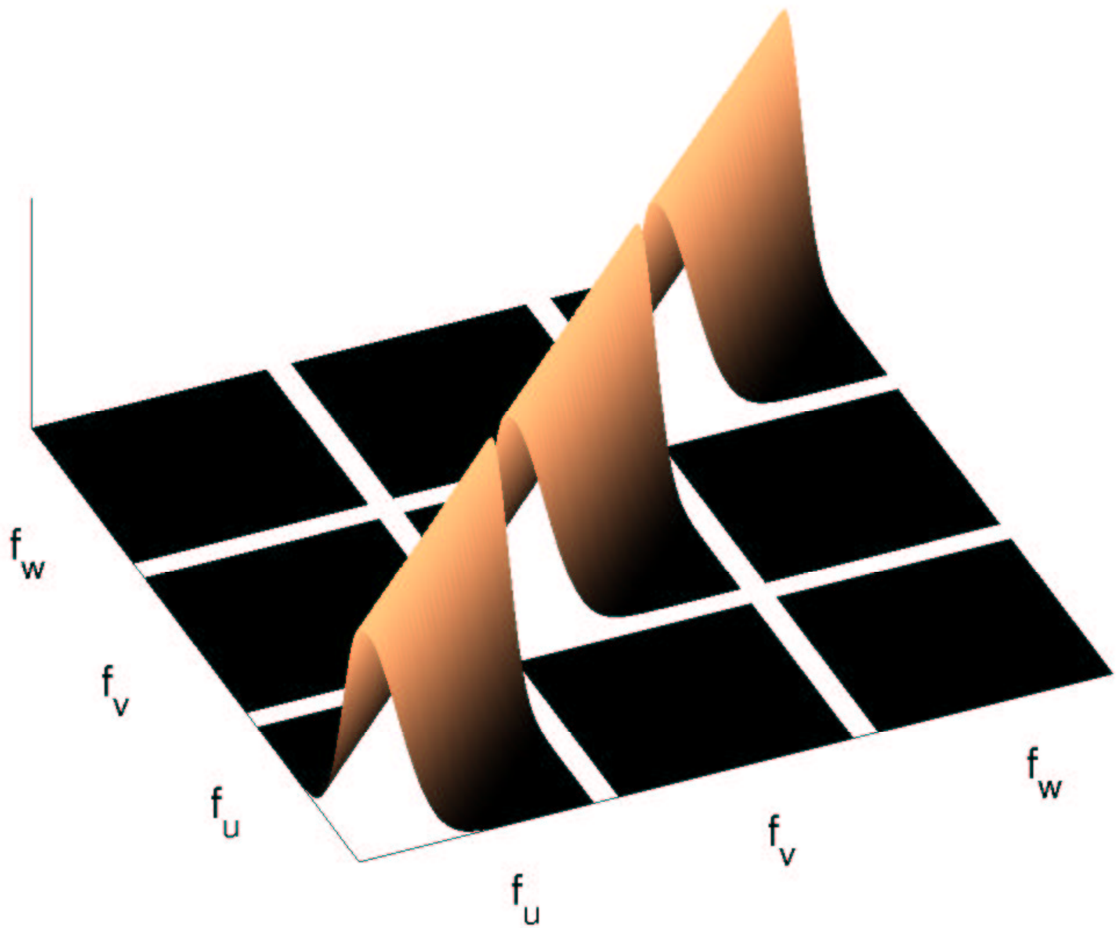
External disturbances

Forcing exciting the flow state:

- Acoustic waves
- Wall roughness
- Free stream turbulence
- ...



