KTH Mechanics

Résonance et contrôle en cavité ouverte

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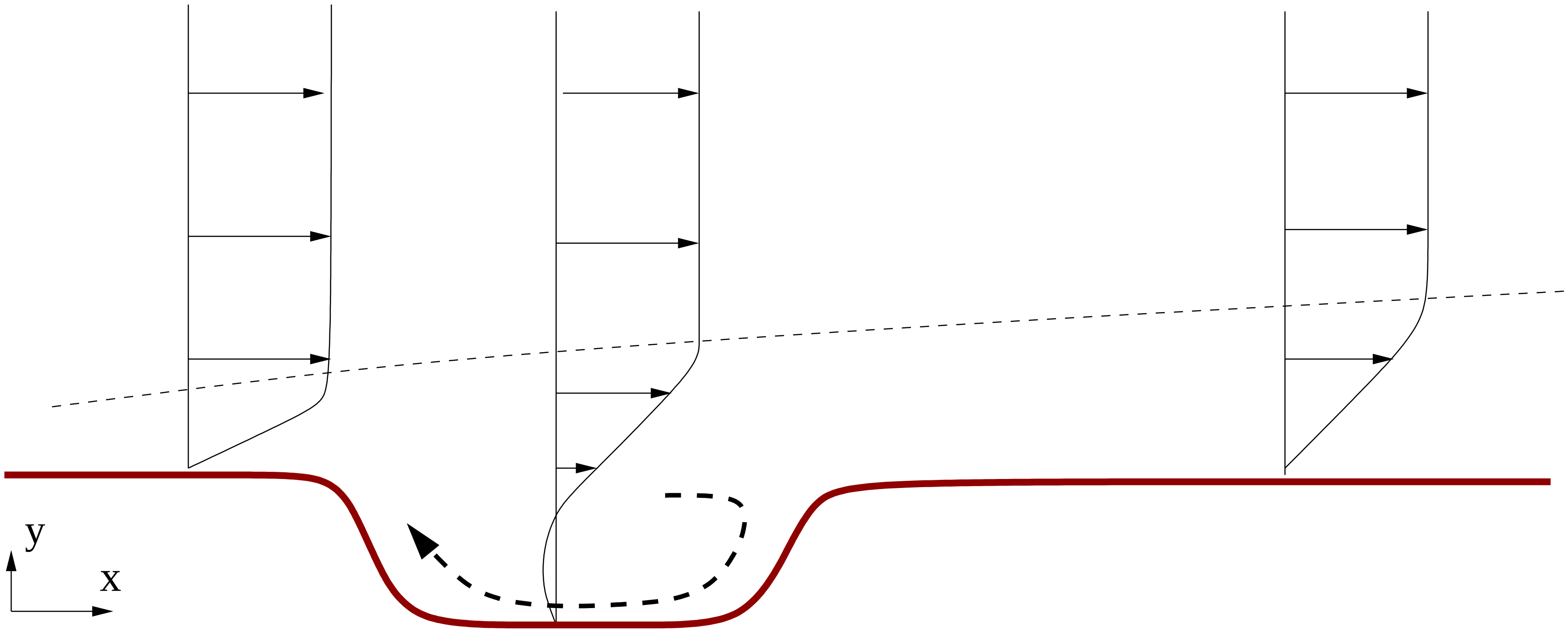
Outline

- The flow case
- Investigation tools
- resonance
- Reduced dynamic model for feedback control
- Control performance

Boundary layer with cavity

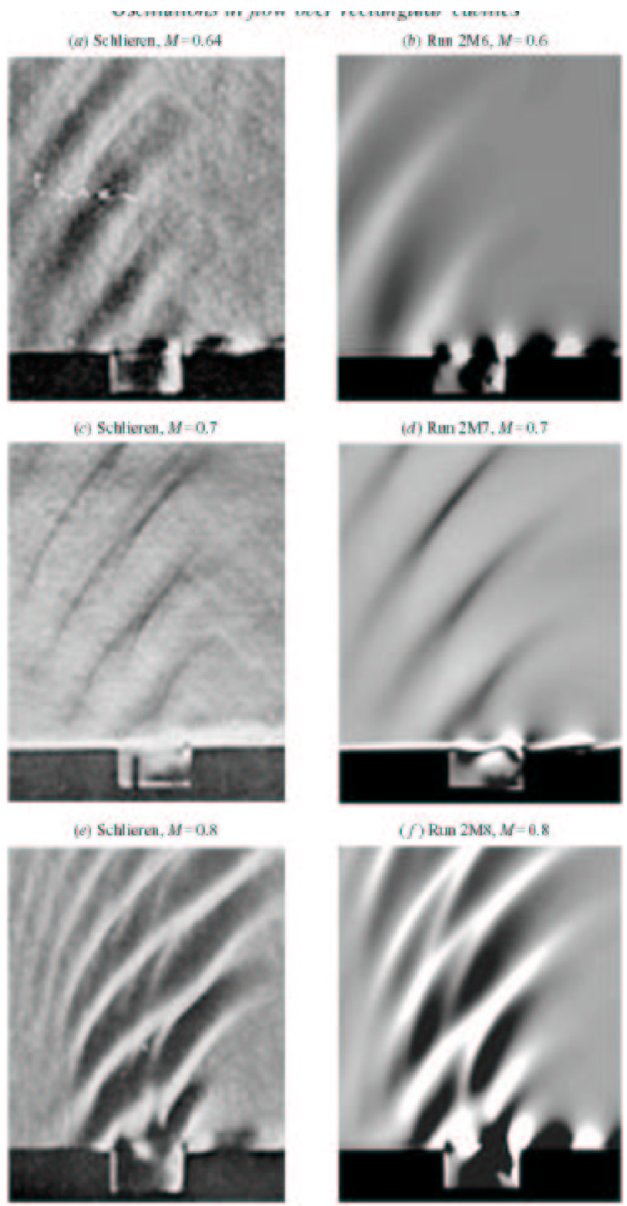


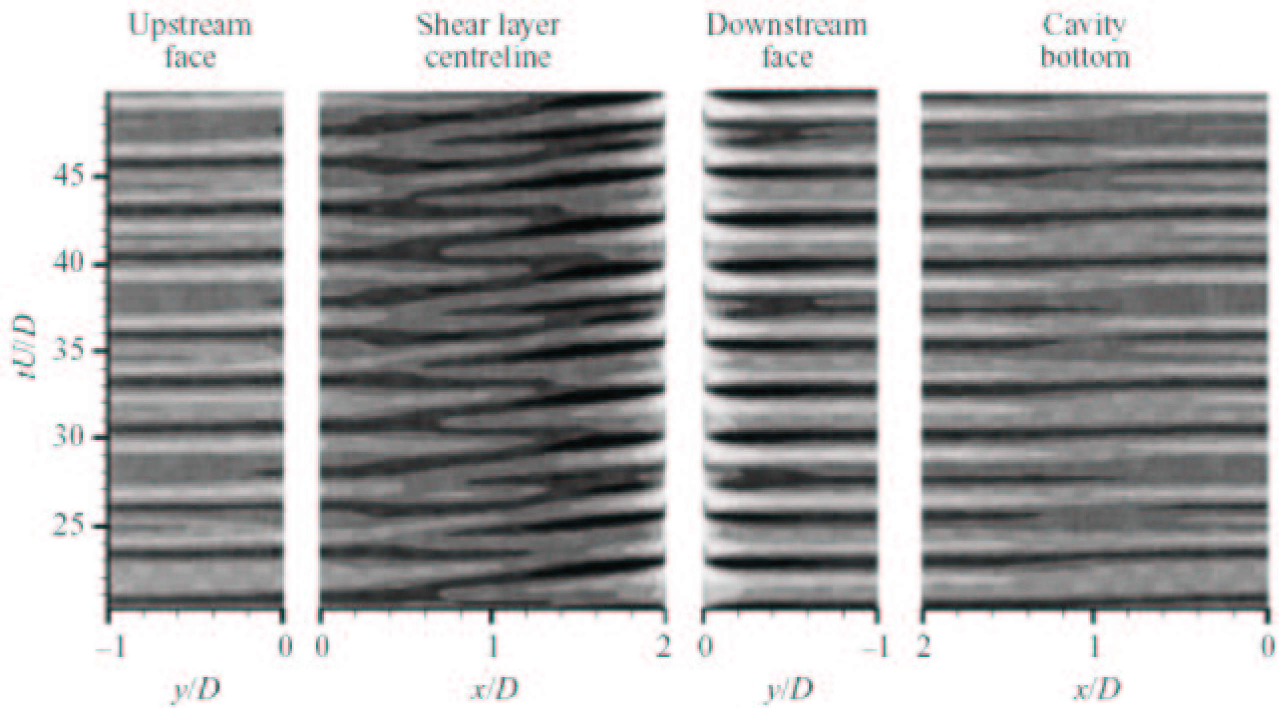
2D flow over a smooth cavity

Inflow: Blasius profile

Reynolds number : 320

Cavity: Shear layer mode (compressible)





From Rowley et al, JFM 2002

Self sustained cycle: perturbation→ growth→ pressure wave→ new perturbation

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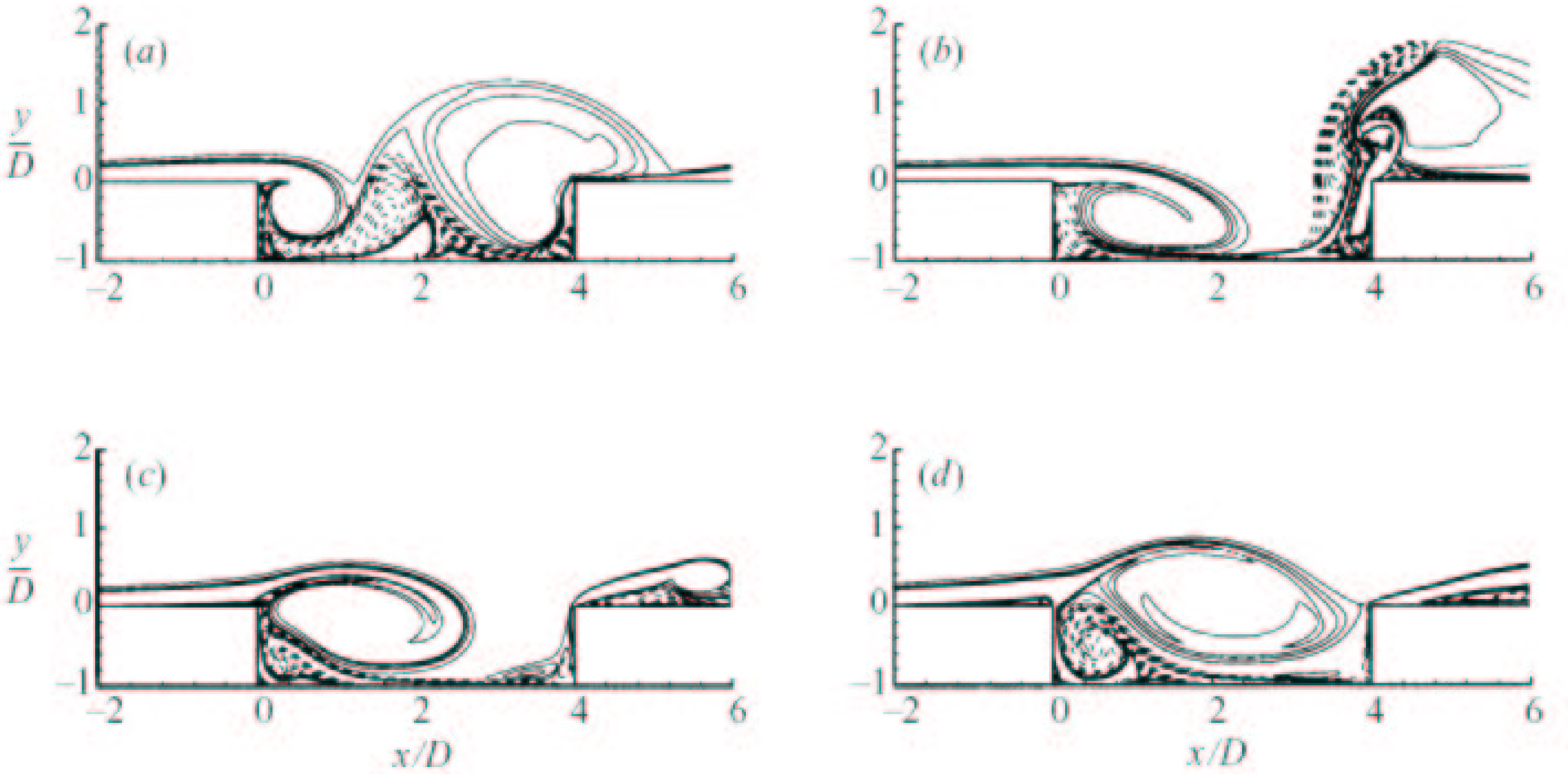
Cavity: Wake mode

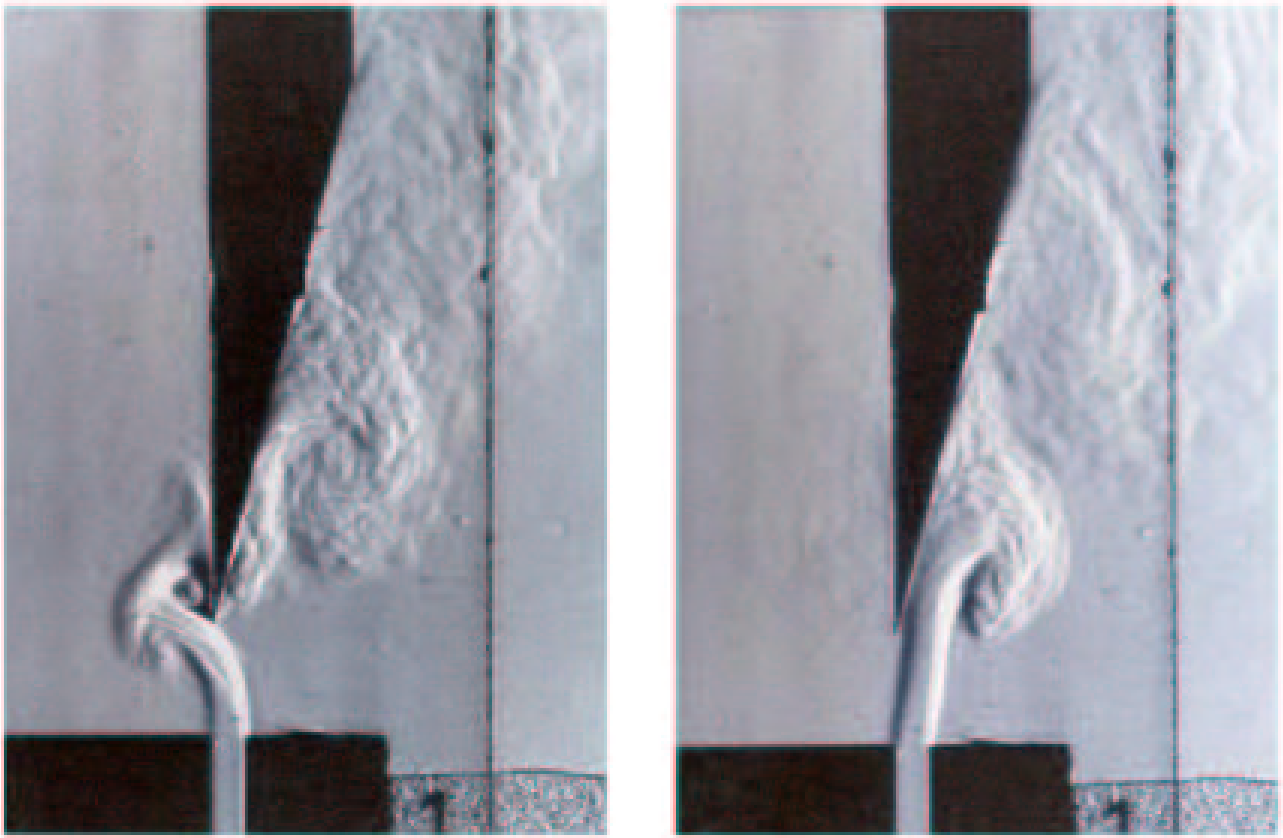


From Rowley et al, JFM 2002

Subcritical bifurcation to oscillating state

Low frequency ejection of large vortices. (for large aspect ratio)

Sound generation in organ pipes



from http://www.fluid.tue.nl/GDY/acous/

Interaction of the jet and the edge generates sound.

Cavity: linear instability of the incompressible flow?

- Stable boundary layer, or convectively unstable
- convectively unstable shear layer

Questions:

- Can we have an globally unstable cavity flow?
- What role does the pressure play in the incompressible case?
- Can we control the cavity flow?

Investigation tools

DNS to compute the base flow:
Chebyshev in wall normal, finite difference in streamwise.

Stability analysis by computation of 2D eigenmodes:
Chebyshev/Chebyshev and Arnoldi

Optimal growth by optimization over initial conditions :
Singular value decomposition, using the reduced model

Control optimization by solution of two Riccati equations :
Using the reduced order model

The eigensolver

2D Navier-Stokes + continuity

$$\begin{cases} -i\omega\hat{u} = -(U\cdot\nabla)\hat{u} - (\hat{\mathsf{u}}\cdot\nabla)U - \dfrac{\partial\hat{p}}{\partial x} + 1/Re\nabla^2\hat{u} \\ -i\omega\hat{v} = -(U\cdot\nabla)\hat{v} - (\hat{\mathsf{u}}\cdot\nabla)V - \dfrac{\partial\hat{p}}{\partial y} + 1/Re\nabla^2\hat{v} \\ 0 = \nabla\cdot\mathsf{u} \end{cases}$$

Generalized eigenproblem:

$$B\omega\mathsf{u} = A\mathsf{u}$$

To be rewritten

$$A^{-1}B\mathsf{u} = \frac{1}{\omega}\mathsf{u}$$

Solved by Arnoldi iterations.

Matrix formulation:

$$\begin{pmatrix} -i\omega\hat{u} \\ -i\omega\hat{v} \\ 0 \end{pmatrix} = \begin{pmatrix} \ldots & \ldots & -\frac{\partial}{\partial x} \\ \ldots & \ldots & -\frac{\partial}{\partial y} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & C \end{pmatrix} \begin{pmatrix} \hat{u} \\ \hat{v} \\ p \end{pmatrix}$$

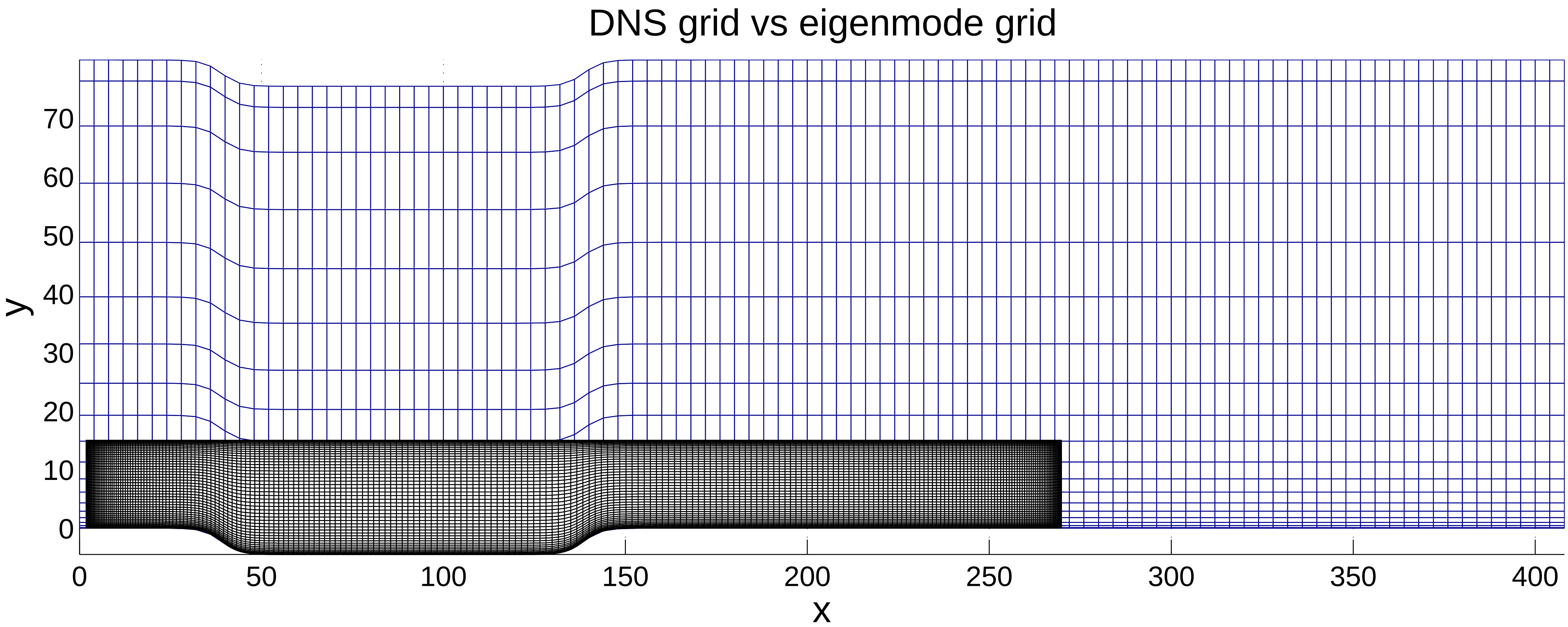
Additional constraints C

Grids & resolution

The resolution are:

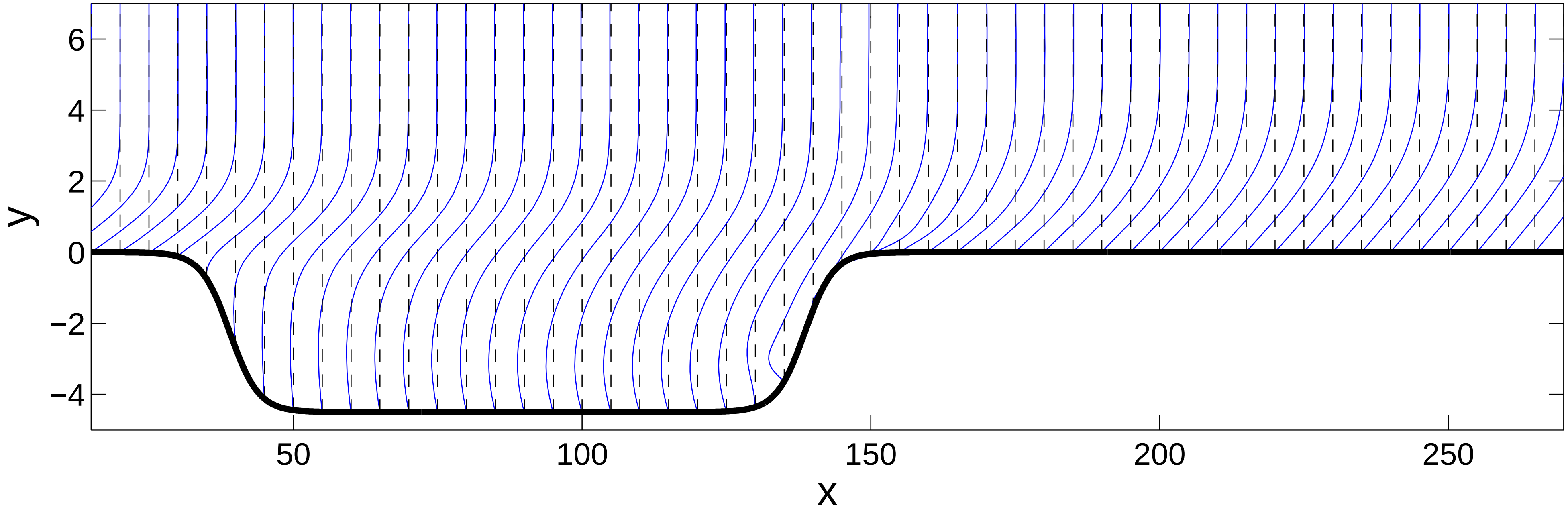
DNS: nx=2048 finite difference, ny=97 Chebyshev, Lx=409, Ly=80

EIG: nx=250 Chebyshev, ny= 50 Chebyshev, Lx=270. Ly=15.



The base flow

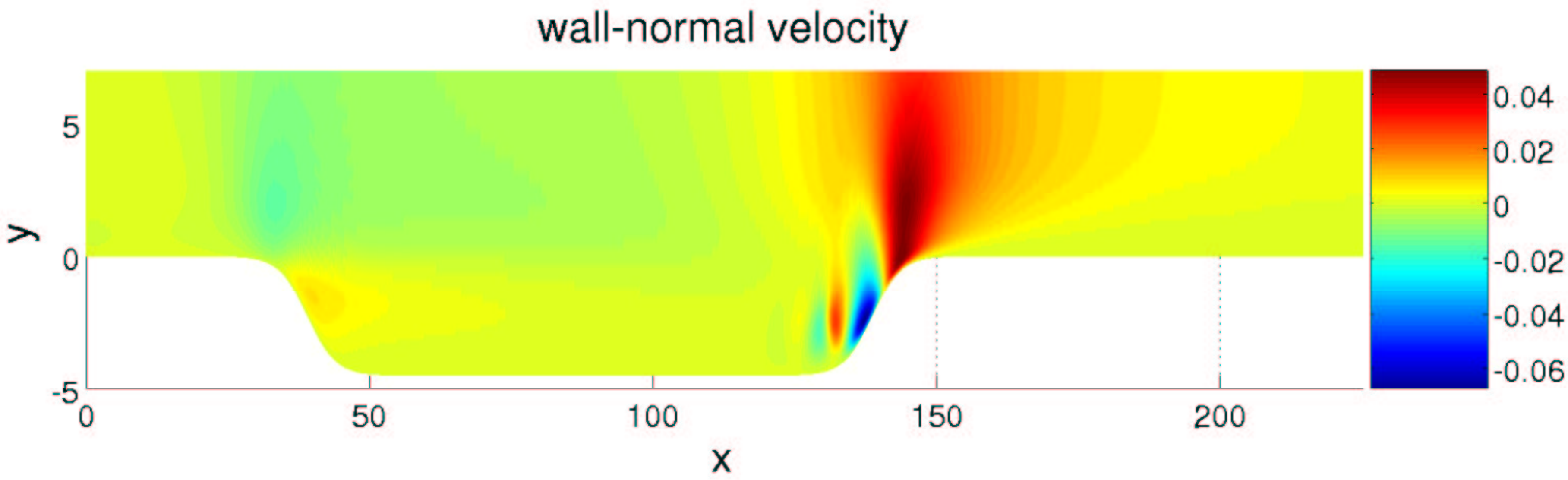
Streamwise velocity profiles u:



Flow is composed of :

- Boundary layers (before and after the cavity)
- Shear-layer over the cavity
- Recirculating zone inside the cavity

The base flow



Globally unstable flow → Base flow obtained from time averaging

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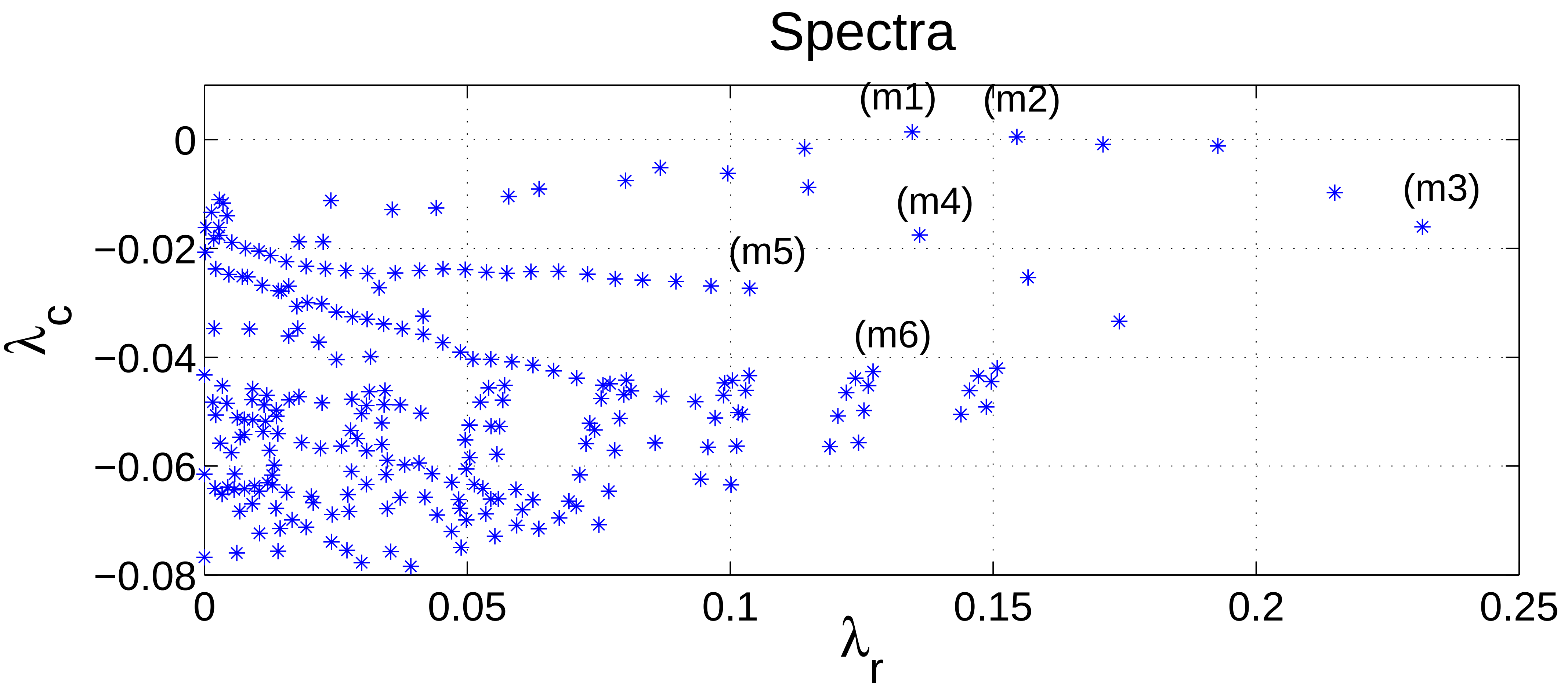
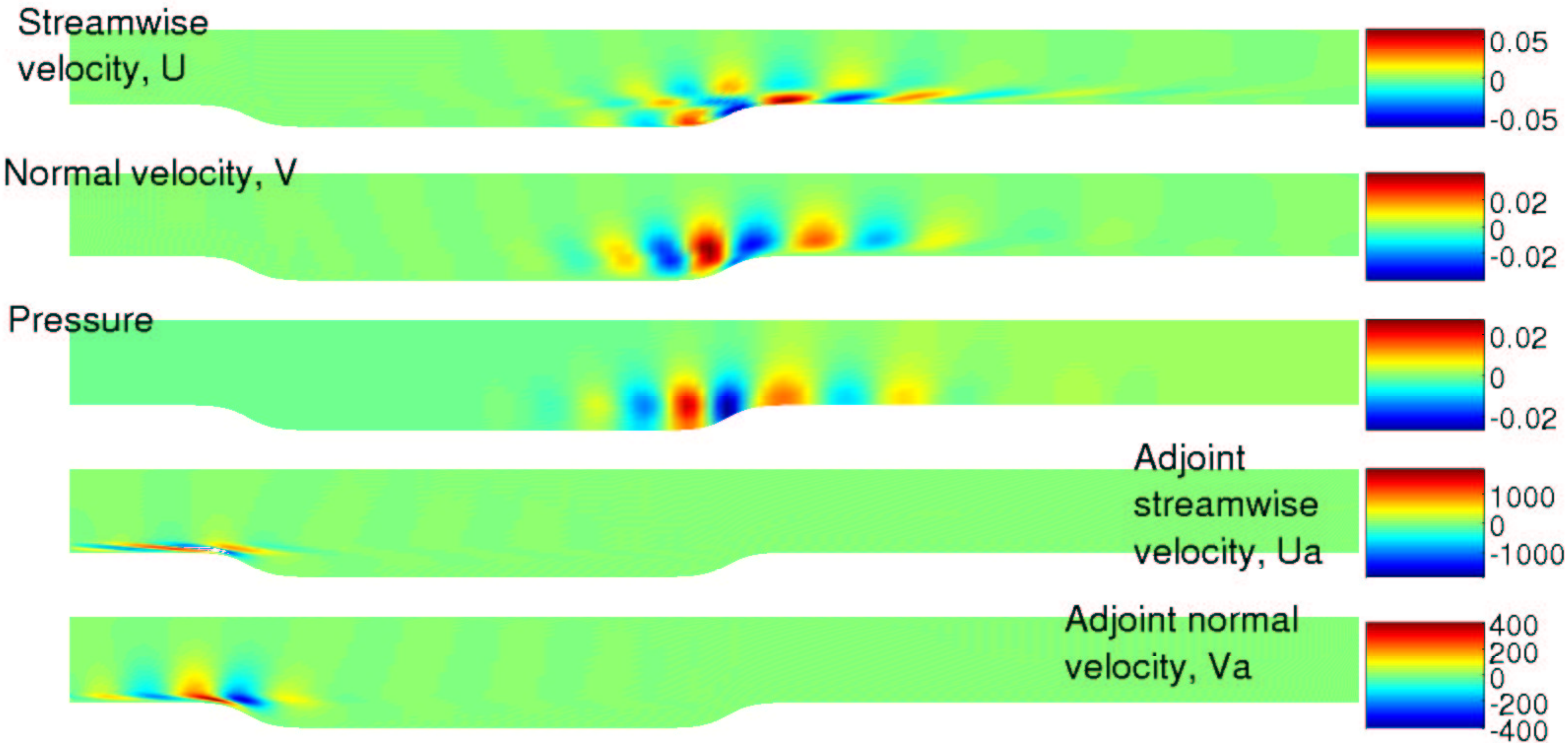
Eigenvalues