Covariance matrix $R_{ff} = E[ff^*]$, for $f = (f_u, f_v, f_w)^T$

Measurement and actuation

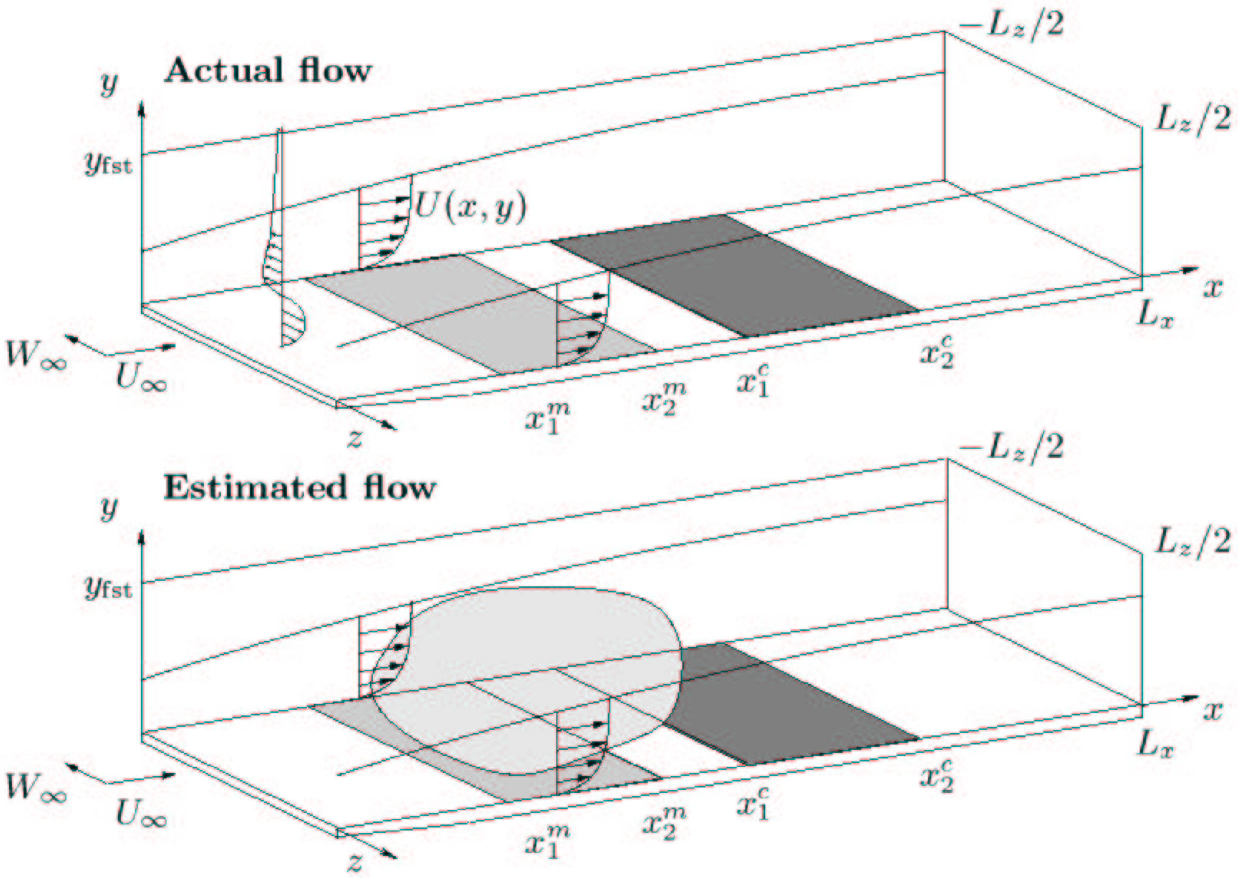
Measure instantaneous wall shear stress and wall pressure.

$$\begin{cases} \tau_x = \tau_{xy}|_{\textit{wall}} = \mu \dfrac{\partial u}{\partial y}\Big|_{\textit{wall}} = \dfrac{i\mu}{k^2}(k_x D^2 v - k_z D\eta)|_{\textit{wall}}\,, \\ \tau_z = \tau_{zy}|_{\textit{wall}} = \mu \dfrac{\partial w}{\partial y}\Big|_{\textit{wall}} = \dfrac{i\mu}{k^2}(k_z D^2 v + k_x D\eta)|_{\textit{wall}}\,, \\ p = p|_{\textit{wall}} = \dfrac{\mu}{k^2} D^3 v|_{\textit{wall}}\,. \end{cases}$$