Spectra: unstable shear layer mode, (m1)



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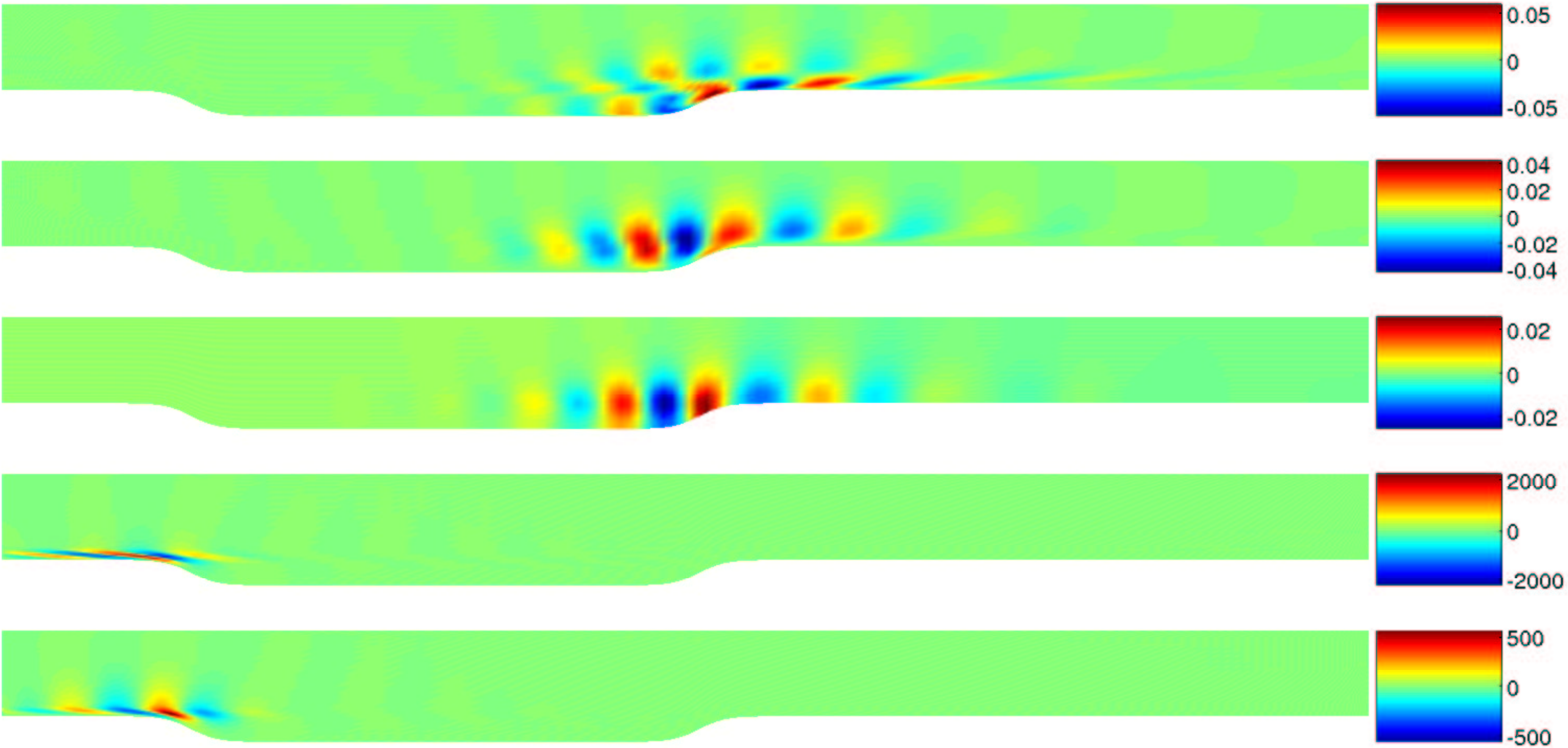
Spectra: unstable shear layer mode, (m2)



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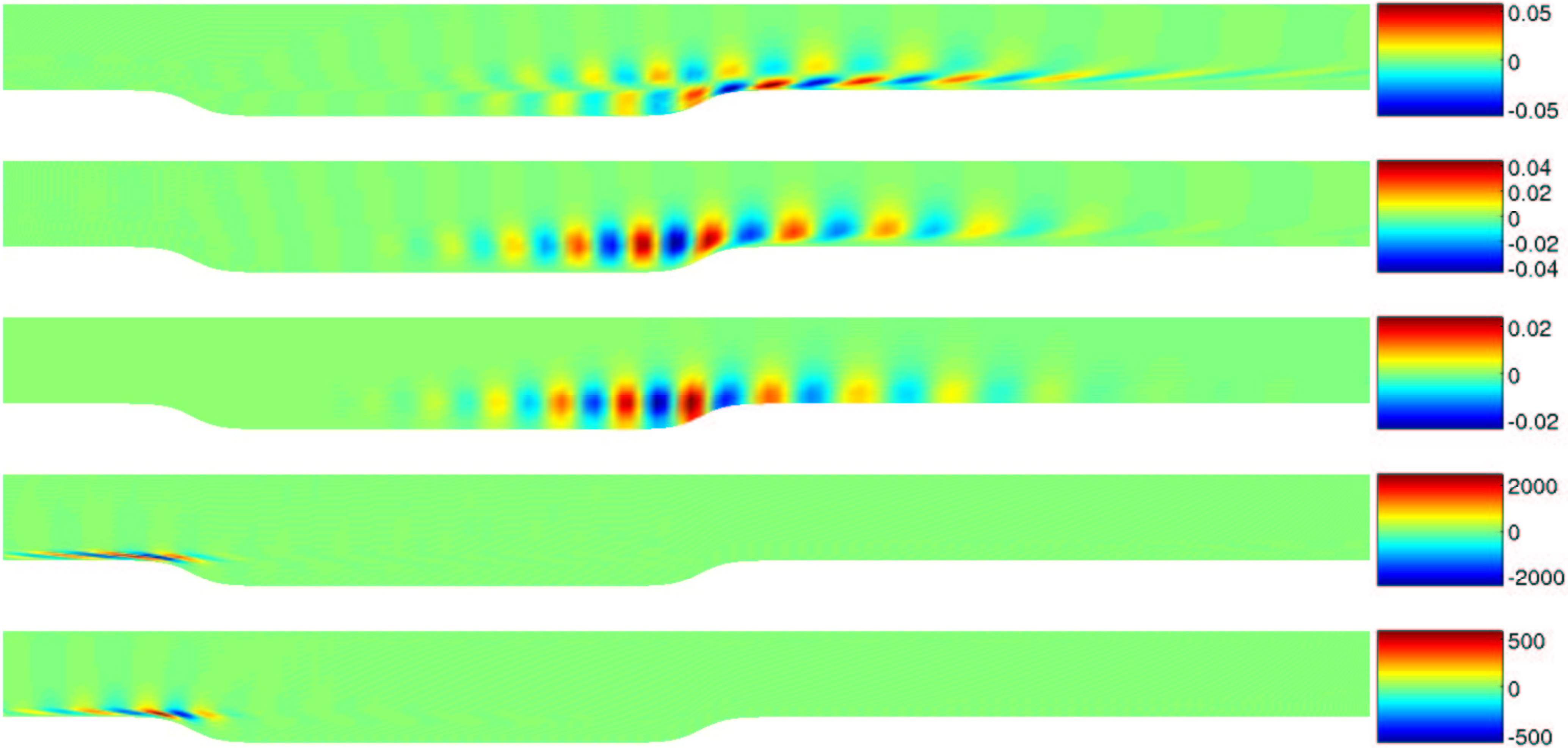
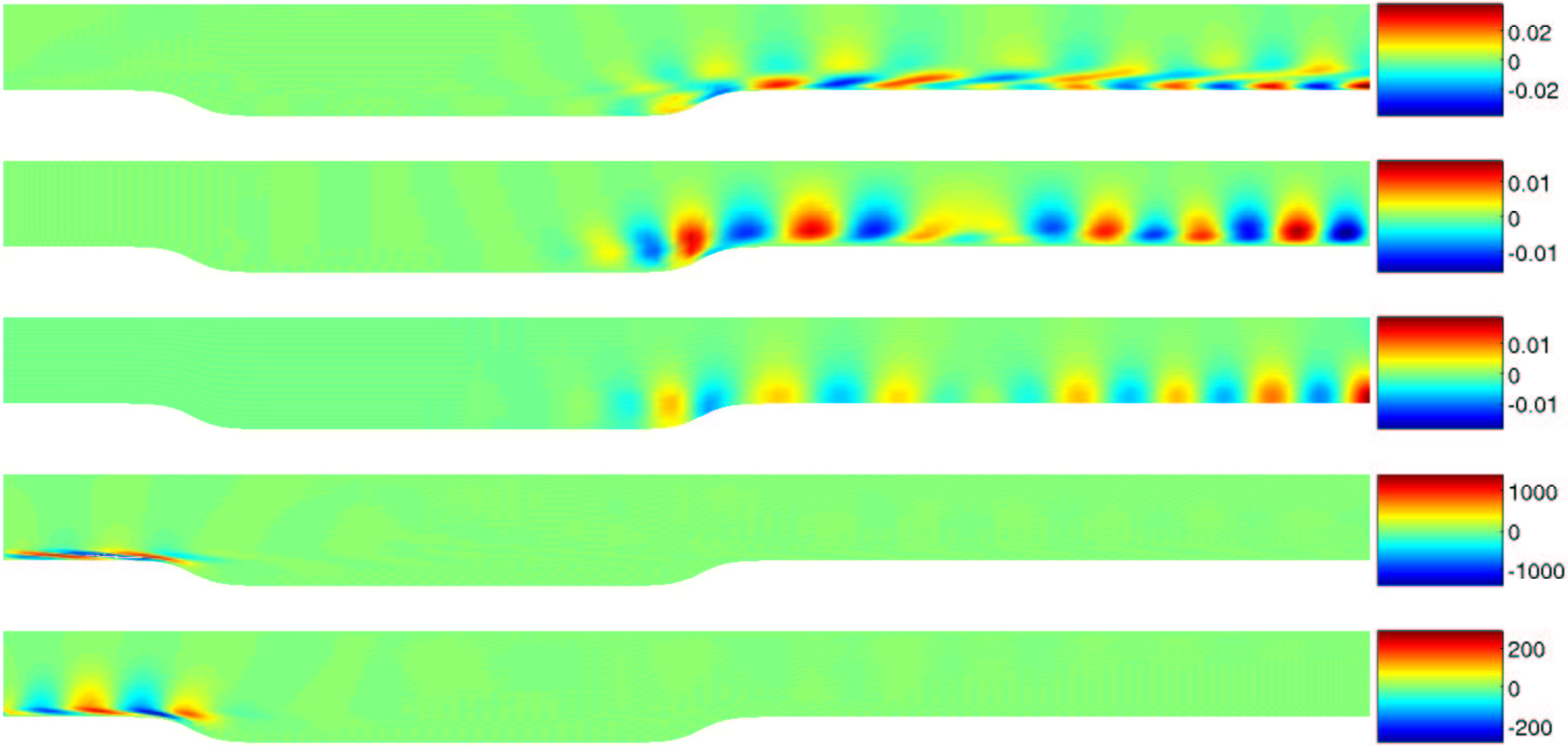
Spectra: higher frequency mode, (m3)