Actuate by means of wall blowing and suction

(boundary conditions on v)

Control procedure



- Get difference in measurements from flows
- Apply estimator forcing in estimated flow
- Compute control signal from estimated flow
- Apply control signal in flow and estimator

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Formulation of the LQG control problem

Flow $\begin{cases} \dot{q} = Aq + B_1 f + B_2 u \\ r = Cq + g. \end{cases}$

Estimator $\begin{cases} \dot{\hat{q}} = A\hat{q} + B_2 u - v \\ \hat{r} = C\hat{q}. \end{cases}$

Feedback $\begin{cases} \text{Control:} \quad v = L(r - \hat{r}) \\ \text{Estimation:} \quad u = K\hat{q}. \end{cases}$

Objective function:

minimize kinetic energy

$$\mathcal{J} = \frac{1}{2}\int_{-1}^{1} (q^* Q q + \ell^2 u^* u)\,\mathrm{d}t,$$

l penalty on control effort

Decouple into an **estimation** problem and a **full information control** problem.

Solve two optimization problems to get the optimal L and K.

Solution of the optimisation

1. Constrained optimisation problem
2. Lagrange multipliers $\longrightarrow$ unconstrained minimisation of a Lagrangian
3. Obtain operator equation : Riccati equation
4. Solve the Riccati equation by spectral factorization

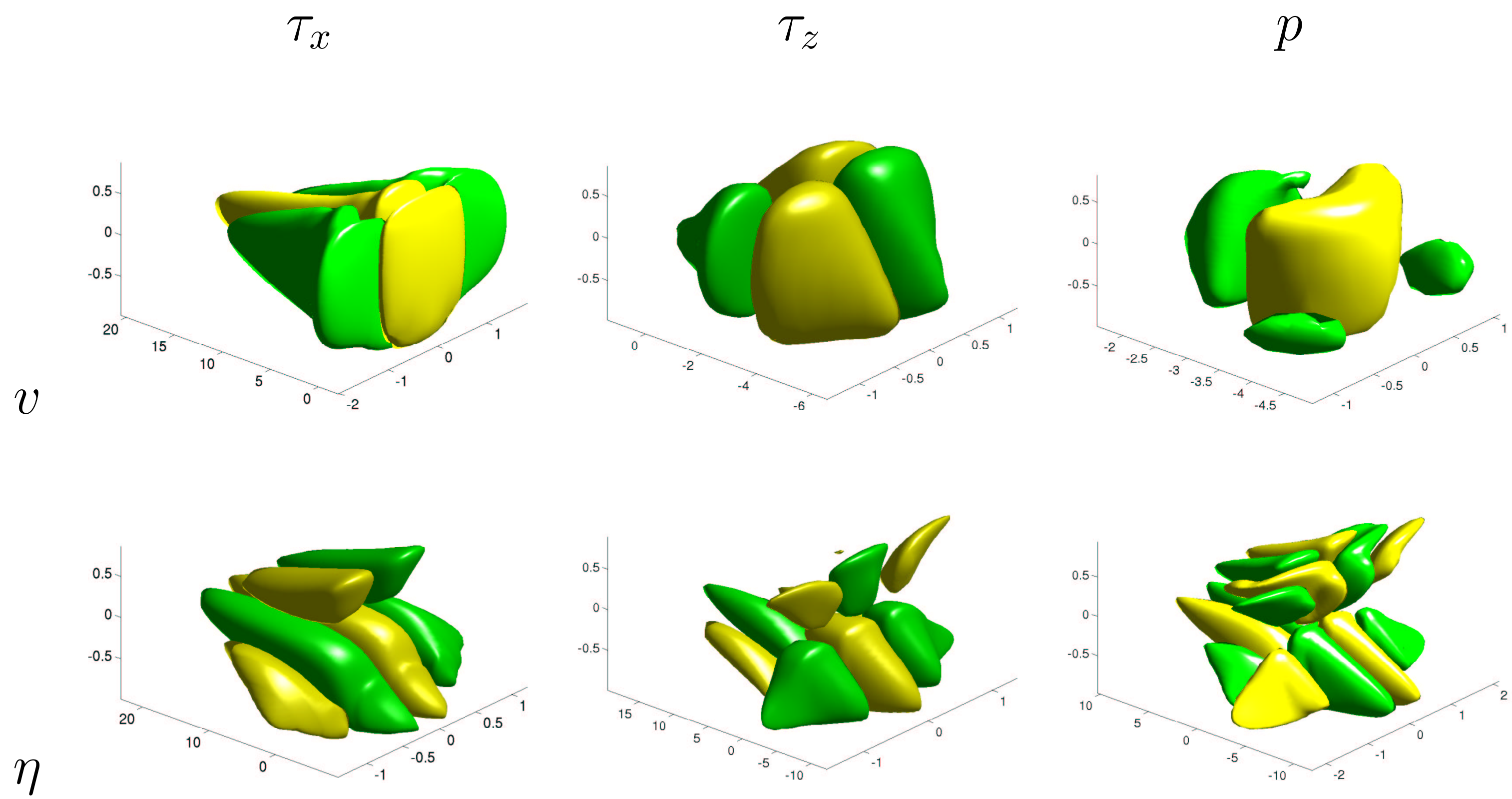
$$Control: \quad \begin{cases} A^*X + XA - \frac{1}{l^2}XB_2B_2^*X + Q = 0, \\ \text{Control gain } K = -\frac{1}{l^2}B^*X, \end{cases}$$