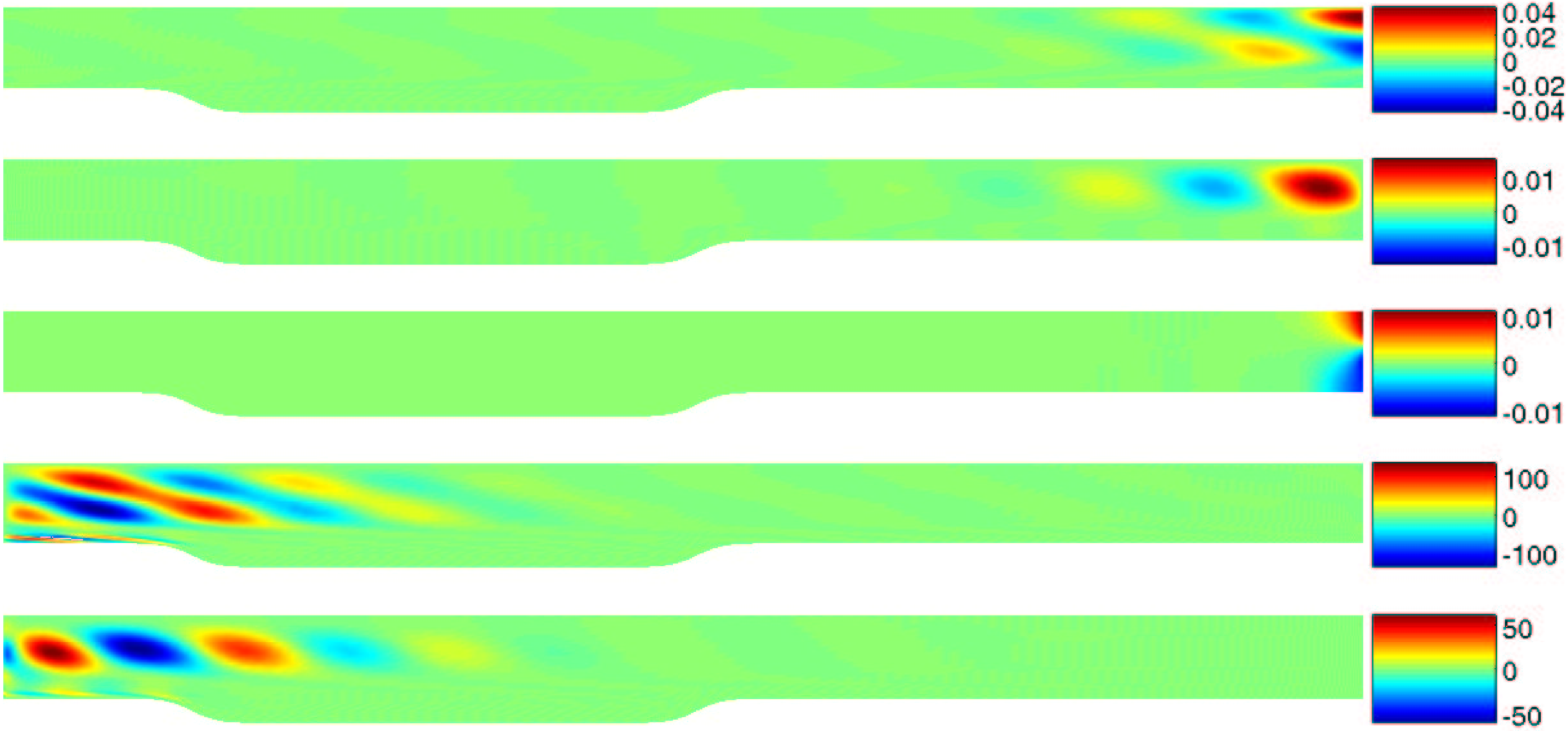


Spectra: more damped, (m4)



Spectra: propagative mode, (m5)

Spectra: propagative mode free-stream, (m6)



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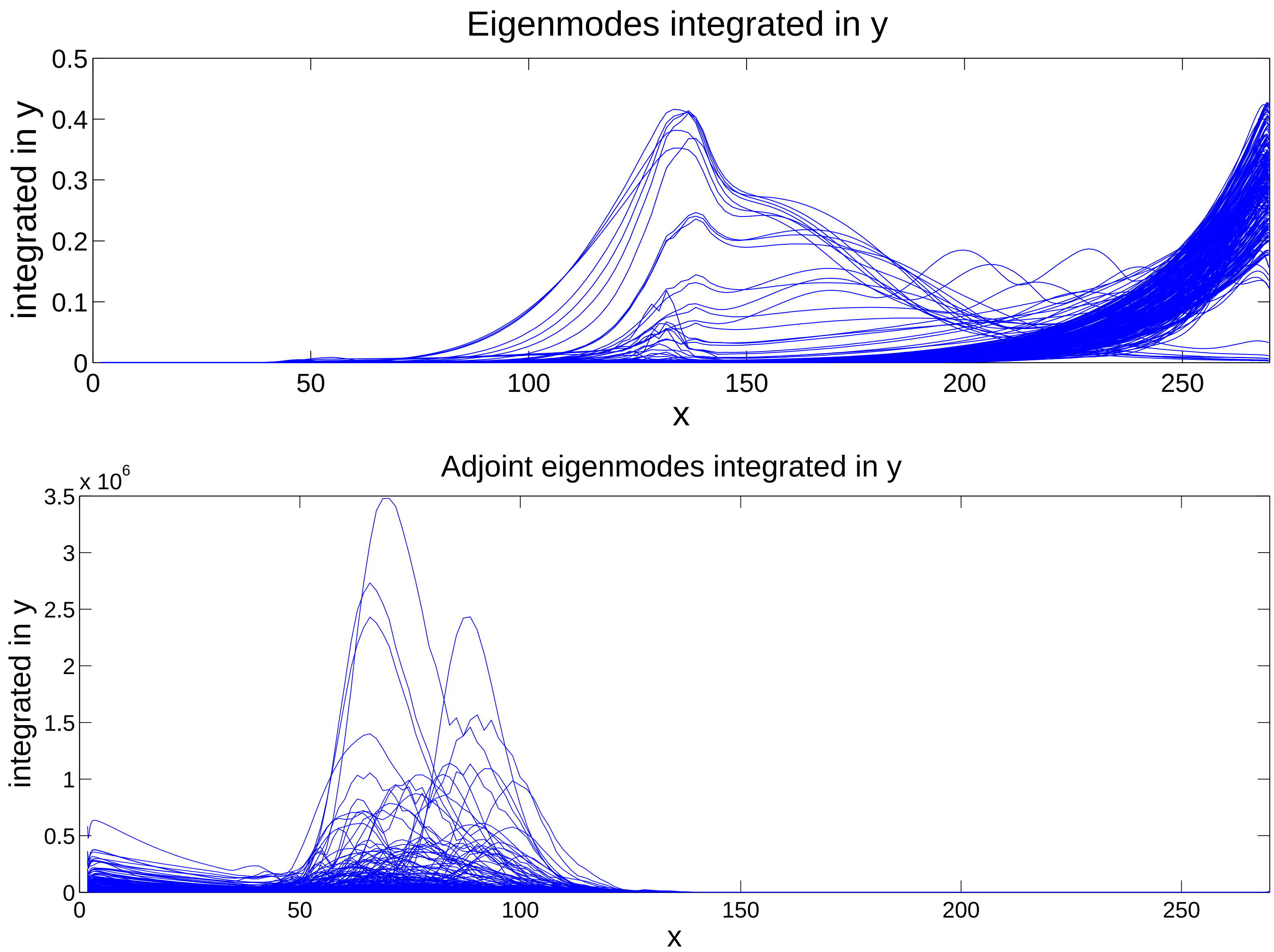
eigenmodes and their adjoint, Integrated



Where are the modes localised and where are they sensitive ?

Optimal transient energy growth from initial conditions

System $x(t) = Ax, \quad \dot{x}(0) = x_0$, with solution

$$x(t) = e^{At} x_0$$

Find the initial condition x_0 maximizing

$$G(t) = max_{x_0} \frac{< x(t), x(t) >}{< x_0, x_0 >}, \qquad \text{adjoint: } < Ax_1, x_2 >=< x_1, A^+ x_2 > \forall x_1, x_2$$

leads to

$$G(t) = max \frac{< e^{At} x_0, e^{At} x_0 >}{< x_0, x_0 >} = max \frac{< e^{A^+ t} e^{At} x_0, x_0 >}{< x_0, x_0 >}$$

$\rightarrow$ Maximum growth at time t: eigenvalue of $e^{A^+ t} e^{At}$.

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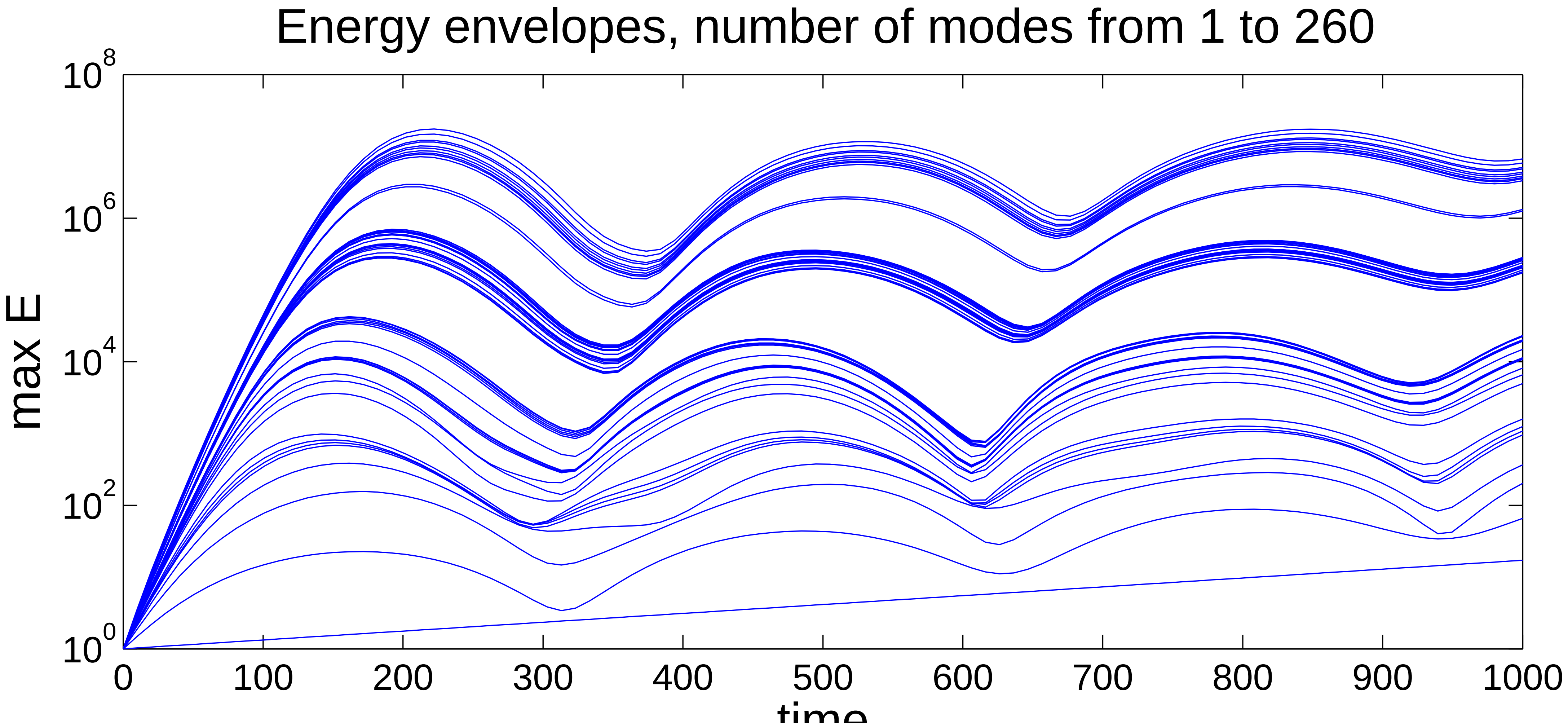
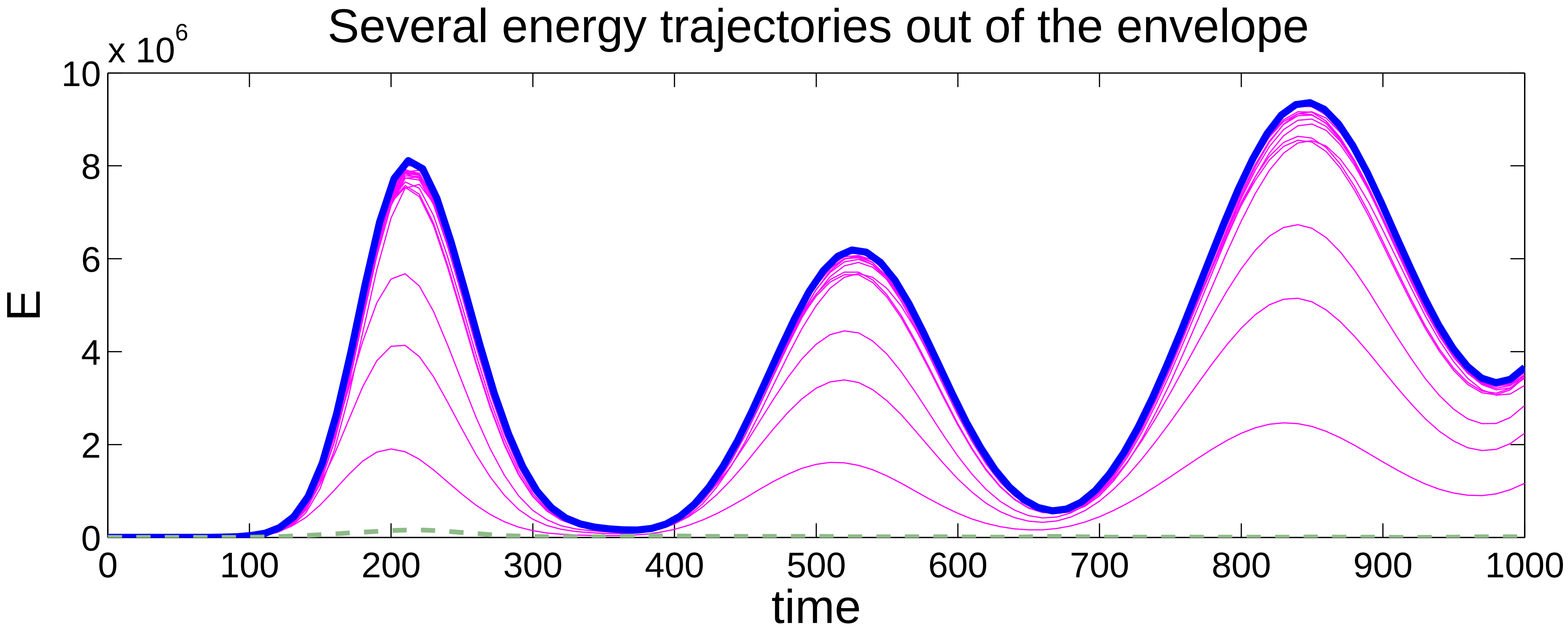
Optimal growth in the cavity

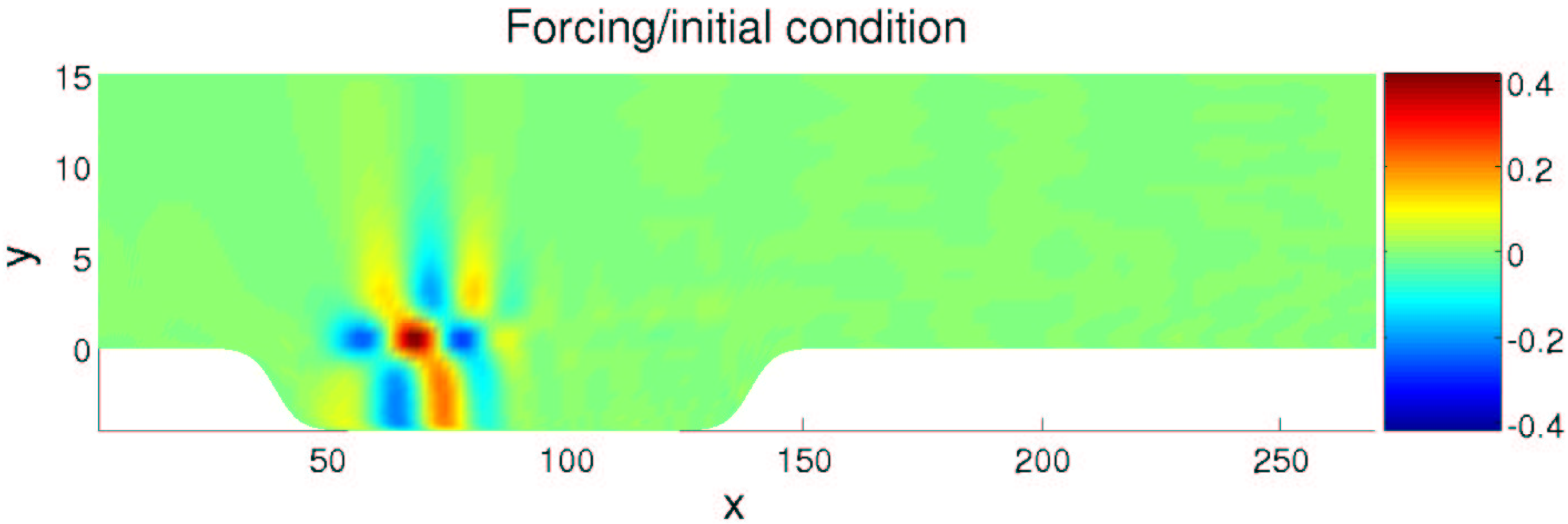
- Global instability
- Potentiality of strong energy growth
- Low frequency cycle



Trajectories from the worst initial conditions



The most dangerous initial condition



A wave packet at the beginning of the shear layer.