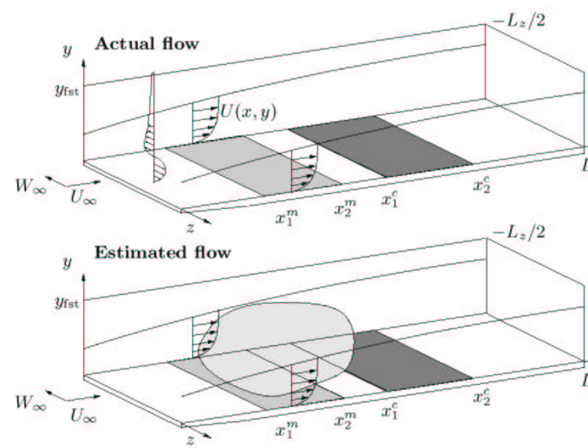
$$Estimation: \quad \begin{cases} AP + PA^* + B_1R_{ff}B_1^* - PC^*G^{-1}CP = 0, \\ \text{Estimation gain } L = -PC^*G^{-1}. \end{cases}$$

Q is the quadratic norm, R_{ff} is the covariance of f

Results

Compact estimation kernels





$v = L(r - \hat{r})$, inverse Fourier transform to obtain convolution kernels

→ **3D forcing in the estimator flow.**

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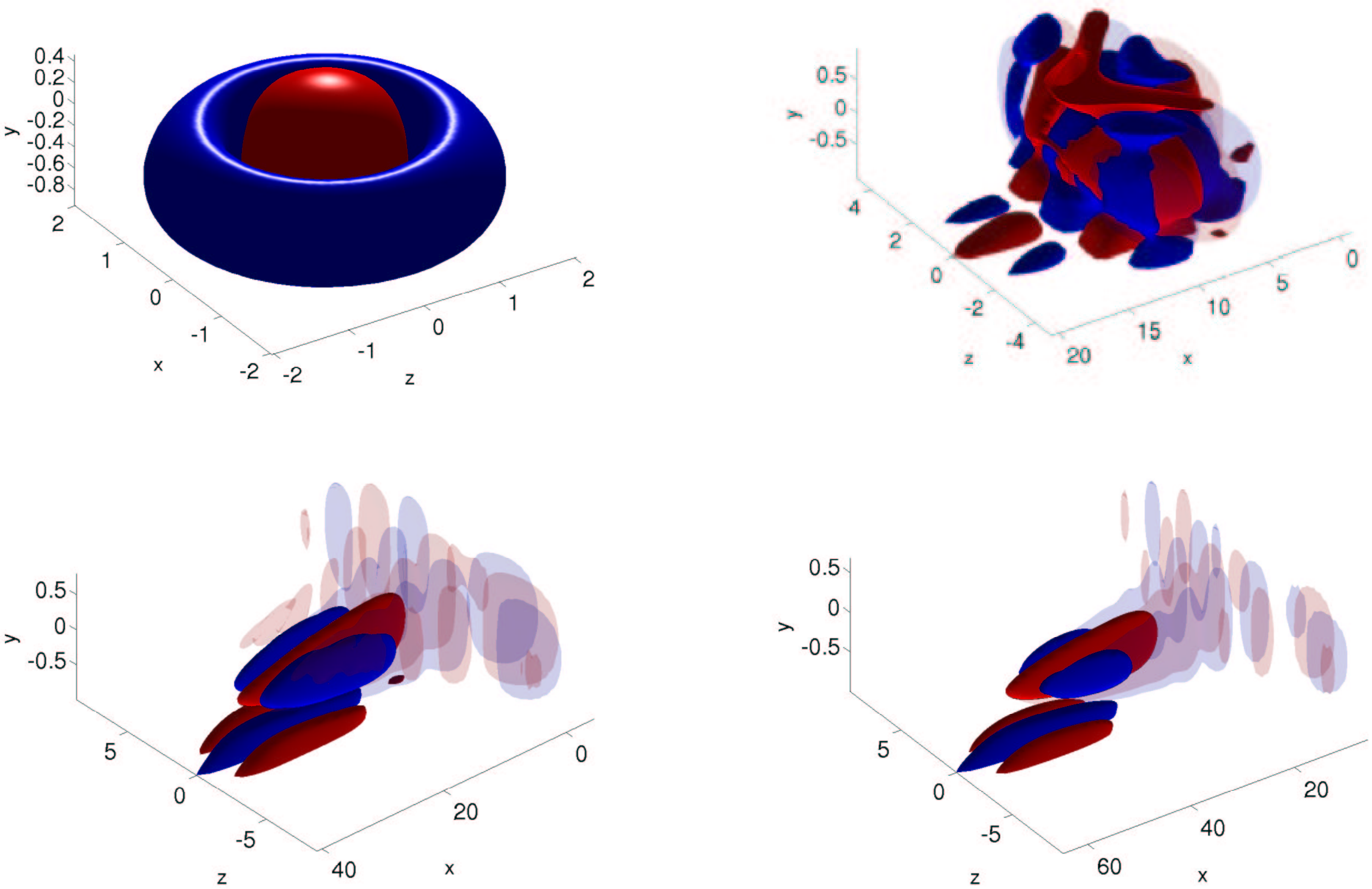
Feedback controlled initial condition

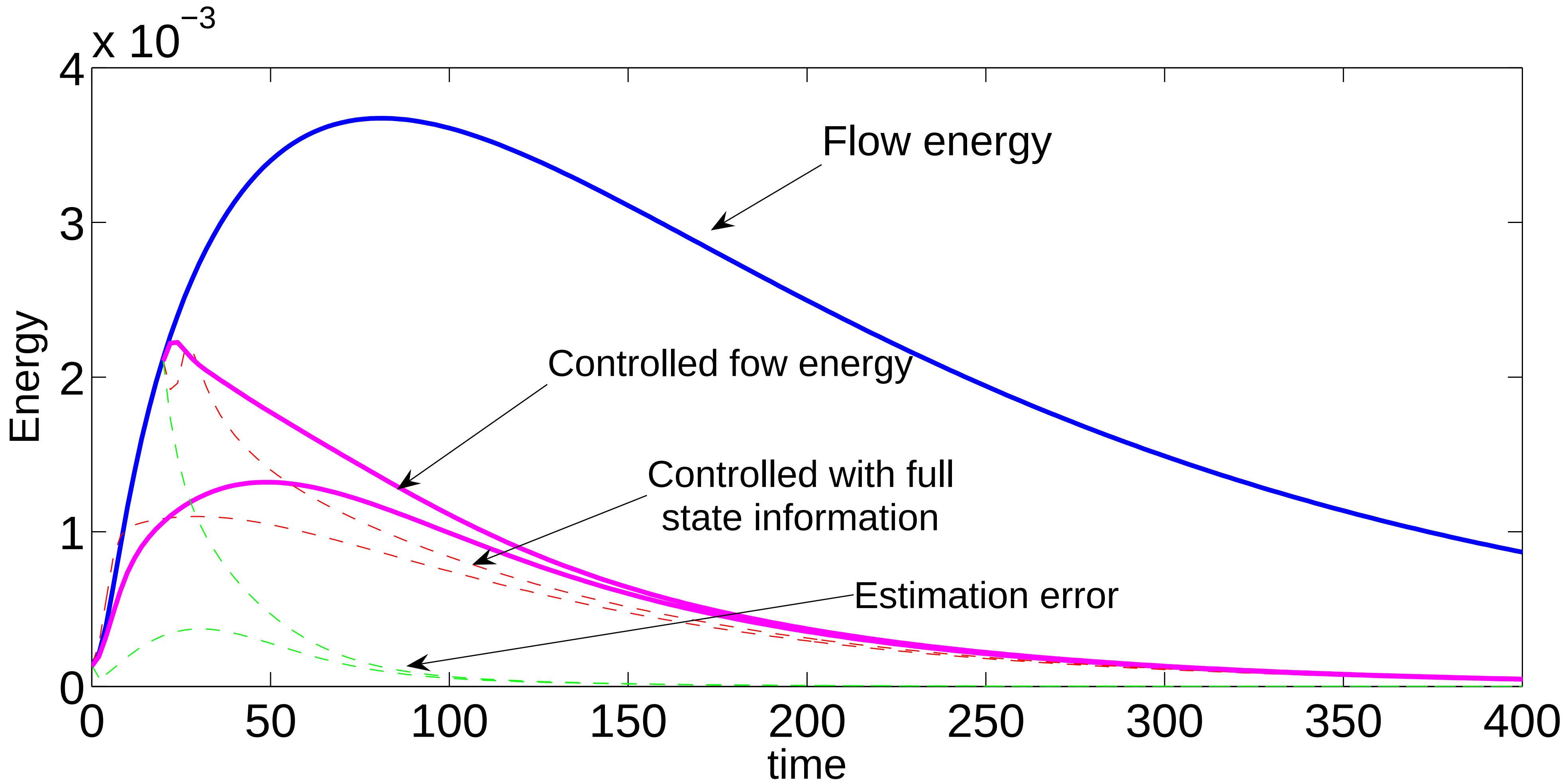
Axisymmetric localised initial condition



Wall normal velocity for original flow and controlled flow, Time 0, 10, 70, 90.