Animation of flow cycle

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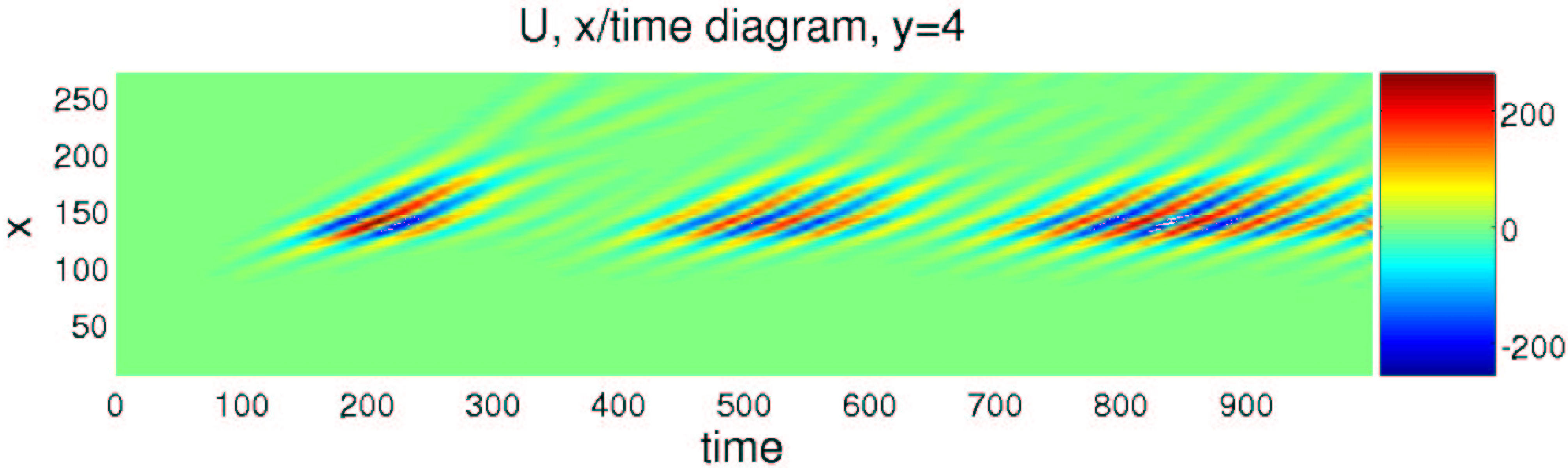
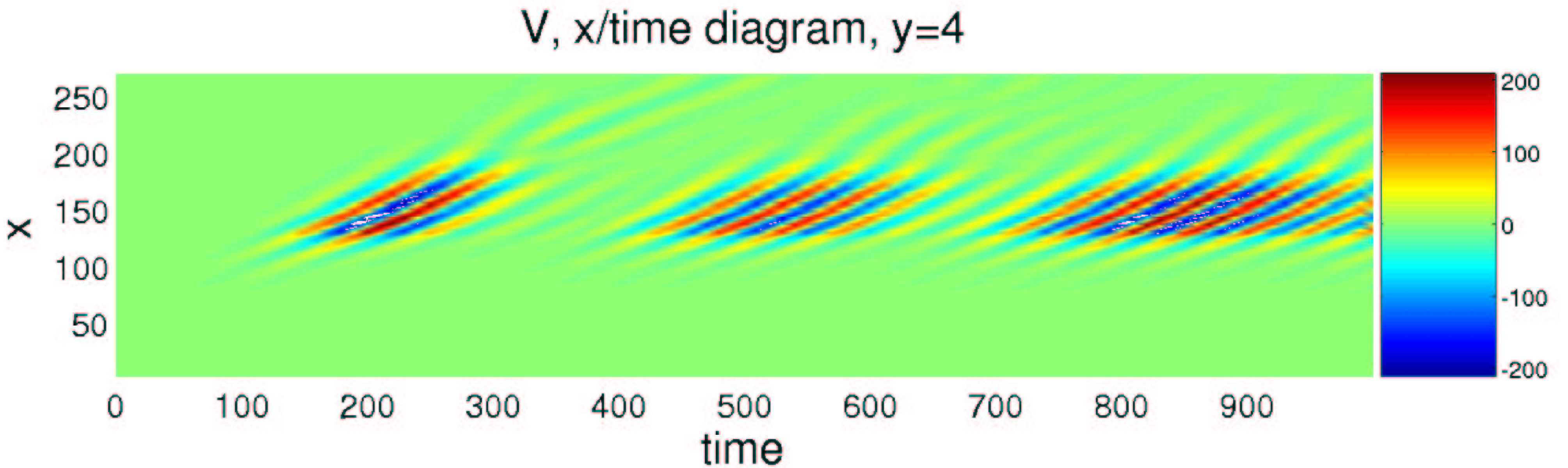
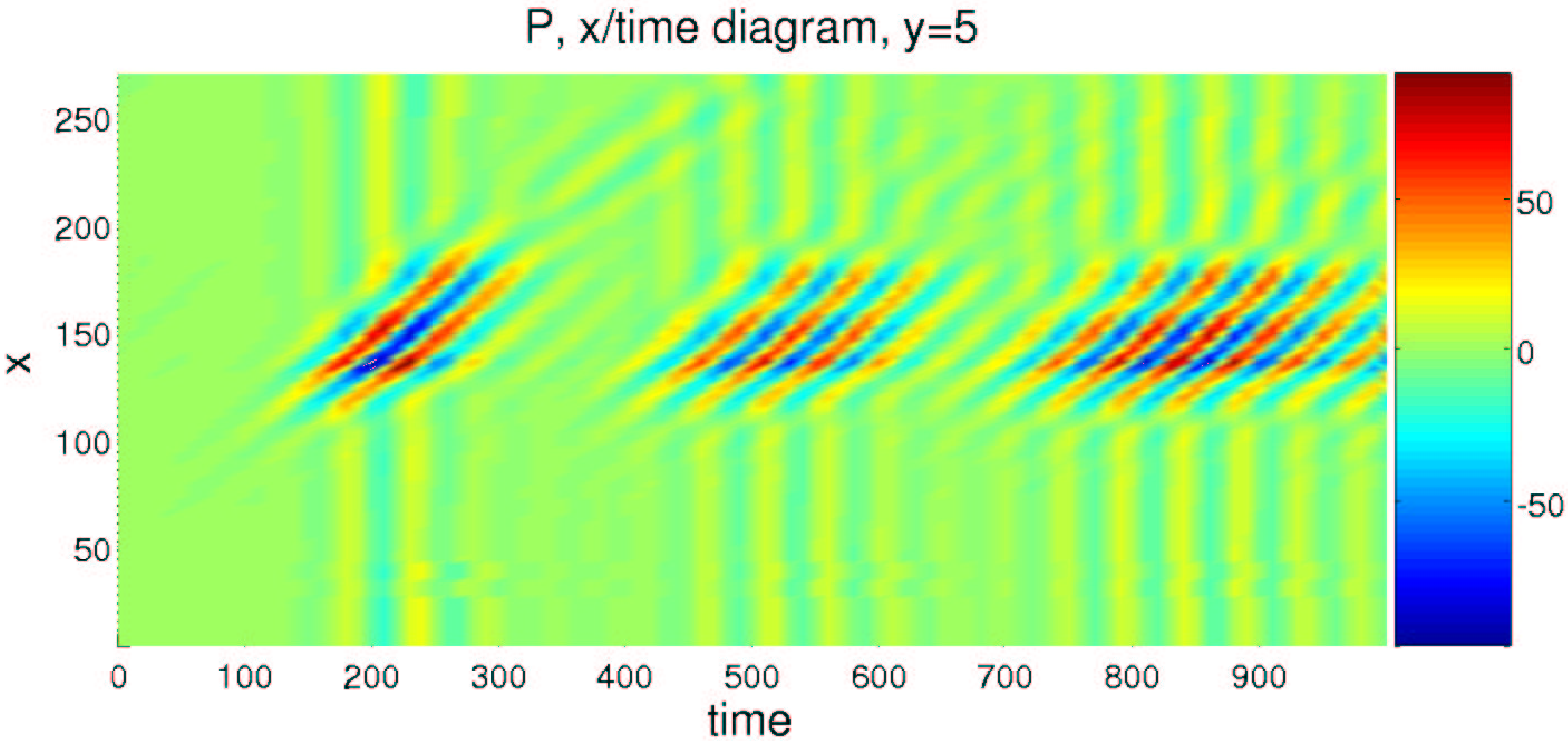
Flow cycle, u and v





Flow cycle, the pressure



Generation of **global pressure change** when the wave-packet impacts on the downstream lip

Regeneration of disturbances when the pressure hits the upstream lip

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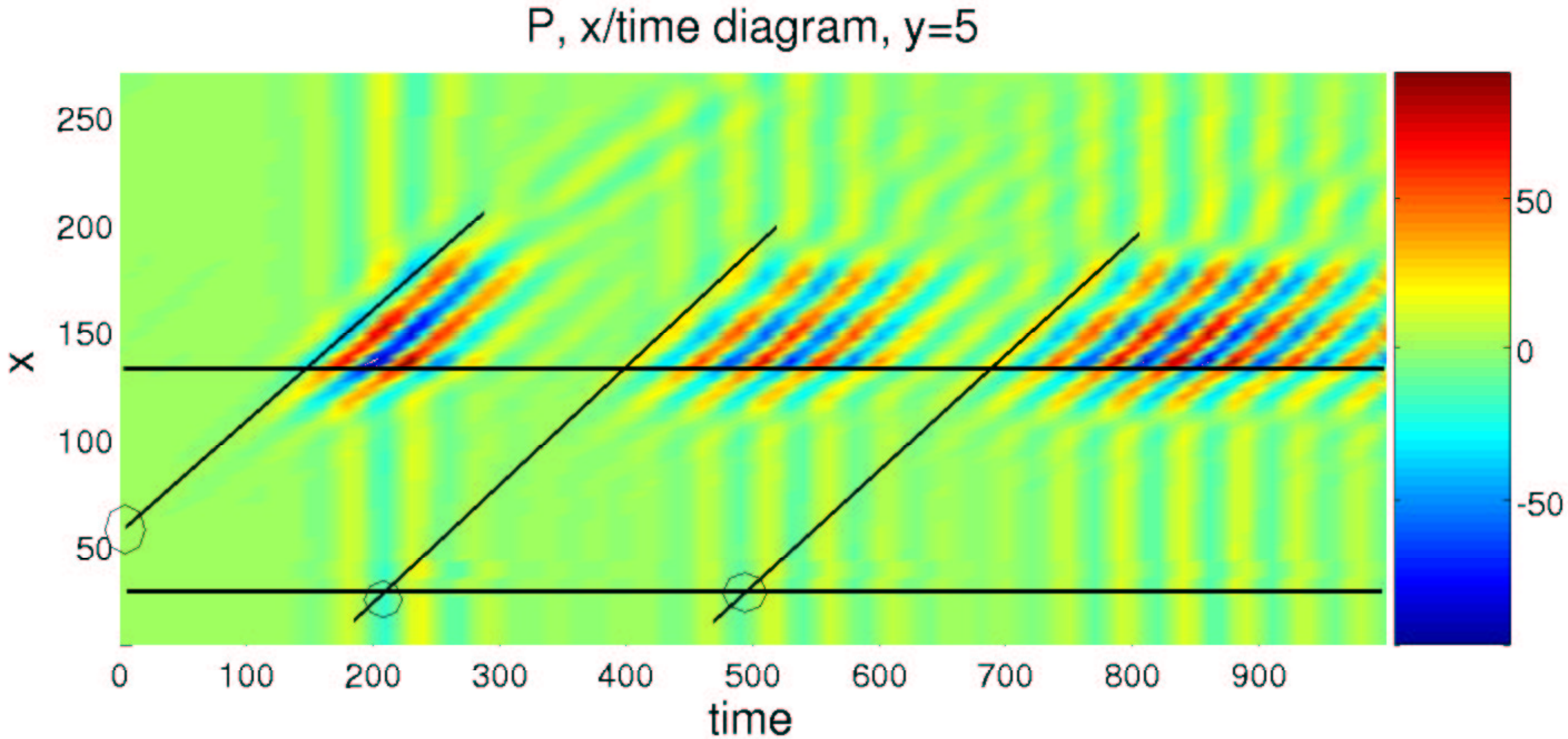
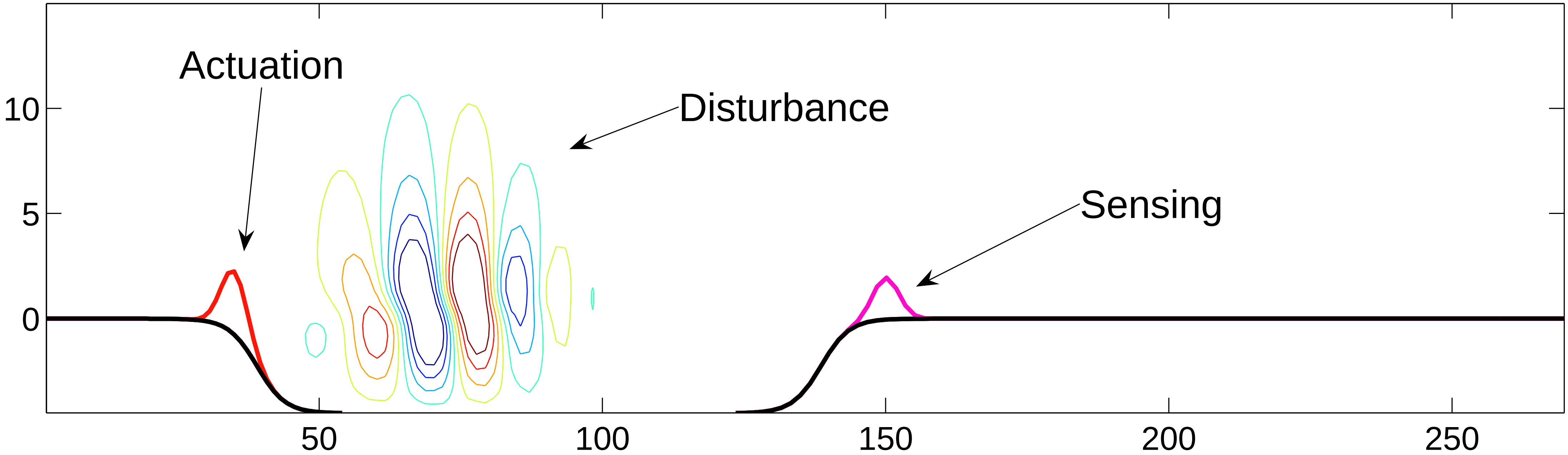
Flow cycle, the pressure



Control

Control

Seek to minimize the energy growth



- One actuator upstream
- One sensor downstream
- Oscillating disturbance in the shear layer

Feedback control

Using a dynamic model of the system:

$$\begin{cases} \dot{x} = Ax + Bu \\ r = Cx \end{cases}$$

One can optimize for the feedback

$$u = \mathcal{G}(r)$$

- The model in 2D is too big for optimization $\longrightarrow$ reduced model .
- For reduction: project the dynamics on the least stable eigenmodes.
- Finally, couple the reduced controller and the flow system

Model reduction

Galerkin projection on least stable eigenmodes:

Physical space:

$$\begin{cases} \dot{x} = Ax + Bu \\ r = Cx \end{cases}$$

Eigenmode space:

$$\begin{cases} \underbrace{P\dot{x}}_{\dot{k}} = \underbrace{PAP^{-1}}_{A^M}\underbrace{Px}_{k} + \underbrace{PB}_{B^M}u \\ r = \underbrace{CP^{-1}}_{C^M}\underbrace{Px}_{k} \end{cases}$$

Projection on eigenmodes $\rightarrow$ **biorthogonal** set of vectors:

- Eigenmodes: q_i,
- Adjoint operator: $A^+ / < Ax_1, x_2 >=< x_1, A^+x_2 >, \forall x_1, x_2$
- Adjoint eigenmodes: q_i^+,
- Biorthogonality: $\delta_{ij} =< q_i, q_j^+ >$, Projection: $k_i =< x, q_i^+ >$

Control terminology

- **Estimation:** From sensor information, recover the instantaneous flow field.
- **Full information control:** From full knowledge of the flow state, apply control.
- **Compensation:** Close the loop by using the estimated flow state for control.
- **Model reduction:** Project the dynamics on a set of selected basis vectors.