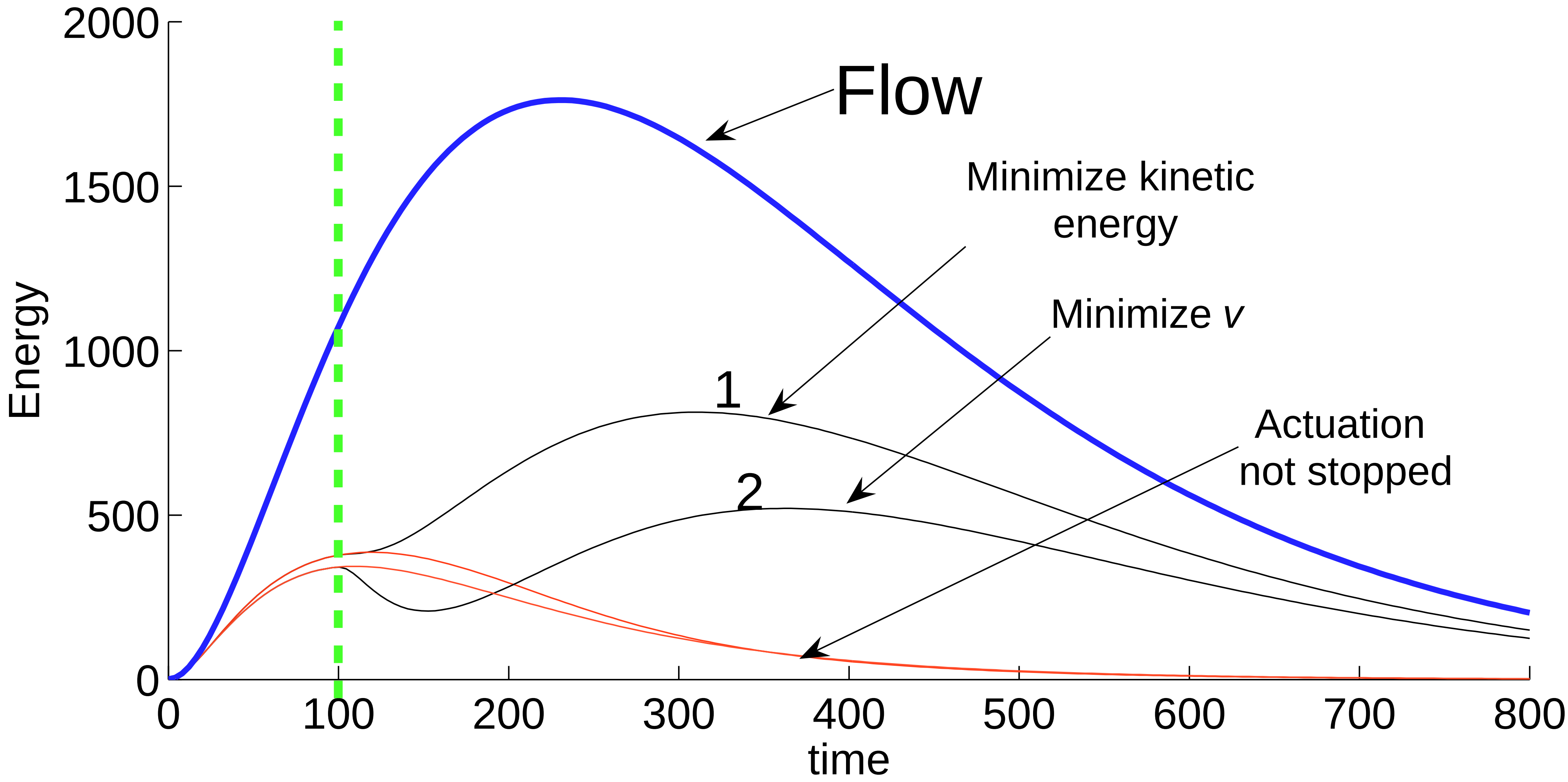
Energy evolution

Turn on the controller at time 0 and time 20



Objective function flexibility

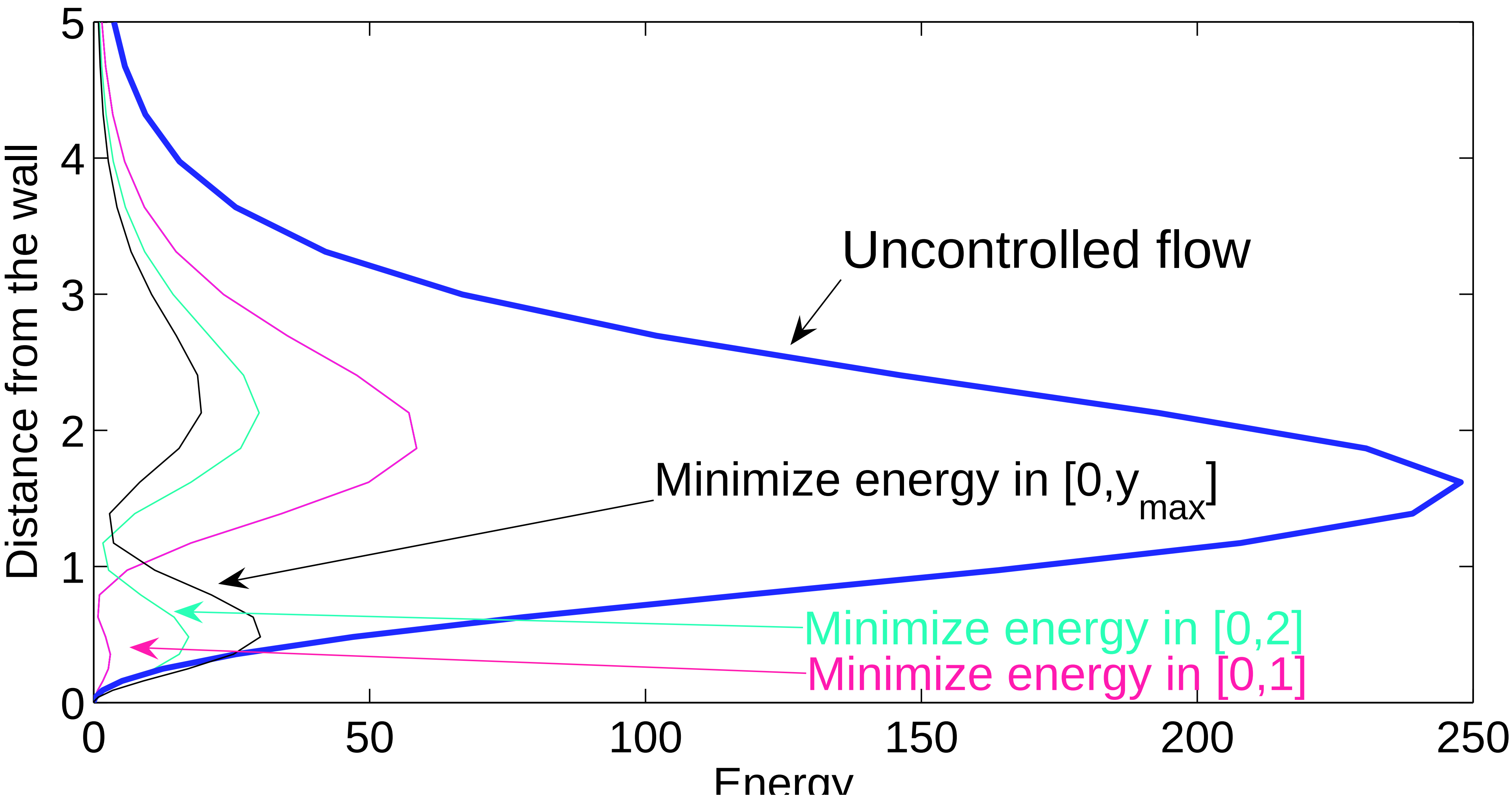
Evolution of a WCD initial condition in a B.L.



The control is turned off at time 100

Objective function flexibility

Flow forced by stochastic external disturbance



The controller seeks to eject the disturbance out of the B.L.

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Conclusions

- Optimal control and estimation applied to linearized Navier–Stokes
- Stochastic description of the external disturbance sources is important
- Localized perturbation controlled using wall measurements and wall actuation only
- Examples of flexibility of the objective function:
  - control effect after the actuation is stopped
  - how to eject the disturbances out of the boundary layer

Related talk

Mattias Chevalier,

Linear control and estimation in boundary layer flows

Day 2, session Flow Control II, 5:50

$\longrightarrow$ Same control scheme

$\longrightarrow$ spatially developing flows