- **Control penalty:** Penalisation of the actuation amplitude.
- **sensor noise:** Uncertainty in the measured signal.

- **Disturbances:** External forcing exciting the flow.
- **Objective function:** Function of the flow state to be minimized.

Central elements of the design

1) From the sensors, estimate the flow state:

- Sensor location
- Sensor noise
- Disturbance model (here perturbations at the inflow)

2) Using the flow state information, apply control :

- Actuator location
- Control penalty
- Objective function

Optimization is done by solving two Riccati equations

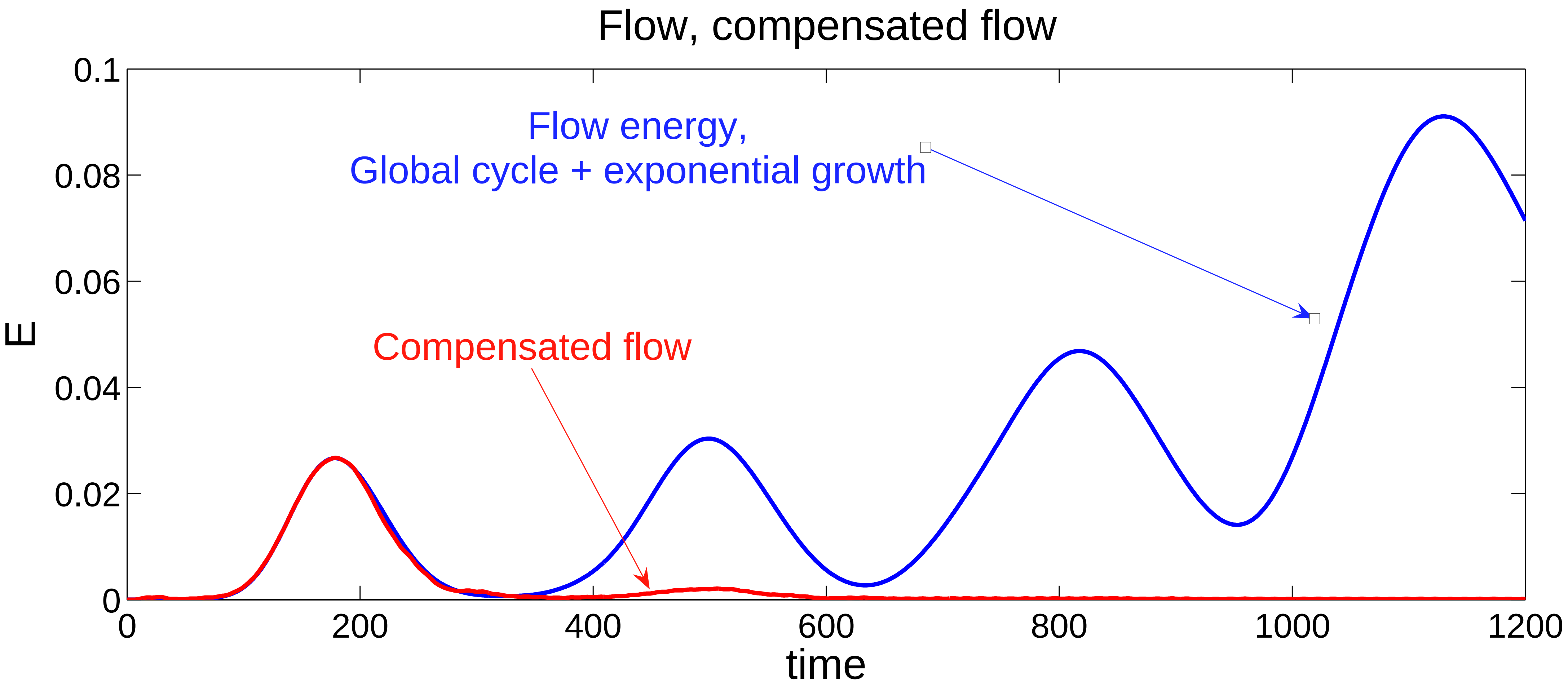
Testing procedure

1. Decide penalties, sensor noise, locations
2. Reduce the model by projection
3. Optimize for the feedback
4. Couple flow system and controller

   The reduced controller (20 states) is applied on the full system (20,000 states)

5. Compute energy of controlled flow

Compensation performance

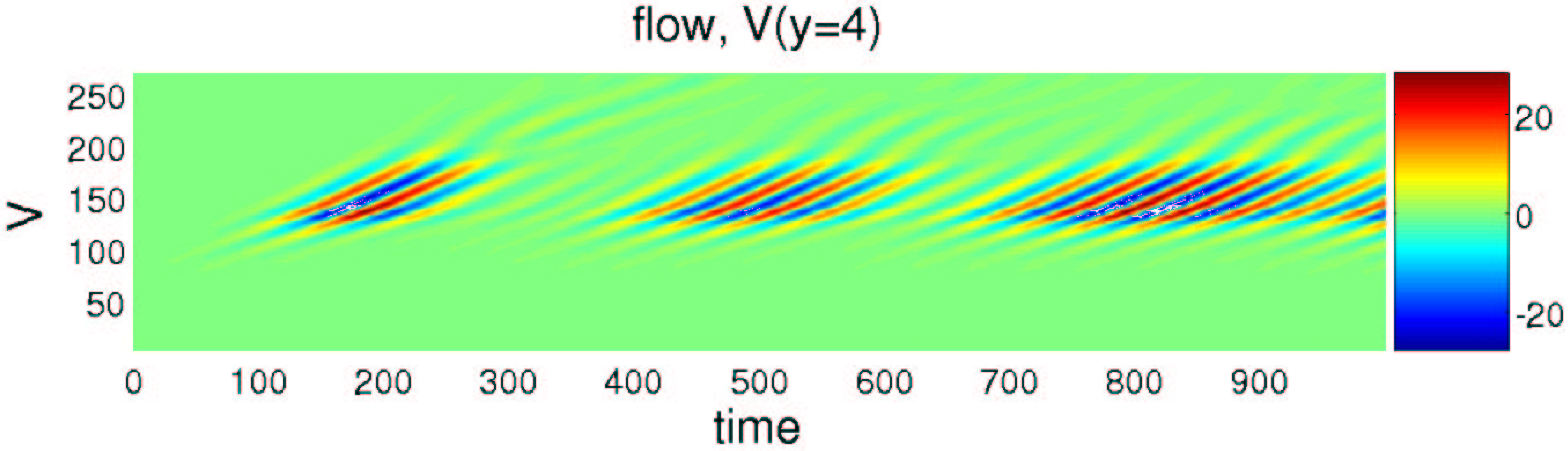


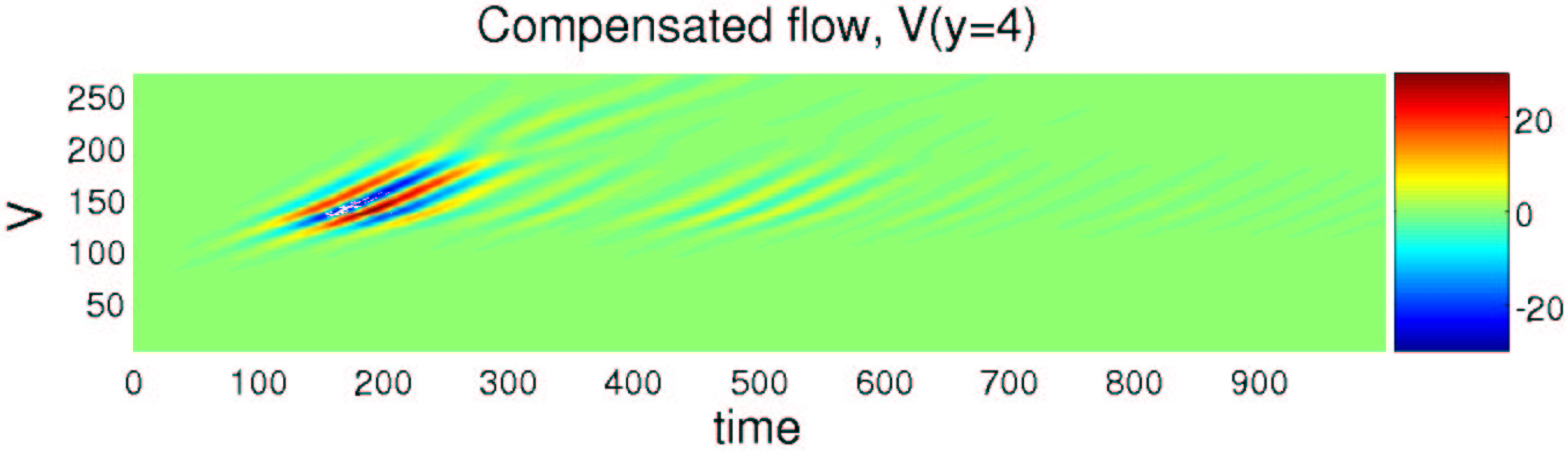
Good control performance from the second cycle

Reduced order model:20 states

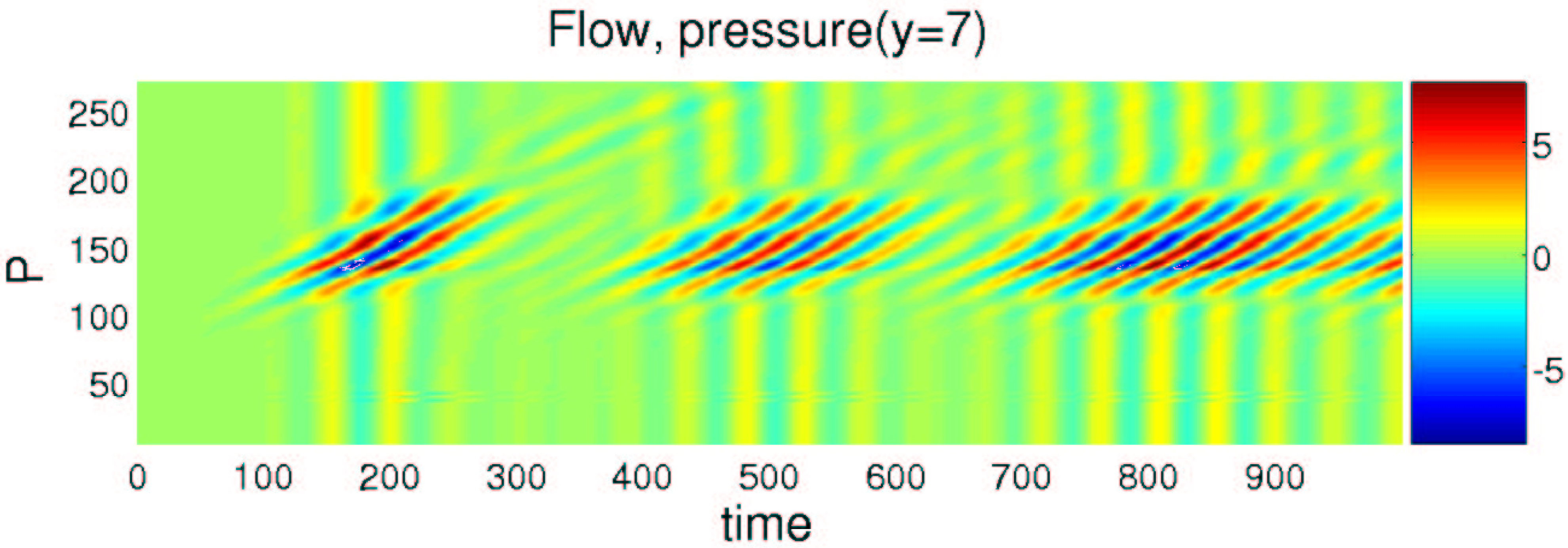
Flow/compensated flow animation

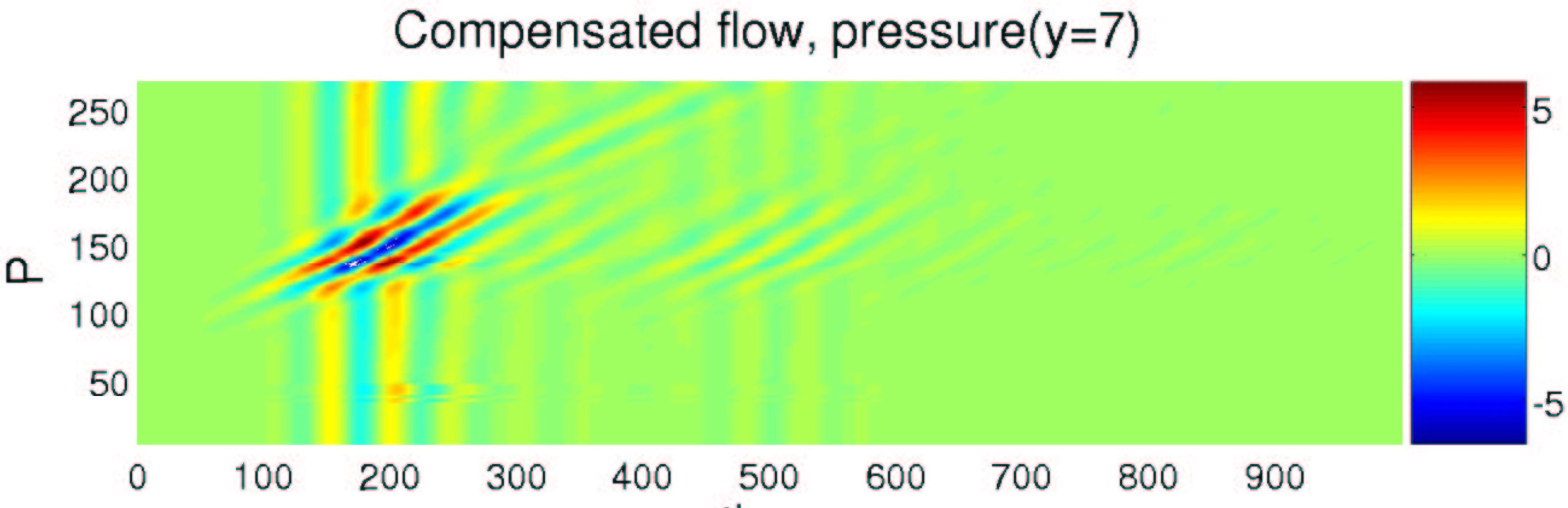
Flow/compensated flow, x/t diagram



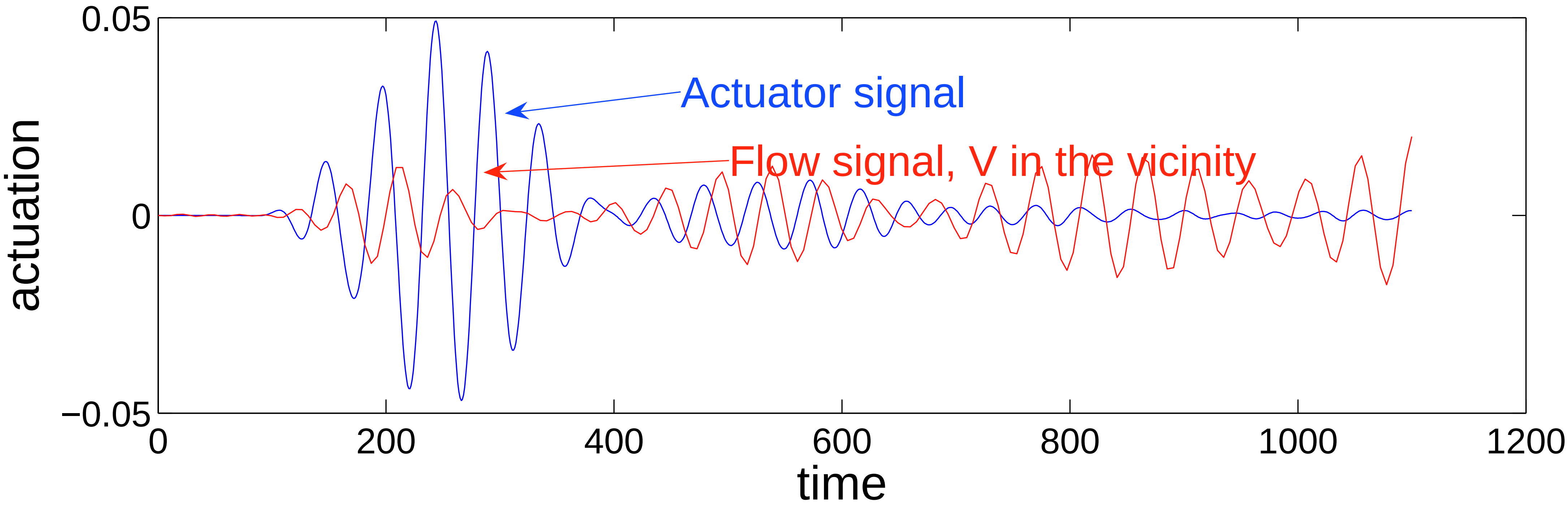


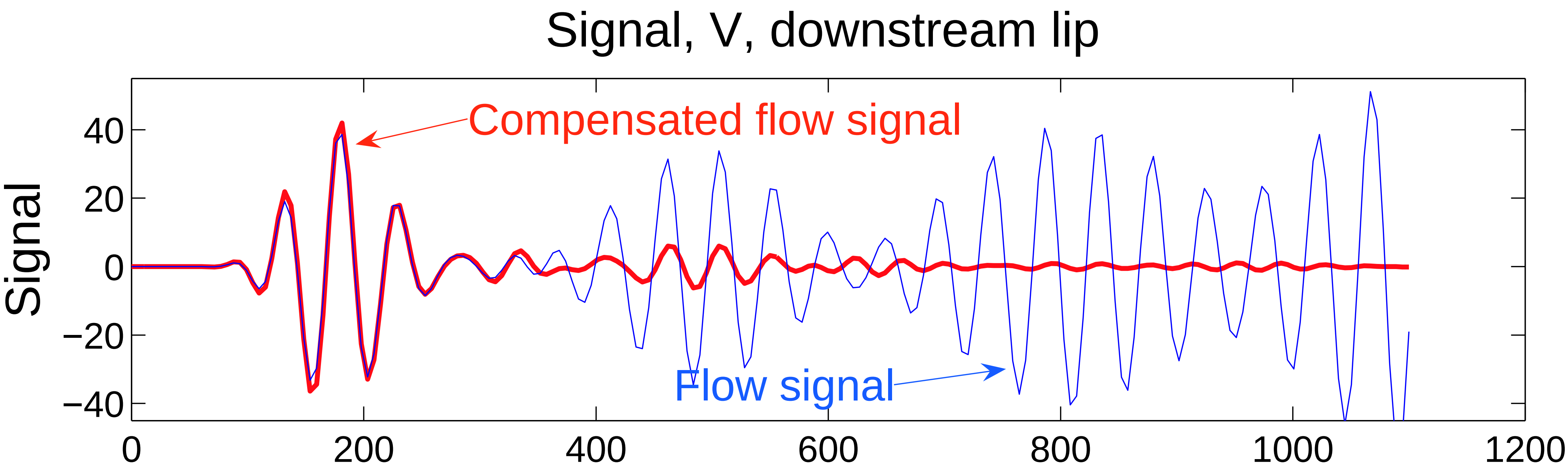
Flow/compensated flow, x/t diagram



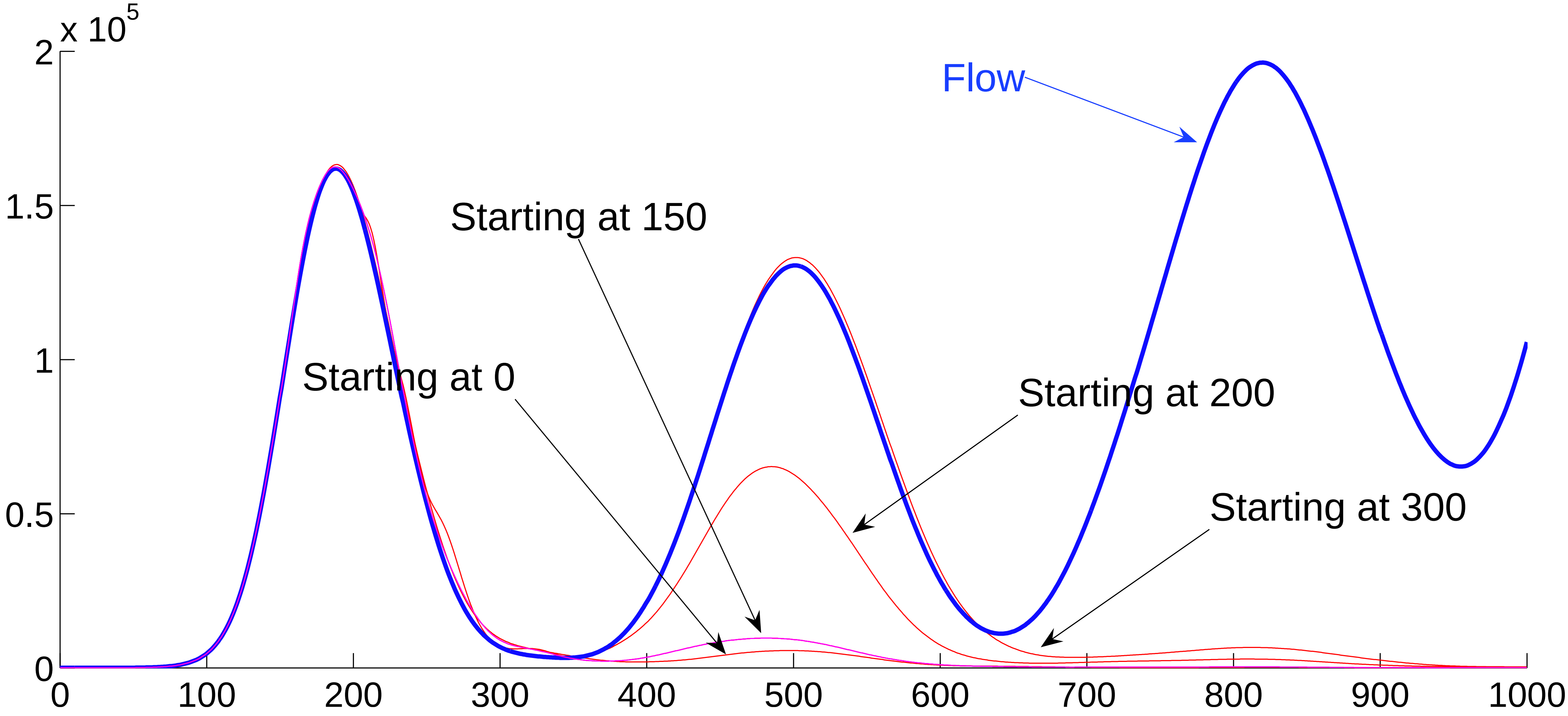


Actuation signal





Starting the compensator at later times



Compensator cannot affect the disturbance propagation

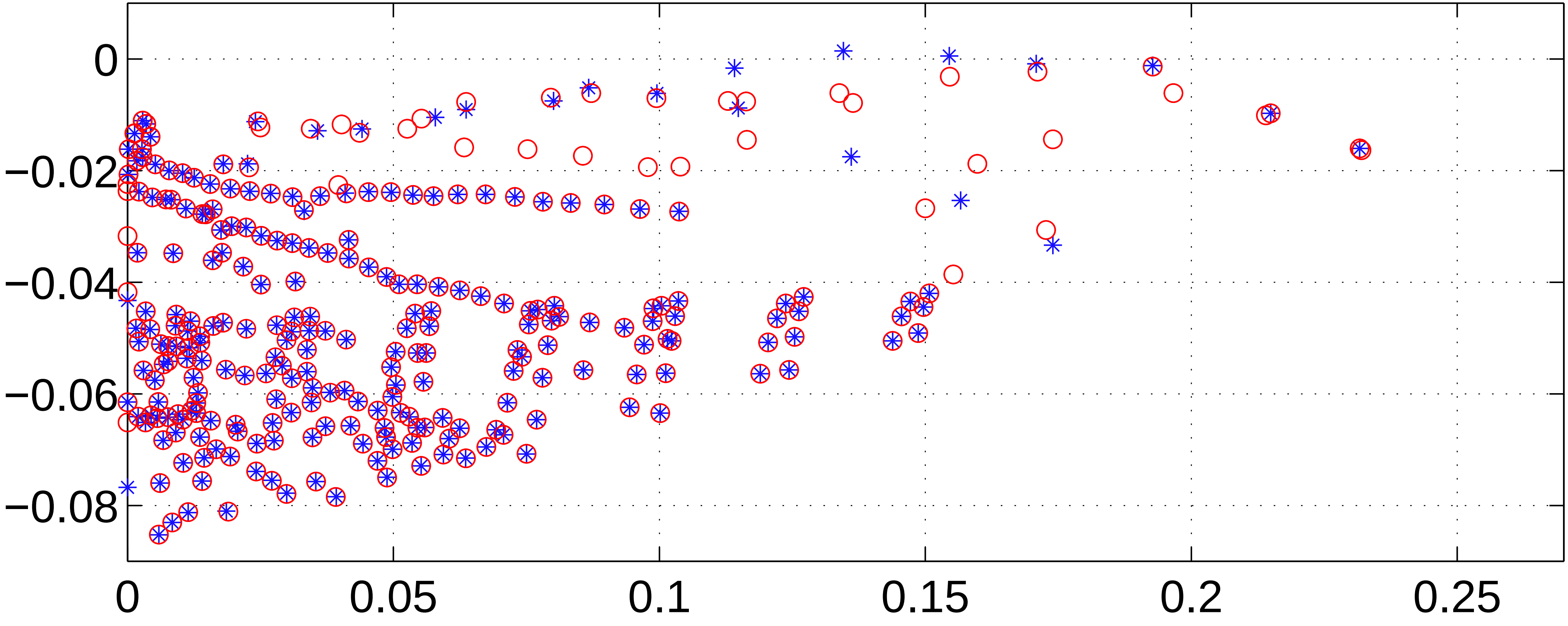
but can affect the disturbance generation

Dynamic distortion

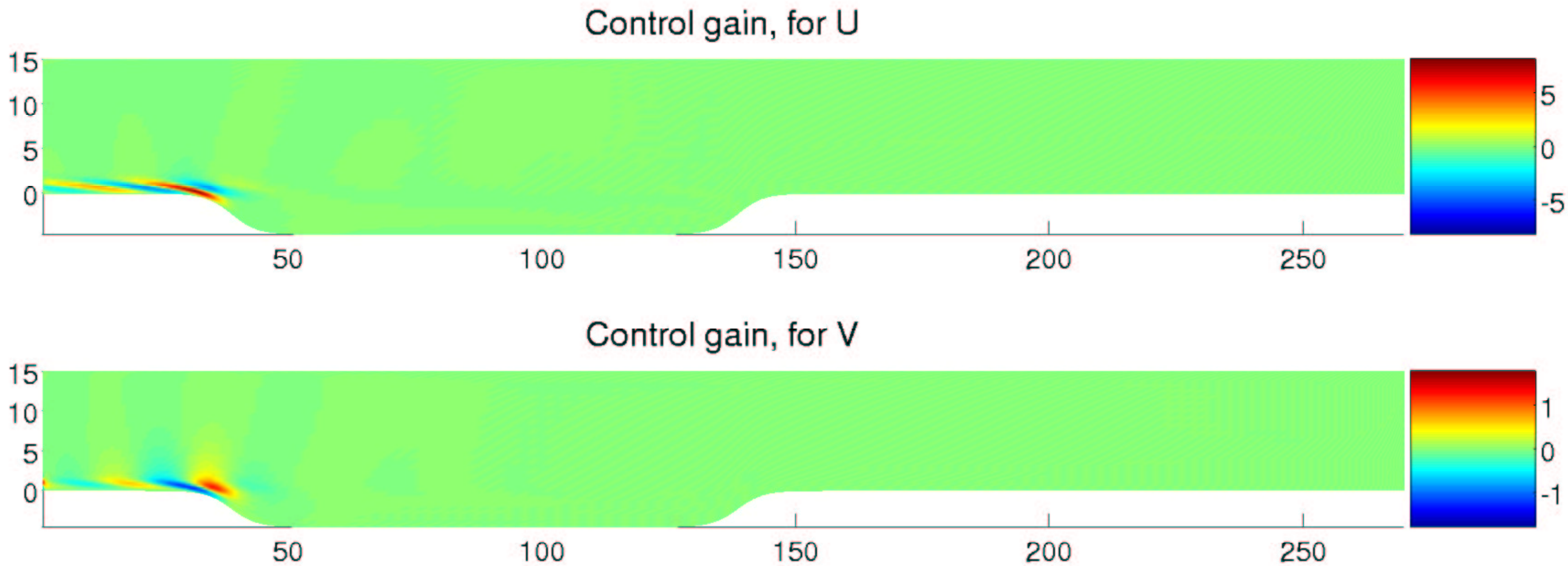
blue :flow

Red :compensated flow

Spectra with and without compensation

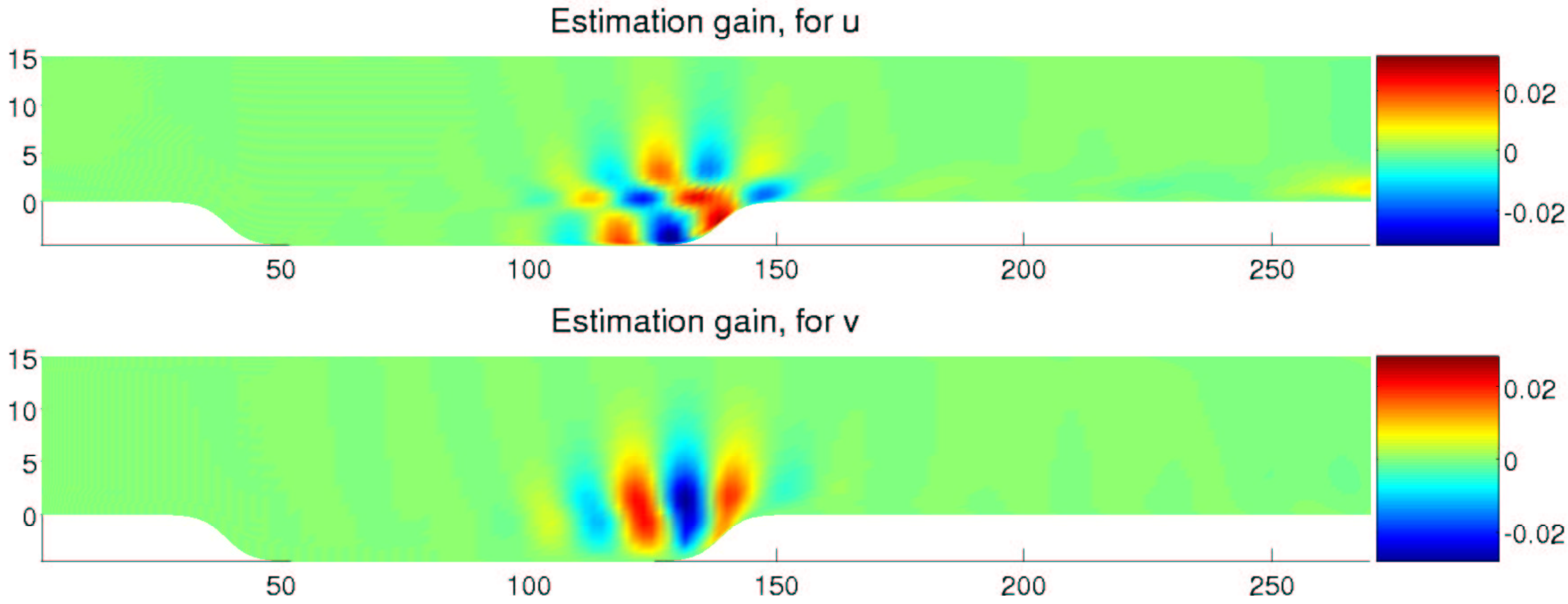


Control gain



Function used to **extract the actuation signal** from the flow

Estimation gain



Function used to **force the estimator** flow

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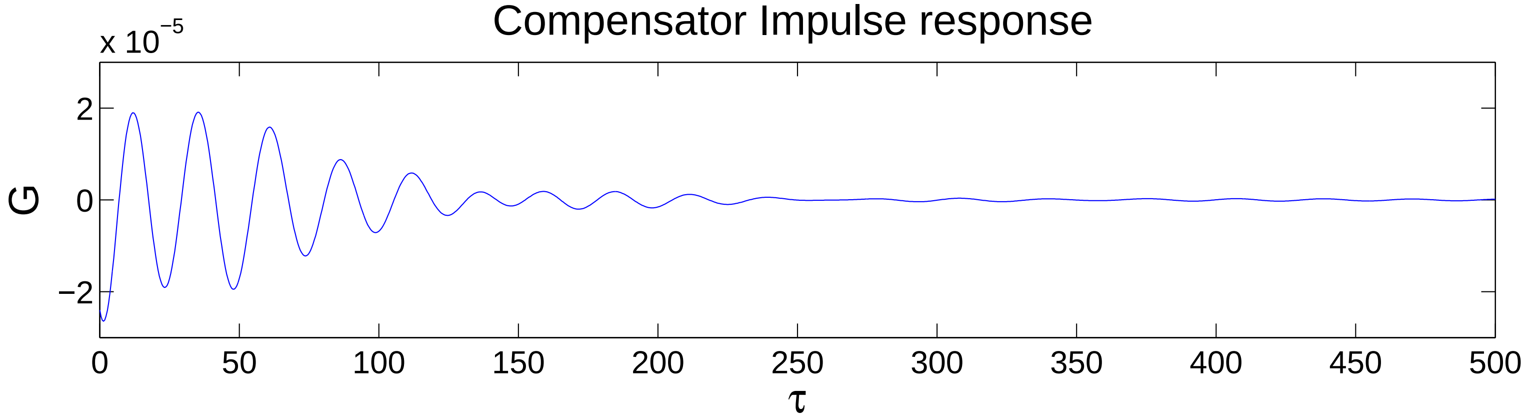
Compensator impulse response

Compensator:

- input (sensor signal, r)
- output (Control signal, u)
- linear system

The input-output relation is described by convolution

$$u(t) = \int_{\tau=0}^{\infty} G(\tau) r(t-\tau)$$



Conclusion

- Found supercritical Hopf bifurcation for long cavity
- Incompressible cavity can have global cycle due to pressure.
- Global eigenmodes can be used for analysis and model reduction.
- Model reduction allows optimal feedback design for large systems.
- Non-parallel effects/global instabilities can be treated.

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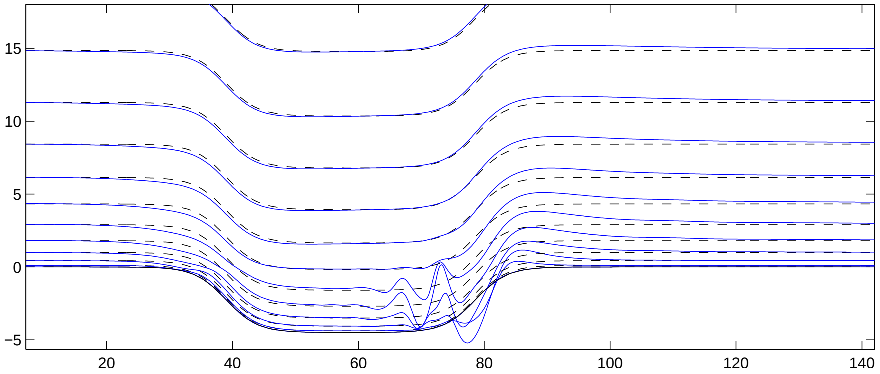
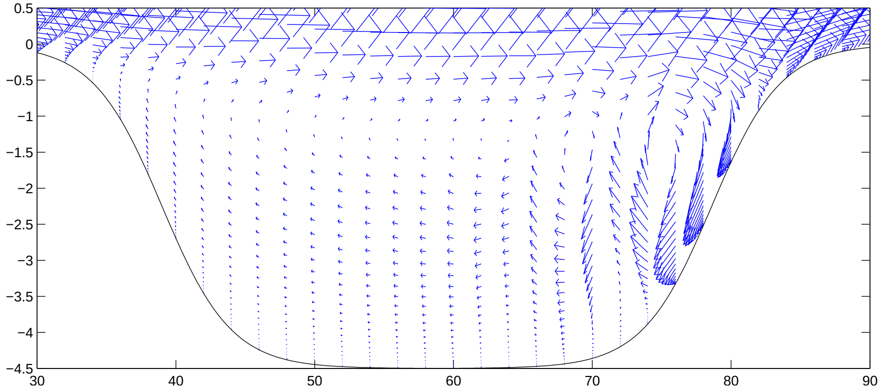
Extra slides

Base flow:

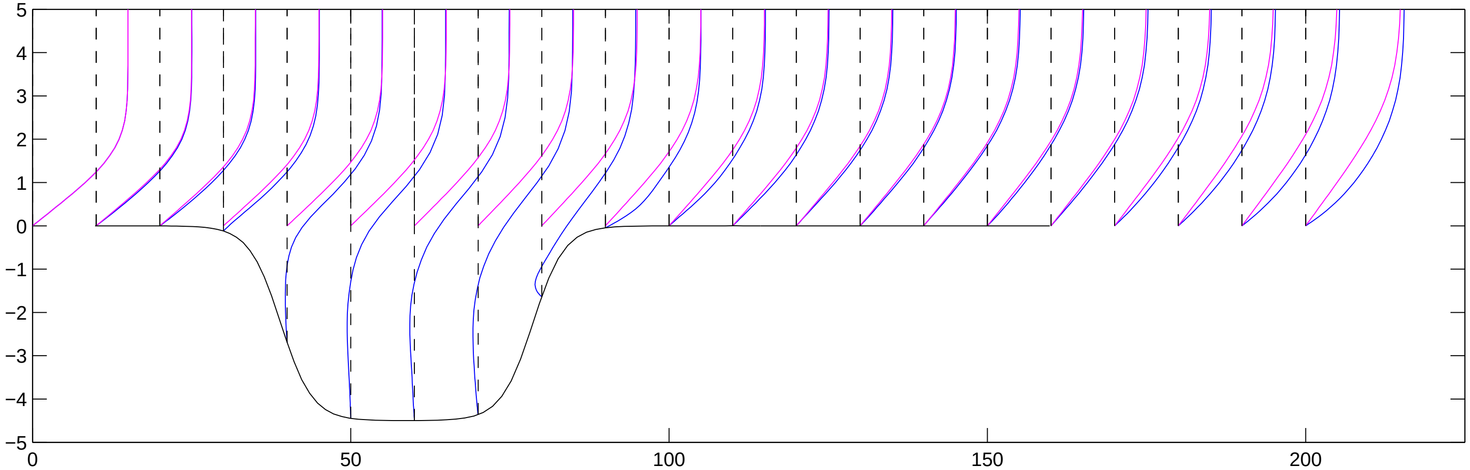


Normal velocity:

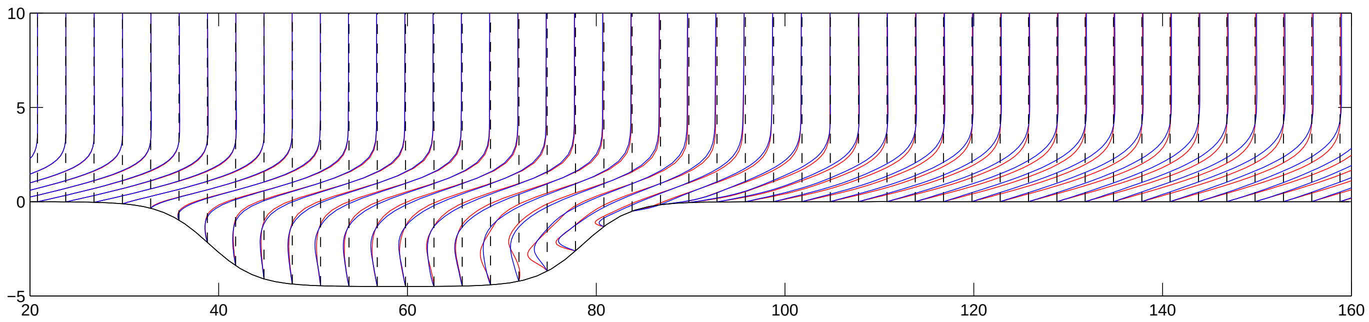
Quiver:



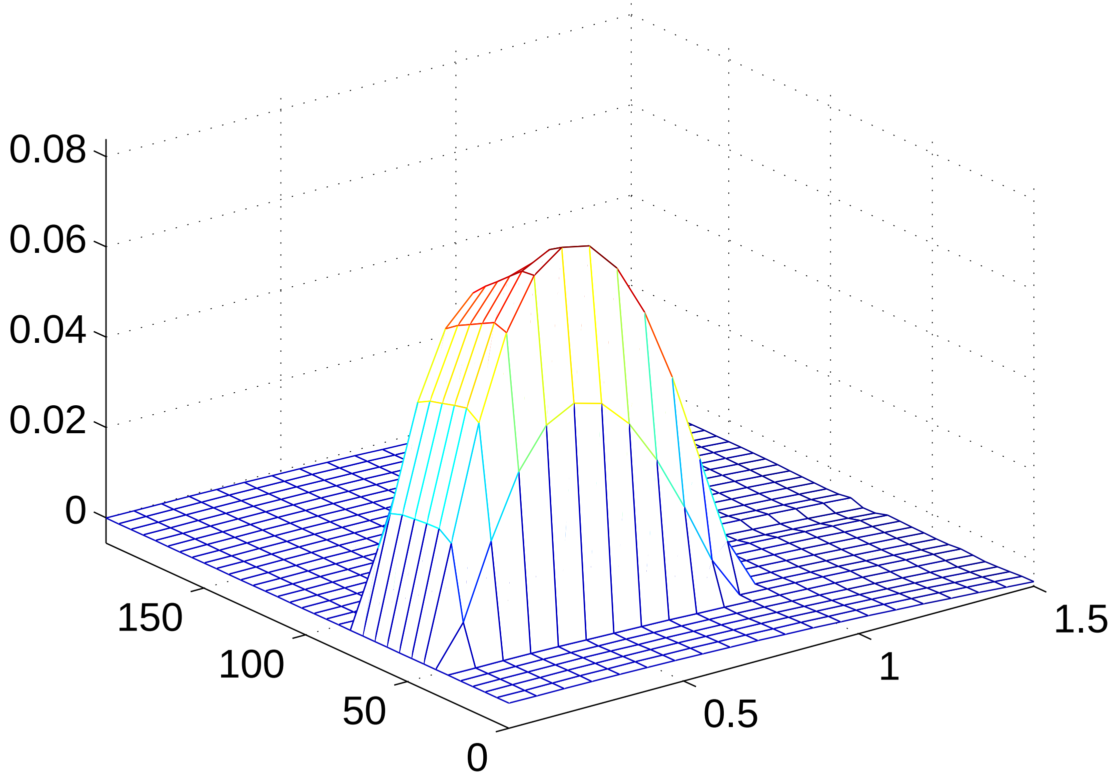
Re500/Blasius:



Re 300/Re 500:



Local stability:



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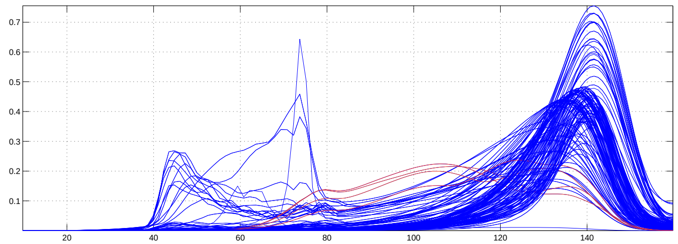
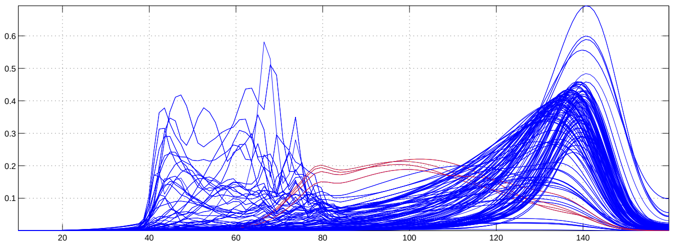
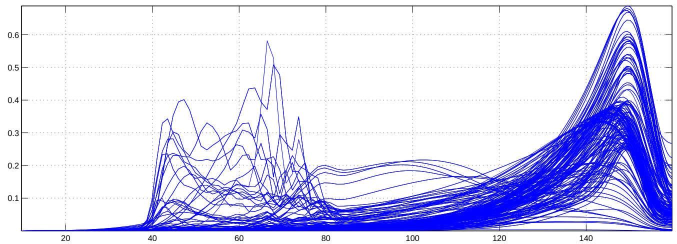
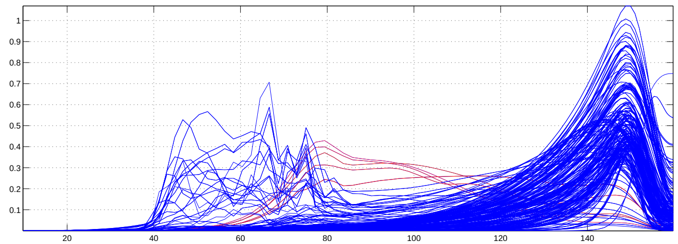
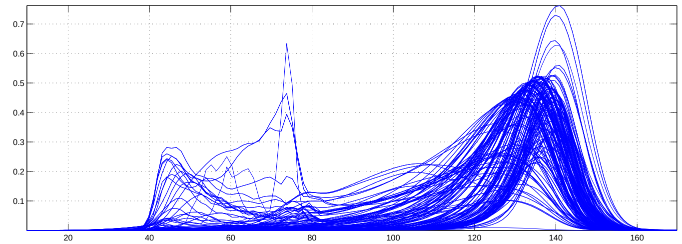
Boundary conditions

Re 300  and re 500:  fringe 20

 low resolution:  long and strong

fringe:  no fringe